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### Coursework 1 Part A

#### 1.a – Calculations to process the first training example

( $x_1 = 0$ ,  $x_2 = 1$ , target = 1)

$w_1: -0.1$ ,  $w_2: -0.3$ ,  $w_3: -0.2$ ,  $w_4: 0.2$ ,  $w_5: 0.1$ ,  $w_6: -0.1$ ,  $w_7: 0.2$ ,  $w_8: -0.3$ ,  $w_9: 0.3$

Weights  $w_{10}$  and  $w_{11}$  are taken as 0 to connect  $x_1$  to sigmoid 3 and  $x_2$  to sigmoid 1 to create a fully connected neural network.

#### 1.a.i – Computed outputs for each network node during the forward pass (4 marks)

1. Sigmoid 1 ( $y_1$ ):  
$$\text{net}_1 = w_1 * x_1 + w_{11} * x_2 = -0.1 * 0 + 0 * 1 = 0$$
$$y_1 = 1 / (1 + e^{(-0)}) = 0.5000$$
2. Sigmoid 2 ( $y_2$ ):  
$$\text{net}_2 = w_3 * x_1 + w_4 * x_2 = -0.2 * 0 + 0.2 * 1 = 0.2000$$
$$y_2 = 1 / (1 + e^{(-0.2)}) \approx 0.5498$$
3. Sigmoid 3 ( $y_3$ ):  
$$\text{net}_3 = w_6 * x_2 + w_{10} * x_1 = -0.1 * 1 + 0 * 0 = -0.1000$$
$$y_3 = 1 / (1 + e^{(0.1)}) \approx 0.4750$$
4. Sum Node (output):  
$$\text{output} = w_2x_1 + w_5x_2 + w_9y_1 + w_8y_2 + w_7y_3$$
$$= -0.30 + 0.11 + 0.30.5000 + (-0.3)0.5498 + 0.20.4750$$
$$\approx 0.1 + 0.1500 - 0.1649 + 0.0950 \approx 0.1801$$

#### 1.a.ii - Computed errors (betas) at each network node during the backward pass (9 marks)

1. Output node:  
$$\beta_{\text{out}} = \text{target} - \text{output} = 1 - 0.1801 = 0.8199$$
2. Sigmoid 3:  
$$\beta_3 = \beta_{\text{out}} * w_7 * y_3 * (1 - y_3)$$
$$= 0.8199 * 0.2 * 0.4750 * (1 - 0.4750) \approx 0.0409$$
3. Sigmoid 2:  
$$\beta_2 = \beta_{\text{out}} * w_8 * y_2 * (1 - y_2)$$
$$= 0.8199 * (-0.3) * 0.5498 * (1 - 0.5498) \approx -0.0609$$
4. Sigmoid 1:  
$$\beta_1 = \beta_{\text{out}} * w_9 * y_1 * (1 - y_1)$$
$$= 0.8199 * 0.3 * 0.5000 * (1 - 0.5000) \approx 0.0615$$

