Worksheet 13 for December 3rd and 8th

1. *Let* $A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$. *Find a diagonal matrix* D *and an orthogonal matrix* Q *such that* $A = QDQ^T$.

2. *Find the limiting values of* y_k *and* z_k *(for* $k \to \infty$*) if*

$$\begin{aligned} y_{k+1} &= .8y_k + .3z_k \qquad & y_0 &= 0 \\ z_{k+1} &= .2y_k + .7z_k \qquad & z_0 &= 5. \end{aligned}$$

Also find formulas for y_k *and* z_k *from* $A^k = S\Lambda^k S^{-1}$.

3. *If* $A = \begin{bmatrix} 4 & 3 \\ 1 & 2 \end{bmatrix}$, *find* A^{100} *by diagonalizing* A.

4. *Decide for or against the positive definiteness of these matrices, and write out the corresponding* $f = x^T Ax$:

(a) $\begin{bmatrix} 1 & 3 \\ 3 & 5 \end{bmatrix}$

(b) $\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} 2 & 3 \\ 3 & 5 \end{bmatrix}$

(d) $\begin{bmatrix} -1 & 2 \\ 2 & -8 \end{bmatrix}$.

The determinant in (b) is zero; along what line is $f(x, y) = 0$*?*

Tutoring Room (443 Altgeld Hall): Mon 4-6 PM, Tue 5-7 PM, Wed 6-8 PM

Final Date: December 17 8-11AM, Conflict December 15, 8-11AM. You are allowed to take the conflict exam if you have more than two examination within 24 hours. Conflict sign up deadline: November 30