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Combinatorial Auction Winner Determination with Branch-and-Price

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www.research.ibm.com/auctions/

Outline

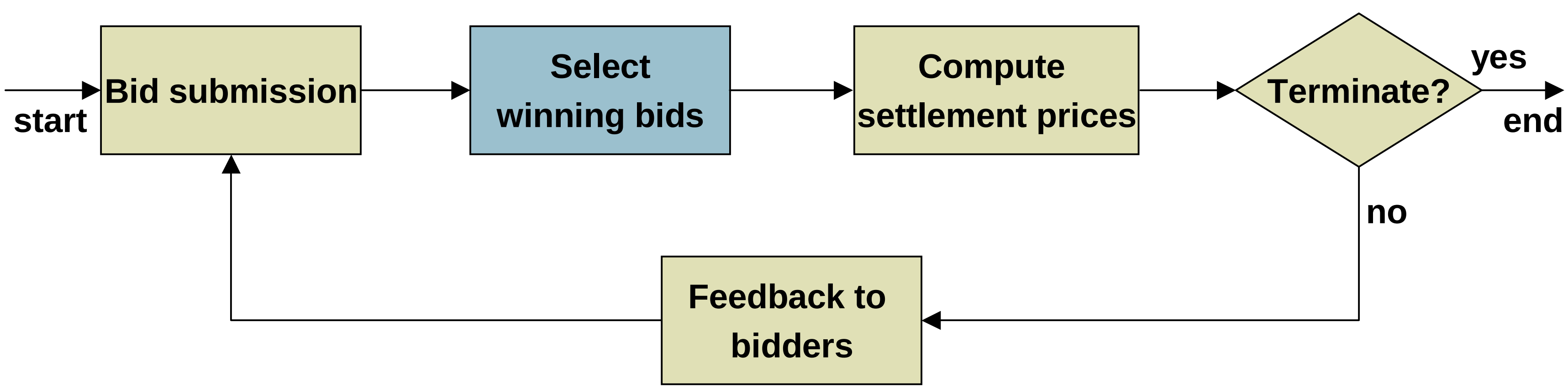
- Role of winner determination in iterative auctions
- Scenario: procurement auction with supply curves
- Modeling the problem as a combinatorial auction
- Solution method of choice: Branch-and-Price
  - Column generation
  - Feasible solution heuristics
  - Branching
- Comparison with naïve models
- Future directions

Motivation

- Procurement auctions are still going strong since powerful buyer can set the rules for its suppliers
- Mutiple items, multiple attributes and business requirements typical for particular industry must be considered
- Need a flexible formulation and efficient solution method
  - Can capture a variety of business requirements
  - Strong relaxations, high scalability
- Branch-and-Price as solution technology
  - Successful in other application areas (transportation, assignment, inventory logistics)
  - BCP framework readily available (www.coin-or.org session TB42)
    - Need to implement only the problem specific components

Iterative auctions

- Auction: negotiation through bidding (forward, reverse, double)
  - Participants: market maker (MM) and agents
- Bids: what and for how much agents want to trade
- Winner Determination: MM selects best allocation of goods
  - May be a subroutine of pricing and feedback



Multi-dimensional auctions in B2B

- Multiple items
  - Necessary (and profitable) if there is *correlation* between goods
- Multiple units
  - Decomposable goods and possibility of aggregation (buy-side, sell-side or both) warrants multi-unit auctions
  - Single unit auctions when items are non-decomposable for technical or marketing reasons (e.g., FCC licence)
- Multiple attributes
  - Goods are very rarely described by price and quantity only
    - Quality, geography, delivery time, etc.
  - Complex business requirements govern how goods can be traded

Combinatorial auctions: multiple items, indivisible bids

Mechanism design and its impact on WD

- How often is the market cleared?
  - Continuous (after each bid) or periodic (e.g., every hour)
  - Very fast response time needed if continuous, but bids change little from round to round
- How are the settlement prices computed?
  - Incentive compatibility can be computationally expensive
- Are approximate solutions acceptable?
  - Value of approximate solution might be close to optimal but the identity of the winners can be very different
  - How quickly is incentive compatibility lost if solutions are approximated?
- How do business requirements influence performance?
  - Even *feasible* allocations might be much more difficult to find

Winner determination

- Given a set of bids …
  - Winning bids from previous round(s) (could also include rejected bids)
  - Bids submitted since last round
- … compute an allocation of goods to bidders …
  - Determine which agents trade what
- … so that market maker's objective is optimized
  - Maximize profit or maximize social welfare
- Usually computationally difficult
  - NP hard and no good (ex ante) bounds on approximability

Scenario: procurement auction with supply curves

- Buyer wishes to purchase multiple items in large quantities to meet long-term need
- Must follow business requirements on allowable trades
- Food manufacturer

