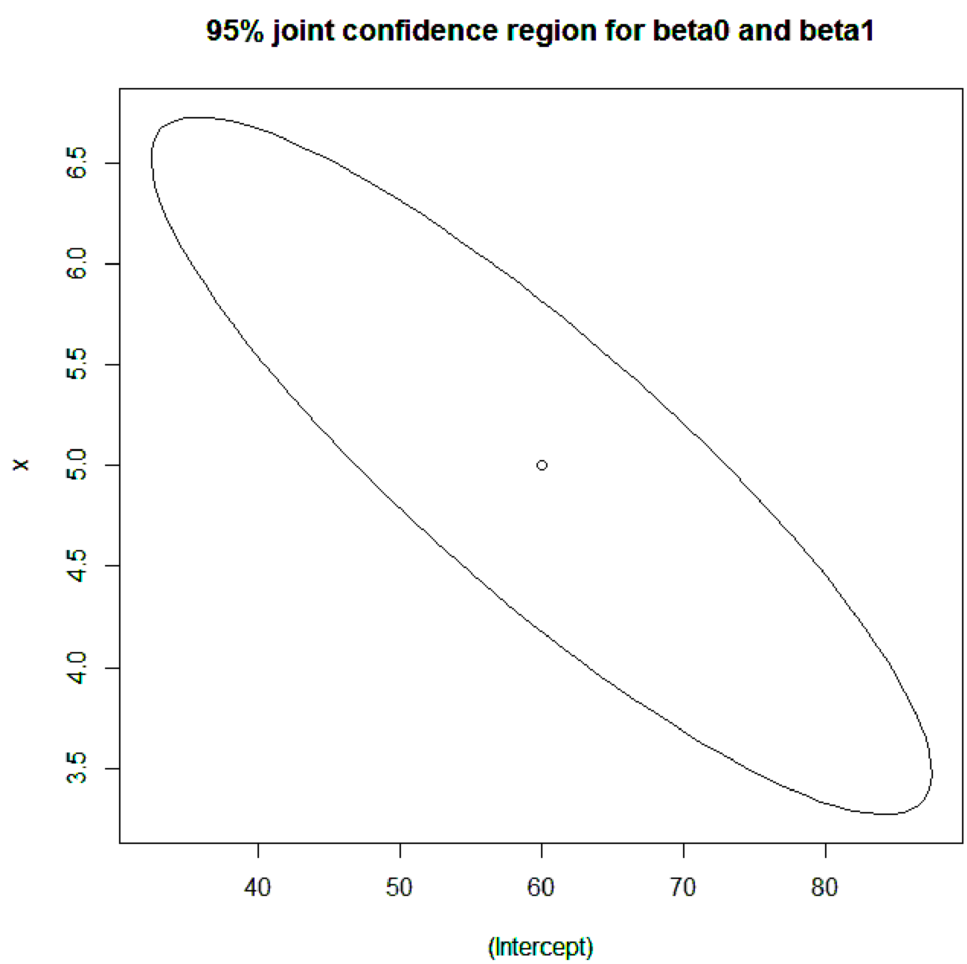
STAT 420

```
> x = c(  2,  6,  8,  8, 12, 16, 20, 20, 22, 26)
> y = c( 58,105, 88,118,117,137,157,169,149,202)
> N = length(x)
>
> Xmat = cbind(rep(1,N), x)
> Xmat
         x
 [1,] 1  2
 [2,] 1  6
 [3,] 1  8
 [4,] 1  8
 [5,] 1 12
 [6,] 1 16
 [7,] 1 20
 [8,] 1 20
 [9,] 1 22
[10,] 1 26
>
> XX = t(Xmat) %*% Xmat
> XX
         x
   10  140
x 140 2528
>
> XXinv = solve(XX)
> XXinv
                          x
   0.44507042 -0.024647887
x -0.02464789  0.001760563
>
> XY = t(Xmat) %*% y
> XY
   [,1]
   1300
x 21040
>
> betahat = XXinv %*% XY
> betahat
  [,1]
    60
x    5
```

$$\begin{bmatrix} Y_1 \\ Y_2 \\ \cdots \\ Y_n \end{bmatrix} = \begin{bmatrix} 1 & x_1 \\ 1 & x_2 \\ \cdots & \cdots \\ 1 & x_n \end{bmatrix} \begin{bmatrix} \beta_0 \\ \beta_1 \end{bmatrix} + \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \cdots \\ \varepsilon_n \end{bmatrix} \qquad \mathbf{Y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\varepsilon}$$

