Today's announcements:

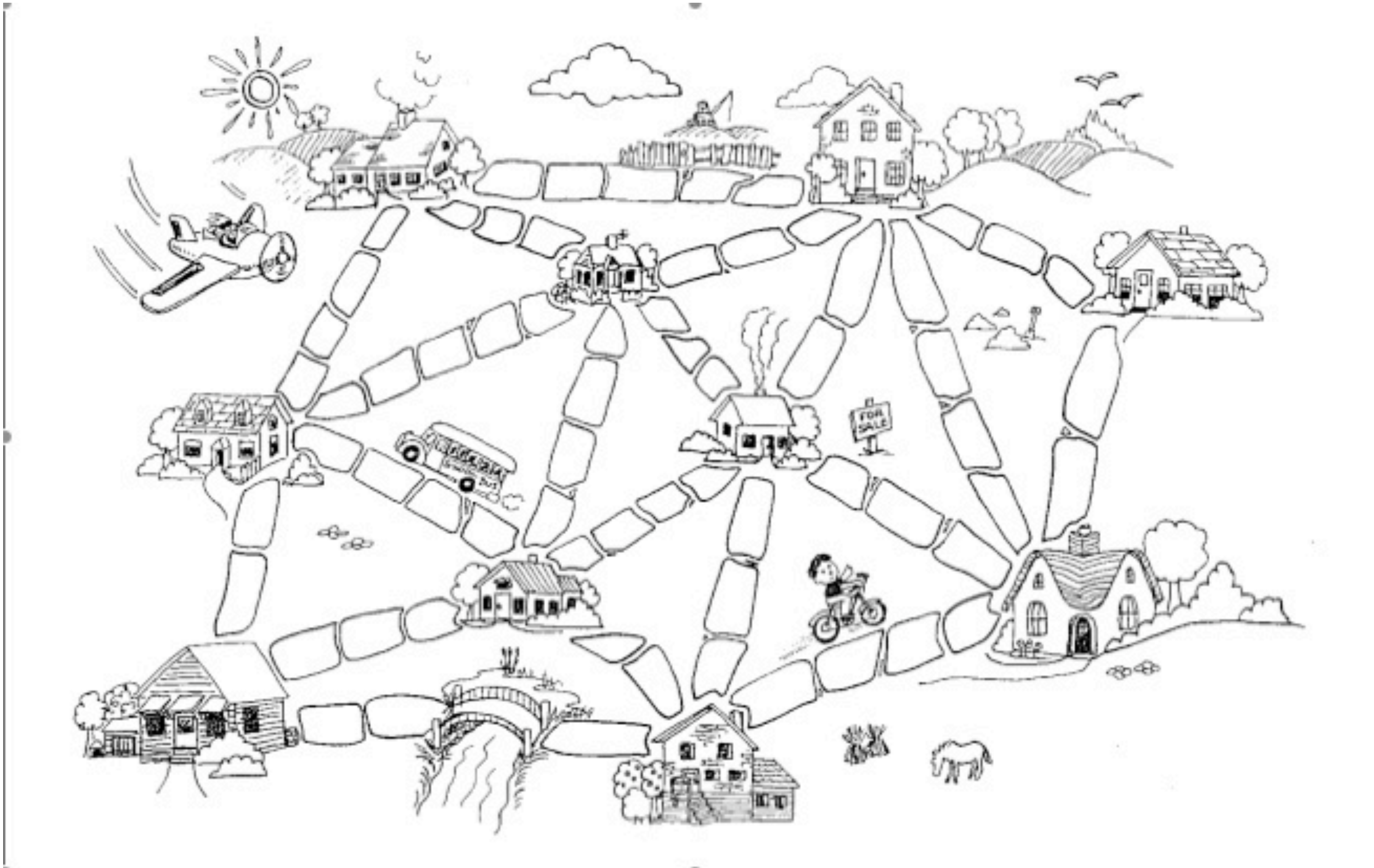
MP7 available, due 4/30, 11:59p.

