Preparation problems for the discussion sections on October 27th and 29th

1. *Let* $W = \operatorname{span}\left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 0 \\ 0 \end{bmatrix} \right\}$ *be a subspace of* $\mathbb{R}^4$.

(i) *Find the closest point to* $\begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}$ *on the subspace* W.

(ii) *Find the projection matrix* P *corresponding to the projection onto* W.

(iii) *Use the projection matrix* P *to find the projection of* $\begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}$ *onto the subspace* W.

2. *Let* $W = \operatorname{span}\left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} \right\}$ *and* $V = \operatorname{span}\left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ -2 \end{bmatrix} \right\}$ *be subspaces of* $\mathbb{R}^3$.

(i) *Find the projection matrices,* P *and* Q*, corresponding to the projections onto* W *and* V*, respectively.*

(ii) *Check that* $PQ = QP$. *Can you interpret* PQ *as a projection matrix?*

3. *Let* $A = \begin{bmatrix} 1 & -1 \\ 1 & 1 \\ 0 & 0 \end{bmatrix}$ *and* $\mathbf{b} = \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}$.

a. *Does* $\mathbf{b}$ *belong to the column space of* A*? Can you solve* $A\mathbf{x} = \mathbf{b}$*?*

b. *What do you expect the projection of* $\mathbf{b}$ *onto* $W = \operatorname{Col}(A)$ *to be?*

c. *Find the projection* $\hat{\mathbf{b}}$ *of* $\mathbf{b}$ *onto* $\operatorname{Col}(A)$*, and then solve* $A\hat{\mathbf{x}} = \hat{\mathbf{b}}$*. (The vector* $\hat{\mathbf{x}}$ *is called* the least square solution *of* $A\mathbf{x} = \mathbf{b}$*.)*

d. *Solve the equation* $A^T A\hat{\mathbf{x}} = A^T\mathbf{b}$*. Compare with your result of the previous part! (This equation is called the* normal equation *of* $A\mathbf{x} = \mathbf{b}$*.)*

e. *Answer these questions for* A *as above but with* $\mathbf{b} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$ *(and then* $\mathbf{b} = \begin{bmatrix} 0 \\ 0 \\ 4 \end{bmatrix}$*).*

4. *Let* $A = \begin{bmatrix} 0 & 1 \\ -2 & 2 \\ 2 & 2 \end{bmatrix}$ *and* $\boldsymbol{b} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$. *Find the least squares solution* $\hat{\mathbf{x}}$ *of* $A\mathbf{x} = \mathbf{b}$.

5. *A scientist tries to find the relation between the mysterious quantities* x *and* y*. She measures the following values:*

x	1	2	3	4
y	2	5	9	17

Tutoring Room (443 Altgeld Hall): Mon 4-6 PM, Tue 5-7 PM, Wed 6-8 PM
Midterm Date: November 19th 7-8:15 PM, Final Date: December 17 8am - 11am)

(i) *Suppose that y is a linear function of x of the form $a+bx$. Set up the system of equations to find the coefficients a and b.*

(ii) *Find the best estimate for the coefficients a and b.*

(iii) *Same question if we suppose that y is a quadratic function of the $a + bx + cx^2$.*

6. *The system of the equations $A\mathbf{x} = \mathbf{b}$ with*

$$A = \begin{bmatrix} 1 & -1 \\ 1 & 0 \\ 1 & 1 \\ 1 & 2 \end{bmatrix} \quad \textit{and} \quad \mathbf{b} = \begin{bmatrix} 5 \\ 0 \\ 5 \\ 10 \end{bmatrix},$$

is not consistent.

(i) *Find the least squares solution $\hat{\mathbf{x}}$ for the equation $A\mathbf{x} = \mathbf{b}$.*

(ii) *Determine the least squares line for the data points $(-1, 5), (0, 0), (1, 5), (2, 10)$.*