Section 1 Solutions

The main theme is that students are memory and TAs are processors. The ideas provided below are in no way exhaustive.

<u>Challenge 1: Parallelism</u>

Imagine a room of 32 students. Each person has an integer (between 1 and 10) and refuses to do any calculation.

The TA needs to add up all of the numbers.

If it takes one second for a TA to ask a question to another person and get a response, it will take 32 seconds to calculate the total.

How long will it take if you had two TAs? (Hint it's not 16 seconds)

16 seconds for each TA to add up numbers from 16 students and 1 second for the 2 TAs to talk to each other (16+1=17).

How long will it take if you had four TAs? Eight TAs?

8 seconds for each TA to add up numbers from 8 students and 1 second for 4 TAs to pair up (2 pairs) resulting in 2 TAs and in turn 1 second for the 1 TA from each pair to talk to each other (8+1+1=10).

Similarly 4+1+1+1=7 seconds for 8 TAs.

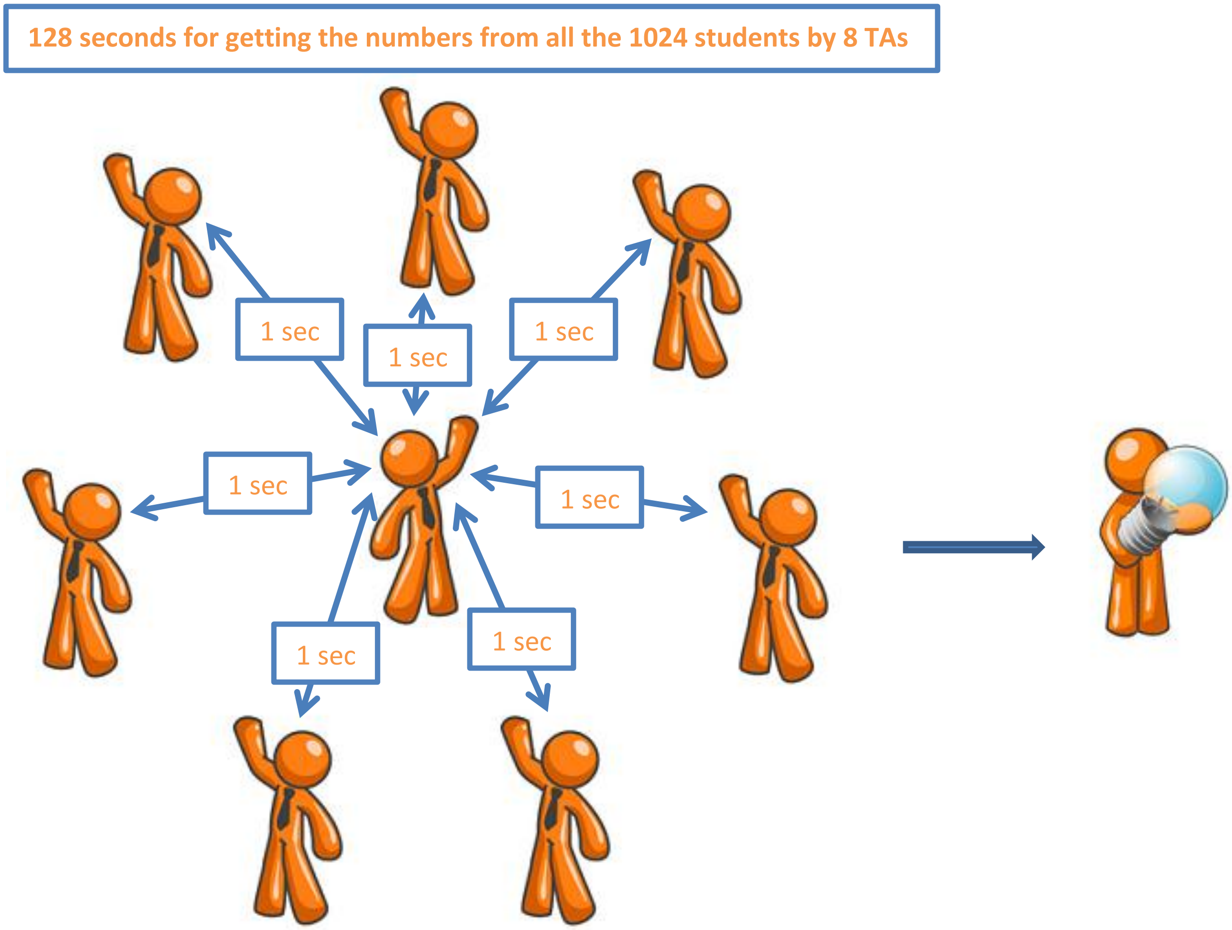
Notice that after adding up the numbers from the students, it takes 1 second for the number of TAs, who have relevant data, to decrease by half. Therefore we can see the logarithm kicking in.

