Quiz 10

1. For $n \geq 0$, let $K_n == \{a^i b^k \mid i \geq n,\ 0 < k < n\}$. Which of the following is true?

   (A) K_n is regular for all values of n

   (B) K_n is not regular for any value of n

   (C) There is an N_0 such that K_n is regular for all $n \leq N_0$ but not regular for $n > N_0$

   (D) The regularity of K_n depends on the value of n and cannot be described in a simple manner.

   Correct answer is (A).

2. For $n \geq 0$, let $W_n == \{a^i b^k \mid i \geq n,\ 0 < k < i\}$. Which of the following is true?

   (A) W_n is regular for all values of n

   (B) W_n is not regular for any value of n

   (C) There is an N_0 such that W_n is regular for all $n \leq N_0$ but not regular for $n > N_0$

   (D) The regularity of W_n depends on the value of n and cannot be described in a simple manner.

   Correct answer is (B).

3. Consider the following proof showing that $L = \mathbf{L}(0^*1^*)$ does not satisfy the pumping lemma. Let p be the pumping length. Consider the string $w = 001^p \in L$. Consider a $x = 0$, $y = 01$ and $z = 1^{p-1}$. Now observe that $xy^2z = 001011^{p-1} \notin L$. Hence, L does not satisfy the pumping lemma.

   (A) This proof demonstrates that L does not satisfy the pumping lemma.

   (B) This proof only shows that one particular w cannot be pumped. That is not enough to show that L does not satisfy the pumping lemma.

   (C) This proof only shows that a specific division of w into x, y, and z cannot be pumped. That is not enough to prove that L does not satisfy the pumping lemma.

   (D) This proof only shows that a specific value of the pumping length p is not correct. That is not enough to show that L does not satisfy the pumping lemma.

   Correct answer is (C).

4. Consider the language $F = \{a^i b^j c^k \mid i, j, k \geq 0 \text{ and if } i = 1 \text{ then } j = k\}$. Here are two proofs about the language F: the first one shows that F is not regular using closure properties, and the second one shows that F satisfies the pumping lemma.

   F is not regular: Consider $A = F \cap L(ab^*c^*) = \{ab^n c^n \mid n \geq 0\}$. Define $h : \{a, b, c\}^* \to \{0, 1\}^*$ where $h(a) = \epsilon$, $h(b) = 0$ and $h(c) = 1$. Then, $h(A) = \{0^n 1^n \mid n \geq 0\} = L_{0n1n}$, which is known to be not regular. Thus, F is not regular as L_{0n1n} was obtained from F by applying a series of regularity preserving operations.

   F satisfies the pumping lemma: Take the pumping length $p = 3$. Consider any $w = a^i b^j c^k \in F$, such that $|w| \geq p$. If $i \neq 2$, then divide w as follows: Take $x = \epsilon$, y to be the first symbol in w, and z to be the rest of the string. Now, $xyz = w$, $|xy| < 3$ and $|y| > 0$. Observe that the string xy^tz, when $t \neq 1$, has the property that the number of as is not 1, and hence $xy^tz \in L$ for any t. If $i = 2$, then divide w as follows: Take $x = aa$, y to be the first symbol after that, and z to be the rest of the string. Again, $w = xyz$, $|xy| \leq 3$, and $|y| > 0$. Further, for any t, xy^tz has 2 leading as, and so belongs to F trivially.

(A) The non-regularity proof using closure properties is incorrect because it relies on non-regular languages being closed under homomorphisms which does not hold!

(B) The pumping lemma proof is incorrect because it picks a specific value for the pumping length p

(C) The pumping lemma proof is incorrect because it picks a specific division of the string w.

(D) Both proofs are correct: F is not regular but it satisfies the pumping lemma.

Correct answer is (D).

5. Let $L \subseteq \Sigma^*$ be a language such that L satisfies the pumping lemma. What can we say about L?

   (A) L is regular.

   (B) L is not regular.

   (C) L may or may not be regular.

   (D) $\Sigma^* \setminus L$ is regular.

   Correct answer is (C).