Math 314 – Final Exam (20 points each) Name:

1)

a. Let p = "Math is fun" and let q ="I am smiling." Write the compound proposition $p \wedge q$.

b. Give an example of a function $f : \mathbb{Z} \to \mathbb{Z}$ that is not one-to-one but is onto.

2)

a. Suppose that A is the set of alligators in a zoo, and B is the set of zoo animals named Bubba. What is in the set $A \cap B$?

b. Given sets E and F, with $|E| = 34, |F| = 35$, and $|E \cup F| = 49$, how big is $|E \cap F|$?

3) Write two formulas for the following sequence, one in closed form and one recursively: 3, 7, 11, 15, 19, …

4) Show that $4n^2 + n$ is $O(n^3)$.

5) I have three punk pigeon stuffed animals. Since they are totally tough, they have piercings. Altogether, they have ten piercings in fact. Prove that there must be at least one pigeon with at least four holes.

6) Use induction to show that the sum of the first n odd numbers is n^2.

7) How many ways can I arrange the letters in the word 'INTERSECTIONALITY,' assuming that I use each letter exactly as many times as it appears in the word?

8) Prove that for natural numbers a, b, and c, if $a|b$ and $a|c$, then $(a+b)|c$.

9) Sketch a proof of the fact that in any planar connected graph, with at least two edges, we have that $3|F| \leq 2|E|$.

10) Describe in English why the fact that the complete graph on five vertices (the graph where every vertex is directly connected to every other vertex) must have a crossing implies that the five farmers cannot divide up their land into five contiguous plots of land, each bordering all others in a length of fence.

11) BONUS (5pts) If I have 800 unicorns and 750 doom crystals, how many different doomicorn armies could I create? To clarify, each doom crystal can transform a unicorn into a doomicorn.