$$\hat{\boldsymbol{\beta}} = (\mathbf{X}^T\mathbf{X})^{-1}\mathbf{X}^T\mathbf{Y} \qquad E(\hat{\boldsymbol{\beta}}) = \boldsymbol{\beta}, \qquad \mathrm{Var}(\hat{\boldsymbol{\beta}}) = \sigma^2(\mathbf{X}^T\mathbf{X})^{-1}.$$

```
> # joint confidence region for the parameters
>
> fit = lm(y ~ x)
>
> library(ellipse)
> plot(ellipse(fit,c(1,2),level=0.95),type="l")
> # plot(ellipse(fit),type="l")
> title("95% joint confidence region for beta0 and beta1")
> points(fit$coeff[1],fit$coeff[2])
```



1. Consider the following data set:

x_1	x_2	y
0	1	11
11	5	15
11	4	13
7	3	14
4	1	0
10	4	19
5	4	16
8	2	8

Consider the model

$$Y_i = \beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + e_i., \quad i = 1, \dots, 8.$$

where e_i's are i.i.d. $N(0, \sigma_\epsilon^2)$.

Then

$$\mathbf{X}^T\mathbf{X} = \begin{bmatrix} 8 & 56 & 24 \\ 56 & 496 & 200 \\ 24 & 200 & 88 \end{bmatrix}, \qquad \mathbf{X}^T\mathbf{Y} = \begin{bmatrix} 96 \\ 740 \\ 336 \end{bmatrix},$$

$$\mathbf{C} = (\mathbf{X}^T\mathbf{X})^{-1} = \begin{bmatrix} 0.7125 & -0.025 & -0.1375 \\ -0.025 & 0.025 & -0.05 \\ -0.1375 & -0.05 & 0.1625 \end{bmatrix}, \quad ,$$

a) Obtain the least-squares estimates $\hat{\beta}_0$, $\hat{\beta}_1$, and $\hat{\beta}_2$.

$$\hat{\boldsymbol{\beta}} = (\mathbf{X}^T\mathbf{X})^{-1}\mathbf{X}^T\mathbf{Y} = \begin{bmatrix} 0.7125 & -0.025 & -0.1375 \\ -0.025 & 0.025 & -0.05 \\ -0.1375 & -0.05 & 0.1625 \end{bmatrix} \begin{bmatrix} 96 \\ 740 \\ 336 \end{bmatrix} = \begin{bmatrix} \mathbf{3.7} \\ \mathbf{-0.7} \\ \mathbf{4.4} \end{bmatrix}. \quad .$$

$SYY = \sum(y - \bar{y})^2 = 240$, $\quad RSS = \sum(y - \hat{y})^2 = 76.4$,

b) Perform the significance of the regression test at a 5% level of significance.

$H_0: \boldsymbol{\beta_1 = \beta_2 = 0}$. $\qquad$ H_a: at least one of β_1 and β_2 is significantly different from 0.

Source	SS	DF	MS	F
Regression	163.6	$p-1=2$	81.8	**5.3534**
Error (Residual)	76.4	$n-p=5$	15.28	
Total	240	$n-1=7$		

Critical Value: $F_{0.05}(2,5) = \mathbf{5.79}$. Reject H_0 if $F > 5.79$.

Decision: **Do NOT Reject H_0.**

c) Test $H_0: \beta_1 = 0$ vs. $H_a: \beta_1 \neq 0$ at $\alpha = 0.10$. Find the p-value.

$\hat{\text{Var}}(\hat{\beta}_1) = C_{11} \times s^2 = 0.025 \times 15.28 = 0.382$.

Test Statistic: $t = \dfrac{-0.7 - 0}{\sqrt{0.382}} \approx \mathbf{-1.1326}$.

$n - p = \mathbf{5}$ d.f.

-1.476	$<$	-1.1326	$<$	-0.727
$-t_{0.10}(5)$	$<$	t	$<$	$-t_{0.25}(5)$

$0.10 <$ left tail < 0.25

p-value = 2 tails. $\mathbf{0.20} <$ p-value $< \mathbf{0.50}$.

(p-value ≈ 0.308765.)

Do NOT Reject H_0.

d) Test $H_0: \beta_2 = 0$ vs. $H_a: \beta_2 \neq 0$ at $\alpha = 0.05$. Find the p-value.

