Matrices to transform geometry

In [1]:

```
import numpy as np
import matplotlib.pyplot as plt
```

In [9]:

```
def parse_squiggle(s):
    numbers = [float(num) for num in s.split()]
    a = np.array(numbers)
    a = a.reshape(-1,2).T
    return a

stickman = parse_squiggle("251.43 286.38 250.93 286.27 250.55 286.04 250.67 286.
61 250.93 286.95 251.31 287.29 251.94 287.63 252.44 287.86 253.33 288.09 254.08
288.32 255.09 288.54 256.11 288.66 257.24 288.77 258.76 289.11 260.02 289.23 261
.54 289.45 262.67 289.57 263.94 289.57 264.82 289.68 265.71 289.68 266.59 289.79
267.09 289.79 267.60 289.91 268.23 290.13 268.61 290.36 269.12 290.70 269.62 290
.93 270.13 291.04 270.76 291.04 271.26 291.04 271.64 291.04 271.26 291.04 271.39
290.70 271.64 290.13 272.02 289.34 272.40 288.43 273.16 287.07 273.79 286.04 274
.42 284.68 275.31 282.97 275.94 281.04 276.69 278.77 277.58 276.38 278.21 274.11
279.09 272.17 279.85 270.81 280.23 269.56 280.99 268.76 281.49 267.85 282.00 266
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290.97 265.92 291.60 267.17 292.11 268.42 292.61 269.67 293.24 271.15 293.62 272
.63 294.13 274.11 294.63 275.36 294.88 276.61 295.39 277.74 295.89 278.65 296.27
279.56 296.65 280.24 296.91 280.81 297.16 281.38 297.66 282.18 298.17 282.86 298
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283.88 328.23 282.29 327.73 280.47 327.35 278.31 326.59 276.15 326.08 273.99 325
.45 271.95 324.95 270.13 324.44 268.42 324.06 266.94 323.68 265.47 323.31 263.99
322.93 262.62 322.67 261.37 322.42 260.35 322.17 259.21 321.92 258.42 321.66 257
.39 321.41 256.48 321.16 255.69 321.03 254.89 320.91 253.98 320.78 253.30 320.53
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239.32 317.24 238.98 317.24 238.64 317.12 238.18 316.99 237.73 316.99 237.16 316
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253.58 152.69 253.33 153.83 253.07 154.74 252.95 155.76 252.95 156.90 252.82 158
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219.54 245.87 219.54 246.38 219.54 246.76 219.54 247.26 219.54 247.77 219.54 248
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.80 217.83 263.31 217.72 263.94 217.49 264.44 217.38 264.82 217.38 265.33 217.26
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.15 219.08 283.52 219.99 282.63 220.56 282.00 221.58 281.12 222.27 280.36 223.06
279.98 223.97 279.47 224.43 279.09 224.99 278.46 225.56 277.83 226.02 277.33 226
.81 276.82 227.49 276.57 228.29 276.32 228.86 275.94 229.31 275.56 229.65 275.18
230.00 274.67 230.56 274.42 231.02 274.04 231.36 273.79 231.93 273.54 232.38 273
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270.88 237.04 270.63 237.50 270.51 238.07 270.25 238.75 270.00 239.20 269.75 239

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.79 253.20 275.36 252.95 276.04 252.69 276.38 252.57 276.95 252.44 277.40 252.32
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.13 250.42 285.59 250.29 285.93 250.29 286.27 250.29 286.61 250.42 286.04 250.93
284.45")
stickman[1] *= -1
```
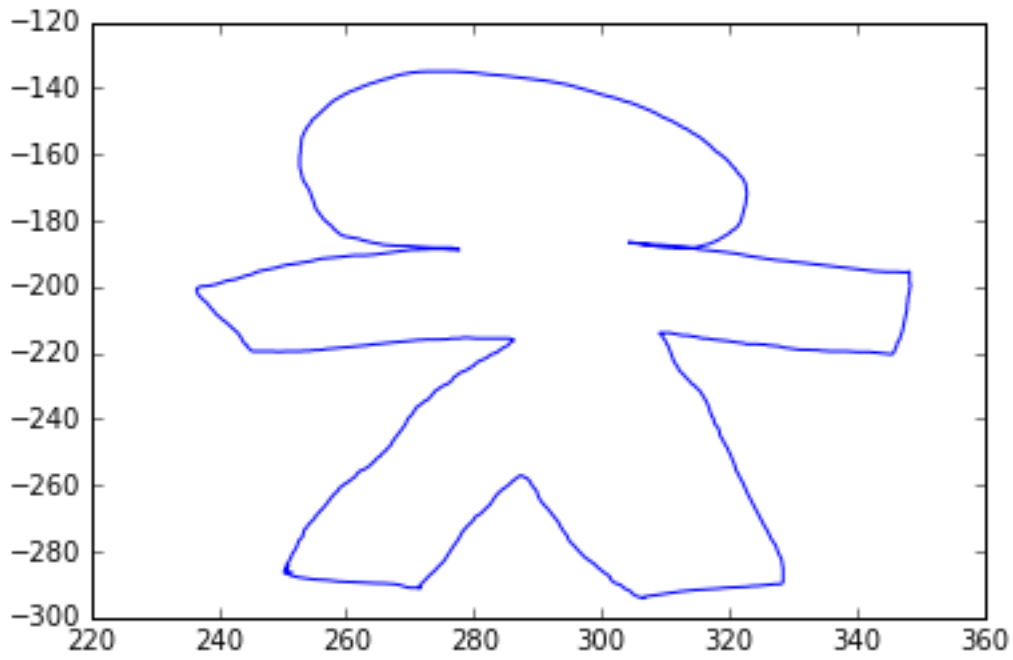

