

Computing the Nullspace

In [16]:

```
#keep  
import numpy as np  
import numpy.linalg as la
```

In [17]:

```
#keep  
n = 5  
np.random.seed(25)  
A = np.random.randn(n, n)  
  
# Decrease the rank  
A[4] = A[0] + 5 * A[2]  
A[1] = 3 * A[0] - 2 * A[3]
```

In [18]:

```
#keep  
from m_echelon import m_echelon
```

In [29]:

```
#keep  
M, U = m_echelon(A.T)
```

In [30]:

```
#keep  
la.norm(  
    M.dot(A.T) - U)
```

Out[30]:

```
1.2599818366099757e-15
```

In [31]:

```
#keep
U.round(3)
```

Out[31]:

```
array([[ 1.027,  0.676, -0.232,  1.202, -0.135],
       [ 0.    , -3.498, -1.468,  1.749, -7.342],
       [ 0.    ,  0.    , -2.14  ,  0.    , -10.699],
       [ 0.    ,  0.    ,  0.    ,  0.    , -0.    ],
       [ 0.    ,  0.    ,  0.    ,  0.    , -0.    ]])
```

Now define NUT as vectors spanning the nullspace of $N(U^T)$.

In [32]:

```
NUT = np.eye(n)[: , 3:]
NUT
```

Out[32]:

```
array([[ 0.,  0.],
       [ 0.,  0.],
       [ 0.,  0.],
       [ 1.,  0.],
       [ 0.,  1.]])
```

Check that it's actually a nullspace:

In [33]:

```
U.T.dot(NUT)
```

Out[33]:

```
array([[ 0.000e+00,  0.000e+00],
       [ 0.000e+00,  0.000e+00],
       [ 0.000e+00,  0.000e+00],
       [ 3.644e-16,  6.163e-33],
       [-1.776e-15, -2.174e-16]])
```

Now define NA as some vectors spanning $N(A)$:

In [34]:

```
NA = M.T.dot(NUT)
```

And check:

In [35]:

```
A.dot(NA)
```

Out[35]:

```
array([[ 1.110e-16, -1.110e-16],
       [ 4.441e-16, -4.441e-16],
       [-2.220e-16, -5.551e-17],
       [ 1.110e-16,  1.110e-16],
       [-8.882e-16, -6.661e-16]])
```

In []: