1. Let the random variables X and Y have the joint pdf

$$f_{XY}(x,y) = \begin{cases} 1, & 0 < x < 1,\ \ 0 < y < 1 \\ 0, & otherwise \end{cases}$$

Also let $U = X + Y$ and $V = X - Y$.

   a. Compute the correlation between *U* and *X*.
   b. Compute the correlation between *U* and *V*.
   c. Find the cdf and pdf for *U.*
   d. Find the cdf and pdf for *V.*

2. Suppose X, Y, U and V are the same as in Problem 1.
   a. Sketch the support for *U* and *V*.
   b. Find the joint pdf for *U* and *V*.
   c. Are *U* and *V* independent?

3. Let the random variables *X* and *Y* have the joint pdf

$$f_{XY}(x,y) = \begin{cases} 2, & 0 < y < x < 1 \\ 0, & otherwise \end{cases}$$

   a. Let $Z = Y / X$. Find the interval of support for *Z*.
   b. Find the cdf $F_Z(z)$ for *Z*.
   c. Find the pdf for *Z.*

4. Suppose X, Y and Z are the same as in Problem 3. Find the joint pdf for (*X*, *Z*).

5. Let *X* and *U* be jointly distributed as,

$$f_{X,U}(x,u) = Ce^{-x}, 0 < x < \infty, 0 < u < 1$$

   a. Sketch the support for *X* and *U*.
   b. Find *C*.
   c. Find the cdf for $Y = \frac{X}{U}$.
   d. Find the pdf for Y.

6. Let X and U be jointly distributed as,

$$f_{X,U}(x,u) = Ce^{-x}, 0 < u < x < \infty, 0 < u < 1$$

   a. Sketch the support for X and U.
   b. Find C.
   c. Find the cdf for $Y = \frac{X}{U}$.
   d. Find the pdf for Y.

7. Dick and Jane have agreed to meet for lunch between noon (12:00 p.m.) and 1:00 p.m. Denote Jane's arrival time by X, Dick's by Y, and suppose X and Y are independent with probability density functions

$$f_X(x) = \begin{cases} 3(1-x)^2 & 0 \le x \le 1 \\ 0 & \text{otherwise} \end{cases} \qquad f_Y(y) = \begin{cases} 2(1-y) & 0 \le y \le 1 \\ 0 & \text{otherwise} \end{cases}$$

   a. Let T_1 denote the arrival time of the person who arrives first. Find the pdf of T_1.
   b. Let T_2 denote the arrival time of the person who arrives second. Find the pdf of T_2.
   c. What is the expected amount of time that the one who arrives first must wait for the person who arrives second?

8. Let X and Y be two independent random variables, with probability density functions $f_X(x)$ and $f_Y(y)$, respectively.

$$f_X(x) = \begin{cases} 3x^2 & 0 \le x \le 1 \\ 0 & \text{otherwise} \end{cases} \qquad f_Y(y) = \begin{cases} 2y & 0 \le y \le 1 \\ 0 & \text{otherwise} \end{cases}$$

   Find the pdf $f_W(w)$ of $W = X + Y$.

9. Let X_1, X_2, X_3 be i.i.d. with probability mass function $p(k) = \frac{k}{10}, k = 1,2,3,4$.
   a. Find the probability mass function of $Y_3 = \max(X_1, X_2, X_3)$.
   b. Find the probability mass function of $Y_1 = \min(X_1, X_2, X_3)$.

10. Let $Y_1 < Y_2 < Y_3 < Y_4$ be the order statistics of a random sample of size 4 from a distribution with pdf $f(x) = 2x, 0 < x < 1$, zero elsewhere.
    a. Find the pdf of Y_4.
    b. Find the joint pdf of Y_3 and Y_4.
    c. Find the conditional pdf of Y_3 given $Y_4 = y_4$.
    d. Evaluate $E(Y_3 | Y_4 = y_4)$.

Extra problems for Graduate Students registered for 4 hours:

11. Given that a nonnegative function $g(z)$ has the property that

$$\int_0^\infty g(z)dz = 1,$$

consider the following joint pdf for the continuous random variables X_1 and X_2,

$$f_{X,Y}(x,y) = \frac{2g\left(\sqrt{x_1^2 + x_2^2}\right)}{\pi\sqrt{x_1^2 + x_2^2}}, 0 < x_1 < \infty, 0 < x_2 < \infty$$

and zero elsewhere.

    a. Let $X_1 = Rcos(W)$ and $X_2 = Rsin(W)$. Find the joint pdf of R and W. Be sure to identify the support for R and W.
    b. Suppose $\int_0^\infty rg(r)dr = \theta$. Find $E(R)$ as a function of θ.
    c. Are R and W independent?

12. Let $Y_1 < Y_2 < \cdots < Y_n$ be the order statistics of a random sample of size n from the exponential distribution with pdf $f(x) = e^{-x}, 0 < x < \infty$, zero elsewhere. Let

$$Z_1 = nY_1,\ Z_2 = (n-1)(Y_2 - Y_1), Z_3 = (n-2)(Y_3 - Y_2), \dots , Z_n = (Y_n - Y_{n-1}).$$

Show that $Z_1, Z_2, \dots Z_n$ are independent and that each of them has the exponential distribution.