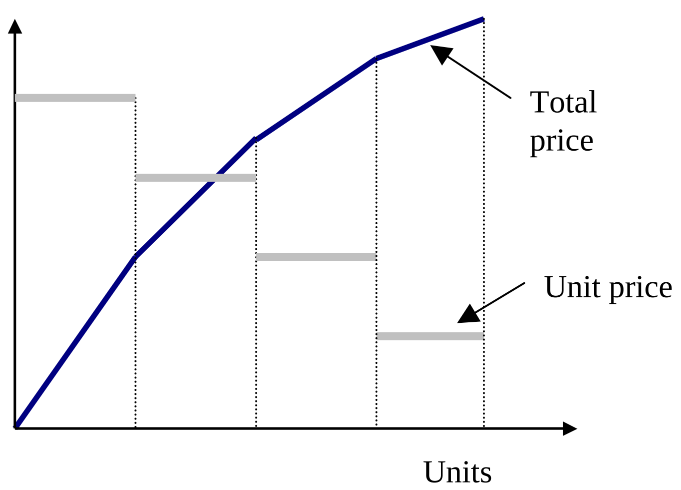
- Determine how much of each item to buy from the suppliers
  - Demand and business constraints are met
  - Cost is minimized

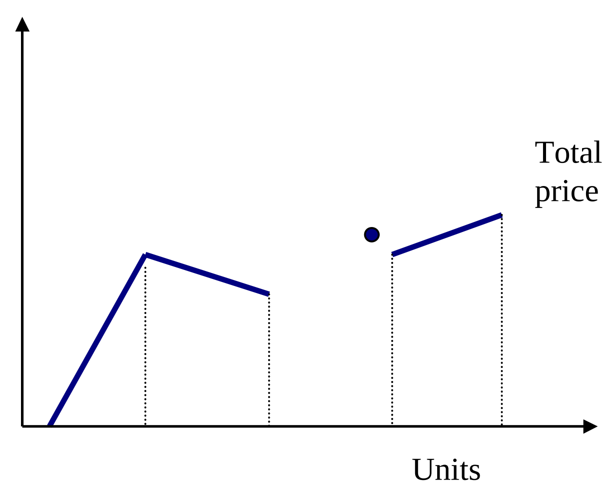
- Sellers provide price-quantity curves
  - Additive separable
  - Piece-wise linear

Examples of supply curves

- “volume discount”: unit price curve is decreasing stepfunction
- continuous
- concave

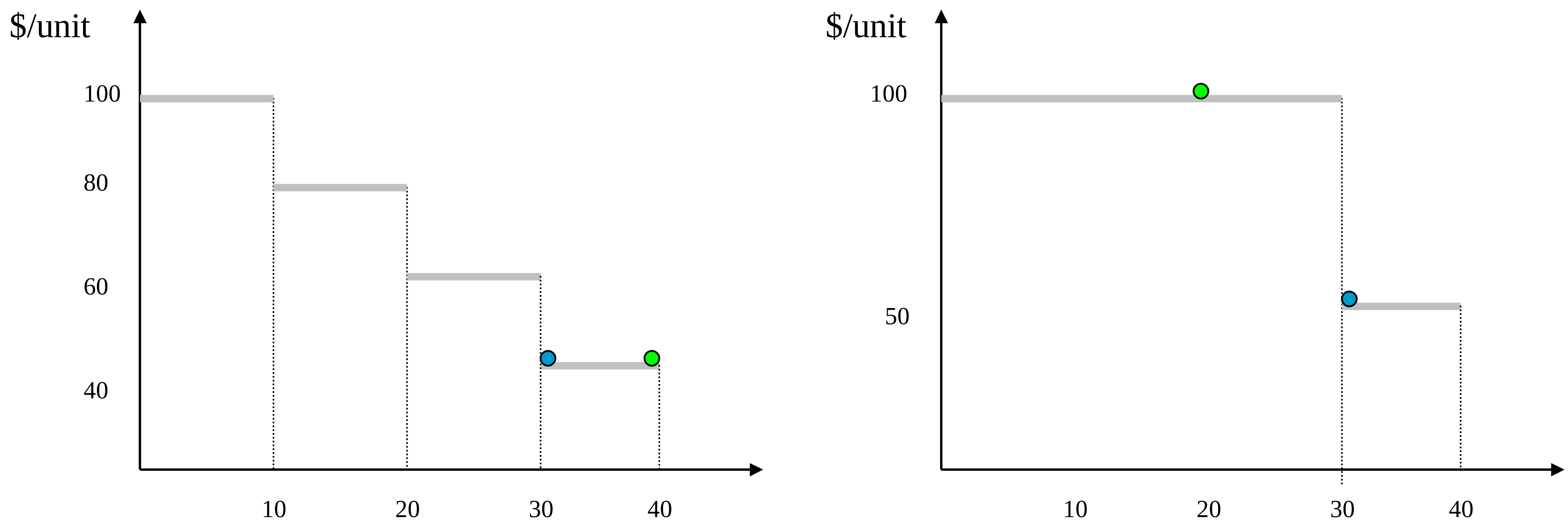
- may be discontinuous
- may have decreasing slopes
- any curve can be approximated by piece-wise linear curves





