Quiz 16

1. Consider the language $L = \{a^i b^j a^i b^j \,|\, i, j \geq 0\}$. Consider the following "proof" that L does not satisfies the pumping lemma. Let p be the pumping length. Choose $z = a^p b a^p b$. Consider a division of z, where $u = a^i$, $v = a^j$, $w = a^k$, $x = a^\ell b$ and $y = a^p b$. Clearly uv^0wx^0y is not in L.

   (A) This is an incorrect proof because all possible pumping lengths have not been considered.
   (B) This is an incorrect proof because all divisions of z have not been considered.
   (C) This is an incorrect proof because all possible z have not been considered.
   (D) This is a correct proof.

   Correct answer is (B).

2. Consider the language $L = \{a^i b^j a^i b^j \mid i, j \geq 0\}$. Consider the following "proof" that L satisfies the pumping lemma. Let p be the pumping length. Choose $z = a^p b a^p b$. Consider a division of z, where $u = a^{p-1}$, $v = a$, $w = b$, $x = a$ and $y = a^{p-1}b$. Clearly uv^iwx^iy is in L for every i.

   (A) This is an incorrect proof because all possible pumping lengths have not been considered.
   (B) This is an incorrect proof because all divisions of z have not been considered.
   (C) This is an incorrect proof because all possible z have not been considered.
   (D) This is a correct proof.

   Correct answer is (C).

3. Consider the language $L = \{a^i b^j c^k d^\ell \mid \text{either } i = 0 \text{ or } j = k = \ell\}$. Consider the following "proof" that L satisfies the pumping lemma. Take $p = 1$. Let $z \in L$ be a string of length at least p. Take $u = \epsilon$, v to be the first symbol of z, $w = x = \epsilon$, and take y to be the rest of the string. Now $uv^iwx^iy \in L$ for every $i \geq 0$.

   (A) This is an incorrect proof because all possible pumping lengths have not been considered.
   (B) This is an incorrect proof because all divisions of z have not been considered.
   (C) This is an incorrect proof because all possible z have not been considered.
   (D) This is a correct proof.

   Correct answer is (D).

4. Here is a faulty proof showing that $L = \{a^n b^n c^n \mid n \geq 0\}$ is context-free. Consider the grammar $G = (\{S\}, \{a, b, c\}, R, S)$ whose rules R are given as

   $$S \to SaSbScS \mid SaScSbS \mid SbSaScS \mid SbScSaS \mid ScSaSbS \mid ScSbSaS \mid SS \mid \epsilon$$

   The proof consists of the following sequence of assertions. Which of the assertions in the proof is incorrect?

   (A) $L(G) = L_{eq}$ where $L_{eq} = \{w \mid w \text{ has an equal number of } a\text{s}, b\text{s, and } c\text{s}\}$
   (B) $\mathbf{L}(a^*b^*c^*)$ is regular.
   (C) $L_{eq} \cap \mathbf{L}(a^*b^*c^*) = \{a^n b^n c^n \mid n \geq 0\} = L$
   (D) $L(G)$ is context-free, and context-free languages are closed under intersection with regular languages. Therefore $L(G) \cap L(a^*b^*c^*)$ is context-free.

   Correct answer is (A).