STAT 420 Fall 2015

Examples for 12/01/15, Part 4

ANOVA and Multiple Linear Regression

Consider a two-factor analysis of variance experiment was performed with $I = 2$, $J = 3$, and $K = 2$ (a 2×3 factorial experiment with 2 replicates):

	B1	B2	B3
A1	Y_{111} Y_{112}	Y_{121} Y_{122}	Y_{131} Y_{132}
A2	Y_{211} Y_{212}	Y_{221} Y_{222}	Y_{231} Y_{232}

A1 – base category $\mathbf{v}_2$ – indicator of A2
(In general, will need $I - 1$ dummy variables for I levels of factor A.)

B1 – base category $\mathbf{w}_2$ – indicator of B2 $\mathbf{w}_3$ – indicator of B3
(In general, will need $J - 1$ dummy variables for J levels of factor B.)

Then will need $(I - 1) \times (J - 1)$ interaction terms $\mathbf{v}_i \, \mathbf{w}_j$.

	β_0	β_1	β_2	β_3	β_4	β_5
Y_{111}	1	0	0	0	0	0
Y_{112}	1	0	0	0	0	0
Y_{121}	1	0	1	0	0	0
Y_{122}	1	0	1	0	0	0
Y_{131}	1	0	0	1	0	0
Y_{132}	1	0	0	1	0	0
Y_{211}	1	1	0	0	0	0
Y_{212}	1	1	0	0	0	0
Y_{221}	1	1	1	0	1	0
Y_{222}	1	1	1	0	1	0
Y_{231}	1	1	0	1	0	1
Y_{232}	1	1	0	1	0	1
	$\mathbf{1}$	$\mathbf{v}_2$	$\mathbf{w}_2$	$\mathbf{w}_3$	$\mathbf{v}_2 \, \mathbf{w}_2$	$\mathbf{v}_2 \, \mathbf{w}_3$

$$\mathbf{Y} = \beta_0 \mathbf{1} + \beta_1 \mathbf{v}_2 + \beta_2 \mathbf{w}_2 + \beta_3 \mathbf{w}_3 + \beta_4 \mathbf{v}_2 \mathbf{w}_2 + \beta_5 \mathbf{v}_2 \mathbf{w}_3 + \boldsymbol{\varepsilon}.$$

$$\begin{aligned}
\mu_{11} &= \beta_0 \\
\mu_{12} &= \beta_0 + \beta_2 \\
\mu_{13} &= \beta_0 + \beta_3 \\
\mu_{21} &= \beta_0 + \beta_1 \\
\mu_{22} &= \beta_0 + \beta_1 + \beta_2 + \beta_4 \\
\mu_{23} &= \beta_0 + \beta_1 + \beta_3 + \beta_5
\end{aligned}$$

Interaction: $\quad H_0: \beta_4 = \beta_5 = 0.$

(In general, $(I-1) \times (J-1)$ parameters.)

Factor A: $\quad H_0: \beta_1 = 0.$

(In general, $I-1$ parameters.)

Factor B: $\quad H_0: \beta_2 = \beta_3 = 0.$

(In general, $J-1$ parameters.)

Residuals DF $\quad n - p$

$$\begin{aligned}
&= IJK - [\,(I-1) + (J-1) + (I-1)\times(J-1) + 1)\,] \\
&= IJK - IJ = IJ(K-1).
\end{aligned}$$