Midterm 2
CS 373: Theory of Computation

Date: Thursday, November 4, 2010.

Instructions:

- This is a closed book exam. No notes, cheat sheets, textbook, or printed material allowed.
- You have 120 minutes to solve this exam.
- This exam has 5 problems each worth 10 points. However, not all problems are of equal difficulty.
- Please write your name on the top of *every* page in the space provided.
- If your solution does not fit in the space provided, and continues onto one of the back sheets, please indicate clearly where we should look for the solution.
- Unless otherwise stated, recall that "prove that", "show that" for a problem means you need to formally prove what you are claiming.
- Answering "I don't know" for a problem *does not receive any points.*
- You may use, without proof, any result that you were asked to prove in the homework or was proved in the lecture. If you use such a result, please explicitly state the result you are using (like " 'Reverse of a regular language is regular' was proved in a homework", rather than saying "this was shown in a homework").

Name	
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Problem	Maximum Points	Points Earned	Grader
1	10		
2	10		
3	10		
4	10		
5	10		
Total	50		

Problem 1. [Category: Comprehension] **True/False.** Decide for each statement whether it is true or false. Circle **T** if the statement is *necessarily true*; circle **F** if it it is not necessarily true. Each correct answer is worth **1 point**.

(a) Let $M = (Q, \Sigma, \delta, q_0, F)$ be the minimal DFA recognizing the language $L(M)$. Suppose M' is same as M except the initial state is changed to $q \neq q_0$, i.e., $M' = (Q, \Sigma, \delta, q, F)$. Assuming all states in Q are reachable from q, M' is the minimal DFA recognizing $L(M')$.
T **F**

(b) Let $M = (Q, \Sigma, \delta, q_0, F)$ be the minimal DFA recognizing the language $L(M)$. Since every pair of states of M is distinguishable, if w is accepted from state q then w is not accepted from state q' ($\neq q$).
T **F**

(c) If L is regular then suffix(L, x) is always regular, no matter what x is. (For a definition of suffix(L, x) see problem 2.)
T **F**

(d) For a decidable language L, L^R may or may not be decidable. (L^R denotes the reverse of language L.)
T **F**

(e) If $L \subseteq \{0\}^*$ then L is decidable.
T **F**

(f) If $L \leq_m \{0^n 1^n \mid n \geq 0\}$ then L is decidable.
T **F**

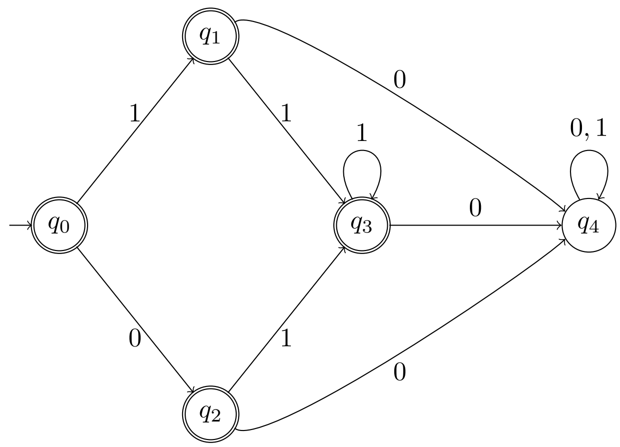
(g) If L is not recursively enumerable then $\overline{L}$ must be recursively enumerable.
T **F**

(h) $L_k = \{M \mid M \text{ halts after at most } k \text{ steps on } \epsilon\}$ is not decidable because of Rice's theorem.
T **F**

(i) If L is recursively enumerable and $L' \subseteq L$ then L' is recursively enumerable because the enumerator for L also enumerates L'.
T **F**

(j) If $A \leq_m B$ then $\overline{A} \leq_m \overline{B}$.
T **F**

Problem 2. [Category: Comprehension+Design] Consider the language $L = L(\epsilon \cup (1 \cup 0)1^*)$ and a DFA M that accepts L:



(a) Recall that for a DFA $M = (Q, \Sigma, \delta, q_0, F)$, suffix$(M, q) = \{w \in \Sigma^* \mid q \xrightarrow{w}_M q' \text{ and } q' \in F\}$. In other words, it is the collection of all words accepted if q were the initial state.

For each state q of M describe the language suffix(M, q), using either regular expressions or formal set notation. **[2.5 Points]**

(b) Recall that for a language $L \subseteq \Sigma^*$, and a string $x \in \Sigma^*$, suffix language of L with respect to x, is defined as

$$\text{suffix}(L, x) = \{y \in \Sigma^* \mid xy \in L\}$$

In other words, $\text{suffix}(L, x)$ is the collection of strings y which when prefixed by x, result in a string in L.

For each of the following values of x, describe $\text{suffix}(L, x)$. *(Hint: You may use the DFA M and the previous problem to simplify your calculations.)* **[3.5 Points]**

(a) $x = \epsilon$

(b) $x = 0$

(c) $x = 1$

(d) $x = 00$

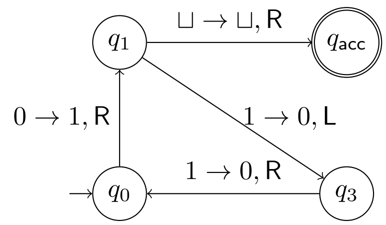
(e) $x = 01$

(f) $x = 10$

(g) $x = 11$

(c) Give a minimal DFA for L. **[4 Points]**

Problem 3. [Category: Comprehension] Consider the following Turing machine M on input alphabet $\{0, 1\}$. All transitions not shown in the diagram below are assumed to go to the reject state q_{rej}.



(a) Give the formal definition of M as a tuple. **[3 Points]**

(b) Describe the computation of M on the input 0111 formally, as a sequence of instantaneous descriptions/configurations. **[5 Points]**

(c) Is there any input on which M does not halt? If so, give an example string. **[1 Point]**

(d) What is the language recognized by M? **[1 Point]**

Problem 4. [Category: Comprehension+Proof]

(a) Suppose A and B are recursively enumerable languages such that $A \cup B$ and $A \cap B$ are both decidable. Prove that A is decidable. **[5 Points]**

(b) Suppose A is recursively enumerable and $A \leq_m \overline{A}$. Prove that A is decidable. **[5 Points]**

Problem 5. [Category: Proof] Let $L = \{M \mid M \text{ is a TM and } L(M) \text{ has at least 11253 strings}\}$. Prove the following facts.

(a) L is undecidable. **[2 Points]**

(b) L is recursively enumerable. **[4 Points]**

(c) $\overline{L}$ is not recursively enumerable. **[4 Points]**

(Additional Space for Problem 5)