Worksheet 8 for October 20th and 22ndth

1. *Let*

$$A = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 1 & 0 & -1 \end{bmatrix}.$$

(a) *Draw a directed graph with numbered edges and nodes, whose edge-node incidence matrix is A.*

(b) *Find a basis for the solutions to $A\mathbf{x} = 0$ in two ways: by using the matrix A, and then by using a property of the graph.*

(c) *Find a basis for the solutions to $A^T\mathbf{y} = 0$ in two ways: by using the matrix A, and then by using a property of the graph.*

(d)* *Conclude from the fundamental theorem that a vector $\mathbf{b}$ is in the column space of A if and only if it satisfies $b_1 + b_2 - b_3 = 0$. What does this condition mean when the b's are potential differences?*

(e)* *Conclude from the fundamental theorem that a vector $\mathbf{f}$ is in the row space of A if and only if satisfies $f_1 + f_2 + f_3 = 0$. What does that mean when the f's are net currents into the nodes?*

($*$): *Questions (d) and (e) will not be on the Midterm.*

2. *Consider the matrix:*

$$A = \begin{bmatrix} -1 & 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 1 & 0 \\ -1 & 0 & 1 & 0 & 0 \\ -1 & 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & -1 \end{bmatrix}$$

(a) *Draw a directed graph with numbered edges and nodes, whose edge-node incidence matrix is A.*

(b) *Use a property of the graph to find a basis for* $\mathrm{Nul}(A)$.

(c) *Use a property of the graph to find a basis for* $\mathrm{Nul}(A^T)$.

3. *Let* $\mathbf{v} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$. *Find the projections $\hat{\mathbf{a}}, \hat{\mathbf{b}}, \hat{\mathbf{c}}$ of the vectors*

$$\mathbf{a} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 2 \\ -1 \\ -1 \end{bmatrix}, \quad \mathbf{c} = \begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix}$$

onto the $\mathbf{v}$. Interpret your results geometrically.

Tutoring Room (443 Altgeld Hall): Mon 4-6 PM, Tue 5-7 PM, Wed 6-8 PM

Midterm Dates: September 29th, October 22nd, November 19th (All Midterms 7-8:15 PM, see `learn.illinois.edu` **for locations)**