$\hat{\text{Var}}(\hat{\beta}_2) = C_{22} \times s^2 = 0.1625 \times 15.28 = 2.483$.

Test Statistic: $t = \dfrac{4.4 - 0}{\sqrt{2.483}} \approx \mathbf{2.792}$.

$n - p = \mathbf{5}$ d.f.

2.517	<	2.792	<	3.365
$t_{0.025}(5)$	<	t	<	$t_{0.01}(5)$

0.01 < right tail < 0.025

p-value = 2 tails. $\quad$ **0.02** < p-value < **0.05**.

(p-value ≈ 0.03834.)

Reject $\mathbf{H_0}$.

a), b), c), d)

```
> x1 = c( 0,11,11, 7, 4,10, 5, 8)
> x2 = c( 1, 5, 4, 3, 1, 4, 4, 2)
> y =  c(11,15,13,14, 0,19,16, 8)
>
> fit = lm(y ~ x1 + x2)
>
> summary(fit)

Call:
lm(formula = y ~ x1 + x2)

Residuals:
   1    2    3    4    5    6    7    8
 2.9 -3.0 -0.6  2.0 -5.3  4.7 -1.8  1.1

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)   3.7000     3.2995   1.121   0.3131
x1           -0.7000     0.6181  -1.133   0.3088
x2            4.4000     1.5758   2.792   0.0383 *
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 3.909 on 5 degrees of freedom
Multiple R-Squared: 0.6817,     Adjusted R-squared: 0.5543
F-statistic: 5.353 on 2 and 5 DF,  p-value: 0.05717
```

b) $\quad \mathrm{H}_0 : \beta_1 = \beta_2 = 0$ vs. H_a : not H_0.

Full: $\quad Y_i = \beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + e_i$,

Null: $\quad Y_i = \beta_0 \qquad\qquad\qquad\quad + e_i$.

```
> anova(lm(y~1),fit)
Analysis of Variance Table

Model 1: y ~ 1
Model 2: y ~ x1 + x2
  Res.Df   RSS Df Sum of Sq      F  Pr(>F)
1      7 240.0
2      5  76.4  2     163.6 5.3534 0.05717 .
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

c) $H_0 : \beta_1 = 0$ vs. $H_a : \beta_1 \neq 0.$

Full: $Y_i = \beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + e_i$,

Null: $Y_i = \beta_0 \qquad\qquad + \beta_2 x_{i2} + e_i$.

```
> fitc = lm(y ~ x2)
> anova(fitc,fit)
Analysis of Variance Table

Model 1: y ~ x2
Model 2: y ~ x1 + x2
  Res.Df  RSS Df Sum of Sq      F Pr(>F)
1      6 96.0
2      5 76.4  1      19.6 1.2827 0.3088
```

d) $H_0 : \beta_2 = 0$ vs. $H_a : \beta_2 \neq 0.$

Full: $Y_i = \beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + e_i$,

Null: $Y_i = \beta_0 + \beta_1 x_{i1} \qquad\qquad + e_i$.

```
> fitd = lm(y ~ x1)
> anova(fitd,fit)
Analysis of Variance Table

Model 1: y ~ x1
Model 2: y ~ x1 + x2
  Res.Df    RSS Df Sum of Sq     F  Pr(>F)
1      6 195.54
2      5  76.40  1    119.14 7.797 0.03834 *
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

$$\mathrm{Y}_i = \beta_0 + \beta_1 x_{i1} + \ldots + \beta_{p-1} x_{i\,p-1} + e_i, \quad i = 1, 2, \ldots, n,$$

$$\mathrm{E}(e_i) = 0, \qquad \mathrm{Var}(e_i) = \sigma^2, \qquad \mathrm{Cov}(e_i, e_j) = 0, \quad i \neq j.$$

$$\mathbf{Y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{e}, \qquad \mathrm{E}(\boldsymbol{e}) = \mathbf{0}, \quad \mathrm{Var}(\boldsymbol{e}) = \left(\!\left(\mathrm{Cov}(e_i, e_j) \right)\!\right)_{ij} = \sigma^2 \mathbf{I}_n.$$

$$\mathbf{Y} = \begin{bmatrix} \mathrm{Y}_1 \\ \mathrm{Y}_2 \\ \cdots \\ \mathrm{Y}_n \end{bmatrix}, \quad \mathbf{X} = \begin{bmatrix} 1 & x_{11} & x_{12} & \cdots & x_{1\,p-1} \\ 1 & x_{21} & x_{22} & \cdots & x_{2\,p-1} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 1 & x_{n1} & x_{n2} & \cdots & x_{n\,p-1} \end{bmatrix}, \quad \boldsymbol{\beta} = \begin{bmatrix} \beta_0 \\ \beta_1 \\ \cdots \\ \beta_{p-1} \end{bmatrix}, \quad \boldsymbol{e} = \begin{bmatrix} e_1 \\ e_2 \\ \cdots \\ e_n \end{bmatrix}.$$

