Interpolation Error

In [1]:

```
#keep
import numpy as np
import numpy.linalg as la
import matplotlib.pyplot as pt
%matplotlib inline
```

Let's fix a function to interpolate:

In [2]:

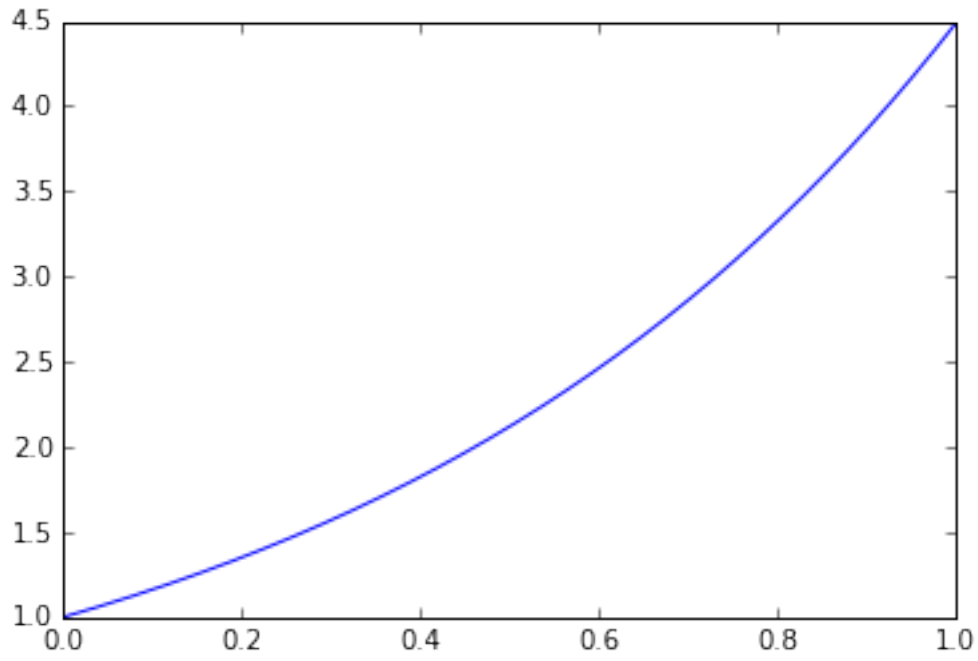
```
#keep

if 1:
    def f(x):
        return np.exp(1.5*x)
elif 0:
    def f(x):
        return np.sin(20*x)
else:
    def f(x):
        return (x>=0.5).astype(np.int).astype(np.float)
```

In [3]:

```
#keep
x_01 = np.linspace(0, 1, 1000)
pt.plot(x_01, f(x_01))
```

Out[3]:

```
[<matplotlib.lines.Line2D at 0x10d4b7cc0>]
```



And let's fix some parameters. Note that the interpolation interval is just $[0, h]$, not $[0, 1]$!

In [4]:

```
#keep
degree = 1
h = 1

nodes = 0.5 + np.linspace(-h/2, h/2, degree+1)
nodes
```

Out[4]:

```
array([ 0.,  1.])
```

Now build the Vandermonde matrix:

In [5]:

```
#keep
V = np.array([
    nodes**i
    for i in range(degree+1)
]).T
```

In [6]:

```
#keep
V
```

Out[6]:

```
array([[ 1.,  0.],
       [ 1.,  1.]])
```

Now find the interpolation coefficients as `coeffs`:

In [7]:

```
coeffs = la.solve(V, f(nodes))
```

Here are some points. Evaluate the interpolant there:

In [8]:

```
#keep
x_0h = 0.5+np.linspace(-h/2, h/2, 1000)
```

In [9]:

```
interp_0h = 0*x_0h
for i in range(degree+1):
    interp_0h += coeffs[i] * x_0h**i
```

