# Newton's method in $n$ dimensions

In [1]:

```
#keep
import numpy as np
import numpy.linalg as la

import scipy.optimize as sopt

import matplotlib.pyplot as pt
from mpl_toolkits.mplot3d import axes3d
%matplotlib inline
```

Here are two functions. The first one is an oblong "bowl-shaped" one made of quadratic functions.

In [2]:

```
#keep
def f(x):
    return 0.5*x[0]**2 + 2.5*x[1]**2

def df(x):
    return np.array([x[0], 5*x[1]])

def ddf(x):
    return np.array([
                     [1,0],
                     [0,5]
                     ])
```

The second one is a challenge problem for optimization algorithms known as Rosenbrock's banana function (https://en.wikipedia.org/wiki/Rosenbrock_function).

In [124]:

```
#keep

def f(X):
    x = X[0]
    y = X[1]
    val = 100.0 * (y - x**2)**2 + (1.0 - x)**2
    return val

def df(X):
    x = X[0]
    y = X[1]
    val1 = 400.0 * (y - x**2) * x - 2 * x
    val2 = 200.0 * (y - x**2)
    return np.array([val1, val2])

def ddf(X):
    x = X[0]
    y = X[1]
    val11 = 400.0 * (y - x**2) - 800.0 * x**2 - 2
    val12 = 400.0
    val21 = -400.0 * x
    val22 = 200.0
    return np.array([[val11, val12], [val21, val22]])
```

Let's take a look at these functions. First in 3D:

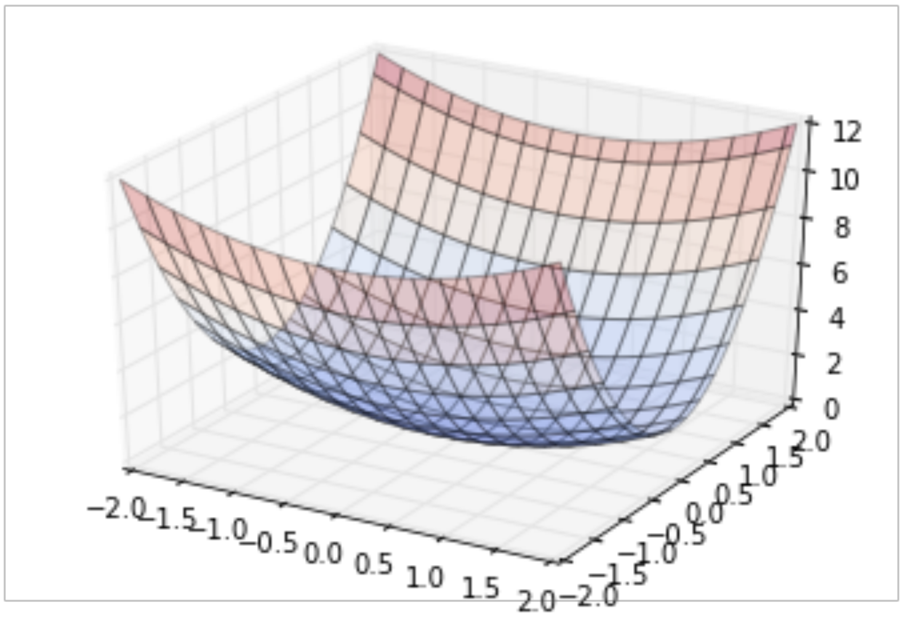
In [3]:

```
#keep
fig = pt.figure()
ax = fig.gca(projection="3d")

xmesh, ymesh = np.mgrid[-2:2:50j,-2:2:50j]
fmesh = f(np.array([xmesh, ymesh]))
ax.plot_surface(xmesh, ymesh, fmesh,
                alpha=0.3, cmap=pt.cm.coolwarm, rstride=3, cstride=3)
```

