STAT 420 Fall 2012

Homework #9

(due Friday, November 16, by 3:00 p.m.)

1. The friendly folks at the Internal Revenue Service (IRS) are always looking for ways to improve the wording and format of its tax return forms. Three new forms have been developed recently. To determine which, if any, are superior to current form, 120 individuals were asked to participate in an experiment. Each of the three new forms and the currently used form were filled out by 30 different people. The amount of time (in minutes) taken by each person to complete the task was recorded and stored in columns 1 through 4 (forms Form1 through Form4, respectively) in file `IRS.csv`.

a) Test for differences in average time required to fill these four forms using the ANOVA F test.

(i) Specify the null and the alternative hypotheses.

(ii) What are the required conditions (assumptions) for this test?

(iii) Show the calculations leading to your conclusion in the form of an ANOVA table.

(iv) What conclusions can be drawn from these data? Use $\alpha = 0.05$.

(i) $Y_{ij} = \mu_i + \varepsilon_{ij}$, $j = 1, 2, \dots, 30$, $i = 1, 2, 3, 4$,

where ε_{ij}'s are i.i.d. $N(0, \sigma^2)$.

H_0: $\mu_1 = \mu_2 = \mu_3 = \mu_4$, where μ_i = the average time required to fill Form i.

H_a: at least two of μ_i's are different.

OR

$Y_{ij} = \mu + \alpha_i + \varepsilon_{ij}$, $j = 1, 2, \dots, 30$, $i = 1, 2, 3, 4$,

where ε_{ij}'s are i.i.d. $N(0, \sigma^2)$, $\sum_{i=1}^{4} \alpha_i = 0$.

H_0: $\alpha_1 = \alpha_2 = \alpha_3 = \alpha_4 = 0$. H_a: Not H_0.

(ii) The time required to fill Form i is normally distributed with mean μ_i and common variance σ^2, $i = 1, 2, 3, 4$. Our data are four independent random samples from these four populations.

(iii)

```
> IRS = read.table("I:\\420f12/IRS.csv", sep=",", header=T)
> Time = c(IRS$Form1, IRS$Form2, IRS$Form3, IRS$Form4)
> Form = c(rep(1,30), rep(2,30), rep(3,30), rep(4,30))
> result = glm(Time ~ factor(Form))
> summary(aov(result))
                Df Sum Sq Mean Sq F value Pr(>F)
factor(Form)     3   8441  2813.6   2.887 0.0386 *
Residuals      116 113065   974.7
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

(iv) p-value = 0.0386 < 0.05.

Reject H_0 at $\alpha = 0.05$.

OR

