Solution

4.18

$$L(\pi) = log[\prod_{i=1}^{N} \binom{n_i}{y_i} \pi^{y_i} (1-\pi)^{n_i - y_i}]$$

$$\frac{\partial L(\pi)}{\partial \pi} = \sum_{i=1}^{N} (\frac{y_i}{\pi} - \frac{n_i - y_i}{1-\pi})$$

Equating this to 0 and we have

$$(1-\pi)\sum_{i=1}^{N} y_i - \pi \sum_{i=1}^{N} (n_i - y_i) = 0$$

$$\sum_{i=1}^{N} y_i = \pi \sum_{i=1}^{N} n_i$$

so $\hat{\pi} = (\sum_i y_i)/(\sum_i n_i)$.

When all $n_i = 1$, we have $\sum_i n_i = N$ and $\hat{\pi} = (\sum_i y_i)/N = \bar{y}$. Then,

$$\begin{aligned}
X^2 &= \sum_{i=1}^{N} \frac{(y_i - \hat{\pi})^2}{\hat{\pi}} + \sum_{i=1}^{N} \frac{[(1-y_i) - (1-\hat{\pi})]^2}{1-\hat{\pi}} \\
&= \frac{\sum_{i=1}^{N} (y_i - \bar{y})^2}{\bar{y}} + \frac{\sum_{i=1}^{N} (y_i - \bar{y})^2}{1-\bar{y}} \\
&= \frac{\sum_{i=1}^{N} {y_i}^2 - N\bar{y}^2}{\bar{y}} + \frac{\sum_{i=1}^{N} {y_i}^2 - N\bar{y}^2}{1-\bar{y}} \\
&= \frac{N\bar{y} - N\bar{y}^2}{\bar{y}} + \frac{N\bar{y} - N\bar{y}^2}{1-\bar{y}} \\
&= N(1-\bar{y}) + N\bar{y} \\
&= N
\end{aligned}$$