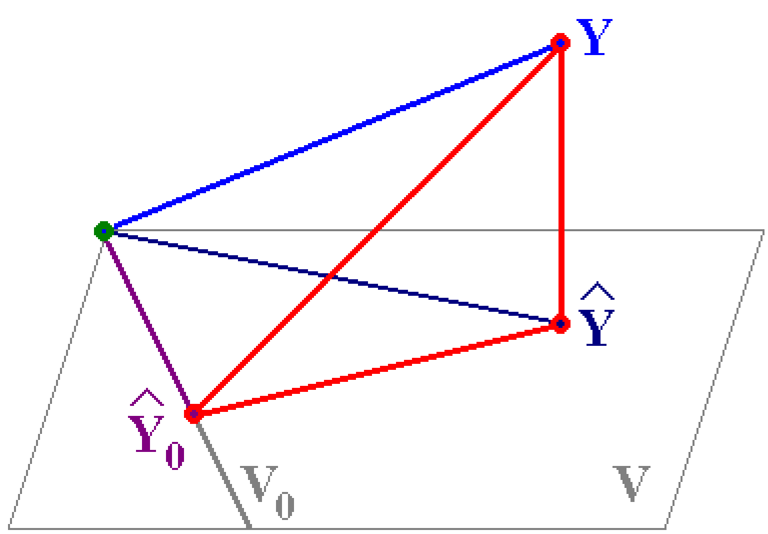
$$\hat{\boldsymbol{\beta}} = (\mathbf{X}^{\mathrm{T}}\mathbf{X})^{-1}\mathbf{X}^{\mathrm{T}}\mathbf{Y}$$

$$\mathrm{E}(\hat{\boldsymbol{\beta}}) = \boldsymbol{\beta}, \qquad \mathrm{Var}(\hat{\boldsymbol{\beta}}) = \sigma^2 (\mathbf{X}^{\mathrm{T}}\mathbf{X})^{-1}.$$

Let $\mathrm{V}_0 = \{ a\,\mathbf{1}, \ a \in \mathbf{R} \}$, where $\mathbf{1} = [1, 1, \ldots, 1]^{\mathrm{T}} \in \mathrm{R}^n$,

$$\mathrm{V} = \{ a_0 \mathbf{1} + a_1 \boldsymbol{x}_{\mathbf{1}} + \ldots + a_{p-1} \boldsymbol{x}_{\boldsymbol{p-1}}, \quad a_0, a_1, \ldots, a_{p-1} \in \mathbf{R} \}.$$

$$\dim(\mathrm{V}_0) = 1, \qquad \dim(\mathrm{V}) = p.$$



$$\sum_{i=1}^{n}\left(Y_i-\hat{Y}_{0i}\right)^2=\sum_{i=1}^{n}\left(Y_i-\hat{Y}_i\right)^2+\sum_{i=1}^{n}\left(\hat{Y}_i-\hat{Y}_{0i}\right)^2 \qquad \hat{\mathbf{Y}}_{\mathbf{0}}=\left[\overline{Y},\overline{Y},\ldots,\overline{Y}\right]^{T}$$

ANOVA table: $\quad H_0:\ \beta_1=\beta_2=\ldots=\beta_{p-1}=0$

Source	SS	df	MS	F
Regr	$\sum_{i=1}^{n}\left(\hat{Y}_i-\hat{Y}_{0i}\right)^2$	$\dim(V)-\dim(V_0)$ $p-1$	$\frac{\text{SS Regr}}{\text{df Regr}}$	$\frac{\text{MS Regr}}{\text{MS Resid}}$
Resid	$\sum_{i=1}^{n}\left(Y_i-\hat{Y}_i\right)^2$	$n-\dim(V)$ $n-p$	$\frac{\text{SS Resid}}{\text{df Resid}}$	– estimator for σ^2
Total	$\sum_{i=1}^{n}\left(Y_i-\hat{Y}_{0i}\right)^2$	$n-\dim(V_0)$ $n-1$		

