STAT 400 | Spring 2015

Discussion 2

1. Of the students who have an internship with *Veridian Dynamics*, 40% are seniors, 30% are juniors, 20% are sophomores, and 10% are freshmen. At the end of each semester, the student interns are evaluated, 20% of the seniors receive "Exceeds Expectations" (EE) rating, so do 24% of juniors, 30% of sophomores, and 38% of freshmen. (The longer you have your internship, the more difficult it is to earn EE rating.)

a) If a student intern selected at random has EE rating, what is the probability he/she is a senior?

b) If a student intern selected at random does not have EE rating, what is the probability he/she is not a senior?

c) At the end of the semester, student interns who either are seniors or have EE rating, or both, receive a coffee mug as a bonus. What proportion of the student interns would receive a coffee mug?

2. The population of Lilliput is divided into Little-Endians and Big-Endians based on which end of a boiled egg they believe should be opened. Suppose that 60% of Lilliputians are Little-Endians, and 40 % are Big-Endians. Suppose also that 16% of Little-Endians and 26% of Big-Endians have blue eyes.

a) What proportion of Lilliput population have blue eyes?

b) You meet a Lilliputian with blue eyes. What is the probability that he is a Little-Endian?

c) What is the probability that a Lilliputian is a Big-Endian, if his eyes are not blue?

d) The first democratically elected Lilliputian president is a Little-Endian with blue eyes. His first day in office he proposed a tax break for Lilliputians who are either Little-Endians or have blue eyes, or both. What proportion of the Lilliput population would receive a tax break under his proposal?

3. Jack, Mike and Tom are roommates, and every Sunday night they split a large pizza for dinner. When there is only one slice left, the probability that Jack wants it is 0.40, the probability that Mike wants it is 0.35, and the probability that Tom wants it is 0.25. Suppose that whether or not each one of them will want the last slice is independent of the other two.

a) What is the probability that only one of the roommates will want the last slice?

b) What is the probability that at least one of the roommates will want the last slice?

c) What is the probability that at most one of the roommates will want the last slice?

4. Suppose that 70% of all customers at a particular store have a Visa credit card, and 50% of the customers have an American Express credit card. It is also known that 40% of customers who have Visa also have American Express.

a) What is the probability that a randomly selected customer would have either a Visa credit card or an American Express credit card, or both?

b) What proportion of customers who have an American Express credit card also have a Visa credit card?

c) Are events {a customer has a Visa credit card} and {a customer has an American Express credit card} independent?

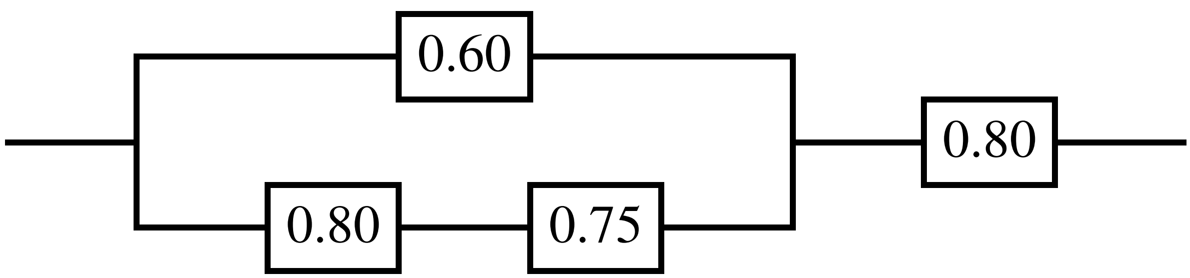
5. Alex learns that his favorite soccer team, Urbana-Champaign United ($\cup\subset\cup$), has a 70% chance of signing one of the best players in the world, Ron Aldo. He immediately runs some computer simulations and discovers that if $\cup\subset\cup$ signs Ron Aldo, it would have a 0.90 probability of winning the American Central Illinois Division championship. Unfortunately, if $\cup\subset\cup$ does not sign Ron Aldo, then the probability of winning the championship is only 0.40. Alex becomes too excited and slips into a coma. He comes out of the coma a year later and finds out that $\cup\subset\cup$ has won the championship. What is the probability that $\cup\subset\cup$ was able to sign Ron Aldo?