```
> qf(0.95,3,116)
[1] 2.682809
```

$F = 2.887 > 2.6828$.

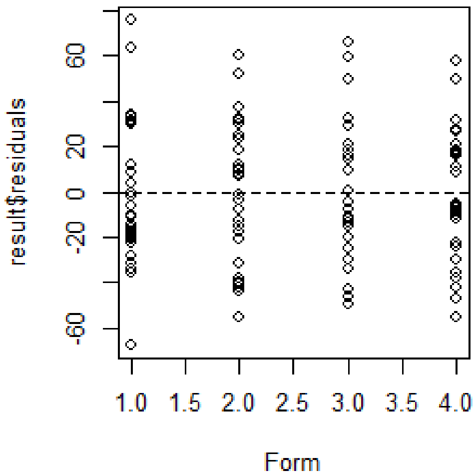
Reject H_0 at $\alpha = 0.05$.

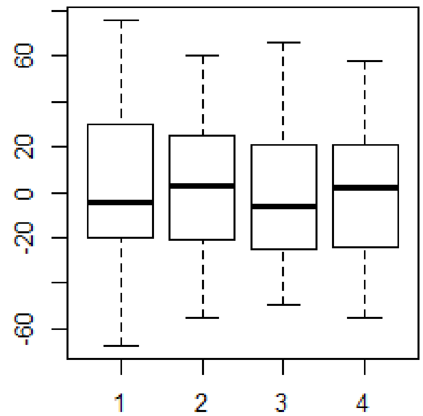
b) (i) Plot the residuals by Form (on one plot) and make four side-by-side boxplots for the residuals (one for each Form) to compare the spreads.

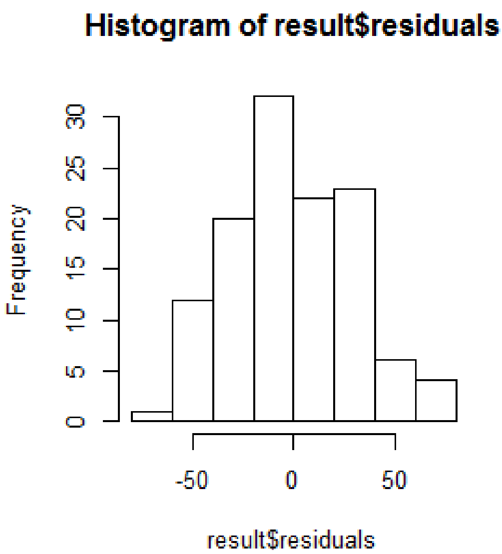
(ii) Make a histogram of the residuals and a normal Q–Q plot of the residuals.

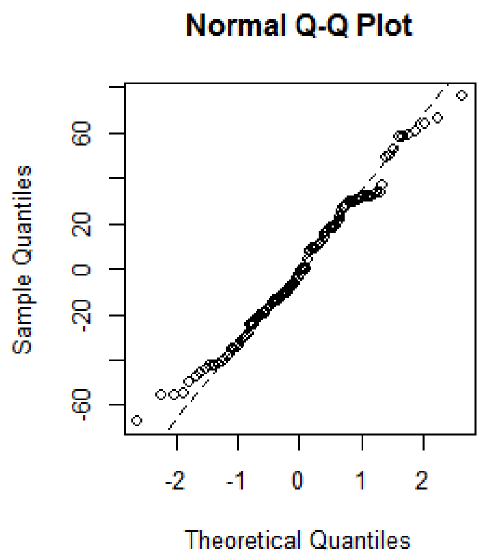
(iii) Comment on the assumptions made in part (a).

```
> # for 4 plots on the same page (and square plots)
> par(mfrow=c(2,2))
> par(pty="s")
> plot(Form,result$residuals)
> abline(h=0,lty=2)
> boxplot(result$residuals ~ Form)
> hist(result$residuals)
> qqnorm(result$residuals)
> qqline(result$residuals,lty=2)
```









The spreads of the residuals (first plot) are similar for the four forms. The four boxplots (second plot) are also similar in size. We have no reason to doubt the assumption that the variances of the times needed to fill out the four forms are equal.

The histogram of the residuals (third plot) is approximately bell-shaped. The points on the Normal Q-Q plot (fourth plot) line up roughly along the straight line. (The probability distribution of the residuals may have lighter tails than Normal distribution.) We have no reason to doubt the assumption that the times needed to fill out the four forms are normally distributed.

c) Use a 95% confidence level and Scheffé's multiple comparison procedure to compare the average time required to fill forms Form1, Form2, and Form3 with the average time required to fill the current form (Form4).

```
> mean(IRS$Form1)
[1] 90.2
> mean(IRS$Form2)
[1] 95.8
> mean(IRS$Form3)
[1] 106.8
> mean(IRS$Form4)
[1] 111.2
```

$$\text{Form4} - \frac{\text{Form1} + \text{Form2} + \text{Form3}}{3}$$

$$c_1 = -\frac{1}{3}, \quad c_2 = -\frac{1}{3}, \quad c_3 = -\frac{1}{3}, \quad c_4 = 1.$$

$$\sum_{j=1}^{4} c_j \, \overline{y}_j \pm \sqrt{2.6828} \cdot \sqrt{974.7} \cdot \sqrt{3 \cdot \sum_{j=1}^{4} \frac{c_j^2}{30}}$$

```
> (111.2-(90.2+95.8+106.8)/3) - sqrt(2.6828)*sqrt(974.7)*sqrt(3*(3*1/270+1/30))
[1] -5.072351
> (111.2-(90.2+95.8+106.8)/3) + sqrt(2.6828)*sqrt(974.7)*sqrt(3*(3*1/270+1/30))
[1] 32.27235
```

13.6 ± 18.67 **(– 5.07 , 32.27)**

d) Use a 95% confidence level and Tukey's pairwise comparison procedure to compare the average time required to fill forms Form1 with the average time required to fill the current form (Form4).