More general: $\quad q<p \qquad H_0:\ \beta_q=\beta_{q+1}=\ldots=\beta_{p-1}=0$

Let $\quad V_0=\{\,a_0\mathbf{1}+a_1\boldsymbol{x}_{\mathbf{1}}+\ldots+a_p\boldsymbol{x}_{\boldsymbol{q-1}},\quad a_0,a_1,\ldots,a_{q-1}\in\mathbf{R}\,\},$

$\quad V=\{\,a_0\mathbf{1}+a_1\boldsymbol{x}_{\mathbf{1}}+\ldots+a_p\boldsymbol{x}_{\boldsymbol{p-1}},\quad a_0,a_1,\ldots,a_{p-1}\in\mathbf{R}\,\}.$

	SS	df	MS	F
Diff.	$\sum_{i=1}^{n}\left(\hat{Y}_i-\hat{Y}_{0i}\right)^2$	$\dim(V)-\dim(V_0)$ $p-q$	…	…
Full	$\sum_{i=1}^{n}\left(Y_i-\hat{Y}_i\right)^2$	$n-\dim(V)$ $n-p$	…	
Null	$\sum_{i=1}^{n}\left(Y_i-\hat{Y}_{0i}\right)^2$	$n-\dim(V_0)$ $n-q$		

d ¼)* Test $H_0: \beta_0 = \beta_2$ vs. $H_a: \beta_0 \neq \beta_2$ at $\alpha = 0.10$. Find the p-value.

$H_0: \beta_0 - \beta_2 = 0$ vs. $H_a: \beta_0 - \beta_2 \neq 0$

$X_0^T = [\ 1 \quad 0 \quad -1\ ]$ $\qquad \beta_0 - \beta_2 = X_0^T \boldsymbol{\beta}$. $\qquad X_0^T \hat{\boldsymbol{\beta}} = -0.7$.

$$X_0^T \mathbf{C} X_0 = [1 \quad 0 \quad -1] \cdot \begin{bmatrix} 0.7125 & -0.025 & -0.1375 \\ -0.025 & 0.025 & -0.05 \\ -0.1375 & -0.05 & 0.1625 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} = 1.15.$$

$$\hat{\mathrm{Var}}(X_0^T \hat{\boldsymbol{\beta}}) = [X_0^T \mathbf{C} X_0]\, s^2 = 1.15 \times 15.28 = 17.572.$$

Test Statistic: $\qquad t = \dfrac{-0.7 - 0}{\sqrt{17.572}} \approx \mathbf{-0.167}$.

$n - p = \mathbf{5}$ d.f.

-0.167	$>$	-0.267
t	$>$	$-t_{0.40}(5)$

left tail > 0.40

p-value = 2 tails. $\qquad$ p-value > **0.80**.

(p-value ≈ 0.8739.)

Do NOT Reject H_0.

```
> x3 = x2 + 1
> x3
[1] 2 6 5 4 2 5 5 3
>
> fite = lm(y ~ x1 + x3 + 0)
> anova(fite,fit)
Analysis of Variance Table

Model 1: y ~ x1 + x3 + 0
Model 2: y ~ x1 + x2
  Res.Df    RSS Df Sum of Sq      F Pr(>F)
1      6 76.826
2      5 76.400  1     0.426 0.0279  0.874
```

d ½)* Test $H_0: 2\beta_1 + \beta_2 = 0$ vs. $H_a: 2\beta_1 + \beta_2 \neq 0$ at $\alpha = 0.05$. Find the p-value.

$X_0^T = [\ 0 \quad 2 \quad 1\ ]$ $\qquad 2\beta_1 + \beta_2 = X_0^T \beta.$ $\qquad X_0^T \hat{\boldsymbol{\beta}} = 3.0.$

$$X_0^T \mathbf{C}\, X_0 = [0 \quad 2 \quad 1] \cdot \begin{bmatrix} 0.7125 & -0.025 & -0.1375 \\ -0.025 & 0.025 & -0.05 \\ -0.1375 & -0.05 & 0.1625 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix} = 0.0625.$$

$$\hat{\mathrm{Var}}(X_0^T \hat{\boldsymbol{\beta}}) = [X_0^T \mathbf{C}\, X_0]\, s^2 = 0.0625 \times 15.28 = 0.955.$$

Test Statistic: $\qquad t = \dfrac{3.0 - 0}{\sqrt{0.955}} \approx \mathbf{3.070}.$

$n - p = \mathbf{5}$ d.f.

