

(6) Finding the nullspace of a matrix A

$$\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 0 & 0 & 0 \end{pmatrix} \quad \begin{pmatrix} 1 & 4 & 0 \\ 2 & 5 & 0 \\ 3 & 6 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$MA = U = \begin{pmatrix} \text{---} \\ 0 \\ \text{---} \\ 0 \end{pmatrix}$$

$$MA^T = U \quad U^T = \begin{pmatrix} \text{---} & 0 \end{pmatrix}$$

finding the
nullspace:

hard

easy.

$$MA^T = U \Leftrightarrow A^T = M^{-1}U$$

$$\Leftrightarrow A = (M^{-1}U)^T \\ = U^T M^{-T}$$

$$U^T = \begin{pmatrix} \text{---} & 0 \end{pmatrix}$$

$$N(U^T) = \text{span} \left(\begin{pmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{pmatrix}, \dots, \begin{pmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{pmatrix} \right)$$

$$N(A) = N(U^T M^{-T})$$

$$y \in N(U^T) : U^T y = 0$$

$$Ax = 0 \Leftrightarrow U^T \underbrace{M^{-T} x}_{= y} = 0$$

$$M^{-T} x = y$$

$$\Leftrightarrow x = M^T y$$

$$N(A) = M^T N(U^T)$$