### 1.a.iii – Computed weight changes (deltas) and weight updates (9 marks)

1.  $\Delta w_1 = \eta * \beta_1 * x_1 = 0.2 * 0.0615 * 0 = 0.0000$   
 $w_1(\text{new}) = -0.1 + 0.0000 = -0.1000$
2.  $\Delta w_2 = \eta * \beta_{\text{out}} * x_1 = 0.2 * 0.8199 * 0 = 0.0000$   
 $w_2(\text{new}) = -0.3 + 0.0000 = -0.3000$
3.  $\Delta w_3 = \eta * \beta_2 * x_1 = 0.2 * (-0.0609) * 0 = 0.0000$   
 $w_3(\text{new}) = -0.2 + 0.0000 = -0.2000$
4.  $\Delta w_4 = \eta * \beta_2 * x_2 = 0.2 * (-0.0609) * 1 \approx -0.0122$   
 $w_4(\text{new}) = 0.2 + (-0.0122) \approx 0.1878$
5.  $\Delta w_5 = \eta * \beta_{\text{out}} * x_2 = 0.2 * 0.8199 * 1 \approx 0.1640$   
 $w_5(\text{new}) = 0.1 + 0.1640 = 0.2640$
6.  $\Delta w_6 = \eta * \beta_3 * x_2 = 0.2 * 0.0409 * 1 \approx 0.0082$   
 $w_6(\text{new}) = -0.1 + 0.0082 = -0.0918$
7.  $\Delta w_7 = \eta * \beta_{\text{out}} * y_3 = 0.2 * 0.8199 * 0.4750 \approx 0.0779$   
 $w_7(\text{new}) = 0.2 + 0.0779 = 0.2779$
8.  $\Delta w_8 = \eta * \beta_{\text{out}} * y_2 = 0.2 * 0.8199 * 0.5498 \approx 0.0902$   
 $w_8(\text{new}) = -0.3 + 0.0902 = -0.2098$
9.  $\Delta w_9 = \eta * \beta_{\text{out}} * y_1 = 0.2 * 0.8199 * 0.5000 \approx 0.0820$   
 $w_9(\text{new}) = 0.3 + 0.0820 = 0.3820$
10.  $\Delta w_{10} = \eta * \beta_3 * x_1 = 0.2 * 0.0409 * 0 = 0.0000$   
 $w_{10}(\text{new}) = 0 + 0.0000 = 0.0000$
11.  $\Delta w_{11} = \eta * \beta_1 * x_2 = 0.2 * 0.0615 * 1 \approx 0.0123$   
 $w_{11}(\text{new}) = 0 + 0.0123 = 0.0123$

Weight Changes:

$w_1$ : 0.0000,  $w_2$ : 0.0000,  $w_3$ : 0.0000,  $w_4$ : -0.0122,  $w_5$ : 0.1640,  $w_6$ : 0.0082,  $w_7$ : 0.0779,  $w_8$ : 0.0902,  $w_9$ : 0.0820,  $w_{10}$ : 0.0000,  $w_{11}$ : 0.0123

Updated Weights After First Input:

**$w_1$ : -0.1000,  $w_2$ : -0.3000,  $w_3$ : -0.2000,  $w_4$ : 0.1878,  $w_5$ : 0.2640,  $w_6$ : -0.0918,  $w_7$ : 0.2779,  $w_8$ : -0.2098,  $w_9$ : 0.3820,  $w_{10}$ : 0.0000,  $w_{11}$ : 0.0123**

## 1.b – Calculations to process the second training example ( $x_1 = 1$ , $x_2 = 0$ , target = 1)

1.b.i – Computed outputs for each network node during the forward pass (4 marks)

1. Sigmoid 1 ( $y_1$ ):

$$\text{net1} = w_1 * x_1 + w_{11} * x_2 = -0.1000 * 1 + 0.0123 * 0 = -0.1000$$

$$y_1 = 1 / (1 + e^{(0.1000)}) \approx 0.4750$$

2. Sigmoid 2 ( $y_2$ ):

$$\text{net2} = w_3 * x_1 + w_4 * x_2 = -0.2000 * 1 + 0.1878 * 0 = -0.2000$$

$$y_2 = 1 / (1 + e^{(0.2000)}) \approx 0.4502$$

3. Sigmoid 3 ( $y_3$ ):

$$\text{net3} = w_6 * x_2 + w_{10} * x_1 = -0.0918 * 0 + 0.0000 * 1 = 0.0000$$

$$y_3 = 1 / (1 + e^{(-0.0000)}) = 0.5000$$

4. Sum Node (output):

$$\text{output} = w_2x_1 + w_5x_2 + w_9y_1 + w_8y_2 + w_7y_3$$

$$= -0.3000 * 1 + 0.2640 * 0 + 0.3820 * 0.4750 + (-0.2098) * 0.4502 + 0.2779 * 0.5000$$