2.571	<	3.070	<	3.365
$t_{0.025}(5)$	<	t	<	$t_{0.01}(5)$

$0.01 <$ right tail < 0.025

p-value = 2 tails. $\qquad \mathbf{0.02} <$ p-value $< \mathbf{0.05}$.

(p-value ≈ 0.0278.)

Reject H_0.

```
> x4 = x1 - 2*x2
> fitf = lm(y ~ x4)
> anova(fitf,fit)
Analysis of Variance Table

Model 1: y ~ x4
Model 2: y ~ x1 + x2
  Res.Df   RSS Df Sum of Sq      F  Pr(>F)
1      6 220.4
2      5  76.4  1     144.0 9.4241 0.02779 *
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

e) Construct a 90% **prediction** interval for the value of Y at $x_{01} = 2$ and $x_{02} = 3$.

$$\mathrm{X}_0^{\mathrm{T}} = [\ 1 \quad 2 \quad 3\ ] \qquad \hat{\mathrm{Y}}_0 = 1 \times 3.7 + 2 \times (-0.7) + 3 \times 4.4 = 15.5.$$

$$\mathrm{X}_0^{\mathrm{T}}\,\mathbf{C}\,\mathrm{X}_0 = [1 \quad 2 \quad 3] \cdot \begin{bmatrix} 0.7125 & -0.025 & -0.1375 \\ -0.025 & 0.025 & -0.05 \\ -0.1375 & -0.05 & 0.1625 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = 0.75.$$

$$[\,1 + \mathrm{X}_0^{\mathrm{T}}\,\mathbf{C}\,\mathrm{X}_0\,]\,s^2 = (1 + 0.75) \times 15.28 = 26.74.$$

$$\mathrm{t}_{0.05}(5) = 2.015. \qquad 15.5 \pm 2.015 \times \sqrt{26.74} \qquad \mathbf{15.5 \pm 10.42}$$

$$\mathbf{(5.08, 25.92)}$$

