Quiz 23

1. Suppose $A \leq_m B$ and $B \leq_m C$. Which of the following is necessarily true?

   (A) $A \leq_m C$.
   (B) $A \leq_m C$, only when A and C are decidable.
   (C) $A \leq_m C$, only when A and B are decidable.
   (D) $A \leq_m C$, only when B and C are decidable.

   Correct answer is (A).

2. Consider languages A and B such that $A \leq_m B$ and A is regular. Which of the following is the strongest statement that necessarily follows?

   (A) B is regular.
   (B) B is not regular.
   (C) B is decidable.
   (D) B may or may not be decidable.

   Correct answer is (D).

3. Consider languages A and B such that $A \leq_m B$ and B is regular. Which of the following is the strongest statement that necessarily follows?

   (A) A is regular.
   (B) A is not regular.
   (C) A is decidable.
   (D) A may or may not be decidable.

   Correct answer is (C).

4. Consider non-empty languages A and B over Σ such that $A \neq \Sigma^*$, $B \neq \Sigma^*$, A is regular, and B is decidable but not regular. Taking $B \not\leq_m A$ to mean that B does not reduce to A, which of the following is the strongest statement that is necessarily true?

   (A) $A \leq_m B$
   (B) $B \leq_m A$
   (C) $A \leq_m B$ and $B \leq_m A$
   (D) $A \leq_m B$ but $B \not\leq_m A$

   Correct answer is (C).

5. Recall that $A_{\text{TM}} = \{\langle M, w\rangle \mid M \text{ accepts } w\}$ and $E_{\text{TM}} = \{\langle M\rangle \mid \mathbf{L}(M) = \emptyset\}$. For languages A and B, $A \not\leq_m B$ will denote that A does not reduce to B. Which of the following is necessarily true?

   (A) $A_{\text{TM}} \leq_m E_{\text{TM}}$
   (B) $E_{\text{TM}} \leq_m A_{\text{TM}}$
   (C) $A_{\text{TM}} \not\leq_m E_{\text{TM}}$ and $E_{\text{TM}} \not\leq_m A_{\text{TM}}$
   (D) None of the above.

   Correct answer is (C).