A small example for one good

- Procurer needs 60 units
- A greedy algorithm results in allocation 40, 20 (green points)
- Optimal solution is 30, 30 (blue points)



Examples of business requirements

- Lower and upper limits on number of winning suppliers
  - Relying on too few suppliers is risky
  - Too many winners increase overhead
- Lower and upper limits on the *total* quantity allocated to a winning supplier
- Lower and upper limits on the quantity *per item* allocated to a winning supplier
  - Too small allocated quantity discourages suppliers
  - Too large allocated quantity makes the buyer dependent on particular suppliers

- Business requirements result in interdependencies between items => need to trade items simultaneously

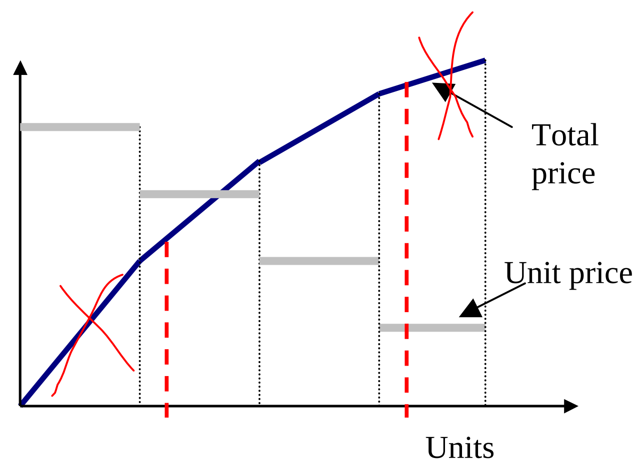
Why is this a combinatorial auction?

- Define *supply pattern* as an array of quantities $s = (a_1^s, \ldots, a_K^s)$
- Pattern is feasible for supplier if it satisfies all supplier-specific business requirements:
  - bounds on quantity for item k supplied by j: $l_k^j,\ u_k^j$
    - Can be handled by "trimming" the supply curve
  - bounds on total amount supplied by j:

$$l^j \le \sum_k a_k^s \le u^j$$

  - denote set for supplier j by S^j
- Cost of supply pattern for supplier j is

$$p^j(s) = \sum_k p_k^j(s)$$



Number of feasible patterns for a supplier might be exponential!

Why is this a combinatorial auction?

- Supplier's bid is represented by the XOR of patterns
- Business requirements that apply across agents are added as side constraints
- Multi-unit reverse combinatorial auction with patterns as bundles

$$\min \sum_j \sum_{s \in S^j} p^j(s) y^s$$ **←minimize total cost**

$$\sum_j \sum_{s \in S^j} a_k^s y^s \geq Q_k \quad \forall k$$ **←satisfy demand**

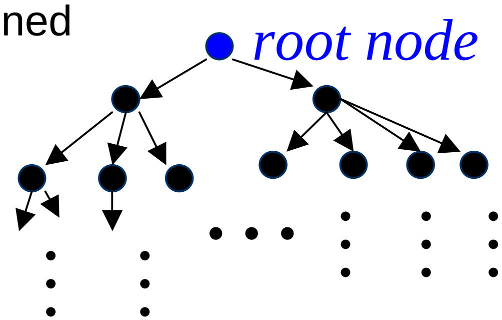
$$\sum_{s \in S^j} y^s \leq 1 \quad \forall j$$ **←at most one pattern per supplier**

$$L \leq \sum_j \sum_{s \in S^j} y^s \leq U$$ **←number of winning suppliers is limited**

$$y^s \in \{0,1\} \quad s \in \bigcup_j S^j$$ **←decision variables indicate which patterns are chosen**

