STAT 420 Spring 2014

Homework 10: Due May 6 by 7:00pm

Exercise 1

Assume the following 10 observations

$$2, -1, -4, 0, 2, 3, 1, -2, -1, 1$$

were generated from a zero-mean AR(1) model: $Y_t = \phi Y_{t-1} + \epsilon_t$.

(a) Given the above sample, calculate the sample estimate of the autocorrelation at lags 1 and 2 (i.e., calculate r_1 and r_2).

(b) Obtain the least-squares estimate of ϕ.
Hint: consider $S_c(\phi) = \sum_{t=2}^{N}(y_t - \phi y_{t-1})^2$
First derive the expression for the least-squares estimator.
Then use expression to find least-squares $\hat{\phi}$ for given sample.

Exercise 2

Consider the AR(2) process

$$Y_t - 0.3Y_{t-1} - 0.1Y_{t-2} = \epsilon_t$$

where ϵ_t is white noise.

(a) Determine whether this model would produce a stationary process. That is, find the roots of $1 - 0.3z - 0.1z^2 = 0$ and check whether they are outside of the unit circle on the complex plane.

(b) Use the Yule-Walker equations to find ρ_1 and ρ_2.
Then find ρ_3 and ρ_4.

Exercise 3

Consider the following MA(1) process: $Y_t - \mu = \epsilon_t - \theta\epsilon_{t-1}$ where $-1 < \theta < 1$.
Based on a series of length $N = 5$ we have

$$y_1 = 3,\ y_2 = 6,\ y_3 = 0,\ y_4 = 9,\ y_5 = 7$$

(a) Given the above sample, calculate the sample estimate of the autocorrelation at lag 1 (i.e., calculate r_1).

(b) Use your answer from part (a) and the method of moments to estimate θ. Round θ to the second decimal place. (Hint: from ACF we know $\rho_1 = \frac{-\theta}{1+\theta^2}$ for $|\theta| < 1$).

(c) Defining $\epsilon_0 = 0$, calculate $S_c(\theta) = \sum_{t=1}^{N} \epsilon_t^2$ for $\theta \in \{-1, -0.99, -0.98, \ldots, 0.98, 0.99, 1\}$. Which value of θ minimizes $S_c(\theta)$?

Hint:

```
> y = c(3,6,0,9,7)
> ydot = y - mean(y)
> theta = seq(-1,1,by=0.01)
> N = length(theta)
> S = rep(0,N)
> e = rep(0,5)
> for (i in 1:N) {
      # fill in some code
+     S[i] = sum(e^2)
+ }
```

(d) If $\tilde{\theta}$ is answer from part (b) and $\epsilon_0 = 0$, forecast y_6 and y_7.
Hint: last residual is $\tilde{\epsilon}_5 = 2.789787$.

(e) If $\hat{\theta}$ is answer from part (c) and $\epsilon_0 = 0$, forecast y_6 and y_7.
Hint: last residual is $\hat{\epsilon}_5 = 2.123771$.

Exercise 4

Consider the ARMA (1,1) model

$$(Y_t - 60) + 0.3(Y_{t-1} - 60) = \epsilon_t - 0.4\epsilon_{t-1}$$

which was fit to a time series where the last 10 values are

$$60, 57, 52, 59, 62, 59, 63, 67, 61, 58$$

and the last residual is $\hat{\epsilon}_N = -2$.

Calculate the forecasts of the next two observations, and indicate how forecasts can be calculated for lead times greater than two. Show what happens to the forecasts as the lead time becomes arbitrarily large.