6. An electronic device has four independent components. Two of those four are new, and have a reliability of 0.80 each, one is old, with 0.75 reliability, and one is very old, and its reliability is 0.60.

a) Suppose that the device works if all four components are functional. What is the probability that the device will work when needed?

b) Suppose that the device works if at least one of the four components is functional. What is the probability that the device will work when needed?

c) Suppose that the four components are connected as shown on the diagram below. Find the reliability of the system.



1. Of the students who have an internship with *Veridian Dynamics*, 40% are seniors, 30% are juniors, 20% are sophomores, and 10% are freshmen. At the end of each semester, the student interns are evaluated, 20% of the seniors receive "Exceeds Expectations" (EE) rating, so do 24% of juniors, 30% of sophomores, and 38% of freshmen. (The longer you have your internship, the more difficult it is to earn EE rating.)

$P(\text{Sr}) = 0.40, \quad P(\text{Jr}) = 0.30, \quad P(\text{So}) = 0.20, \quad P(\text{Fr}) = 0.10,$

$P(\text{EE} \mid \text{Sr}) = 0.20, \quad P(\text{EE} \mid \text{Jr}) = 0.24, \quad P(\text{EE} \mid \text{So}) = 0.30,$

$P(\text{EE} \mid \text{Fr}) = 0.38.$

a) If a student intern selected at random has EE rating, what is the probability he/she is a senior?

Law of Total Probability:

$$P(\text{EE}) = P(\text{Sr}) \times P(\text{EE} \mid \text{Sr}) + P(\text{Jr}) \times P(\text{EE} \mid \text{Jr}) + P(\text{So}) \times P(\text{EE} \mid \text{So}) + P(\text{Fr}) \times P(\text{EE} \mid \text{Fr})$$

$$= 0.40 \times 0.20 + 0.30 \times 0.24 + 0.20 \times 0.30 + 0.10 \times 0.38 = 0.25.$$

Bayes' Theorem:

$$P(\text{Sr} \mid \text{EE}) = \frac{P(\text{Sr} \cap \text{EE})}{P(\text{EE})} = \frac{P(\text{Sr}) \times P(\text{EE} \mid \text{Sr})}{P(\text{EE})} = \frac{0.40 \times 0.20}{0.25} = \frac{0.08}{0.25} = \mathbf{0.32}.$$

	Sr	Jr	So	Fr	
EE	0.08	0.072	0.06	0.038	0.25
EE′	0.32	0.228	0.14	0.062	0.75
	0.40	0.30	0.20	0.10	1.00

b) If a student intern selected at random does not have EE rating, what is the probability he/she is not a senior?

$$P(Sr' \mid EE') = \frac{P(Sr' \cap EE')}{P(EE')} = \frac{0.228 + 0.14 + 0.062}{0.75} = \frac{0.43}{0.75} = \mathbf{\frac{43}{75}} \approx 0.57333.$$

c) At the end of the semester, student interns who either are seniors or have EE rating, or both, receive a coffee mug as a bonus. What proportion of the student interns would receive a coffee mug?

$$P(Sr \cup EE) = P(Sr) + P(EE) - P(Sr \cap EE) = 0.40 + 0.25 - 0.08 = \mathbf{0.57}.$$

2. The population of Lilliput is divided into Little-Endians and Big-Endians based on which end of a boiled egg they believe should be opened. Suppose that 60% of Lilliputians are Little-Endians, and 40 % are Big-Endians. Suppose also that 16% of Little-Endians and 26% of Big-Endians have blue eyes.

P(Little) = 0.60, P(Big) = 0.40,

P(Blue | Little) = 0.16, (16% <u>of Little-Endians</u> have blue eyes),

P(Blue | Big) = 0.26. (26% <u>of Big-Endians</u> have blue eyes).

a) What proportion of Lilliput population have blue eyes?

	Blue	Blue′		
Little	$0.60 \cdot \mathbf{0.16}$ 0.096	0.504	**0.60**	P(Blue | Little) = 0.16
Big	$0.40 \cdot \mathbf{0.26}$ 0.104	0.296	**0.40**	P(Blue | Big) = 0.26
	0.20	0.80	1.00	

OR

$$P(\text{ Blue }) = P(\text{ Little }) \cdot P(\text{ Blue} \mid \text{Little }) + P(\text{ Big }) \cdot P(\text{ Blue} \mid \text{Big })$$
$$= 0.60 \cdot 0.16 + 0.40 \cdot 0.26 = 0.096 + 0.104 = \mathbf{0.20}.$$

b) You meet a Lilliputian with blue eyes. What is the probability that he is a Little-Endian?

$$P(\text{ Little} \mid \text{Blue }) = \frac{P(\text{ Little} \cap \text{Blue })}{P(\text{ Blue })} = \frac{0.096}{0.20} = \mathbf{0.48}.$$

c) What is the probability that a Lilliputian is a Big-Endian, if his eyes are not blue?

$$P(\text{ Big} \mid \text{Blue}') = \frac{P(\text{ Big} \cap \text{Blue}')}{P(\text{ Blue}')} = \frac{0.296}{0.80} = \mathbf{0.37}.$$

d) The first democratically elected Lilliputian president is a Little-Endian with blue eyes. His first day in office he proposed a tax break for Lilliputians who are either Little-Endians or have blue eyes, or both. What proportion of the Lilliput population would receive a tax break under his proposal?

$$P(\text{ Little} \cup \text{Blue }) = P(\text{ Little }) + P(\text{ Blue }) - P(\text{ Little} \cap \text{Blue })$$
$$= 0.60 + 0.20 - 0.096 = \mathbf{0.704}.$$

OR

$$P(\text{ Little} \cup \text{Blue }) = P(\text{ Little} \cap \text{Blue }) + P(\text{ Little} \cap \text{Blue}') + P(\text{ Big} \cap \text{Blue })$$
$$= 0.096 + 0.504 + 0.104 = \mathbf{0.704}.$$

OR

$$P(\text{ Little} \cup \text{Blue }) = 1 - P(\text{ Big} \cap \text{Blue}') = 1 - 0.296 = \mathbf{0.704}.$$

3. Jack, Mike and Tom are roommates, and every Sunday night they split a large pizza for dinner. When there is only one slice left, the probability that Jack wants it is 0.40, the probability that Mike wants it is 0.35, and the probability that Tom wants it is 0.25. Suppose that whether or not each one of them will want the last slice is independent of the other two.

(Jack) = 0.40,	P(Jack′) = 0.60,
(Mike) = 0.35,	P(Mike′) = 0.65,
(Tom) = 0.25,	P(Tom′) = 0.75.

a) What is the probability that only one of the roommates will want the last slice?

only Jack	Jack	Mike′	Tom′	$0.40 \times 0.65 \times 0.75 = 0.1950$,
only Mike	Jack′	Mike	Tom′	$0.60 \times 0.35 \times 0.75 = 0.1575$,
only Tom	Jack′	Mike′	Tom	$0.60 \times 0.65 \times 0.25 = 0.0975$.

P(only one wants the last slice) = P(only Jack **or** only Mike **or** only Tom)

= P(only Jack) + P(only Mike) + P(only Tom)

= 0.1950 + 0.1575 + 0.0975 = **0.45**.

b) What is the probability that at least one of the roommates will want the last slice?

P(at least one wants the last slice) = 1 − P(no one wants the last slice)

$= 1 - P(\text{Jack}' \cap \text{Mike}' \cap \text{Tom}')$

$= 1 - 0.60 \times 0.65 \times 0.75$

= 1 − 0.2925 = **0.7075**.

c) What is the probability that at most one of the roommates will want the last slice?

P(at most one wants the last slice)

= P(only one wants the last slice) + P(no one wants the last slice)

= 0.45 + 0.2925 = **0.7425**.

4. Suppose that 70% of all customers at a particular store have a Visa credit card, and 50% of the customers have an American Express credit card. It is also known that 40% of customers who have Visa also have American Express.

$P(V) = 0.70, \qquad P(AE) = 0.50, \qquad P(AE \mid V) = 0.40.$

a) What is the probability that a randomly selected customer would have either a Visa credit card or an American Express credit card, or both?