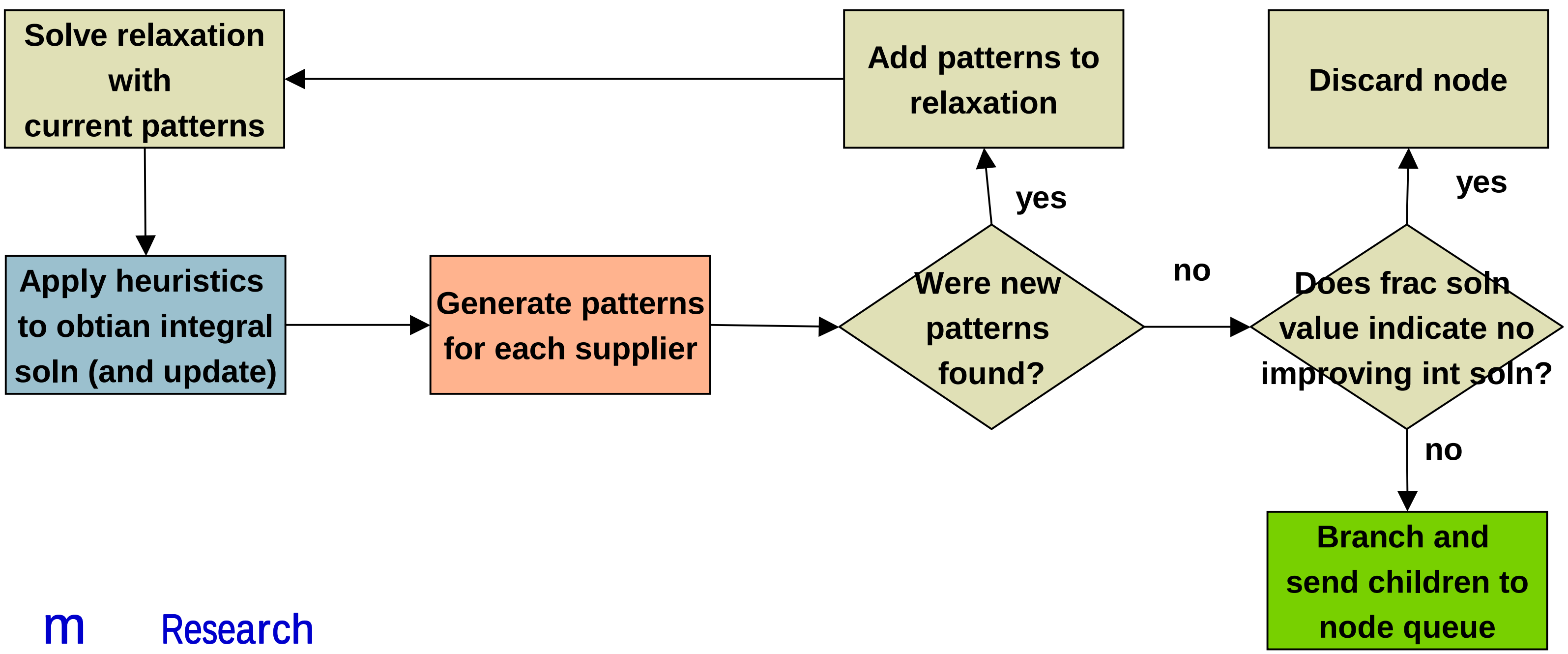
How to solve: Branch-and-Price

- Two difficulties:
  - too many variables to enumerate all before optimization
  - integrality requirement on variables
- Method of choice: Branch-and-Price
  - Branch-and-Bound backbone…
    - Solve model with variables *relaxed from integral to continuous*
    - If solution is not integral subdivide the feasible region (cut off frac opt)
    - Maintain best integral soln found
    - Branches provably w/o improving soln are pruned
  - …with generating patterns in each search tree node as needed
    - Start with an initial set of patterns
    - Generate patterns that improve objective
    - Branch if no patterns can be generated



Processing one node of the search tree

- Search tree nodes in a queue (initially populated with root only)
- Best integral solution found is maintained globally
- Main tasks: int soln heuristics, pattern generation, branching
- Rest of the tasks are taken care of BCP framework (coin-or.org)

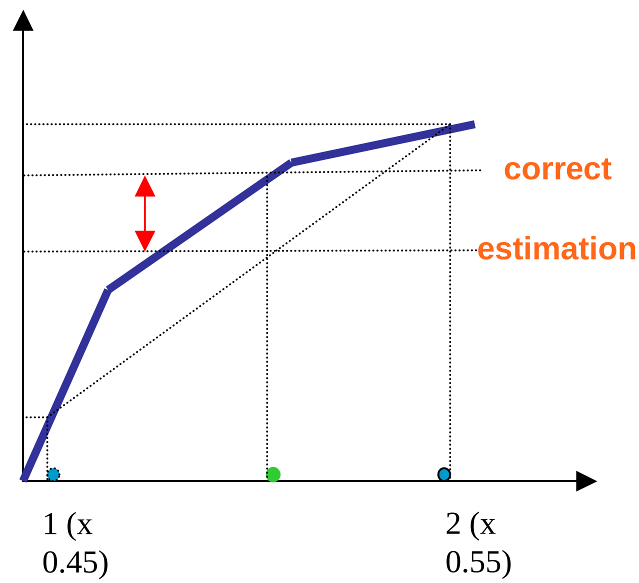


I: Integer solution heuristics

- Relaxed problem results in soln with fractional-valued patterns (primal solution to the LP)
- Goal is to find a solution with integer-valued patterns whose value is close to the value of the fractional solution
- Accomplished in two steps:
  - Rounding heuristics: construct soln with integer-valued patterns
    - Idea: “weight” patterns with their respective solution value; combine patterns with these weights for each supplier
  - Local improvement heuristics: improve an integer-valued soln
    - Idea: look for opportunities where some suppliers could form a “circle” and swap around a small quantity of some items while maintaining the feasibility of the patterns and the solution itself

