

Gaussian elimination with elimination matrices

In [56]:

```
#keep
import numpy as np
import numpy.linalg as la
```

In [57]:

```
#keep
n = 3

np.random.seed(15)
A = np.round(5*np.random.randn(n, n))

A
```

Out[57]:

```
array([[ -2.,   2.,  -1.],
       [ -3.,   1.,  -9.],
       [ -5.,  -5.,  -2.]])
```

U is the copy of A that we'll modify:

In [58]:

```
#keep
U = A.copy()
```

Now eliminate $U[1,0]$:

In [59]:

```
M1 = np.eye(n)
M1[1,0] = -U[1,0]/U[0,0]
M1
```

Out[59]:

```
array([[ 1. ,  0. ,  0. ],
       [-1.5,  1. ,  0. ],
       [ 0. ,  0. ,  1. ]])
```

In [60]:

```
#keep  
U = M1.dot(U)  
U
```

Out[60]:

```
array([[ -2. ,  2. , -1. ],  
       [  0. , -2. , -7.5],  
       [-5. , -5. , -2. ]])
```

Now eliminate $U[2,0]$:

In [61]:

```
M2 = np.eye(n)  
M2[2,0] = -U[2,0]/U[0,0]
```

In [62]:

```
#keep  
U = np.dot(M2, U)  
U
```

Out[62]:

```
array([[ -2. ,  2. , -1. ],  
       [  0. , -2. , -7.5],  
       [  0. , -10. ,  0.5]])
```

Now eliminate $U[2,1]$:

In [63]:

```
M3 = np.eye(n)  
M3[2,1] = -U[2,1]/U[1,1]
```

In [64]:

```
#keep  
U = M3.dot(U)  
U
```

Out[64]:

```
array([[ -2. ,  2. , -1. ],  
       [  0. , -2. , -7.5],  
       [  0. ,  0. , 38. ]])
```

Try inverting one of the Ms:

In [65]:

```
#keep
print(M2)
print(la.inv(M2))
```

```
[[ 1.  0.  0.]
 [ 0.  1.  0.]
 [-2.5 0.  1.]]
[[ 1. -0. -0.]
 [ 0.  1.  0.]
 [ 2.5 0.  1.]]
```

So we've built $M3 * M2 * M1 * A = U$. Test:

In [66]:

```
#keep
U2 = M3.dot(M2.dot(M1.dot(A)))
U2
```

Out[66]:

```
array([[ -2. ,  2. , -1. ],
       [  0. , -2. , -7.5],
       [  0. ,  0. , 38. ]])
```

In [67]:

```
#keep
U
```

Out[67]:

```
array([[ -2. ,  2. , -1. ],
       [  0. , -2. , -7.5],
       [  0. ,  0. , 38. ]])
```

Now define L:

In [68]:

```
L = la.inv(M1).dot(la.inv(M2).dot(la.inv(M3)))  
L
```

Out[68]:

```
array([[ 1. ,  0. ,  0. ],  
       [ 1.5,  1. ,  0. ],  
       [ 2.5,  5. ,  1. ]])
```

Observations? (Shape? Diagonal values?)

In [69]:

```
#keep  
np.dot(L, U) - A
```

Out[69]:

```
array([[ 0.,  0.,  0.],  
       [ 0.,  0.,  0.],  
       [ 0.,  0.,  0.]])
```