Data Fitting with Least Squares

In [11]:

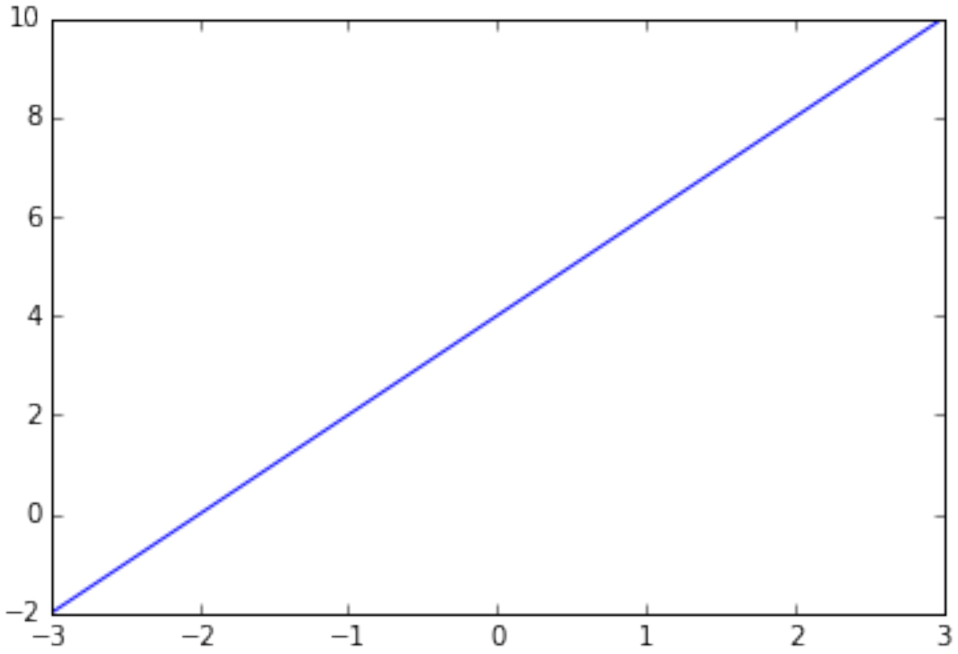
```
#keep
import numpy as np
import numpy.linalg as npla
import scipy.linalg as spla
import matplotlib.pyplot as pt
%matplotlib inline
```

In [12]:

```
#keep
a = 4
b = 2

def f(x):
    return a + b*x

plot_grid = np.linspace(-3, 3, 100)

pt.plot(plot_grid, f(plot_grid))
```

Out[12]:

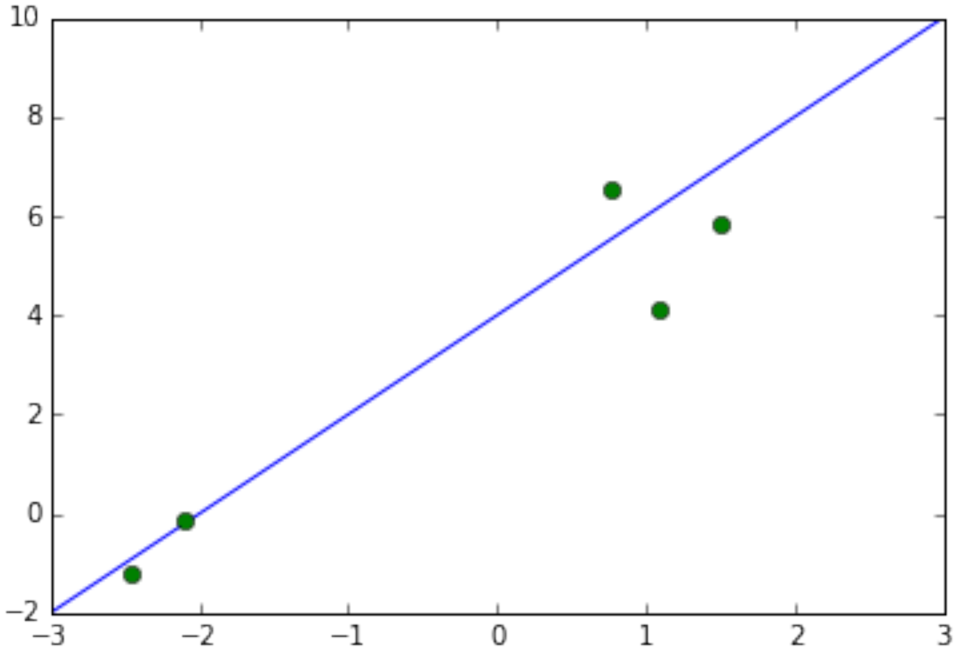
```
[<matplotlib.lines.Line2D at 0x109d52898>]
```



```
In [13]:
```

```
#keep
npts = 5

np.random.seed(22)
points = np.linspace(-2, 2, npts) + np.random.randn(npts)
values = f(points) + 0.3*np.random.randn(npts)*f(points)

pt.plot(plot_grid, f(plot_grid))
pt.plot(points, values, "o")
```

```
Out[13]:

[<matplotlib.lines.Line2D at 0x109c79b00>]
```



What's the system of equations for a and b?

Now build the system matrix A:

In [14]:

```
A = np.array([
    1+0*points,
    points,
    ]).T
A
```

Out[14]:

```
array([[ 1.        , -2.09194992],
       [ 1.        , -2.46335065],
       [ 1.        ,  1.08179168],
       [ 1.        ,  0.76067483],
       [ 1.        ,  1.50887086]])
```

What's the right-hand side vector?

Now solve the least-squares system:

In [15]:

```
Q, R = npla.qr(A, "complete")
```

In [16]:

```
#keep
print(A.shape)
print(Q.shape)
print(R.shape)

m, n = A.shape
```

```
(5, 2)
(5, 5)
(5, 2)
```

Determine x. Use `spla.solve_triangular(A, b, lower=False)`.

In [17]:

```
x = spla.solve_triangular(R[:n], Q.T.dot(values)[:n], lower=False)
```

Recover the computed a, b:

In [18]:

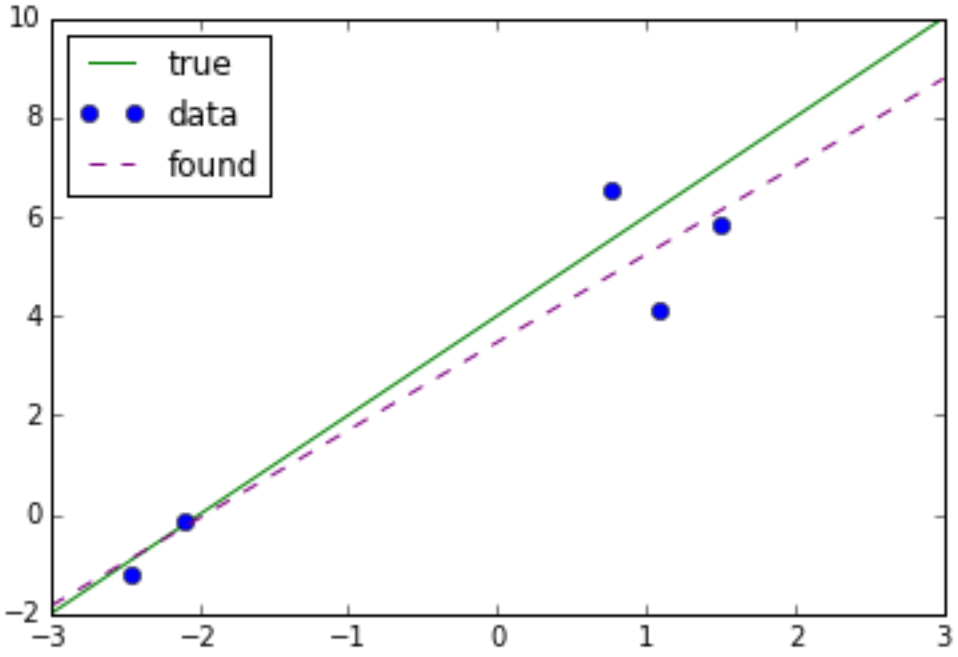
```
#keep
a_c, b_c = x
```

In [19]:

```
#keep
def f_c(x):
    return a_c + b_c * x

pt.plot(plot_grid, f(plot_grid), label="true", color="green")
pt.plot(points, values, "o", label="data", color="blue")
pt.plot(plot_grid, f_c(plot_grid), "--", label="found",color="purple",)

if 0:
    # show residual components
    pt.vlines(points,
              np.minimum(f_c(points), values),
              np.maximum(f_c(points), values),
              color="red", lw=2)

pt.legend(loc="best")
```

Out[19]:

```
<matplotlib.legend.Legend at 0x109f14f28>
```



- If we enable 'show residual components above', what will appear?
- Is it possible for the residual to involve the 'true data'?

- What should happen if we change the number of data points?
- What happens if there are lots of outliers?
- What should happen if we don't add noise?
- What about a bigger model?
- What about different functions in the model?