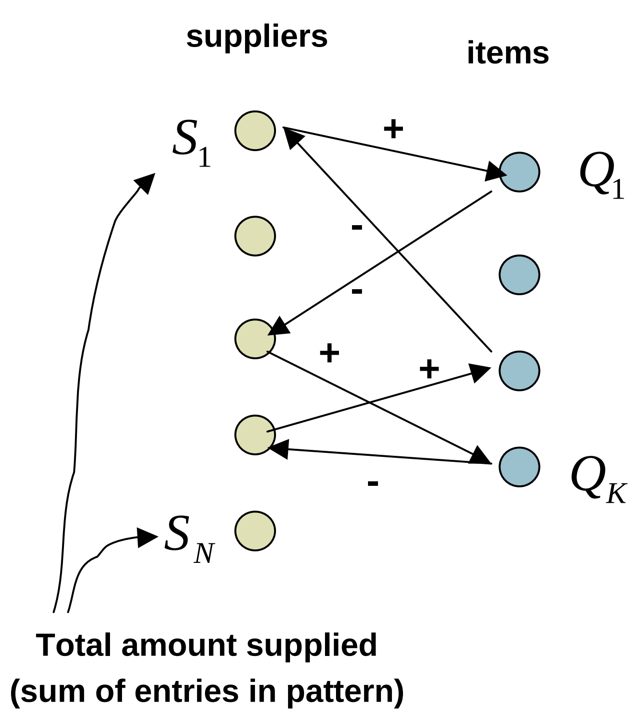
I: Rounding heuristics

- Weight of pattern: corresponding solution value y^s
- Make sure total weight for each supplier is either one or zero
  - Interpreted as supplier is a winner or not
  - Can be obtained by solving an easier Mixed Integer Program
- Create a single pattern for each winner as weighted combination of his patterns
  - Will be a feasible pattern if lower bounds on supplied quantities are zeros and the price curve has no discontinuities
  - Solution consisting of these patterns will be feasible
- Problem: value of solution will be far from value of fractional soln → local improvement heuristics



I: Local improvement heuristics

- Given a set of winners and their patterns, modify the entries in the patterns to obtain a better (less costly) solution

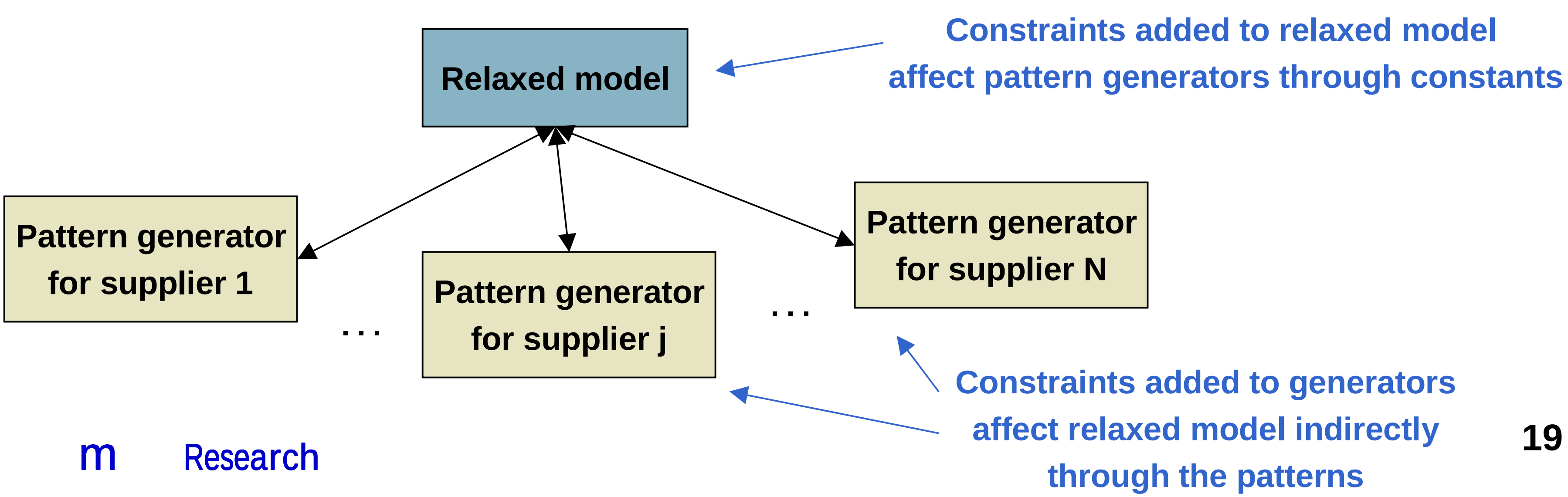


- If total supplied amounts are fixed: transportation problem with
  - Non-linear edge costs
  - Capacity limits on the edges
- NP-hard (unless cost fn is convex)
  - Here: concave fns (volume discount)
- Given patterns correspond to a feasible solution of this transportation problem
- Find improving solution by looking for negative cost cycles (circulation) in the residual graph and pushing flow around them

  (disclaimer: need some additional tricks b/c of non-linearity of cost functions – see tech report for details)