$$\approx -0.3000 + 0 + 0.1815 - 0.0945 + 0.1390 \approx -0.0740$$

## 1.b.ii - Computed errors (betas) at each network node during the backward pass (9 marks)

1. Output node:

$$\beta_{\text{out}} = \text{target} - \text{output} = 1 - (-0.0740) = 1.0740$$

2. Sigmoid 3:

$$\beta_3 = \beta_{\text{out}} * w_7 * y_3 * (1 - y_3)$$

$$= 1.0740 * 0.2779 * 0.5000 * (1 - 0.5000) \approx 0.0746$$

3. Sigmoid 2:

$$\beta_2 = \beta_{\text{out}} * w_8 * y_2 * (1 - y_2)$$

$$= 1.0740 * (-0.2098) * 0.4502 * (1 - 0.4502) \approx -0.0558$$

4. Sigmoid 1:

$$\beta_1 = \beta_{\text{out}} * w_9 * y_1 * (1 - y_1)$$

$$= 1.0740 * 0.3820 * 0.4750 * (1 - 0.4750) \approx 0.1023$$

### 1.b.iii – Computed weight changes (deltas) and weight updates (9 marks)

$$1. \quad \Delta w_1 = \eta * \beta_1 * x_1 = 0.2 * 0.1023 * 1 \approx 0.0205$$

$$w_1(\text{new}) = -0.1000 + 0.0205 = -0.0795$$

$$2. \quad \Delta w_2 = \eta * \beta_{\text{out}} * x_1 = 0.2 * 1.0740 * 1 \approx 0.2148$$

$$w_2(\text{new}) = -0.3000 + 0.2148 = -0.0852$$

$$3. \quad \Delta w_3 = \eta * \beta_2 * x_1 = 0.2 * (-0.0558) * 1 \approx -0.0112$$

$$w_3(\text{new}) = -0.2000 + (-0.0112) = -0.2112$$

$$4. \quad \Delta w_4 = \eta * \beta_2 * x_2 = 0.2 * (-0.0570) * 0 = 0.0000$$

$$w_4(\text{new}) = 0.1878 + 0.0000 = 0.1878$$

$$5. \quad \Delta w_5 = \eta * \beta_{\text{out}} * x_2 = 0.2 * 1.0740 * 0 = 0.0000$$

$$w_5(\text{new}) = 0.2640 + 0.0000 = 0.2640$$

$$6. \quad \Delta w_6 = \eta * \beta_3 * x_2 = 0.2 * 0.0745 * 0 = 0.0000$$

$$w_6(\text{new}) = -0.0918 + 0.0000 = -0.0918$$

$$7. \quad \Delta w_7 = \eta * \beta_{\text{out}} * y_3 = 0.2 * 1.0740 * 0.5000 \approx 0.1074$$

$$w_7(\text{new}) = 0.2779 + 0.1074 = 0.3853$$

$$8. \quad \Delta w_8 = \eta * \beta_{\text{out}} * y_2 = 0.2 * 1.0740 * 0.4502 \approx 0.0967$$

$$w_8(\text{new}) = -0.2098 + 0.0967 = -0.1131$$

$$9. \quad \Delta w_9 = \eta * \beta_{\text{out}} * y_1 = 0.2 * 1.0740 * 0.4750 \approx 0.1020$$

$$w_9(\text{new}) = 0.3820 + 0.1020 = 0.4840$$

$$10. \quad \Delta w_{10} = \eta * \beta_3 * x_1 = 0.2 * 0.0746 * 1 \approx 0.0149$$

$$w_{10}(\text{new}) = 0.0000 + 0.0149 = 0.0149$$

$$11. \quad \Delta w_{11} = \eta * \beta_1 * x_2 = 0.2 * 0.0927 * 0 = 0.0000$$

$$w_{11}(\text{new}) = 0.0123 + 0.0000 = 0.0123$$

#### Updated Weights After Second Input:

**w1: -0.0795, w2: -0.0852, w3: -0.2112, w4: 0.1878, w5: 0.2640, w6: -0.0918, w7: 0.3853, w8: -0.1131, w9: 0.4840, w10: 0.0149, w11: 0.0123**