In [11]:

```
plt.plot(stickman[0], stickman[1])
```

Out[11]:

```
[<matplotlib.lines.Line2D at 0x106bd4750>]
```



Now define A to be a rotation matrix:

In [12]:

```
alpha = 0.1*np.pi
```

In [13]:

```
A = np.array([
    [np.cos(alpha), np.sin(alpha)],
    [-np.sin(alpha), np.cos(alpha)]
])
```

Why does this matrix act as a rotation?

- Think: What happens to $(1, 0)^T$ and $(0, 1)^T$ when they're multiplied by `A`?

Now multiply the geometry by this matrix, reassign to `stickman`, and plot:

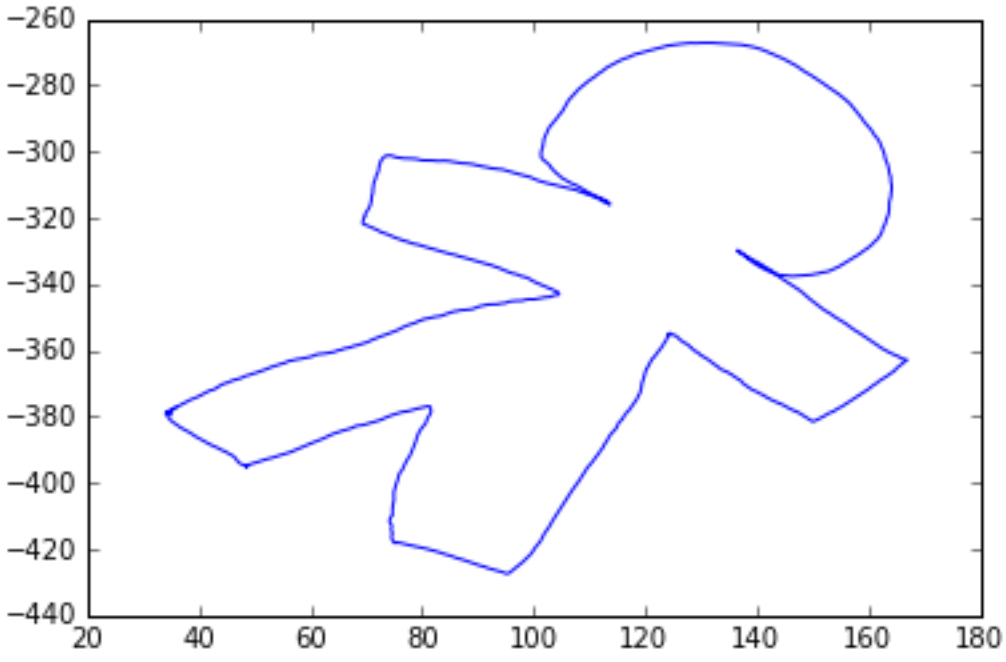
(Also try to evaluate this cell multiple times with Ctrl+Enter)

In [15]:

```
stickman = A.dot(stickman)
plt.plot(stickman[0], stickman[1])
```

Out[15]:

```
[<matplotlib.lines.Line2D at 0x106d39950>]
```



- Observe that this is, nominally, matrix-matrix multiplication. (`stickman` is a 2D array)
- However, `stickman` is really more 'array of vertices' than 'matrix'.
- The math doesn't care either way though.