```
> qtukey(0.95,4,116)
[1] 3.686381
```

$$\left(\overline{y}_i - \overline{y}_j\right) \pm \frac{q_{\gamma, J, N-J}}{\sqrt{2}} \cdot \sqrt{\text{MSW}} \cdot \sqrt{\frac{1}{n_i} + \frac{1}{n_j}}$$

$$(111.2 - 90.2) \pm \frac{3.686381}{\sqrt{2}} \cdot \sqrt{974.7} \cdot \sqrt{\frac{1}{30} + \frac{1}{30}}$$

21 ± 21.0123717 **(– 0.01237 , 41.01237)**

OR

```
> TukeyHSD(aov(glm(Time ~ factor(Form))))
  Tukey multiple comparisons of means
    95% family-wise confidence level

Fit: aov(formula = glm(Time ~ factor(Form)))

$`factor(Form)`
    diff          lwr      upr     p adj
2-1  5.6 -15.41237134 26.61237 0.8989325
3-1 16.6  -4.41237134 37.61237 0.1727559
4-1 21.0  -0.01237134 42.01237 0.0501949
3-2 11.0 -10.01237134 32.01237 0.5240458
4-2 15.4  -5.61237134 36.41237 0.2293505
4-3  4.4 -16.61237134 25.41237 0.9474978
```

2. An engineer is investigating the strength of concrete beams made from four types of cement and employing three curing processes. The breaking strengths are measured and are presented below.

Curing Process	Cement Type			
	A	B	C	D
1	14	6	22	10
2	15	8	20	7
3	10	7	18	7

a) Specify an appropriate model for this situation and the underlying assumptions.

$Y_{ij} = \mu + \alpha_i + \beta_j + \varepsilon_{ij}$, $\quad i = 1, 2, 3, \quad j = 1, 2, 3, 4,$

$\alpha_1 + \alpha_2 + \alpha_3 = 0$, $\quad \beta_1 + \beta_2 + \beta_3 + \beta_4 = 0$,

(Curing Process) $\quad$ (Cement Type)

ε_{ij} are independent $N(0, \sigma^2)$ random variables.

b) Construct the ANOVA table and test for differences in cement types and curing processes. Assume a 5-percent significance level.

```
> Str <- c(14, 6,22,10,15, 8,20, 7,10, 7,18, 7)
> Cur <- c( 1, 1, 1, 1, 2, 2, 2, 2, 3, 3, 3, 3)
> Cem <- c( 1, 2, 3, 4, 1, 2, 3, 4, 1, 2, 3, 4)
>
> summary(aov(glm(Str ~ factor(Cur) + factor(Cem))))
            Df Sum Sq Mean Sq F value    Pr(>F)
factor(Cur)  2  14.00    7.00   2.625 0.1517037
factor(Cem)  3 318.00  106.00  39.750 0.0002361 ***
Residuals    6  16.00    2.67
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Curing Process:

$H_0: \alpha_1 = \alpha_2 = \alpha_3 = 0.$ $F = 2.625.$

P-value = 0.151704. Do NOT Reject $H_0: \alpha_1 = \alpha_2 = \alpha_3 = 0$ at $\alpha = 0.05$.

(Critical value: $F_{0.05}(2, 6) = 5.14$)

Cement Type:

$H_0: \beta_1 = \beta_2 = \beta_3 = \beta_4 = 0.$ $F = 39.75.$

P-value = 0.000236. Reject $H_0: \beta_1 = \beta_2 = \beta_3 = \beta_4 = 0$ at $\alpha = 0.05$.

