STAT 400
Lecture CL1

Examples for 1.5

Spring 2015
A. Stepanov

1. In Neverland, men constitute 60% of the labor force. The rates of unemployment are 6.0% and 4.5% among males and females, respectively. A person is selected at random from Neverland's labor force.

$P(M) = 0.60$, $P(U \mid M) = 0.06$, $P(U \mid F) = 0.045$.

a) What is the probability that the person selected is a male and is unemployed?

$P(M \cap U) = P(M) \times P(U \mid M) = 0.60 \times 0.06 = \mathbf{0.036}$.

b) What is the probability that the person selected is a female and is unemployed?

$P(F \cap U) = P(F) \times P(U \mid F) = 0.40 \times 0.045 = \mathbf{0.018}$.

	Unemployed	**Employed**	Total
Male	0.036	0.564	0.60
Female	0.018	0.382	0.40
Total	0.054	0.946	1.00

c) What is the probability that the person selected is unemployed?

$P(U) = 0.036 + 0.018 = \mathbf{0.054}$.

OR

Law of Total Probability:

$P(U) = P(M) \times P(U \mid M) + P(F) \times P(U \mid F) = 0.60 \times 0.06 + 0.40 \times 0.045 = \mathbf{0.054}$.

d) Suppose the person selected is unemployed. What is the probability that a male was selected?

$P(M \mid U) = {}^{0.036}/_{0.054} = \mathbf{{}^{2}/_{3}}$.

OR

Bayes' Theorem:

$$P(M \mid U) = \frac{P(M) \times P(U \mid M)}{P(M) \times P(U \mid M) + P(F) \times P(U \mid F)} = \frac{0.60 \times 0.06}{0.60 \times 0.06 + 0.40 \times 0.045} = \mathbf{\frac{2}{3}}.$$

2. In a presidential race in Neverland, the incumbent Democrat (D) is running against a field of four Republicans (R_1, R_2, R_3, R_4) seeking the nomination. Political pundits estimate that the probabilities of R_1, R_2, R_3, and R_4 winning the nomination are 0.40, 0.30, 0.20, and 0.10, respectively. Furthermore, results from a variety of polls are suggesting that D would have a 55% chance of defeating R_1 in the general election, a 70% chance of defeating R_2, a 60% chance of defeating R_3, and an 80% chance of defeating R_4. Assuming all these estimates to be accurate, what are the chances that D will be a two-term president?

$P(R_1) = 0.40$, $P(R_2) = 0.30$, $P(R_3) = 0.20$, $P(R_4) = 0.10$,

$P(W \mid R_1) = 0.55$, $P(W \mid R_2) = 0.70$, $P(W \mid R_3) = 0.60$, $P(W \mid R_4) = 0.80$.

Law of Total Probability:

$$
\begin{aligned}
P(W) &= P(W \cap R_1) + P(W \cap R_2) + P(W \cap R_3) + P(W \cap R_4) \\
&= P(R_1)\,P(W \mid R_1) + P(R_2)\,P(W \mid R_2) \\
&\quad + P(R_3)\,P(W \mid R_3) + P(R_4)\,P(W \mid R_4) \\
&= 0.40 \cdot 0.55 + 0.30 \cdot 0.70 + 0.20 \cdot 0.60 + 0.10 \cdot 0.80 = \mathbf{0.63}.
\end{aligned}
$$

	R_1	R_2	R_3	R_4	
W	0.40 · 0.55 0.22	0.30 · 0.70 0.21	0.20 · 0.60 0.12	0.10 · 0.80 0.08	**0.63**
L	0.18	0.09	0.08	0.02	0.37
	0.40	0.30	0.20	0.10	1.00

3. In Anytown, 10% of the people leave their keys in the ignition of their cars. Anytown's police records indicate that 4.2% of the cars with keys left in the ignition are stolen. On the other hand, only 0.2% of the cars without keys left in the ignition are stolen. Suppose a car in Anytown is stolen. What is the probability that the keys were left in the ignition?

$P(\text{ Keys }) = 0.10,$ $\qquad$ $P(\text{ Keys}') = 1 - 0.10 = 0.90.$

$P(\text{ Stolen } | \text{ Keys }) = 0.042.$ $\qquad$ $P(\text{ Stolen } | \text{ Keys}') = 0.002.$

Need $P(\text{ Keys } | \text{ Stolen }) = ?$

Bayes' Theorem:

$$P(\text{Keys} \mid \text{Stolen}) = \frac{P(\text{Keys}) \times P(\text{Stolen} \mid \text{Keys})}{P(\text{Keys}) \times P(\text{Stolen} \mid \text{Keys}) + P(\text{Keys}') \times P(\text{Stolen} \mid \text{Keys}')}$$