Adjust the pseudocode below to 1) count components 2) detect cycles.

```
Algorithm DFS(G)
    Input: graph G
    Output: labeling of the
    edges of G as discovery
    edges and back edges

For all u in G.vertices()
  setLabel(u, UNEXPLORED)
For all e in G.edges()
  setLabel(e, UNEXPLORED)
For all v in G.vertices()
  if getLabel(v) = UNEXPLORED
    DFS(G,v)
```

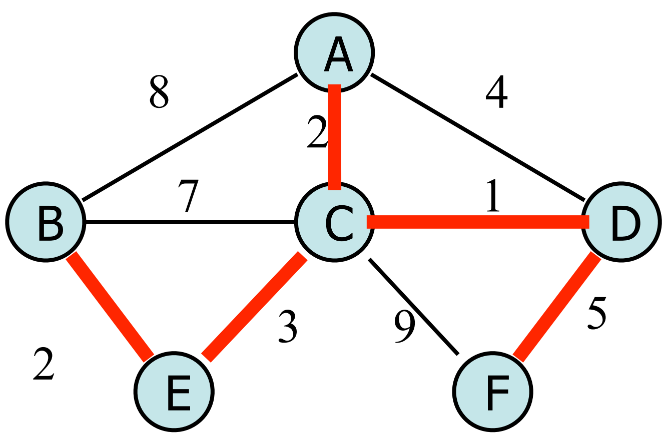
```
Algorithm DFS(G,v)
    Input: graph G and start vertex v
    Output: labeling of the edges of G in
    the connected component of v as
    discovery edges and back edges

setLabel(v, VISITED)
For all w in G.adjacentVertices(v)
    if getLabel(w) = UNEXPLORED
      setLabel((v,w),DISCOVERY)
      DFS(G,w)
    else if getLabel((v,w)) = UNEXPLORED
      setLabel(e,BACK)
```

Pause for an example:

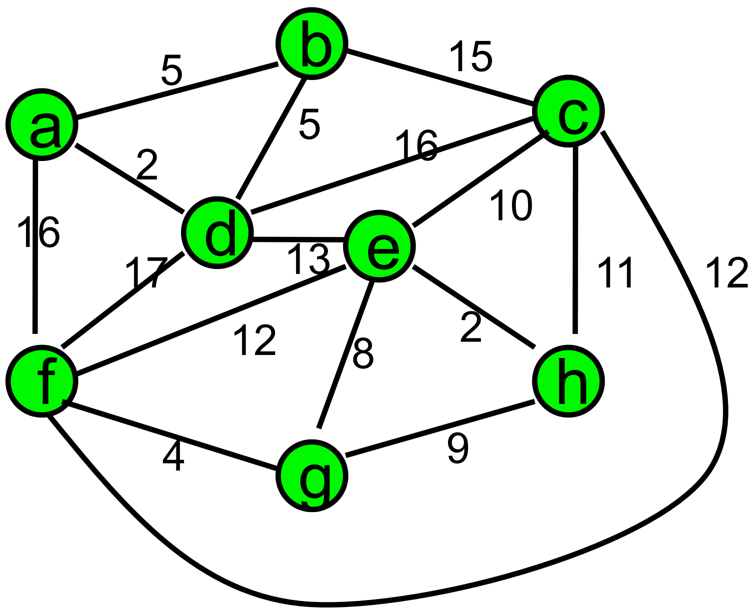


Minimum Spanning Tree Algorithms:

- Input: connected, undirected graph G with unconstrained edge weights
- Output: a graph G' with the following characteristics -
  - G' is a spanning subgraph of G
  - G' is connected and acyclic (a tree)
  - G' has minimal total weight among all such spanning trees -