(Critical value: $F_{0.05}(3, 6) = 4.76$)

Cement type affects the breaking strength.

3. Referring back to Problem 2, suppose that two additional beams are made for each combination of cement type and curing process and the breaking strengths determined. The complete layout follows.

Curing Process	Cement Type			
	A	B	C	D
1	14, 14, 10	6, 9, 7	22, 25, 20	10, 11, 13
2	15, 12, 10	8, 4, 11	20, 15, 19	7, 9, 10
3	10, 8, 10	7, 4, 7	18, 21, 22	7, 6, 6

a) Specify an appropriate model for this situation and the underlying assumptions.

$$Y_{ijk} = \mu + \alpha_i + \beta_j + (\alpha\beta)_{ij} + \varepsilon_{ijk}, \qquad i = 1, 2, 3, \quad j = 1, 2, 3, 4, \quad k = 1, 2, 3.$$

ε_{ijk} are independent $N(0, \sigma^2)$ random variables,

$\alpha_1 + \alpha_2 + \alpha_3 = 0,$ $\qquad \beta_1 + \beta_2 + \beta_3 + \beta_4 = 0,$

$(\alpha\beta)_{1j} + (\alpha\beta)_{2j} + (\alpha\beta)_{3j} = 0,$ $\qquad j = 1, 2, 3, 4,$

$(\alpha\beta)_{i1} + (\alpha\beta)_{i2} + (\alpha\beta)_{i3} + (\alpha\beta)_{i4} = 0,$ $\qquad i = 1, 2, 3.$

b) Construct the ANOVA table and test for the interaction first and then, if appropriate, the main effects. Assume a 5-percent significance level.

```
> Str <- c(14,14,10,  6, 9, 7, 22,25,20, 10,11,13,
+           15,12,10,  8, 4,11, 20,15,19,  7, 9,10,
+           10, 8,10,  7, 4, 7, 18,21,22,  7, 6, 6)
> Cur <- c(rep(1,12), rep(2,12), rep(3,12))
> Ce <- c(rep(1,3), rep(2,3), rep(3,3), rep(4,3))
> Cem <- c(rep(Ce,3))
> summary(aov(glm(Str ~ factor(Cur) * factor(Cem))))
                        Df Sum Sq Mean Sq F value    Pr(>F)
factor(Cur)              2  51.72   25.86  5.8187  0.008703 **
factor(Cem)              3 928.97  309.66 69.6729 5.463e-12 ***
factor(Cur):factor(Cem)  6  38.94    6.49  1.4604  0.233748
Residuals               24 106.67    4.44
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Interaction:

$H_0: (\alpha\beta)_{11} = (\alpha\beta)_{12} = \ldots = (\alpha\beta)_{34} = 0.$ $\quad$ $F = 1.4604.$

P-value = 0.233748. $\quad$ Do NOT Reject H_0 at $\alpha = 0.05$.

(Critical value: $F_{0.05}(6, 24) = 2.51$)

Curing Process:

$H_0: \alpha_1 = \alpha_2 = \alpha_3 = 0.$ $\quad$ $F = 5.8187.$

P-value = 0.008703. $\quad$ Reject H_0 at $\alpha = 0.05$.

(Critical value: $F_{0.05}(2, 24) = 3.40$)

Cement Type:

$H_0: \beta_1 = \beta_2 = \beta_3 = \beta_4 = 0.$ $\quad$ $F = 69.6729.$

P-value = 5.463×10^{-12}. $\quad$ REJECT H_0 at $\alpha = 0.05$.

(Critical value: $F_{0.05}(3, 24) = 3.01$)

Cement type and curing process do not seem to interact.

Both cement type and curing process affect the breaking strength.

c) Make an interaction plot.

```
> interaction.plot(Cur,Cem,Str)    > interaction.plot(Cem,Cur,Str)
```