Formula for the fastest time : $\frac{(\boldsymbol{Number\ of\ Students})}{(\boldsymbol{Number\ of\ TAs})} + \mathbf{log_2}(\boldsymbol{Number\ of\ TAs})$

The green part gives the time taken for the Student-TA interaction.

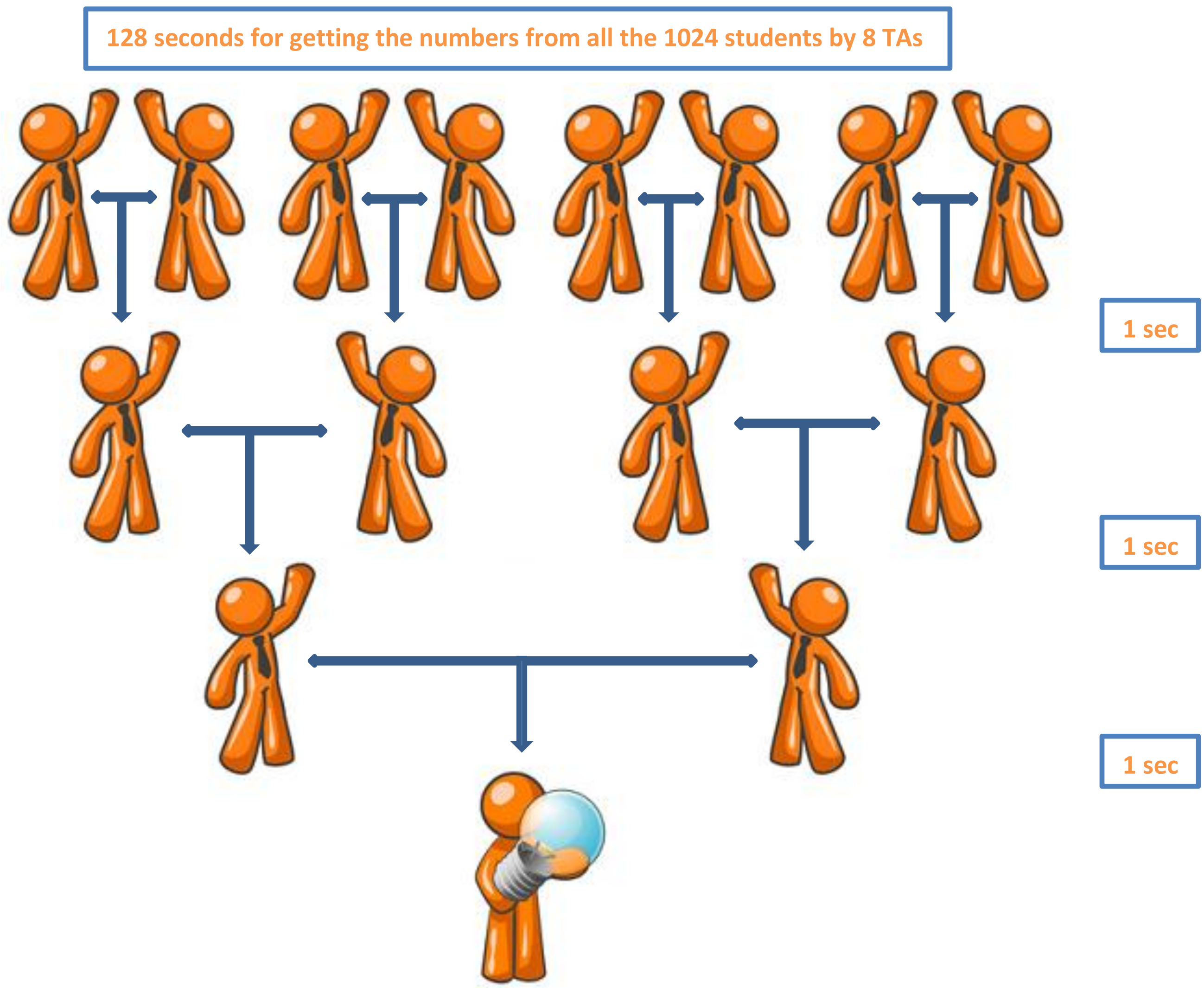
The red part gives the time taken for TA-TA interaction.