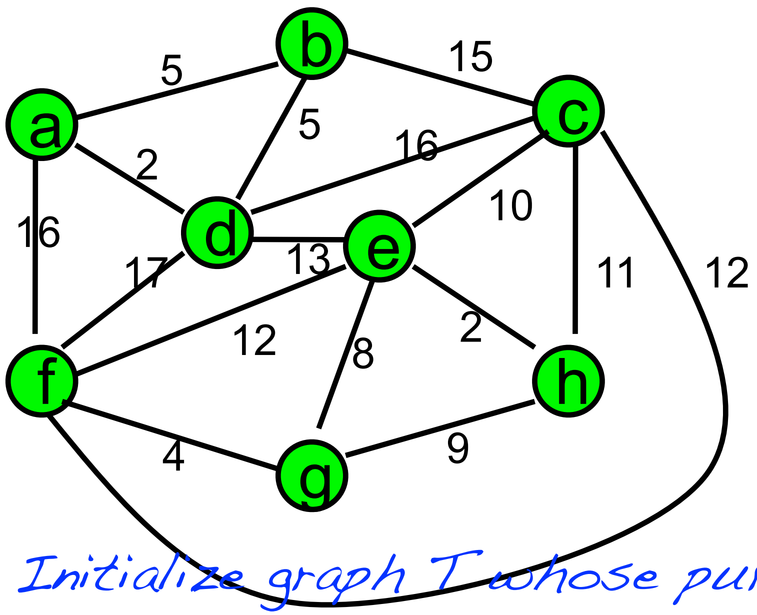


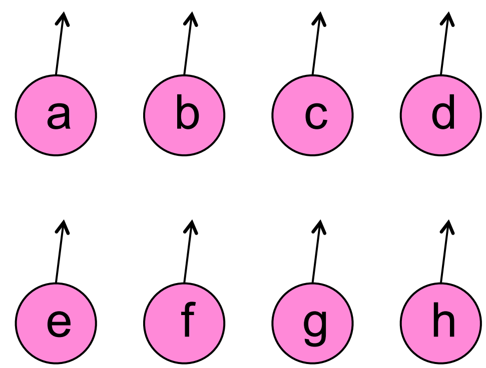
Kruskal's Algorithm



(a,d)
(e,h)
(f,g)
(a,b)
(b,d)
(g,e)
(g,h)
(e,c)
(c,h)
(e,f)
(f,c)
(d,e)
(b,c)
(c,d)
(a,f)
(d,f)

Kruskal's Algorithm (1956)





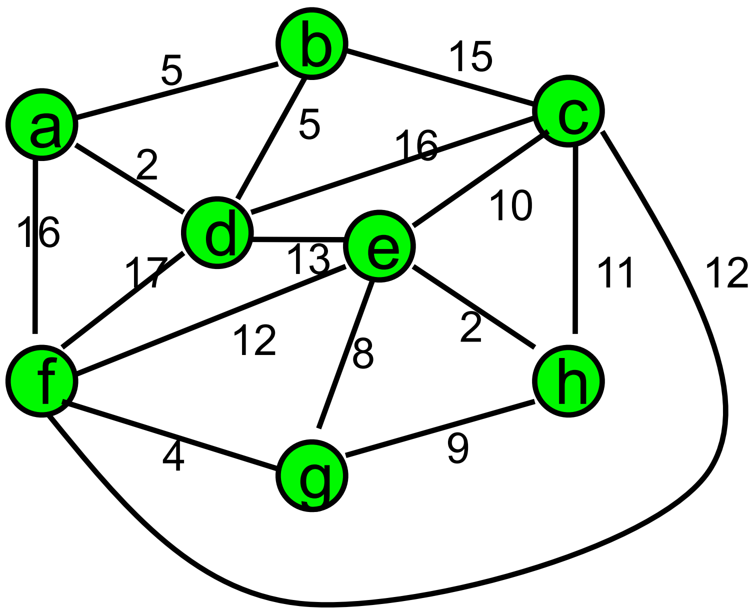
(a,d)
(e,h)
(f,g)
(a,b)
(b,d)
(g,e)
(g,h)
(e,c)
(c,h)
(e,f)
(f,c)
(d,e)
(b,c)
(c,d)
(a,f)
(d,f)

1. Initialize graph T whose purpose is to be our output. Let it consist of all n vertices and no edges.

2. Initialize a disjoint sets structure where each vertex is represented by a set.

3. RemoveMin from PQ. If that edge connects 2 vertices from different sets, add the edge to T and take Union of the vertices' two sets, otherwise do nothing. Repeat

Kruskal's Algorithm - preanalysis



Algorithm ***KruskalMST*(*G*)**

disjointSets forest;
for each vertex ***v*** in ***V*** **do**
  forest.makeSet(v);