```
> predict.lm(fit,data.frame(x1=2,x2=3),interval=c("prediction"),level=0.90)
      fit      lwr      upr
[1,] 15.5 5.080037 25.91996
```

f) Construct a 90% **prediction** interval for the value of Y at $x_{01} = 8$ and $x_{02} = 5$.

$$\mathrm{X}_0^{\mathrm{T}} = [\ 1 \quad 8 \quad 5\ ] \qquad \hat{\mathrm{Y}}_0 = 1 \times 3.7 + 8 \times (-0.7) + 5 \times 4.4 = 20.1.$$

$$\mathrm{X}_0^{\mathrm{T}}\,\mathbf{C}\,\mathrm{X}_0 = [1 \quad 8 \quad 5] \cdot \begin{bmatrix} 0.7125 & -0.025 & -0.1375 \\ -0.025 & 0.025 & -0.05 \\ -0.1375 & -0.05 & 0.1625 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 8 \\ 5 \end{bmatrix} = 0.60.$$

$$[\,1 + \mathrm{X}_0^{\mathrm{T}}\,\mathbf{C}\,\mathrm{X}_0\,]\,s^2 = (1 + 0.60) \times 15.28 = 24.448.$$

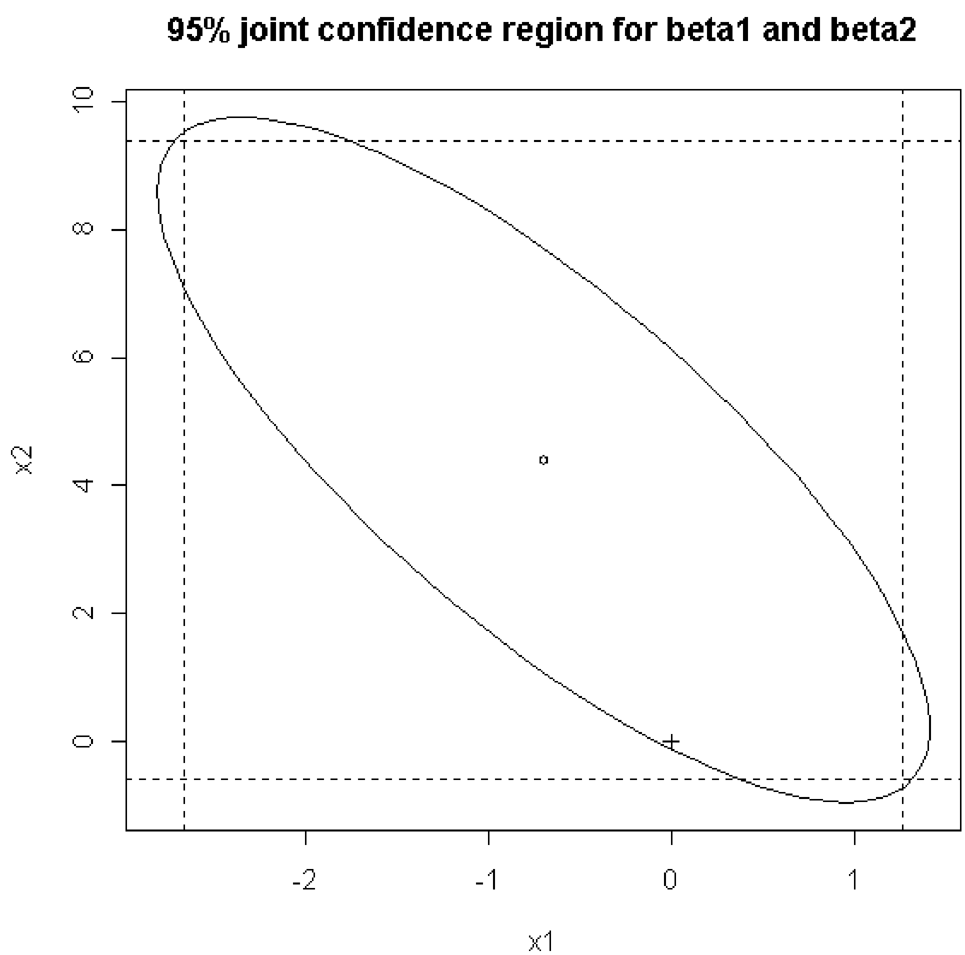
$$\mathrm{t}_{0.05}(5) = 2.015. \qquad 20.1 \pm 2.015 \times \sqrt{24.448} \qquad \mathbf{20.1 \pm 9.96}$$

$$\mathbf{(10.14, 30.06)}$$

```
> predict.lm(fit,data.frame(x1=8,x2=5),interval=c("prediction"),level=0.90)
      fit      lwr      upr
[1,] 20.1 10.13661 30.06339
```

```
> x1 = c( 0,11,11, 7, 4,10, 5, 8)
> x2 = c( 1, 5, 4, 3, 1, 4, 4, 2)
> y =  c(11,15,13,14, 0,19,16, 8)
>
> X = cbind( rep(1,8), x1, x2 )
> X
       x1 x2
[1,] 1  0  1
[2,] 1 11  5
[3,] 1 11  4
[4,] 1  7  3
[5,] 1  4  1
[6,] 1 10  4
[7,] 1  5  4
[8,] 1  8  2
>
> t(X) %*% X
       x1  x2
    8  56  24
x1 56 496 200
x2 24 200  88
>
> solve( t(X) %*% X )
                x1      x2
    0.7125 -0.025 -0.1375
x1 -0.0250  0.025 -0.0500
x2 -0.1375 -0.050  0.1625
>
> t(X) %*% y
   [,1]
     96
x1  740
x2  336
>
> solve( t(X) %*% X ) %*% t(X) %*% y
   [,1]
    3.7
x1 -0.7
x2  4.4
```