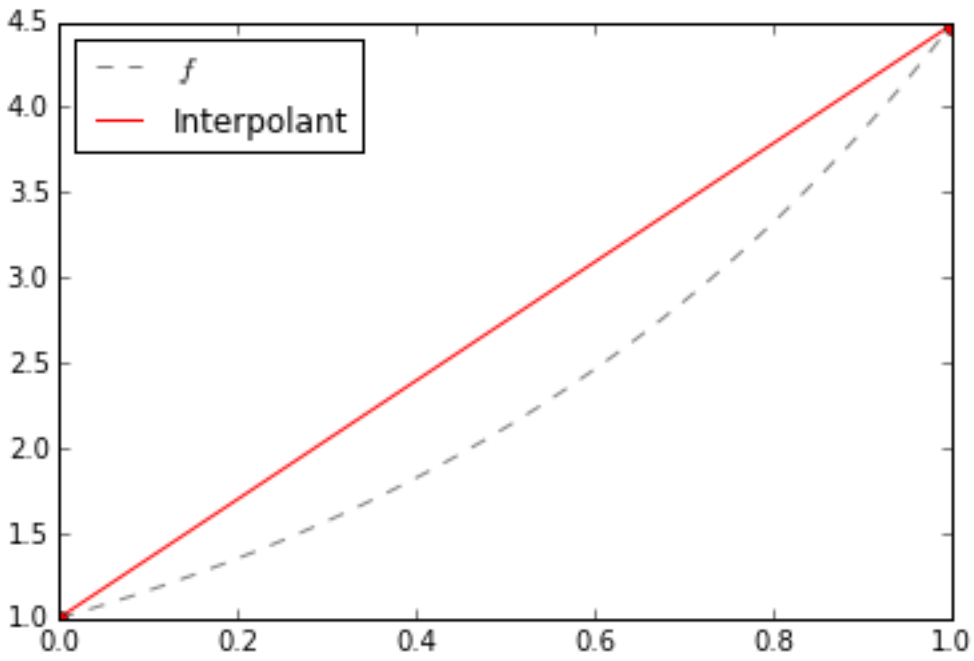
Now plot the interpolant with the function:

In [10]:

```
#keep
pt.plot(x_01, f(x_01), "--", color="gray", label="$f$")
pt.plot(x_0h, interp_0h, color="red", label="Interpolant")
pt.plot(nodes, f(nodes), "or")
pt.legend(loc="best")
```

Out[10]:

```
<matplotlib.legend.Legend at 0x10d5cc0b8>
```

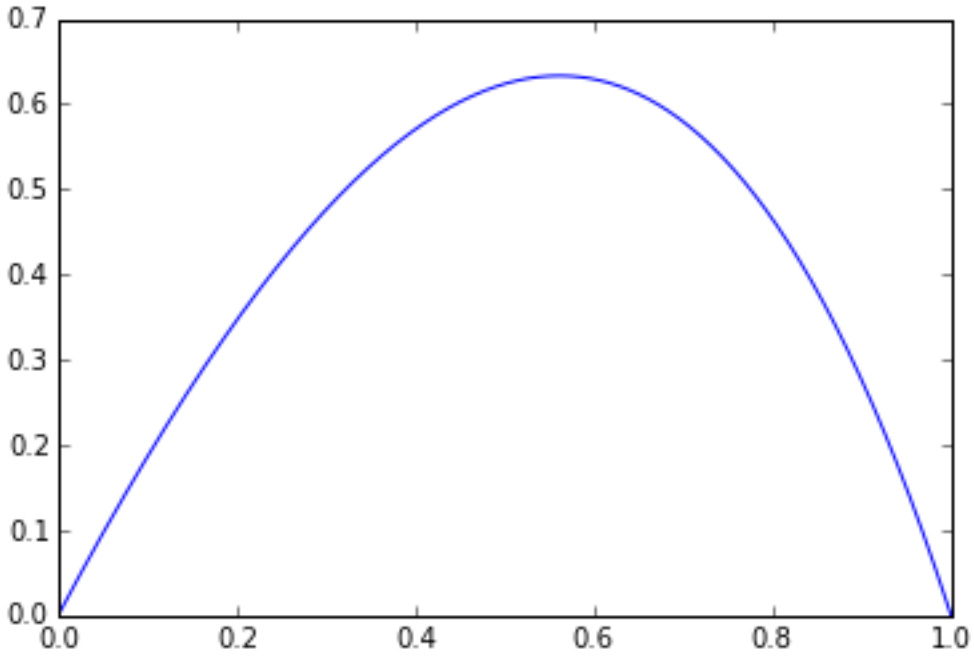


Also plot the error:

In [11]:

```
#keep
error = interp_0h - f(x_0h)

pt.plot(x_0h, error)

print("Max error: %g" % np.max(np.abs(error)))
```

```
Max error: 0.633384
```



- What does the error look like? (Approximately)
- How will the error react if we shrink the interval?
- What will happen if we increase the polynomial degree?