Computing the SVD

In [1]:

```
#keep
import numpy as np
import numpy.linalg as la
```

In [2]:

```
#keep
np.random.seed(15)
n = 5
A = np.random.randn(n, n)
```

Now compute the eigenvalues and eigenvectors of $A^T A$ as `eigvals` and `eigvecs` using `la.eig` or `la.eigh` (symmetric):

In [3]:

```
eigvals, eigvecs = la.eigh(A.T.dot(A))
```

In [4]:

```
#keep
eigvals
```

Out[4]:

```
array([ 0.08637178,  0.457892  ,  2.04177547,  2.34383161,  8.370001
84])
```

Eigenvalues are real and positive. Coincidence?

In [5]:

```
#keep
eigvecs.shape
```

Out[5]:

```
(5, 5)
```

Check that those are in fact eigenvectors and eigenvalues:

In [6]:

```
B = A.T.dot(A)
B - eigvecs.dot(np.diag(eigvals)).dot(la.inv(eigvecs))
```

Out[6]:

```
array([[ -8.88178420e-16,   4.44089210e-16,   4.44089210e-16,
         -4.44089210e-16,   6.66133815e-16],
       [ -1.77635684e-15,  -3.10862447e-15,  -2.22044605e-15,
         -2.22044605e-15,  -1.94289029e-16],
       [ -2.22044605e-15,  -1.77635684e-15,  -8.88178420e-16,
         -1.11022302e-15,   3.33066907e-16],
       [ -1.33226763e-15,  -8.88178420e-16,  -4.44089210e-16,
         -4.44089210e-16,   2.22044605e-16],
       [ -4.44089210e-16,  -8.04911693e-16,  -6.66133815e-16,
         -4.44089210e-16,   1.11022302e-16]])
```

`eigvecs` are orthonormal! (Why?)

Check:

In [7]:

```
eigvecs.T.dot(eigvecs) - np.eye(n)
```

Out[7]:

```
array([[  6.66133815e-16,   1.83230524e-16,  -6.80556436e-17,
         -1.00482385e-16,  -3.40005801e-16],
       [  1.83230524e-16,   1.11022302e-15,  -1.41968548e-16,
          2.53432919e-16,   4.67507977e-16],
       [ -6.80556436e-17,  -1.41968548e-16,   4.44089210e-16,
         -2.25729902e-16,   1.04083409e-17],
       [ -1.00482385e-16,   2.53432919e-16,  -2.25729902e-16,
          4.44089210e-16,   2.35922393e-16],
       [ -3.40005801e-16,   4.67507977e-16,   1.04083409e-17,
          2.35922393e-16,  -1.11022302e-16]])
```

Now piece together the SVD:

In [19]:

```
Sigma = np.diag(np.sqrt(eigvals))
```

In [20]:

```
V = eigvecs
```

In [38]:

```
U = A.dot(V).dot(la.inv(Sigma))
```

Check orthogonality of `U`:

In [39]:

```
U.dot(U.T) - np.eye(n)
```

Out[39]:

```
array([[ -2.10942375e-15,  -1.08246745e-15,  -8.46545056e-16,
          8.32667268e-17,   8.25728375e-16],
       [ -1.08246745e-15,   4.44089210e-16,  -1.38777878e-16,
         -5.55111512e-17,  -1.38777878e-16],
       [ -8.46545056e-16,  -1.38777878e-16,   2.22044605e-16,
          9.85322934e-16,  -3.95516953e-16],
       [  8.32667268e-17,  -5.55111512e-17,   9.85322934e-16,
          2.22044605e-16,   4.99600361e-16],
       [  8.25728375e-16,  -1.38777878e-16,  -3.95516953e-16,
          4.99600361e-16,  -2.22044605e-16]])
```