```
> plot(ellipse(fit,c(2,3),level=0.95),type="l")
> title("95% joint confidence region for beta1 and beta2")
> points(fit$coeff[2],fit$coeff[3])
> points(0,0,pch=3)
>
> confint(fit,"x1",level=0.975)
       1.25 %   98.75 %
x1 -2.655164 1.255164
> confint(fit,"x2",level=0.975)
        1.25 % 98.75 %
x2 -0.5847103 9.38471
> abline(v=confint(fit,"x1",level=0.975),lty=2)
> abline(h=confint(fit,"x2",level=0.975),lty=2)
```



Consider $\mathbf{H_0 : \beta_1 = \beta_2 = 0}$ vs. H_1 : at least one of β_1 and β_2 is not zero.

Recall part (b): **Do NOT Reject $\mathbf{H_0}$** at $\alpha = 0.05$ (p-value = 0.05717).

Note that the origin (0, 0) lies **inside** the 95% joint confidence region for β_1 and β_2.

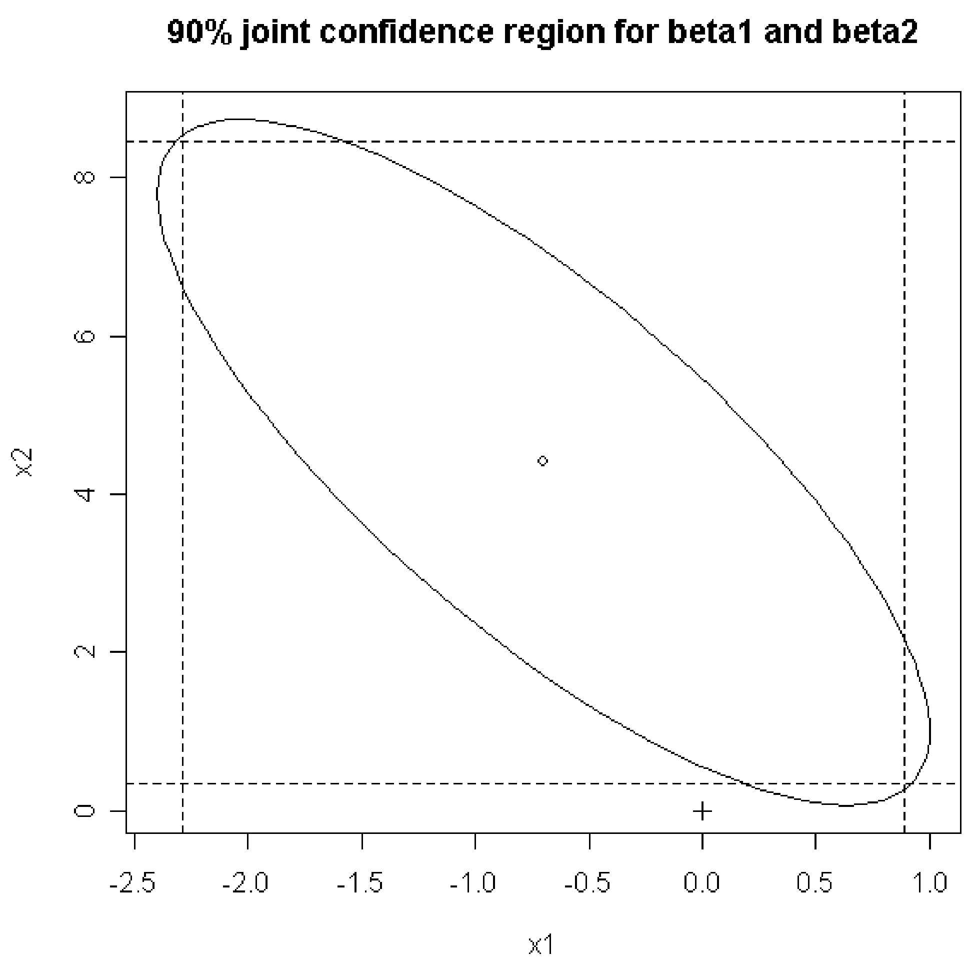
```
> plot(ellipse(fit,c(2,3),level=0.9),type="l")
> title("90% joint confidence region for beta1 and beta2")
> points(fit$coeff[2],fit$coeff[3])
> points(0,0,pch=3)
>
> confint(fit,"x1",level=0.95)
        2.5 %     97.5 %
x1 -2.288778 0.8887776
```

$\Rightarrow$ Do NOT Reject $H_0: \beta_1 = 0$ at $\alpha = 0.05$

```
> confint(fit,"x2",level=0.95)
        2.5 %    97.5 %
x2 0.3493959 8.450604
```

$\Rightarrow$ Reject $H_0: \beta_2 = 0$ at $\alpha = 0.05$

```
> abline(v=confint(fit,"x1",level=0.95),lty=2)
> abline(h=confint(fit,"x2",level=0.95),lty=2)
```



Reject $H_0: \beta_1 = \beta_2 = 0$ at $\alpha = 0.10$.

Note that the origin $(0, 0)$ lies **outside** the 90% joint confidence region for β_1 and β_2.