Matrix norms

In [1]:

```
#keep
import numpy as np
import numpy.linalg as la
import matplotlib.pyplot as pt
%matplotlib inline
```

Here's a matrix of which we're trying to compute the norm:

In [2]:

```
#keep
n = 2
A = np.random.randn(n, n)
```

Recall:

$$||A|| = \max_{\|x\|=1} \|Ax\|,$$

where the vector norm must be specified, and the value of the matrix norm $\|A\|$ depends on the choice of vector norm.

For instance, for the p-norms, we often write:

$$||A||_2 = \max_{\|x\|=1} \|Ax\|_2,$$

and similarly for different values of p.

We can approximate this by just producing very many random vectors and evaluating the formula:

In [3]:

```
#keep
xs = np.random.randn(n, 1000)
```

First, we need to bring all those vectors to have norm 1. First, compute the norms:

In [4]:

```
#keep
p = 2
norm_xs = np.sum(np.abs(xs)**p, axis=0)**(1/p)
norm_xs.shape
```

Out[4]:

```
(1000,)
```

Then, divide by the norms and assign to `normalized_xs`:

In [5]:

```
normalized_xs = xs/norm_xs
la.norm(normalized_xs[:, 316], p)
```

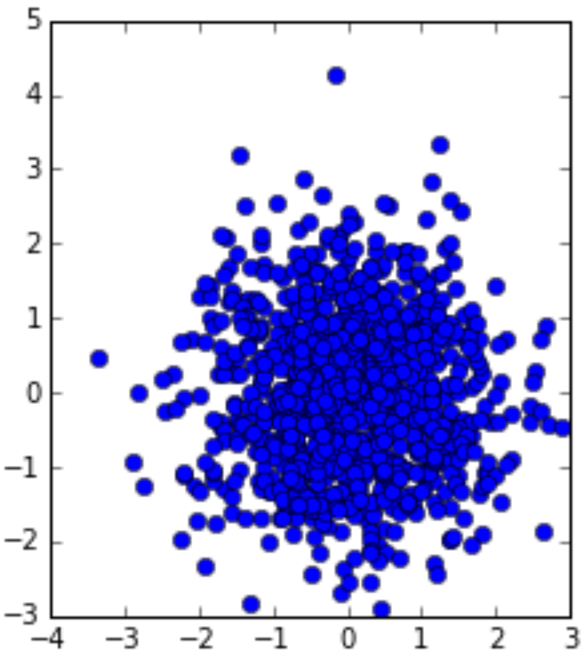
Out[5]:

```
1.4190346549699246
```

Let's take a look:

In [6]:

```
#keep
pt.plot(normalized_xs[0], normalized_xs[1], "o")
pt.gca().set_aspect("equal")
```



Now apply A to these normalized vectors:

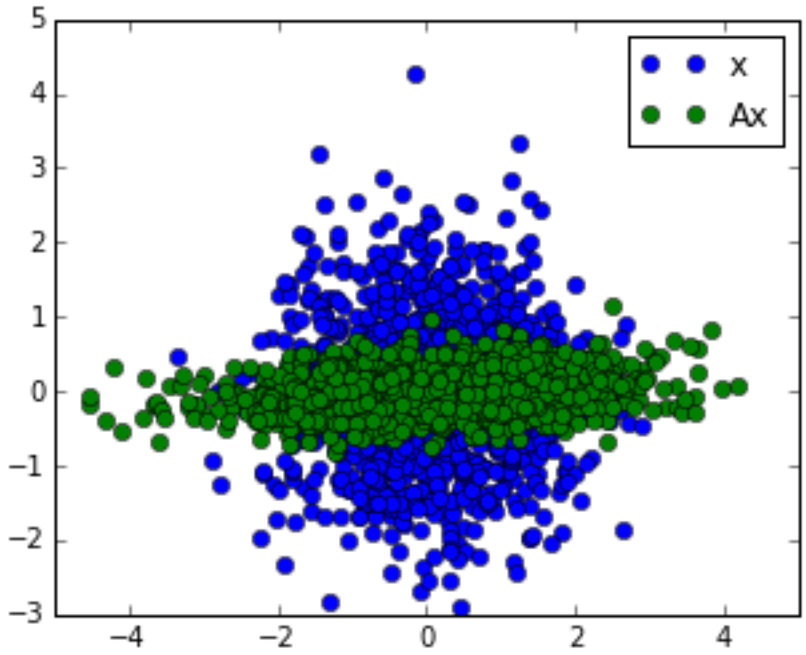
In [7]:

```
A_nxs = A.dot(normalized_xs)
```

Let's take a look again:

In [8]:

```
#keep
pt.plot(normalized_xs[0], normalized_xs[1], "o", label="x")
pt.plot(A_nxs[0], A_nxs[1], "o", label="Ax")
pt.legend()
pt.gca().set_aspect("equal")
```



Next, compute norms of the Ax vectors:

In [9]:

```
norm_Axs = np.sum(np.abs(A_nxs)**p, axis=0)**(1/p)
norm_Axs.shape
```

Out[9]:

```
(1000,)
```

What's the biggest one?

In [10]:

```
np.max(norm_Axs)
```

Out[10]:

```
1.0
```

Compare that with what `numpy` thinks the matrix norm is:

In [11]:

```
la.norm(A, p)
```

Out[11]:

```
1.4987148313922474
```

In [12]:

```
A = np.arange(9).reshape(3,3)
A
```

Out[12]:

```
array([[0, 1, 2],
       [3, 4, 5],
       [6, 7, 8]])
```

In [13]:

```
np.sum(A)
```

Out[13]:

```
36
```

In [ ]:

```

```

In [ ]:

```

```