Midterm 2
CS 373: Theory of Computation

Date: Thursday, November 8, 2012.

Instructions:

- This is a closed book exam. No notes, cheat sheets, textbook, or printed material allowed.
- You have 90 minutes to solve this exam.
- This exam has 4 problems. Problems 1 and 4 are worth 10 points, while problems 2 and 3 are worth 15 points. The points are not a measure of the relative difficulty of the problems.
- Please write your name on the top of *every* page in the space provided.
- If your solution does not fit in the space provided, and continues onto one of the back sheets, please indicate clearly where we should look for the solution.
- Unless otherwise stated, "prove that", "show that" for a problem means you need to formally prove what you are claiming.
- You may use, without proof, any result that you were asked to prove in the homework or was proved in the lecture. If you use such a result, please explicitly state the result (like " 'Perfect shuffle of regular languages is regular' was proved in a homework", instead of "this was shown in a homework").

Name	
Netid	

Discussion: T 2:00–2:50 T 3:00–3:50 W 1:00–1:50 W 4:00–4:50 W 5:00–5:50

Problem	Maximum Points	Points Earned	Grader
1	10		
2	15		
3	15		
4	10		
Total	50		

Problem 1. [Category: Comprehension] **True/False.** Decide for each statement whether it is true or false. Circle **T** if the statement is *necessarily true*; circle **F** if it it is not necessarily true. Each correct answer is worth **1 point**.

(a) The language $L_1 = \{a^i b^j c^k d^\ell \mid i, j, k, \ell \geq 0,\ i = \ell \text{ and } j = k\}$ is not context-free.
T **F**

(b) The language $L_2 = \{a^i b^j c^k d^\ell \mid i, j, k, \ell \geq 0,\ i + \ell = j + k\}$ is context-free.
T **F**

(c) If P is a PDA and $w \in \mathbf{L}(P)$ then P's stack is empty when it accepts w.
T **F**

(d) In order to remove all the useless symbols in grammar G, we need to first remove all the non-generating variables and then all the unreachable variables.
T **F**

(e) Let $G = (V, \Sigma, R, S)$ be a CFG without ϵ-productions, unit productions, and useless variables, where the length of the right-hand-side of any rule in R be at most k. Suppose $G' = (V', \Sigma, R', S')$ is the grammar in Chomsky Normal form constructed by the algorithm discussed in class. The $|R'| = O(k|R| + |\Sigma|)$.
T **F**

(f) Let G, with start symbol S, be a grammar in Chomsky normal form. Suppose $S \stackrel{*}{\Rightarrow} w_1$ in k_1 steps and $S \stackrel{*}{\Rightarrow} w_2$ in k_2 steps such that $k_1 \neq k_2$. Then $w_1 \neq w_2$.
T **F**

(g) Suppose $L \subseteq \Sigma^*$ is non-context-free language and $h : \Sigma^* \to \Delta^*$ is a homomorphism. Then $h^{-1}(h(L))$ is also not context-free.
T **F**

(h) Since context-free languages are not closed under complementation, that means that if $L \subseteq \Sigma^*$ is context-free then $\Sigma^* \setminus L$ is not context-free.
T **F**

(i) Suppose context-free languages are closed under an operation `op`. Since every context-free language can be described by a Type 1 grammar, languages describable by Type 1 grammars are also closed under `op`.
T **F**

(j) There are languages that can be described using Type 1 grammars that cannot be described by context-free grammars.
T **F**

Problem 2. [Category: Comprehension+Proof] Consider the context-free grammar $G = (V = \{S, A, B\}, \Sigma = \{a, b, c, d\}, R, S)$ where the rules are given by

$$S \rightarrow aSb \mid aAb \qquad A \rightarrow cAd \mid B \qquad B \rightarrow aBb \mid \epsilon$$

(a) For each of the following strings, answer whether or not they belong to the language $\mathbf{L}(G)$, and if they do then give a derivation: $aabb$, $ccabdd$, $acabdb$. **[3 points]**

(b) For a variable $C \in V$, define $\mathbf{L}_G(C) = \{w \in \Sigma^* \mid C \stackrel{*}{\Rightarrow} w\}$. Fill in the blanks for each of the variables in G. **[3 points]**

$\mathbf{L}_G(B) =$ ______________________________

$\mathbf{L}_G(A) =$ ______________________________

$\mathbf{L}_G(S) =$ ______________________________

(c) Prove that your answer for $\mathbf{L}_G(B)$ given in part (b) is correct. **[6 points]**

(Additional space for part (c))

(d) Prove that grammar G is ambiguous by giving an example string $w \in \mathbf{L}(G)$ such that w has two different parse trees. **[3 points]**

Problem 3. [Category: Design+Comprehension+Proof] Given $L \subseteq \Sigma^*$, define an operation FLIP as follows:

$$\mathsf{FLIP}(L) = \{st \mid s, t \in \Sigma^*, \text{ and } st^R \in L\}.$$

In this problem you will show that context free languages are not closed under this operation.

(a) Show that the language $A = \{x\#y\# \mid x, y \in \{0,1\}^*, \text{ and } x = y^R\}$ is a CFL over the alphabet $\{0, 1\#\}$, by giving a CFG for A. You need not prove that your grammar is correct. **[4 points]**

(b) For each of the following strings w, give a reason why $w \in \mathsf{FLIP}(A)$. In other words, find s, t such that $w = st$ and $st^R \in A$. **[6 points]**

(a) $w = x\#x^R\#$ where $x \in \{0,1\}^*$.

(b) $w = x\#\#x$ where $x \in \{0,1\}^*$.

(c) $w = x\#x\#$ where $x \in \{0,1\}^*$.

(d) $w = u\#ux\#x^R$ where $u, x \in \{0,1\}^*$.

(c) Give a regular language R and a homomorphism $h : \{0,1,\#\}^* \to \{0,1\}^*$ such that $h(\mathsf{FLIP}(A) \cap R) = \{vv \mid v \in \{0,1\}^*\}$. **[4 points]**

(d) Using the above, show that CFLs are not closed under FLIP. You can use the fact that the language $\{vv \mid v \in \{0,1\}^*\}$ is not context-free. **[1 point]**

Problem 4. [Category: Proof] Consider the language $B \subseteq \{a, b\}^*$ defined as

$$B = \{babaabaaab \cdots ba^{n-1}ba^n b \mid n \geq 1\}$$

Prove that B is not context-free. If needed, you may use the fact that the language $\{a^{n^2} \mid n \geq 0\}$ is not context-free. **[10 points]**