Out[3]:

```
<mpl_toolkits.mplot3d.art3d.Poly3DCollection at 0x10473b080>
```



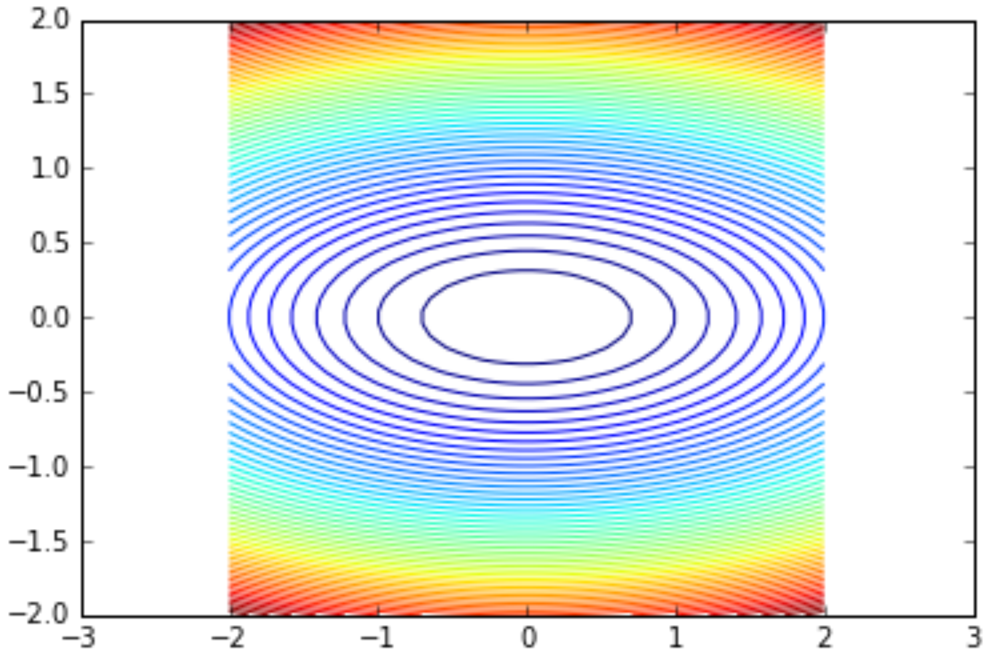

Then as a "contour plot":

In [4]:

```
#keep
pt.axis("equal")
pt.contour(xmesh, ymesh, fmesh, 50)
```

Out[4]:

```
<matplotlib.contour.QuadContourSet at 0x10473ae48>
```




- You may need to add contours to seee more detail.
- The function is *not* symmetric about the y axis!

## Newton

First, initialize:

In [5]:

```
#keep
# Initialize the method

guesses = [np.array([2, 2./5])]
```

Then evaluate this cell lots of times:

In [15]:

```
x = guesses[-1]
s = la.solve(ddf(x), df(x))
next_guess = x - s
print(f(next_guess), next_guess)

guesses.append(next_guess)
```

```
0.0 [ 0.  0.]
```

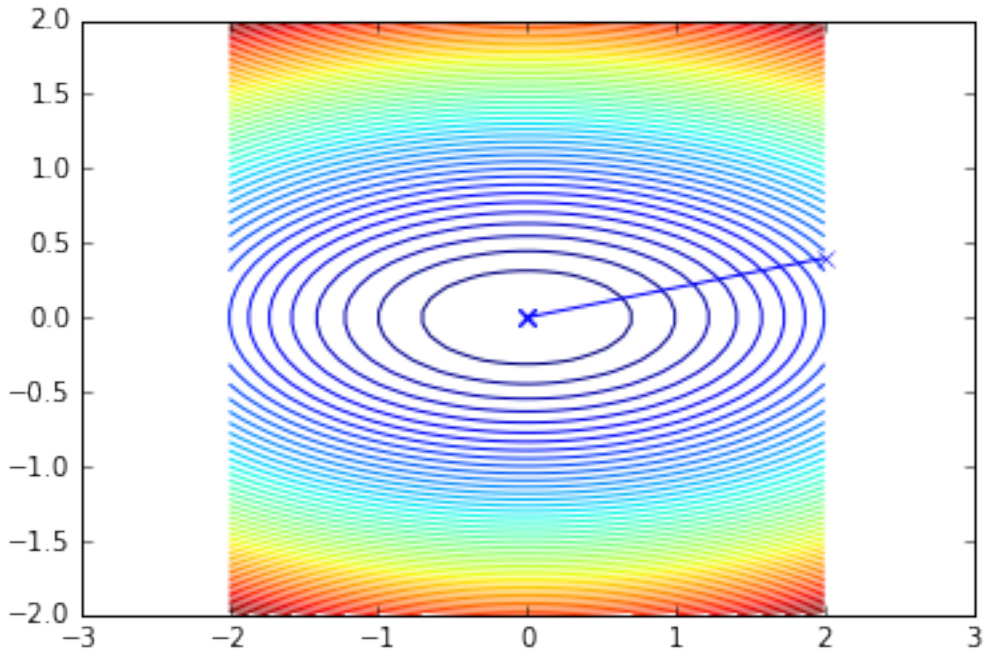
Here's some plotting code to see what's going on:

In [17]:

```
#keep
pt.axis("equal")
pt.contour(xmesh, ymesh, fmesh, 50)
it_array = np.array(guesses)
pt.plot(it_array.T[0], it_array.T[1], "x-")
```

Out[17]:

```
[<matplotlib.lines.Line2D at 0x105423358>]
```




In [ ]: