Numerical Methods (CS 357)

Worksheet

Part 1. Zeros in Backward Substitution

Suppose you encounter a zero on the diagonal of an upper triangular matrix U on which you are using backward substitution to solve $Ux = b$.

What happens?

(A) The algorithm completes as designed and finds a solution x of $Ux = b$.

(B) The algorithm takes more steps to complete.

(C) For most right-hand side vectors b, there is no solution to $Ux = b$.

(D) The algorithm fails to find the correct solution (which exists).

Part 2. Elimination matrices

Consider two 10×10 elimination matrices M_4 and M_7.

- M_4 only has off-diagonal entries (below the diagonal) in column 4.
- M_7 only has off-diagonal entries (below the diagonal) in column 7.

Which of the following is true?

(A) $M_4 = M_7$

(B) $M_4 M_7 = M_4 + M_7 - I$ (where I is the identity matrix)

(C) $M_7 M_4 = M_4 + M_7 - I$ (where I is the identity matrix)

(D) None of these

Part 3. Forward substitution

You're given a 2×2 real-valued, lower triangular matrix `L` and a right-hand side vector `b`. Use forward substitution to find a vector `x` so that `L.dot(x)` equals `b` (at least approximately).

(Since this is a small matrix with a known size, you shouldn't need `for` loops or other complicated things.)

```
import numpy as np
```