II: Pattern generation for a supplier

- Relaxed problem also yields price information (dual solution)
- For each supplier find pattern(s) that would give a positive surplus or prove that none exists; add patterns to relaxed problem
  - i.e., find column w/ smallest reduced cost
- Pattern generator separately for each supplier
  - Don’t need to be identical (different feasibility requirements)



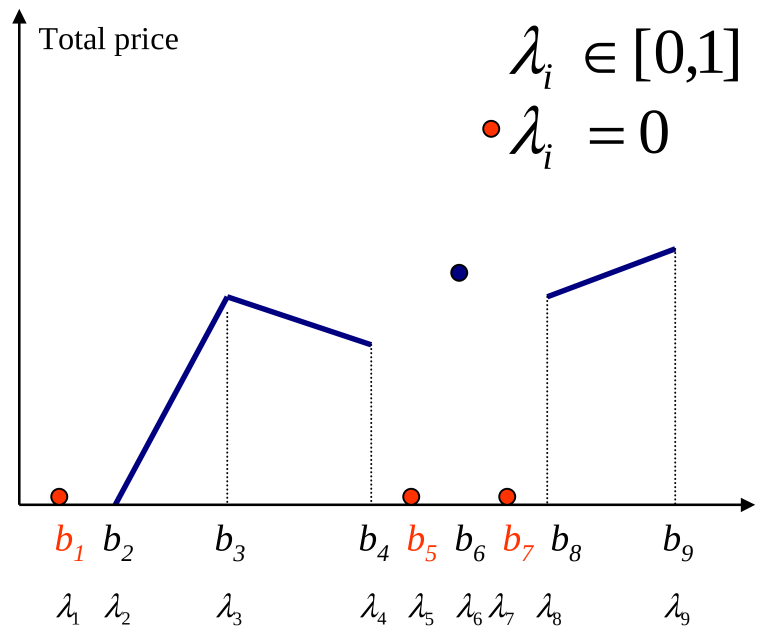
II: Pattern generation for a supplier

- Any value in a piece-wise linear function's domain can be represented as a convex combination of two neighboring breakpoints
  - That is, weights in convex combination form an SOS Type 2 set
  - Include dummy breakpoints at discontinuities

$$x = \sum_i \lambda_i b_i$$

$$\sum_i \lambda_i = 1$$

λ_i -s form an SOS Type 2 set



- The value of the fn can be expressed as the same convex combination: $p(x) = \sum_i \lambda_i p(b_i)$

II: Pattern generation for a supplier

- The unknowns are the entries in the pattern we seek
- Consider the breakpoint representation for each item this supplier bids on
- Represent each entry in the pattern with the set of weights corresponding to the breakpoints
- Minimize reduced cost so that pattern is feasible:

$$\min \sum_k \sum_i \left(p_k^j (b_{ki}^j) - \pi_k b_{ki}^j \right) \lambda_{ki}^j - \rho_j - \tau$$

$$l^j \le \sum_k \sum_i b_{ki}^j \lambda_{ki}^j \le u^j$$ ← **supplied amount between bounds**

$$\sum_i \lambda_{ki}^j = 1, \ \forall k$$

$$\lambda_{ki}^j \text{ -s form an SOS Type 2 set}, \forall k$$

π_k, ρ_j, τ

Dual prices from relaxed problem

III: Branching

- When no more patterns can be found at a search tree node the feasible region is subdivided
- Traditional (variable) branching:
  - For a variable at non-integral value: 2 branches, set variable to 0 or 1
  - 1-branch sets this supplier to be a winner but 0-branch carries little additional information (feasible region is split into uneven chunks)
  - Pattern generation must avoid a set of “forbidden” patterns – this is difficult to achive
- Pattern generation and branching must be consistent
- Branching should split feasible region more-or-less equally between children
- So what should we branch on?

III: Branching

- If there are suppliers for whom total weight of patterns is not one or zero then branch on whether supplier is a winner or not
  - Set supplier's XOR constraint to =1 or to =0
  - Effect on pattern generation: do not generate patterns for supplier in 0-branch
- Otherwise find a supplier and an item so that in two of the supplier's patterns the item is sold in quantities with different unit price (i.e., different intervals)
  - Branch on what unit price the item should have
  - Branches are specified by new bounds on supplied quantity – pattern generation is the same problem
- If neither of above: weighted combination of patterns gives optimal solution