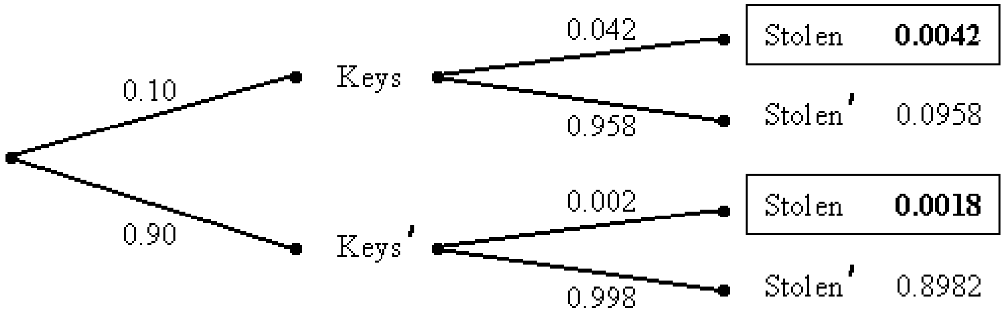
$$= \frac{0.10 \times 0.042}{0.10 \times 0.042 + 0.90 \times 0.002} = \mathbf{0.70}.$$

OR

	Stolen	Stolen′	
Keys	*0.042 · 0.10* 0.0042	0.0958	0.10
Keys′	*0.002 · 0.90* 0.0018	0.8982	0.90
	0.0060	0.9940	1.00

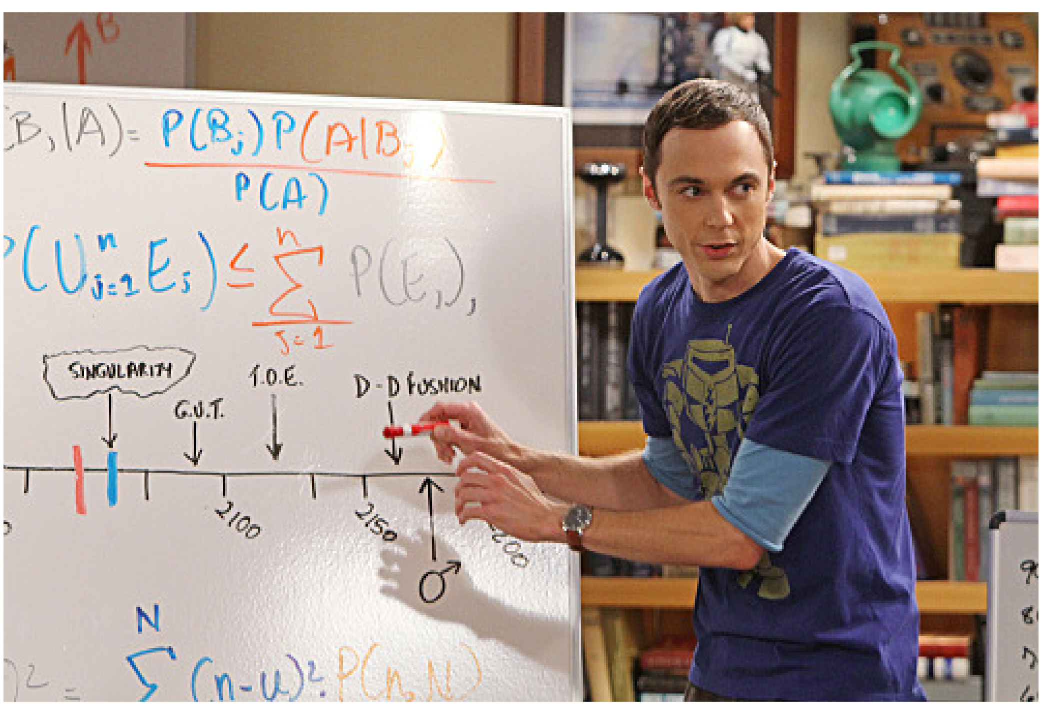
$$P(\text{ Keys } | \text{ Stolen }) = \frac{P(\text{ Keys } \cap \text{ Stolen })}{P(\text{ Stolen })} = \frac{0.0042}{0.0060} = \mathbf{0.70}.$$

OR



$$P(\text{ Stolen }) = 0.0042 + 0.0018 = 0.0060.$$

$$P(\text{ Keys } \mid \text{ Stolen }) = \frac{P(\text{ Keys } \cap \text{ Stolen })}{P(\text{ Stolen })} = \frac{0.0042}{0.0060} = \mathbf{0.70}.$$



3¼. A warehouse receives widgets from three different manufacturers, A (50%), B (30%), and C (20%). Suppose that 2% of the widgets coming from A are defective, as are 4% of the widgets coming from B, and 7% coming from C.

a) Find the probability that a widget selected at random at this warehouse is defective.

