Math 415 - Lecture 39

Review

Wednesday December 6th 2015

Final Information:

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- Thursday December 17th, 8:00-11:00AM.

Bring university ID, pencils and erasers, there will be a part multiple choice.

Final Information:

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  - 101 Armory: AD3,ADG,ADU,ADW

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After Exam 3

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- Diagonalization,

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- Singular Value Decomposition of A from spectral theorem for $A^T A$, and AA^T.
- Approximation of a matrix A according to the singular values: image compression.

Big Topics

Big topics for the Final

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- Solving Systems $A\mathbf{x} = \mathbf{b}$

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  - Pivots, free variables, parametric form of general solution.
  - Inconsistent system, unique solution or infinitely many solutions.

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- Vectors and Matrices

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  - LU factorization, solving $Ax = b$ by $Lc = b$, $Ux = c$.
  - Inverse of a square matrix, Gauss-Jordan calculation of A^{-1} (Big Augmented Matrix).

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  - Coordinate matrix with respect to input/output bases.

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  - Orthogonal and orthonormal basis.

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    - Dimensions of the subspaces and bases, from echelon form.
    - Networks and fundamental subspaces.

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  - Orthogonal decomposition: x can be written as $x = x_W + x_{W^\perp}$ for $x_W \in W$, $x_{W^\perp} \in W^\perp$.

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  - The projection of $\mathbf{b}$ on the subspace $\mathrm{Col}(A)$ is $A\hat{\mathbf{x}}$.

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  - Data Fitting

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  - From arbitrary basis get orthonormal basis.

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  - Orthogonal matrix Q: $Q^T Q = I$.

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  - Cofactor expansion.

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  - Characteristic polynomial.
  - Eigenspace.
  - Eigenbasis and diagonalization.
  - Sum and product of eigenvectors and trace and det of A.
  - Powers of A.
  - Discrete Dynamical systems: state vector $\mathbf{x}_t$ evolves in time by $\mathbf{x}_{t+1} = A\mathbf{x}_t$.

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- Symmetric matrices and spectral theorem.

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  - if $A = A^T$ then eigenvalues of A are real
  - A has an orthonormal basis of eigenvectors.

Random Examples

Example

Let

$$a_1 = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}, \quad a_2 = \begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix}, \quad a_3 = \begin{bmatrix} 3 \\ 3 \\ 3 \end{bmatrix}, \quad b = \begin{bmatrix} 3 \\ 4 \\ 4 \end{bmatrix}$$

Is b a linear combination of a_1, a_2, a_3? Explain!

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Solution

We need to solve a system $Ax = b$, with augmented matrix

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 3 \\ 2 & 1 & 3 & 4 \\ 1 & 2 & 3 & 4 \end{array}\right] \simeq \left[\begin{array}{ccc|c} 1 & 2 & 3 & 3 \\ 0 & -1 & -3 & -2 \\ 0 & 0 & 0 & 1 \end{array}\right].$$

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- Find the LU factorization.
- Find two descriptions of the column space: $\mathrm{Col}(A)$ is the span of which vectors, and if $b \in \mathrm{Col}(A)$ give equations for b.

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- Find the LU factorization.
- Find two descriptions of the column space: Col(A) is the span of which vectors, and if $b \in$ Col(A) give equations for b.
- If $b \in$ Col(A) is the general set of solutions $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ of $Ax = b$ a point, a line, a plane or all of $\mathbb{R}^3$?

Solution

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 4 & 2 & 3 \\ 6 & 3 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & -1 & 1 \end{bmatrix} \begin{bmatrix} 2 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}.$$

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We can also give equations for $b \in \mathrm{Col}(A)$*:*

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If $b \in \mathrm{Col}(A)$ *how many solutions of* $Ax = b$, *how many free variables? Get point, line, plane....?*

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- If A is the 2×3 zero matrix, then $\text{Nul}(A) = \left\{ \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \right\}$. True or false?

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