# Quiz 1

1. Let $A = \{1, 2, 3\}$ and $B = \{\emptyset, \{1\}, \{2\}, \{3\}, \{1, 2, 3\}\}$. Which of the following statements is true?

   (A) $A \subseteq B$

   (B) $A \in B$

   (C) $A \cap B \neq \emptyset$

   (D) $A \cup B = B$

   Correct answer is (B).

2. Let $A$, $B$, and $C$ be (finite) sets such that $|A| = |B|$. (For a (finite) set $S$, $|S|$ denotes the number of elements in $S$.) Which of the following statements is necessarily true? (For sets $S_1$ and $S_2$, $S_1 \times S_2$ denotes the "Cartesian product" of sets $S_1$ and $S_2$, and $S_1 \setminus S_2 = \{x \in S_1 \mid x \notin S_2\}$.)

   (A) $|A \times C| = |B \times C|$

   (B) $|A \cup C| = |B \cup C|$

   (C) $|A \cap C| = |B \cap C|$

   (D) $|A \setminus C| = |B \setminus C|$

   Correct answer is (A).

3. Consider the set $X$ defined inductively as follows: (1) $(3, 5) \in X$, (2) if $(x, y) \in X$ then $(x + 2, y) \in X$, and (3) if $(x, y) \in X$ then $(y, x) \in X$. Which of the following pairs is a member of $X$?

   (A) $(222, 402)$

   (B) $(1, 7)$

   (C) $(151, 1171)$

   (D) $(6, 3)$

   Correct answer is (C).

4. Consider the following "proof" of the statement "$1/4 > 1/2$".

$$2 > 1 \tag{1}$$
$$2\log_{10}(\frac{1}{2}) > 1\log_{10}(\frac{1}{2}) \tag{2}$$
$$\log_{10}((\frac{1}{2})^2) > \log_{10}(\frac{1}{2}) \tag{3}$$
$$\frac{1}{4} = \frac{1}{2}^2 > \frac{1}{2} \tag{4}$$

   Which of the below options correctly identifies the mistake in the above proof?

   (A) $2 > 1$ is not correct.

   (B) $2 > 1$ does not imply $2\log_{10}(\frac{1}{2}) > 1\log_{10}(\frac{1}{2})$.

   (C) $2\log_{10}(\frac{1}{2}) > 1\log_{10}(\frac{1}{2})$ does not imply $\log_{10}((\frac{1}{2})^2) > \log_{10}(\frac{1}{2})$.

(D) $\log_{10}((\frac{1}{2})^2) > \log_{10}(\frac{1}{2})$ does not imply $\frac{1}{2}^2 > \frac{1}{2}$.

Correct answer is (B).

5. Consider the following "proof" of the statement "If $a, b$ are real numbers such that $a = b$ then $a = 0$".

$$a = b \tag{5}$$
$$a^2 = ab \tag{6}$$
$$a^2 - b^2 = ab - b^2 \tag{7}$$
$$(a-b)(a+b) = (a-b)b \tag{8}$$
$$a + b = b \tag{9}$$
$$a = 0 \tag{10}$$

Which of the below options correctly identifies the mistake in the above proof?

(A) $a = b$ does not imply $a^2 = ab$.

(B) $a^2 = ab$ does not imply $a^2 - b^2 = ab - b^2$.

(C) $a^2 - b^2 = ab - b^2$ does not imply $(a-b)(a+b) = (a-b)b$.

(D) $(a-b)(a+b) = (a-b)b$ does not imply $a + b = b$.

Correct answer is (D).