1. Suppose $P(A) = 0.45,$ $P(B) = 0.40,$ $P(C) = 0.50,$
$P(A \cap B) = 0.15,$ $P(A \cap C) = 0.20,$ $P(B \cap C) = 0.25,$
$P(A \cap B \cap C) = 0.05.$

a) Find $P(A \cup B \cup C)$.

b) Find $P(A \cup (B \cap C))$.

c) Find $P(A \cap (B \cup C))$.

2. Find the value of p that would make this a valid probability model.

a) Suppose $S = \{ 0, 2, 4, 6, 8, \ldots \}$ (even non-negative integers) and

$$P(0) = p, \qquad P(k) = \frac{1}{3^k}, \quad k = 2, 4, 6, 8, \ldots .$$

b) Suppose $S = \{ 1, 2, 3, 4, \ldots \}$ (positive integers) and

$$P(1) = p, \qquad P(k) = \frac{(\ln 3)^k}{k!}, \quad k = 2, 3, 4, \ldots .$$

3. Suppose $S = \{ 3, 4, 5, 6, \ldots \}$ and $P(k) = \frac{C}{5^k}, \quad k = 3, 4, 5, 6, \ldots .$

a) Find the value of C that would make this a valid probability model.

b) Find $P(\text{even outcome})$.

4. At *Initech*, 60% of all employees surf the Internet during work hours. 24% of the employees surf the Internet and play MMORPG during work hours. It is also known that 72% of the employees either surf the Internet or play MMORPG (or both) during work hours.

a) What proportion of the employees play MMORPG during work hours?

b) If it is known that an employee surfs the Internet during work hours, what is the probability that he/she also plays MMORPG ?

c) Suppose an employee does not play MMORPG during work hours. What is the probability that he/she surfs the Internet?

5. Consider the following experiment:

A letter is chosen at random from the word **STATISTICS**.

a) List all possible outcomes and their probabilities.

b) What is the probability that the letter selected is a vowel?

6. If $P(A) = 0.7$, $P(B) = 0.5$, and $P(A' \cap B') = 0.1$, find …

a) $P(A \cap B)$; b) $P(A \mid B)$; c) $P(B \mid A)$.

7. 500 people, all of whom drive approximately 10,000 miles per year, were classified according to age and the number of auto accidents each has had during the last three years:

Number of Accidents	*Age (in years)*	
	Under 40	**Over 40**
0	170	80
1	80	70
More than 1	50	50

A person is selected at random from those 500.

a) What is the probability that the person selected is over 40 and has had more than 1accident?

b) What is the probability that the person selected is either over 40 or has had more than 1 accident (or both)?

c) Find the probability that the person selected is over 40 given that he/she has had more than 1 accident.

d) Suppose that the person selected is over 40. What is the probability that he/she has had more than 1 accident?

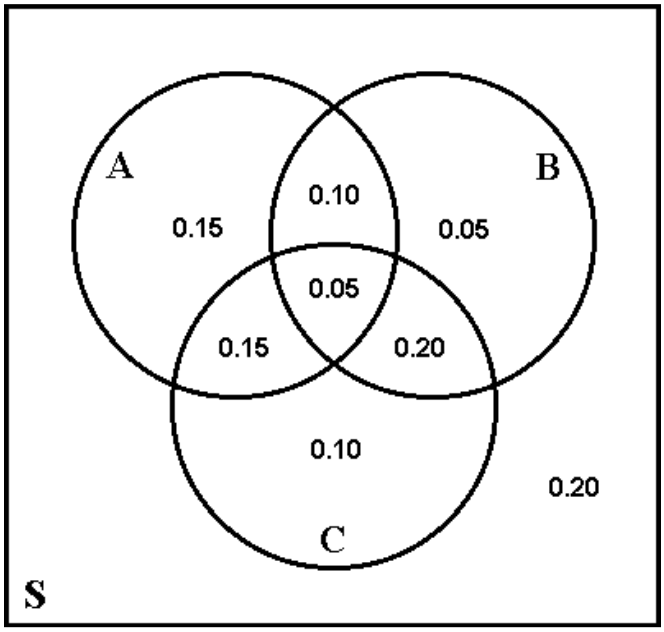
e) Find the probability that the person selected is over 40 given that he/she has had at most 1 accident.

f) Find the probability that the person selected has had more than 1 accident given that he/she has had at least one accident.

1. Suppose $P(A) = 0.45$, $P(B) = 0.40$, $P(C) = 0.50$,

$P(A \cap B) = 0.15$, $P(A \cap C) = 0.20$, $P(B \cap C) = 0.25$,

$P(A \cap B \cap C) = 0.05$.



a) Find $P(A \cup B \cup C)$.

