Quiz 19

1. Suppose a (single tape) Turing machine M on input w of length n, takes k steps and halts. Recall that a configuration of M is a string that describes the contents of tape cells until the rightmost non-blank cell, position of the head and the control state. What is the best upper bound on the length of the configuration at any step during the computation of M on w?

   (A) $O(\max(n, k))$

   (B) $O(n)$

   (C) $O(k^2)$

   (D) $O(2^k)$

   Correct answer is (A).

2. Let M be a 2-tape Turing machine, and let single(M) be the 1-tape Turing machine that simulates M as described in the proof of Theorem 1 in lecture 20 (pages 3 to 6). Consider a configuration (q, t_1, t_2), where t_1 (and t_2) describes the contents of tape 1 (of tape 2) and the head position. Suppose $|t_1| \leq k$ and $|t_2| \leq k$. What is the best upper bound on the number of steps single(M) will take in order to simulate one step of M from configuration (q, t_1, t_2)?

   (A) $O(k)$

   (B) $O(k^2)$

   (C) $O(2^k)$

   (D) $O(2^{2^k})$

   Correct answer is (A).

3. Let M be a 2-tape Turing machine, and let single(M) be the 1-tape Turing machine that simulates M as described in the proof of Theorem 1 in lecture 20 (pages 3 to 6). Suppose on input w of length n, M takes k-steps and halts, where $n \leq k$. What is the best upper bound on the number of steps single(M) will take before halting on input w?

   (A) $O(k)$

   (B) $O(k^2)$

   (C) $O(2^k)$

   (D) $O(2^{2^k})$

   Correct answer is (B).

4. Let N be a nondeterministic (single-tape) Turing machine such that from any configuration N has *at most* 2 possible choices for the next configuration. Suppose N on input w takes at most k steps before halting no matter what sequence of nondeterministic choices it makes. That is, the computation tree (as shown on page 8) of N on w is binary and has height k. Let det(N) be the deterministic 3-tape Turing machine that simulates N as described in the proof of Theorem 2 in lecture 20 (pages 7 through 9). Let us assume that given a choice sequence of length ℓ, det(N) can write the lexicographically next choice sequence (step 4 of algorithm on page 9) in $O(\ell)$ steps. What is the best upper bound on the number of steps det(N) will take before halting on input w?

(A) $O(k)$

(B) $O(k^2)$

(C) $O(2^k)$

(D) $O(2^{2^k})$

Correct answer is (C).