Taylor Series

1分

We have seen in HW1 that a function defined by an infinite series can sometimes be approximated well with a truncated finite series. In HW1, this was shown for the exponential function

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

We will explore the same concept in HW2, but with the $sin(x)$ function.

Some important questions to answer are:

- Is e^x special, in that a finite series approximation worked well and will this work for $sin(x)$?
- In general, is there a class of functions that this will always work for?
- How do we derive a finite series approximation if we are not given the infinite series (e.g. the function e^x given above)?

The answers to these questions lie in understanding the *Taylor Series* expansion.

Read the Intro, Definition and first-half of the section on Analytic functions for *Taylor Series* (https://en.wikipedia.org/wiki/Taylor_series). Then answer the following question. You may also want to recall the definition of a *Power Series* (https://en.wikipedia.org/wiki/Power_series).

Which of the following statements about Taylor series are true? You may select more than one answer.

多项选择*

- [ ] If a function $f(x)$ can be represented by an infinite power series, then it has a Taylor series representation.
- [ ] If a Taylor series converges locally for a function, then it always converges globally for the same function.
- [ ] Finite Taylor series approximations are useful because one does not need to know the derivative (analytic or otherwise) of the function to be approximated $f(x)$, to create the finite Taylor approximation.
- [ ] A finite Taylor series approximation is *local*, meaning it does depend on a point of evaluation.

正确答案:

- If a function $f(x)$ can be represented by an infinite power series, then it has a Taylor series representation.
- A finite Taylor series approximation is *local*, meaning it does depend on a point of evaluation.

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