You can calculate 1024 student numbers with Eight TAs in 135 seconds. How?



Using the first part of the formula, 1024/8=128. Now allowing all 7 TAs talking to 1 TA takes 7 seconds (128+7=135).

You can calculate 1024 student numbers with Eight TAs in 131 seconds. How?



Using the formula, $1024/8+\log_2 8$=128+3=131 seconds.

Challenge 2: Debugging

You give all 32 students the same number and you have 4 TAs to find the total.

Strangely the last TA consistently gives you the wrong answer. There's a bug!

Think of 5 reasons why.

Write them down:

1. The last TA can't count above 7. (Notice that each student has a number from 1 to 10) (upper limit of datatype – overflow)
2. Bad communication between the last TA and student. (network/hardware latency)
3. Student or the last TA is lying. (virus)
4. Student holds a number different from what was given. (memory corrupt)
5. The last TA could not understand the student's language. (protocol mismatch)

Come up with test strategies for each reason.

i.e. How would you narrow down where the fault occurred (and by who).

i.e. You need to diagnose which one of your hypothesis is true.

Following steps elucidate the strategy:

i. Replace the last TA by one of the other TAs.
ii. Now if the present TA's answer matches with the other TAs, the past TA was buggy.
iii. If the present TA's answer matches with past TA, there is a bug among students.
iv. Use any of the other TAs to single out the buggy student/s by asking the value from each student of the last TA's group.

Challenge 3: Testing

You're ready to start selling your 4 TA-powered parallel-adding-machine.

Before shipping your TA-machine you decide you better have a set of test cases, to make sure it's working properly.

For the first test, you set all the students' values to one and check the output is 32. Is this a sufficient test?

Would it catch all possible ways in which your machine might be broken?

No. What if

- there are 2 students – 1 holding a value of '0' and another '2'.
- the machine gives 32 no matter what values the students hold.

What test cases would you use to try to ensure your machine was working properly?

Try to enumerate at least 5

i. Give all students a value of 1. (Tests lower limit)
ii. Give all students a value of 10. (Tests upper limit or overflow)
iii. Give all students a value of 0. (Illegal input)
iv. Give 1, 2, 3, 4, 5, 6, 7, 8 to the students, with respect to order, under each TA.
v. Randomized values to all students.

If you wanted to enumerate all possible input values, how long would it take?

10^{32} is the number of combinations of input values (1-10) that can be given to the set of 32 students. Using the above formula, the fastest time for a 4-TA-32-student scenario is 10 seconds. Therefore the fastest time to test all the 10^{32} scenarios is $10^{32} * 10 = 10^{33}$seconds.

There are roughly 10^7 seconds in a year. So 10^{33} seconds is roughly 10^{26} years.
The Universe is roughly 10 billion years old (10^{10} years).

So 10^{26} years = 10^{16} or 10 quadrillion or 10,000,000,000,000,000 times the age of the universe....