1. Find the probability mass function (pmf) for a random variable with the following moment generating function (mgf),

$$M(t) = \frac{1}{2}e^{-t} + \frac{1}{4} + \frac{1}{5}e^{t} + \frac{1}{20}e^{5t}$$

2. Complete the following integrals to obtain simplified functions of t, and determine the range of t for which each of these integrals is defined:
   a. $M(t) = \int_0^1 e^{xt}\, dx$
   b. $M(t) = \int_0^\infty e^{x(t-1)}\, dx$
   c. Determine the probability density functions (pdfs) corresponding to the moment generating functions in parts a) and b).

3. Consider the following joint probability distribution $p(x, y)$ for random variables X and Y:

$x \setminus y$	0	1	2	
0	0.10	0.15	0.05	0.30
1	0.10	0.20	0.40	0.70
	0.20	0.35	0.45	

   a. Find $P(X = Y)$
   b. Find $P(Y > 0 \mid X > 0)$
   c. Find $E(X + Y)$
   d. Find $E(Y|X = 1)$

4. Suppose X and Y are jointly distributed random variables with density,

$$f_{XY}(x, y) = C, 0 < x < y < 1$$

   a. Sketch the support for X and Y.

   b. Find C.

   c. Find $P\left(X > \frac{Y}{2}\right)$.

5. Let $f_{X,Y}(x, y) = e^{-x-y}, 0 < x < \infty, 0 < y < \infty$ be the pdf of X and Y.
   a. Let $Z = X + Y$. Compute $P(Z \leq 0)$.

b. Find the pdf of Z.
c. Find $M_{X,Y}(t_x, t_y)$.

6. Consider the joint pdf,

$$f_{XY}(x, y) = C, 0 < x < y < 2x, x + y < 1$$

a. Sketch the support for X and Y.
b. Find C.
c. Find $P\left(X + Y > \frac{1}{2}\right)$.

7. Reconsider the pdf in problem 6.
a. Find the marginal distribution for X, $f_X(x)$.
b. Find the marginal distribution for Y, $f_Y(y)$.
c. Are X and Y independent?

8. Let (X, Y) be jointly distributed random variables supported on (0, 0), (1, 1), (1, 0), (1, -1) with probabilities 0.5, 0.125, 0.25 and 0.125 respectively.
a. Compute $E(Y|X = 0)$ and $E(Y|X = 1)$
b. Compute the correlation between X and Y.
c. Determine whether X and Y are independent or not.

9. Let X and Y be independent random variables each with mean = 0 and variance =1.
a. Find $Cov(2X + 3Y, X - Y)$
b. Find an expression for $E(X + Y \mid X)$ (this is a random variable)
c. Show that $E(X + Y) = E\big(E(X + Y|X)\big) = E(E(X + Y|Y))$

10. Let X be a random variable such that $P(X > 0) = 1$ and $\mu = E(X)$ exists and is finite. Show that $(X \geq 2\mu) \leq \frac{1}{2}$.

Extra problems for Graduate Students registered for 4 hours:

11. Use Markov's theorem to show,

$$P\left[\frac{(x-\mu)^4}{\sigma^4} \geq d^4\right] \leq \frac{\kappa}{d^4}$$

where $\kappa = \frac{E[(x-\mu)^4]}{\sigma^4}$ is a measure of kurtosis (see 1.9.15).

12. Use Jensen's inequality to prove that if the fourth moment is finite then the first and second moments are also finite:
    a. $[E(X)]^4 \leq E(X^4)$
    b. $[E(X^2)]^2 \leq E(X^4)$