III: Branching

- … find a supplier and an item so that in two of the supplier's patterns the item is sold in quantities for different unit price (i.e., different intervals)
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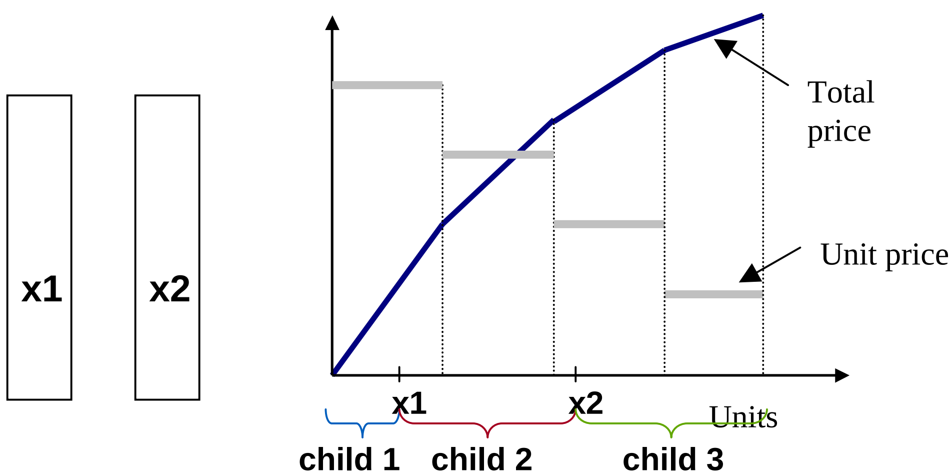


If neither of above: weighted combination of patterns gives optimal solution

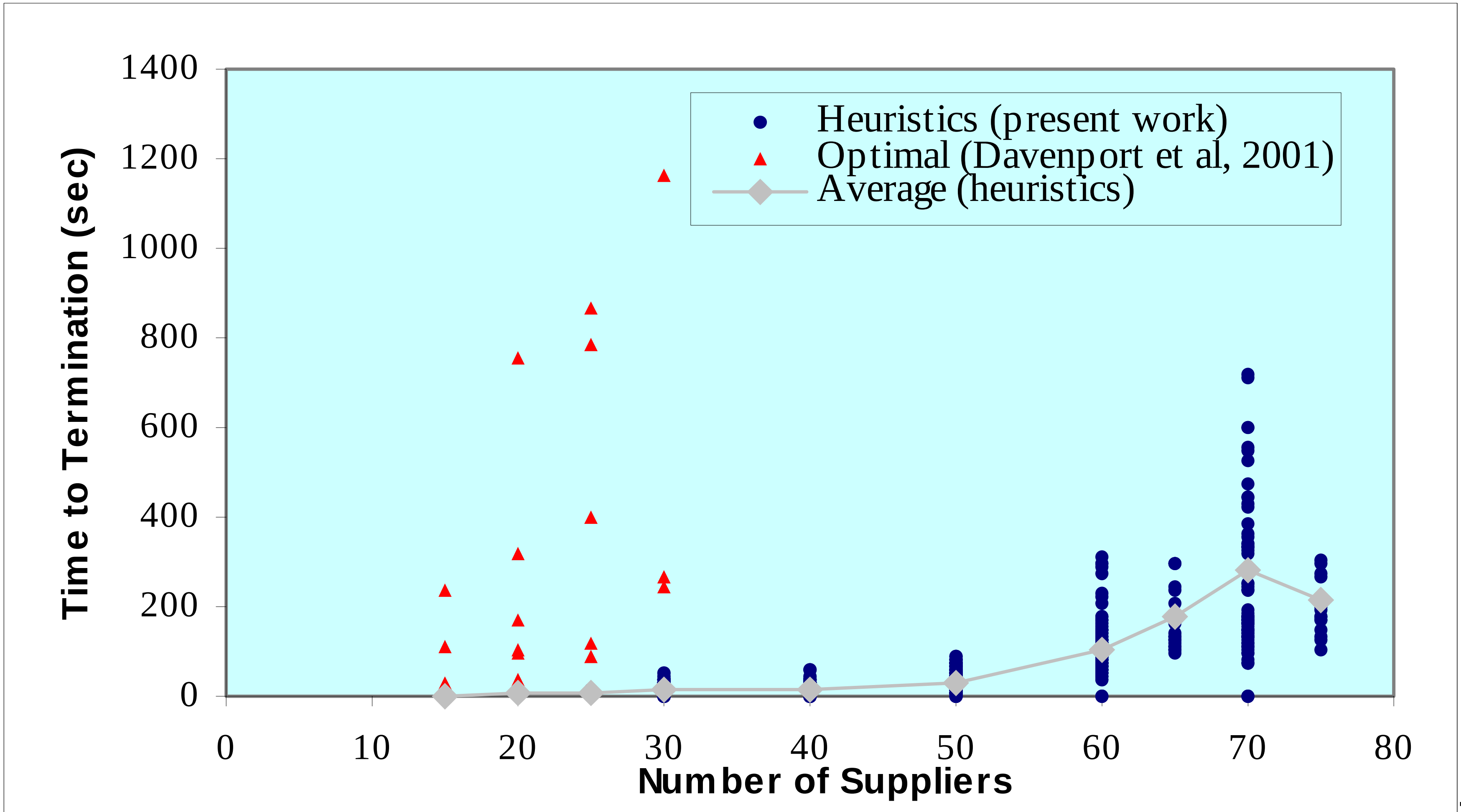
Comparison with naïve model

- Naïve model (multiple choice knapsack)
  - Variables specify the amount purchased from each supplier of each item, decision variables specify which suppliers are winners
  - Constraints for meeting demand, sets of constraints for each business requirement, constraints to define variables
  - New business constraints may require introduction of new variables
  - Having different requirements for suppliers is complicated to model
  - Solve to optimality with a commercial solver (CPLEX)
- We achive:
  - Stronger lower bounds and about the same integrality gap in a few seconds (even before branching) than naïve model in 10 minutes
  - Our model scales much better, most large problems take <2 mins, while naïve formulation almost always exhausts available

