STAT 420 Spring 2014

Homework 3: Due February 18 by 7:00pm

Exercise 1

DO NOT use a computer for this problem.

An employee claims that drinking beer has no effect on the amount of time it takes for him to perform a particular task. The following data show how many seconds he took to perform the task after consuming various quantities of beer, measured in ounces:

Beer consumption (x)	0	12	24	36	48	60	72
Task time (y)	62	50	59	74	59	83	68

Consider the simple linear regression model: $y_i = b_0 + b_1 x_i + e_i$ with $e_i \overset{\text{iid}}{\sim} \text{N}(0, \sigma^2)$.

From Homework 2 Solutions we know that

$$\sum x = 252, \quad \sum y = 455, \quad \sum x^2 = 13104, \quad \sum y^2 = 30295, \quad \sum xy = 17388,$$
$$\sum (x - \bar{x})^2 = 4032, \quad \sum (y - \bar{y})^2 = 720, \quad \sum (x - \bar{x})(y - \bar{y}) = \sum (x - \bar{x}) y = 1008,$$
$$\hat{y} = \hat{b}_0 + \hat{b}_1 x = 56 + 0.25x, \quad \sum (y - \hat{y})^2 = 468, \quad \hat{\sigma} = \sqrt{468/5} = 9.6747, \quad R^2 = 0.35$$

(h) Construct a 95% confidence interval for b_1.

(i) Test the employee's claim at a 10% level of significance. That is, test $H_0 : b_1 = 0$ versus $H_1 : b_1 > 0$ at a 10% level of significance.

(j) Test $H_0 : b_1 = 0$ versus $H_1 : b_1 \neq 0$ at a 10% level of significance using t-test.

(k) Test $H_0 : b_1 = 0$ versus $H_0 : b_1 \neq 0$ at a 10% level of significance using F-test.

(l) The supervisor believes that when the employee is not drinking beer, it takes him on average 60 seconds to perform the task. Test $H_0 : b_0 = 60$ versus $H_0 : b_0 \neq 60$ at a 10% level of significance.

(m) The supervisor claims that after the employee drinks 48 oz of beer, it takes him on average over 60 seconds to perform the task. Test $H_0 : E(Y|x = 48) = 60$ versus $H_1 : E(Y|x = 48) > 60$ at a 10% level of significance.

(n) Construct a 90% confidence interval for the mean time the employee needs to perform the task after consuming 144 ounces of beer.

(o) Construct 90% limits of prediction for the time the employee needs to perform the task after consuming 144 ounces of beer.

Exercise 2

DO use a computer for this problem. Use the data from Exercise 1.

Double check 1 (h), (j), (k), (n), (o). Please include a printout and mark (circle or highlight) the intervals (3), the test statistics (2), and the p-values (2).

Exercise 3

The more beer you drink, the more your blood alcohol level (BAL) rises:

# 12 oz beers: x	5	2	9	8	3	7	3	5	3	5
BAL (%): y	0.1	0.03	0.19	0.12	0.04	0.095	0.07	0.06	0.02	0.05

Consider the simple linear regression model: $y_i = b_0 + b_1 x_i + e_i$ with $e_i \overset{\text{iid}}{\sim} \text{N}(0, \sigma^2)$.

(a) Make a scatterplot with the regression line.

(b) Test the hypothesis that one beer raises your BAL by 0.02% (on average) against the alternative that it raises it less. Use a 5% level of significance. Find the test statistic and p-value for this test.

(c) Test the hypothesis that the y-intercept is 0 against the two-sided alternative. Use a 5% level of significance. Find the test statistic and p-value for this test.

Exercise 4

Suppose that the amount of beer a person has consumed (X) and the person's blood alcohol level (Y) in a certain bar follow a bivariate normal distribution with

$$\mu_X = 60, \qquad \sigma_X = 15, \qquad \mu_Y = 0.072, \qquad \sigma_Y = 0.02, \qquad \rho = 0.60$$

(a) What is the probability that the person's blood alcohol level is above 0.08? That is, find $P(Y > 0.08)$.

(b) Given that someone has consumed 45 oz of beer, what is the probability that the person's blood alcohol level is above 0.08? That is, find $P(Y > 0.08|X = 45)$.

(c) Given that someone has consumed 80 oz of beer, what is the probability that the person's blood alcohol level is above 0.08? That is, find $P(Y > 0.08|X = 80)$.

(d) Given that someone has blood alcohol level 0.10, what is the probability that this person has consumed over 90 oz of beer? That is, find $P(X > 90|Y = 0.10)$.