Need $P(V \cup AE) = P(V) + P(AE) - P(V \cap AE)$.

$P(V \cap AE) = P(V) \times P(AE \mid V) = 0.70 \times 0.40 = 0.28.$

(28% of all customers have Visa and American Express)

$P(V \cup AE) = P(V) + P(AE) - P(V \cap AE) = 0.70 + 0.50 - 0.28 = \mathbf{0.92}.$

b) What proportion of customers who have an American Express credit card also have a Visa credit card?

$$P(V \mid AE) = \frac{P(V \cap AE)}{P(AE)} = \frac{0.28}{0.50} = \mathbf{0.56}.$$

c) Are events {a customer has a Visa credit card} and {a customer has an American Express credit card} independent?

$P(V \cap AE) = 0.28. \qquad P(V) \times P(AE) = 0.70 \times 0.50 = 0.35.$

Since $P(V \cap AE) \neq P(V) \times P(AE)$, V and AE are **NOT Independent**.

OR

$P(AE \mid V) = 0.40. \qquad P(AE) = 0.50.$

Since $P(AE \mid V) \neq P(AE)$, V and AE are **NOT Independent**.

OR

$P(V \mid AE) = 0.56. \qquad P(V) = 0.70.$

Since $P(V \mid AE) \neq P(V)$, V and AE are **NOT Independent**.

5. Alex learns that his favorite soccer team, Urbana-Champaign United ($\cup\subset\cup$), has a 70% chance of signing one of the best players in the world, Ron Aldo. He immediately runs some computer simulations and discovers that if $\cup\subset\cup$ signs Ron Aldo, it would have a 0.90 probability of winning the American Central Illinois Division championship. Unfortunately, if $\cup\subset\cup$ does not sign Ron Aldo, then the probability of winning the championship is only 0.40. Alex becomes too excited and slips into a coma. He comes out of the coma a year later and finds out that $\cup\subset\cup$ has won the championship. What is the probability that $\cup\subset\cup$ was able to sign Ron Aldo?

$P(\mathrm{RA}) = 0.70,$ $\quad P(\mathrm{W} \mid \mathrm{RA}) = 0.90,$ $\quad P(\mathrm{W} \mid \mathrm{RA}') = 0.40.$

Bayes' Theorem:

$$P(\mathrm{RA} \mid \mathrm{W}) = \frac{P(\mathrm{RA}) \times P(\mathrm{W} \mid \mathrm{RA})}{P(\mathrm{RA}) \times P(\mathrm{W} \mid \mathrm{RA}) + P(\mathrm{RA}') \times P(\mathrm{W} \mid \mathrm{RA}')}$$

$$= \frac{0.70 \times 0.90}{0.70 \times 0.90 + 0.30 \times 0.40} = \frac{0.63}{0.63 + 0.12} = \frac{0.63}{0.75} = \mathbf{0.84}.$$

OR

	W	W′	
RA	0.90 · 0.70 0.63	0.07	0.70
RA′	0.40 · 0.30 0.12	0.18	0.30
	0.75	0.25	1.00

$$P(\mathrm{RA} \mid \mathrm{W}) = \frac{0.63}{0.75} = \mathbf{0.84}.$$

6. An electronic device has four independent components. Two of those four are new, and have a reliability of 0.80 each, one is old, with 0.75 reliability, and one is very old, and its reliability is 0.60.

Let $A_i = \{\, i^{\text{th}}$ component is functional $\}$.

Then $P(A_1) = P(A_2) = 0.80, \quad P(A_3) = 0.75, \quad P(A_4) = 0.60.$

a) Suppose that the device works if all four components are functional. What is the probability that the device will work when needed?

"all four" = "1st **and** 2nd **and** 3rd **and** 4th" = intersection.

$P(A_1 \cap A_2 \cap A_3 \cap A_4)$ since the components are independent

$$= P(A_1) \cdot P(A_2) \cdot P(A_3) \cdot P(A_4) = (0.80) \cdot (0.80) \cdot (0.75) \cdot (0.60) = \mathbf{0.288}.$$

b) Suppose that the device works if at least one of the four components is functional. What is the probability that the device will work when needed?