priorityQueue Q;
Insert edges into ***Q***, keyed by weights

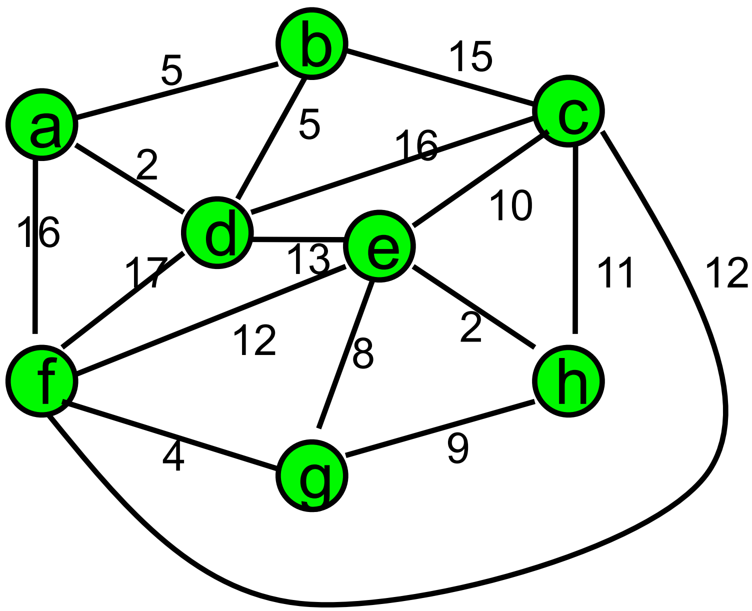
graph T = (V,E) with ***E = ∅;***

while ***T*** has fewer than ***n***-1 edges **do**
  edge ***e*** = ***Q.removeMin()***
  Let ***u, v*** be the endpoints of *e*
  if ***forest.find(v) ≠ forest.find(u)*** **then**
    Add edge ***e*** to ***E***
    forest.smartUnion
      (forest.find(v),forest.find(u))

return ***T***

Priority Queue:	Heap	Sorted Array
To build		
Each removeMin		

Kruskal's Algorithm - analysis



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priorityQueue Q;
Insert edges into ***Q***, keyed by weights

graph T = *(V,E)* with ***E*** = $\emptyset$;

while ***T*** has fewer than ***n***-1 edges **do**
edge ***e*** = ***Q.removeMin()***
Let ***u***, ***v*** be the endpoints of *e*
if ***forest.find(v)*** $\neq$ ***forest.find(u)*** **then**
Add edge ***e*** to ***E***
forest.smartUnion
(forest.find(v),forest.find(u))

return ***T***

Priority Queue:	Total Running time:
Heap	
Sorted Array	

Prim's algorithms (1957) is based on the Partition Property:

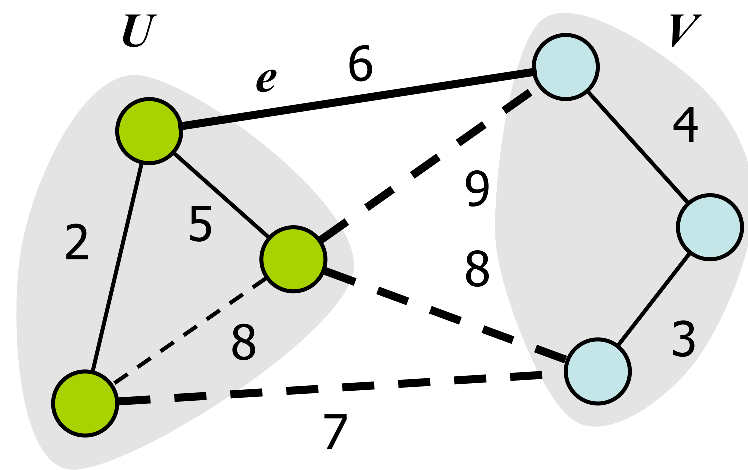
Consider a partition of the vertices of G into subsets U and V.

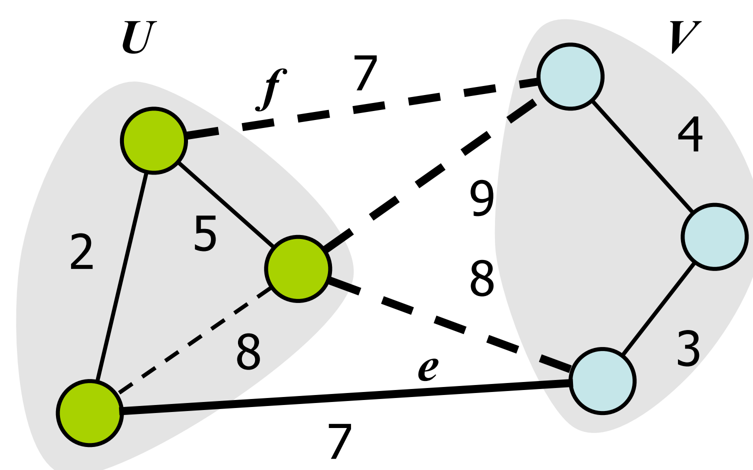
Let e be an edge of minimum weight across the partition.