Law of Total Probability:

$$\begin{aligned} P(D) &= P(A) \times P(D \mid A) + P(B) \times P(D \mid B) + P(C) \times P(D \mid C) \\ &= 0.50 \times 0.02 + 0.30 \times 0.04 + 0.20 \times 0.07 \\ &= 0.010 + 0.012 + 0.014 = \mathbf{0.036}. \end{aligned}$$

b) Suppose a widget that was selected at random is found to be defective. What is the probability that it came from manufacturer A? Manufacturer B? Manufacturer C?

$$P(A \mid D) = \frac{0.010}{0.036} = \mathbf{\frac{5}{18}}.$$

$$P(B \mid D) = \frac{0.012}{0.036} = \mathbf{\frac{6}{18}}.$$

$$P(C \mid D) = \frac{0.014}{0.036} = \mathbf{\frac{7}{18}}.$$

3½. Seventy percent of the light aircraft that disappear while in flight in Neverland are subsequently discovered. Of the aircraft that are discovered, 60% have an emergency locator, whereas 90% of the aircraft not discovered do not have such a locator. Suppose a light aircraft that has just disappeared has an emergency locator. What is the probability that it will not be discovered?

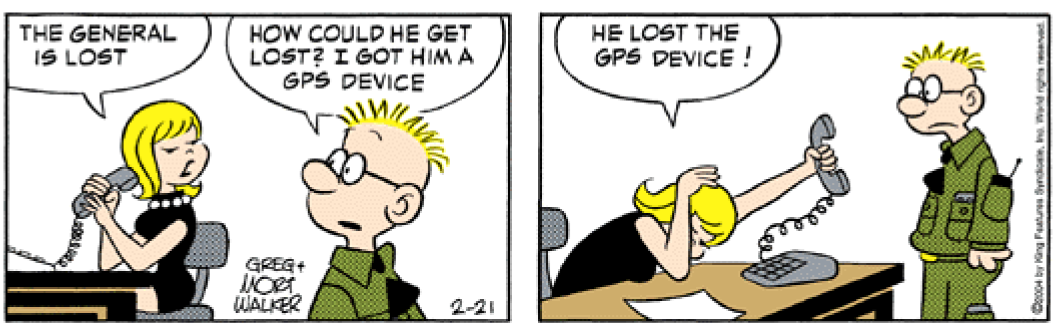
$P(\text{Discovered}) = 0.70,$ $\quad$ $P(\text{Discovered}') = 1 - 0.70 = 0.30.$

$P(\text{Locator} \mid \text{Discovered}) = 0.60.$ $\quad$ $P(\text{Locator}' \mid \text{Discovered}') = 0.90.$

Need $P(\text{Discovered}' \mid \text{Locator}) = ?$

	Locator	Locator ′	
Discovered	*0.60 · 0.70* 0.42	0.28	0.70
Discovered ′	0.03	*0.90 · 0.30* 0.27	0.30
	0.45	0.55	1.00

$$P(\text{Discovered}' \mid \text{Locator}) = \frac{P(\text{Discovered}' \cap \text{Locator})}{P(\text{Locator})} = \frac{0.03}{0.45} = \mathbf{\frac{1}{15}} \approx \mathbf{0.06667}.$$



4. In a certain population, the proportion of individuals who have a particular disease is 0.025. A test for the disease is positive in 94% of the people who have the disease and in 4% of the people who do not.

$P(D) = 0.025$, $P(+ \mid D) = 0.94$, $P(+ \mid D') = 0.04$.

a) Find the probability of receiving a positive reaction from this test.

Need $P(+) = ?$

	+	−	
D	$0.025 \cdot 0.94$ 0.0235	0.0015	0.025
D′	$0.975 \cdot 0.04$ 0.0390	0.9360	0.975
	0.0625	0.9375	1.000

OR

Law of Total Probability:

$P(+) = P(D) \times P(+ \mid D) + P(D') \times P(+ \mid D') = 0.025 \times 0.94 + 0.975 \times 0.04 = \mathbf{0.0625}$.

b) If a person received a positive reaction from this test, what is the probability that he/she has the disease?

$P(D \mid +) = {}^{0.0235}/_{0.0625} = \mathbf{0.376}$.

OR

Bayes' Theorem:

$$P(D \mid +) = \frac{0.025 \times 0.94}{0.025 \times 0.94 + 0.975 \times 0.04} = \mathbf{0.376}.$$

c) If a person received a negative reaction from this test, what is the probability that he/she doesn't have the disease?

$P(D' \mid -) = {}^{0.9360}/_{0.9375} = \mathbf{0.9984}$.

OR

Bayes' Theorem:

$$P(D' \mid -) = \frac{0.975 \times 0.96}{0.025 \times 0.06 + 0.975 \times 0.96} = \mathbf{0.9984}.$$