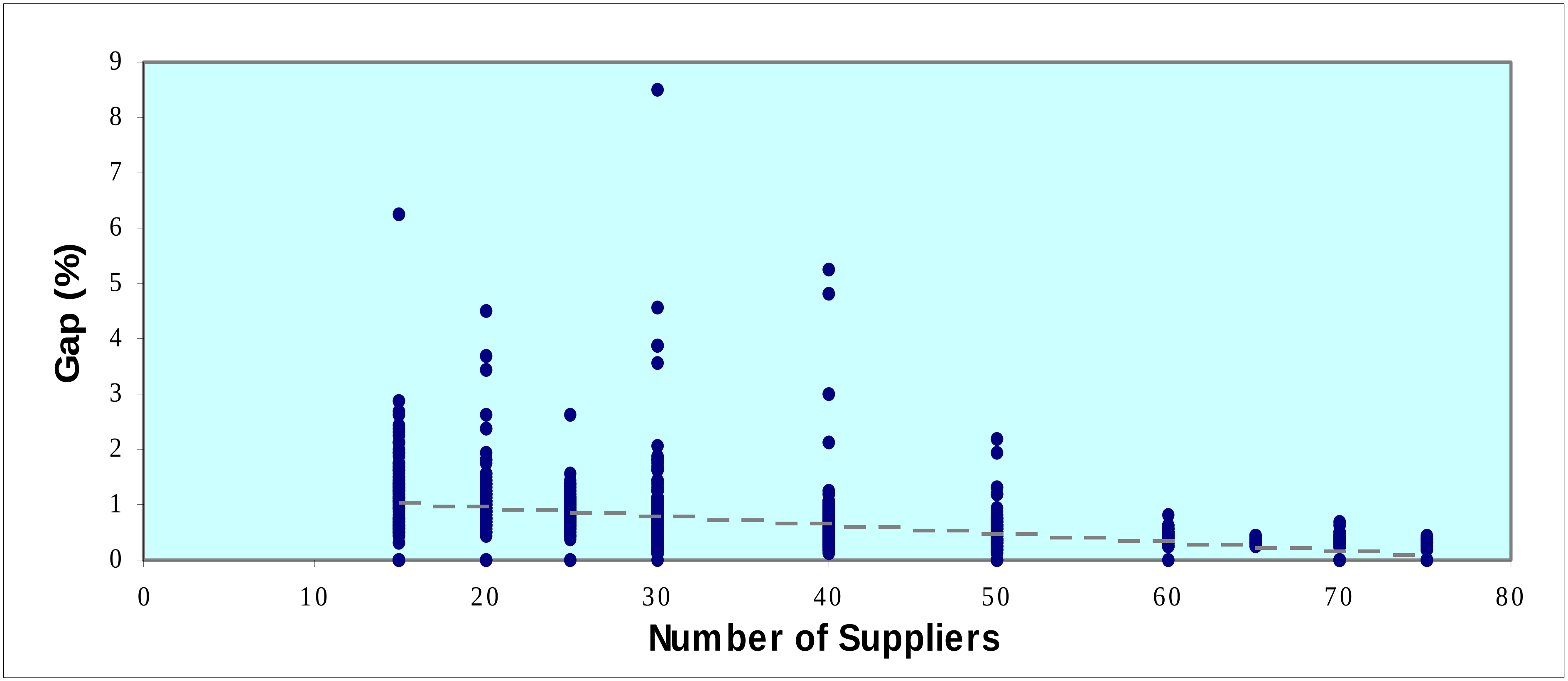
Test data generation

- Implemented problem generator
  - Data sets are assured to be feasible
  - Tightness can be adjusted via parameters
- Number of suppliers 15-75, number of items 10-60
- 4 different tightness settings
- Solve problems without limit on the number of winning suppliers
- Then make problems more difficult by setting upper limit on winning suppliers to 2 less than one obtained above
- Run commercial solver on naïve formulation and evaluating the root node for the new formulation (10 mins limit)

Experimental results



Experimental results



Upper Bound on Optimality Gap Vs Number of Suppliers

Future directions

- Warmstart next round with solution from previous round
- Technique can be applied to a variety of problems
  - Map out other industries and other business requirements
- Now that a robust solution method is available construct experiments to test impact of solving approximately on incentive compatibility

Technical report available from

http://www.research.ibm.com/auctions/publications.htm