Then e is part of some minimum spanning tree.

Proof:

See cs473

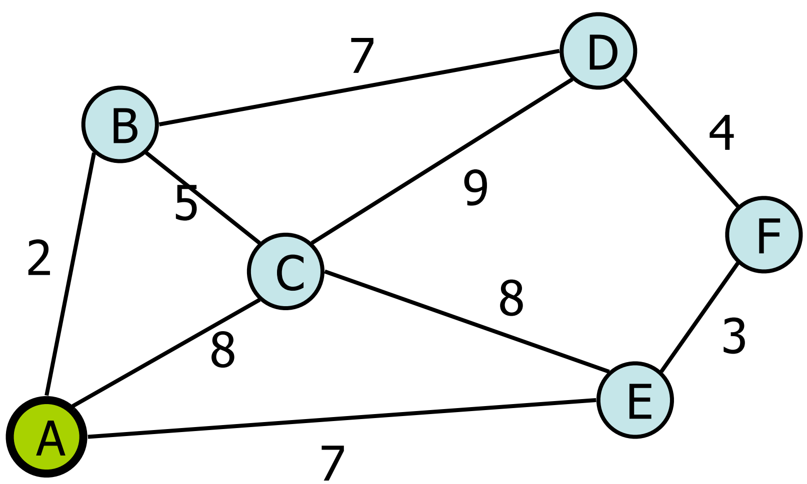




Example of Prim's algorithm -

Initialize structure:

1. For all v, d[v] = “infinity”, p[v] = null
2. Initialize source: d[s] = 0
3. Initialize priority (min) queue
4. Initialize set of labeled vertices to ∅.



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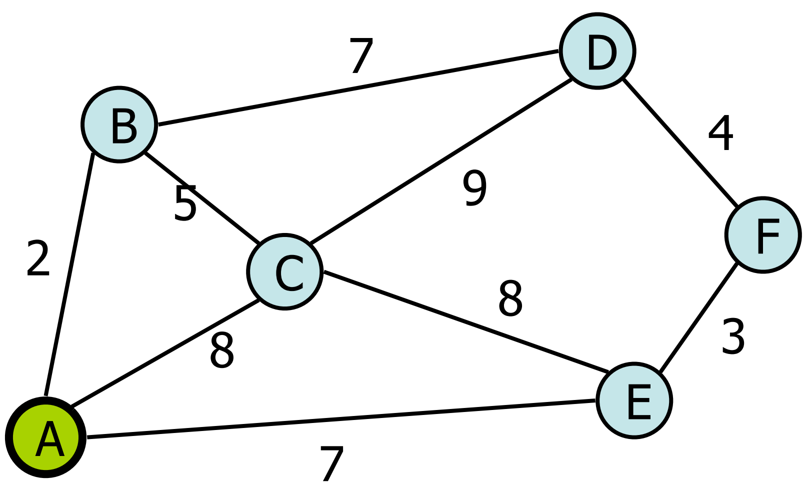

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Repeat these steps n times:

- Find & remove minimum d[] unlabelled vertex: v
- Label vertex v
- For all unlabelled neighbors w of v,

    If cost(v,w) < d[w]

        d[w] = cost(v,w)

        p[w] = v

Prim's Algorithm (undirected graph with unconstrained edge weights):

Initialize structure:

1. For all v, d[v] = "infinity", p[v] = null
2. Initialize source: d[s] = 0
3. Initialize priority (min) queue
4. Initialize set of labeled vertices to ∅.

Repeat these steps n times:

- Remove minimum d[] unlabelled vertex: v
- Label vertex v (set a flag)
- For all unlabelled neighbors w of v,

  If cost(v,w) < d[w]

  d[w] = cost(v,w)

  p[w] = v

	adj mtx	adj list
heap		
Unsorted array		

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Initialize structure:

1. For all v, d[v] = "infinity", p[v] = null
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Unsorted array		

Which is best?

Depends on density of the graph:

Sparse

Dense