"at least one" = "either 1st **or** 2nd **or** 3rd **or** 4th **or** 5th" = union.

P(at least one) = 1 – P(none).

"none" = "not 1st **and** not 2nd **and** not 3rd **and** not 4th **and** not 5th".

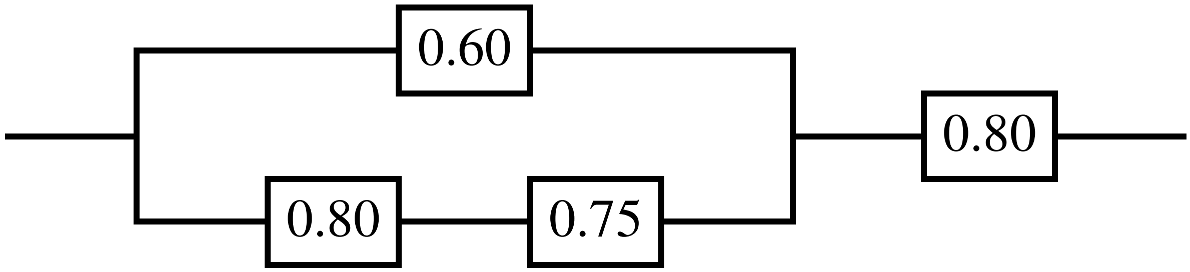
$$P(A_1 \cup A_2 \cup A_3 \cup A_4) = 1 - P(A_1' \cap A_2' \cap A_3' \cap A_4')$$

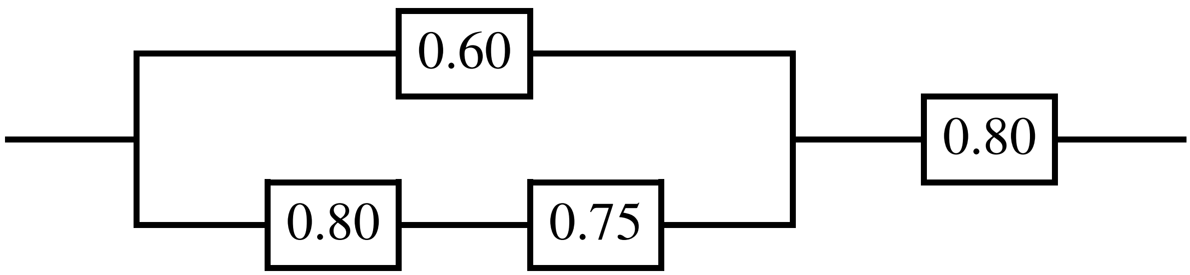
since the companies are independent

$$= 1 - P(A_1') \cdot P(A_2') \cdot P(A_3') \cdot P(A_4')$$

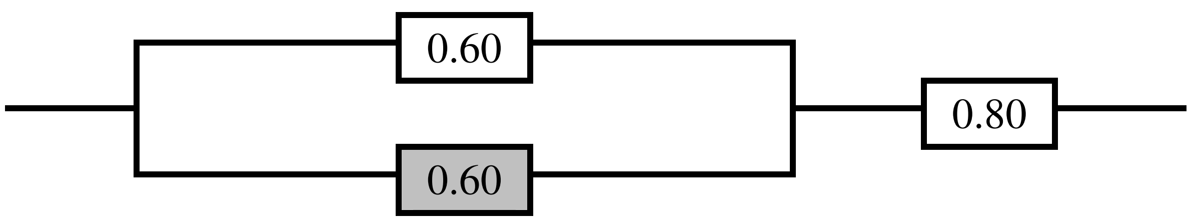
$$= 1 - (0.20) \cdot (0.20) \cdot (0.25) \cdot (0.40) = 1 - 0.004 = \mathbf{0.996}.$$

c) Suppose that the four components are connected as shown on the diagram below. Find the reliability of the system.





$0.80 \times 0.75 = 0.60$.



$1 - 0.40 \times 0.40 = 0.84$. OR $0.60 + 0.60 - 0.60 \times 0.60 = 0.84$.

$0.84 \times 0.80 = \mathbf{0.672}$.