# Quiz 11

1. Let $\equiv$ be an equivalence relation and let $E$ be an equivalence class of $\equiv$. Which of the following is the strongest statement that is necessarily true about $E$?

   (A) There are $a, b \in E$ such that $a \equiv b$.
   (B) For every $a, b \in E$, $a \equiv b$.
   (C) There are $a, b \in E$ and $c \notin E$ such that $a \equiv b$ and $a \not\equiv c$
   (D) For every $a, b \in E$ and every $c \notin E$, $a \equiv b$ and $a \not\equiv c$.

   Correct answer is (D).

2. Recall that for any language $L$, we say $x \equiv_L y$ iff $xz \in L \leftrightarrow yz \in L$. Let $L = \mathbf{L}((0 \cup 1)^*11(0 \cup 1)^*)$. Then, $[1]_{\equiv_L}$ is the set

   (A) $\mathbf{L}((0 \cup 1)^*1(0 \cup 1)^*)$
   (B) $\mathbf{L}((0 \cup 1)^*1)$
   (C) $\mathbf{L}((0 \cup \epsilon)(10 \cup 0)^*1)$
   (D) $\mathbf{L}((01 \cup 0)^*1)$

   Correct answer is (C).

3. Recall that for any language $L$, we say $x \equiv_L y$ iff $xz \in L \leftrightarrow yz \in L$. Let $L = \mathbf{L}((0 \cup 1)^*11(0 \cup 1)^*)$. Then, $\#(\equiv_L)$ is

   (A) 1
   (B) 2
   (C) 3
   (D) 4

   Correct answer is (C).

4. Recall that for any language $L$, we say $x \equiv_L y$ iff $xz \in L \leftrightarrow yz \in L$. Let $L$ be a language such that $\#(\equiv_L)$ is infinite. What is necessarily true about $L$?

   (A) $L$ is regular.
   (B) $L$ is not regular.
   (C) $L$ may or may not be regular.
   (D) $L$ is regular if the alphabet of $L$ is unary.

   Correct answer is (B).

5. Recall that for any language $L$, we say $x \equiv_L y$ iff $xz \in L \leftrightarrow yz \in L$. Let $L$ be a language such that $\#(\equiv_L) = 5$. Which of the following is necessarily true?

   (A) Any NFA recognizing $L$ must have at least 5 states.
   (B) Any NFA recognizing $L$ must have at most 5 states.
   (C) Any DFA recognizing $L$ must have at most 5 states.
   (D) The DFA with the fewest states that recognizes $L$ has 5 states.

   Correct answer is (D).