STAT 420 Spring 2014

Homework 1: Due February 4 by 7:00pm

Exercise 1

(a) At Anytown College, the heights of female students are normally distributed with mean 66 inches and standard deviation 1.5 inches. The heights of male students are also normally distributed with mean 69 inches and standard deviation 2 inches. For Homecoming, a male student and a female student are selected independently at random to be the King and the Queen. What is the probability that the female student selected to be the Queen is taller than the male student selected to be the King?

(b) Suppose that a population of married couples in Anytown have heights in inches, X for the wife, and Y for the husband. Suppose that (X, Y) has a bivariate normal distribution with parameters $\mu_X = 66$, $\sigma_X = 1.5$, $\mu_Y = 69$, $\sigma_Y = 2$, $\rho = 0.44$. What is the probability that the wife is taller than her husband?

Exercise 2

Every month, the government of Neverland spends X million dollars purchasing guns and Y million dollars purchasing butter. Assume X and Y jointly follow a bivariate normal distribution with parameters

$$\mu_X = 370, \qquad \sigma_X = 50, \qquad \mu_Y = 270, \qquad \sigma_Y = 40, \qquad \rho = -0.80$$

(a) Find the probability that more than 250 million was spent on butter during a particular month. That is, find $P(Y > 250)$.

(b) Suppose the government of Neverland spent 450 million dollars on guns during a particular month. Find the probability that more than 250 million was spent on butter during the same month. That is, find $P(Y > 250|X = 450)$.

(c) Find the probability that the government of Neverland spends more on guns than on butter during a given month. That is, find $P(X > Y)$.

(d) Find the probability that the government of Neverland exceeds the 700-million spending limit during a given month. That is, find $P(X + Y > 700)$.

Exercise 3

In Anytown, the price of a gallon of milk (X) and the price of a package of Oreo cookies (Y) vary from day to day according to a bivariate normal distribution with

$$\mu_X = \$3.38, \qquad \sigma_X = \$0.16, \qquad \mu_Y = \$3.17, \qquad \sigma_Y = \$0.10, \qquad \rho = 0.50$$

(a) Find the probability that the price of a package of Oreo cookies is above \$3.33. That is, find $P(Y > 3.33)$.

(b) Find the probability that the price of a package of Oreo cookies is above \$3.33, if the price of a gallon of milk is \$3.54. That is, find $P(Y > 3.33|X = 3.54)$.

(c) Find the probability that on a given day, the price of a package of Oreo cookies is higher than the the price of a gallon of milk. That is, find $P(Y > X)$.

(d) Alex is planning a Milk-and-Oreos party for his imaginary friends. He buys 5 gallons of milk and 7 packages of Oreo cookies. Find the probability that he paid less than \$40. That is, find $P(5X + 7Y < 40)$.

Exercise 4

The data below are a random sample of 17 "3/4-pound" burgers at Burger Queen. (Assume the weights are normally distributed.)

$$12.15, 11.93, 12.04, 11.80, 12.02, 11.92, 12.18, 11.65, 11.73,$$
$$12.15, 11.68, 11.85, 11.97, 11.70, 11.75, 11.87, 12.08$$

(a) Compute the sample mean $\bar{x}$ and the sample standard deviation s.

(b) Make a Normal Q-Q plot for the data. Comment on the normality assumption.

(c) Construct a 90% (two-sided) confidence interval for the overall average weight of beef in a 3/4-pound burger at Burger Queen.

(d) Burger Queen claims that "3/4-pound" burgers contain an average of at least 12 ounces of beef. Find the p-value for the test $H_0 : \mu \geq 12$ versus $H_1 : \mu < 12$.

Exercise 5

Assume that the distributions of X and Y are $\text{N}(\mu_1, \sigma^2)$ and $\text{N}(\mu_2, \sigma^2)$, respectively. Given the $n = 4$ observations of X,

$$105, \quad 130, \quad 135, \quad 150$$

and the $m = 6$ observations of Y,

$$126, \quad 141, \quad 146, \quad 156, \quad 166, \quad 171$$

find the p-value for the test $H_0 : \mu_1 = \mu_2$ versus $H_1 : \mu_1 \neq \mu_2$.

Note: assume the population variances are equal.

Exercise 6

Generate $S = 1000$ random samples of size $n = 16$ from a normal distribution with mean of 10 and standard deviation of 2.

For each of the $s \in \{1, \ldots, 1000\}$ datasets, compute the sample mean $\bar{x}$.

Make a histogram for the 1000 values of $\bar{x}$.

What is the proportion of values of $\bar{x}$ (among the 1000 samples) that are greater than 11?