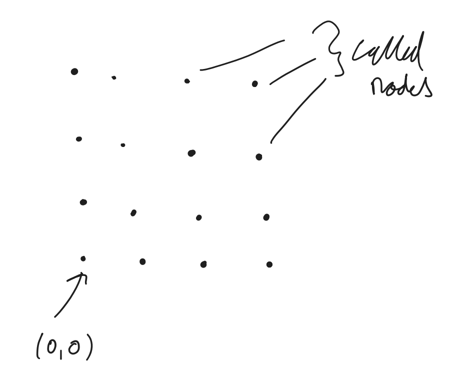
Why is Pascal's $\triangle$ combinatorial?

$$
\begin{array}{ccccccc}
 & & & 1 & & & \\
 & & 1 & & 1 & & \\
 & 1 & & 2 & & 1 & \\
1 & & 3 & & 3 & & 1 \\
 & & & \vdots & & &
\end{array}
$$

Consider the lattice:



Q. How many ways* to get from $(0,0)$ to (n,m)?

* a path along the nodes using only "right" or "up" steps: "increasing path".

Some examples:

$(0,0)$ to $(0,0)$: 1 way
$(0,0)$ to $(1,0)$: 1 way
$(0,0)$ to $(0,1)$: 1 way
$(0,0)$ to $(1,1)$: 2 ways

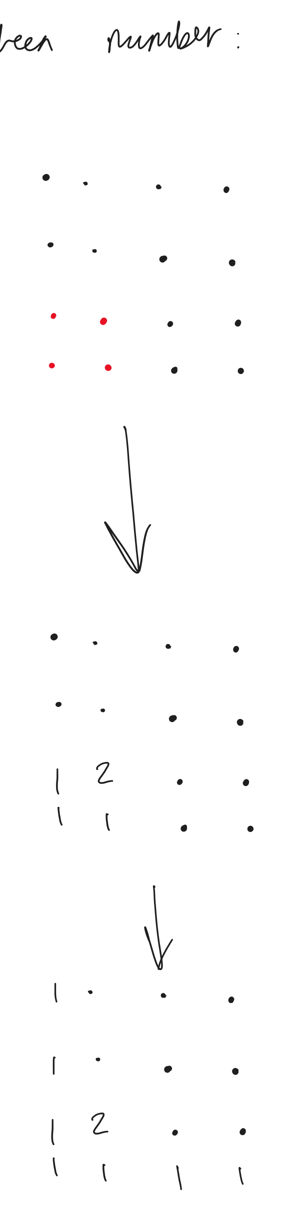
} do by inspection

Ok, what about in general?
Notice that to get from $(0,0)$ to (n,m), we need exactly n right steps and m up steps. So just need to find the number of ways of combining these.

Hmmm, a bit difficult at first. We can note that along the edges of the lattice, i.e., for all n, $(n,0)$ and for all m $(0,m)$, the number of possible paths is only 1.

$(0,0)$ to $(0,0)$: 1
$(0,0)$ to $(1,0)$: 1
$(0,0)$ to $(0,1)$: 1
$(0,0)$ to $(1,1)$: 2
$\vdots$

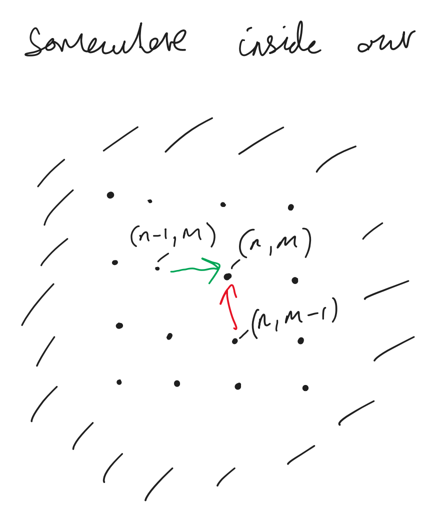
on our lattice, replace the node at the red point with our corresponding green number:



(edges are 1)

Looks familiar?
Let's make a recurrence relation:

Somewhere inside our lattice:

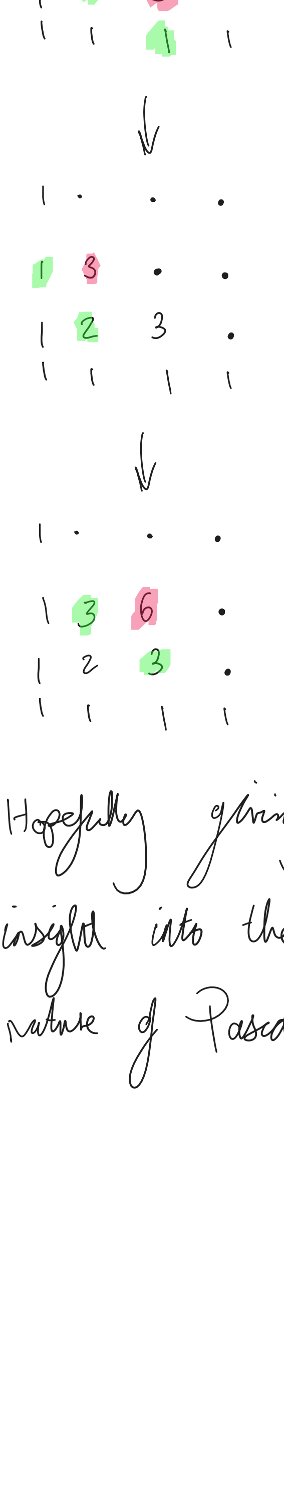


From $(n-1,m)$ to (n,m), only one way. Likewise, from $(n,m-1)$ to (n,m), only one way.

So, from $(0,0)$ to (n,m), is the sum of the ways from

$$\left.\begin{array}{l}(0,0) \text{ to } (n-1,m)\\ \quad + \\ (0,0) \text{ to } (n,m-1)\end{array}\right\} = (0,0) \text{ to } (n,m)$$

So now we are equipped to deal with any general point in the lattice:



Hopefully giving some insight into the combinatorial nature of Pascal's $\triangle$.