$P(A \cup B \cup C) = \mathbf{0.80}$.

OR

$P(A \cup B \cup C) = P(A) + P(B) + P(C)$

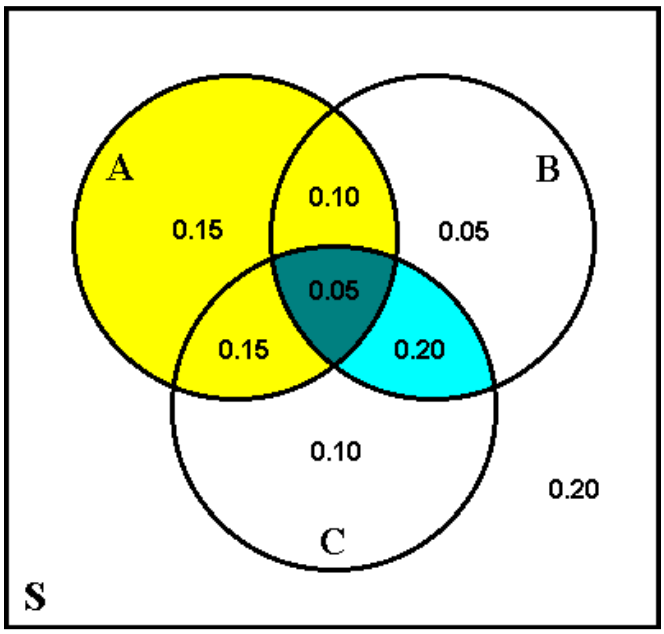
$- P(A \cap B) - P(A \cap C) - P(B \cap C)$

$+ P(A \cap B \cap C)$

$= 0.45 + 0.40 + 0.50 - 0.15 - 0.20 - 0.25 + 0.05 = \mathbf{0.80}$.

b) Find $P(A \cup (B \cap C))$.

$P(A \cup (B \cap C)) = \mathbf{0.65}$.



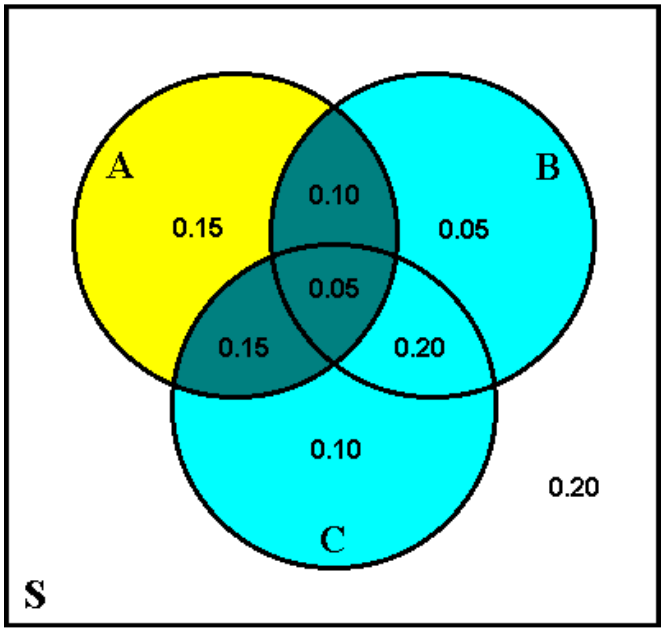
OR

$$P(A \cup (B \cap C)) = P(A) + P(B \cap C) - P(A \cap (B \cap C)) =$$

$$= 0.45 + 0.25 - 0.05 = \mathbf{0.65}.$$

c) Find $P(A \cap (B \cup C))$.

$P(A \cap (B \cup C)) = \mathbf{0.30}$.



2. Find the value of p that would make this a valid probability model.

a) Suppose S = { 0, 2, 4, 6, 8, … } (even non-negative integers) and

$$P(0) = p, \qquad P(k) = \frac{1}{3^k}, \quad k = 2, 4, 6, 8, \ldots .$$

$$1 = p(0) + p(2) + p(4) + p(6) + p(8) + \ldots$$

$$= p + \frac{1}{3^2} + \frac{1}{3^4} + \frac{1}{3^6} + \frac{1}{3^8} + \ldots = p + \frac{\frac{1}{9}}{1 - \frac{1}{9}} = p + \frac{1}{8}. \qquad \Rightarrow \qquad p = \mathbf{\frac{7}{8}}.$$

b) Suppose S = { 1, 2, 3, 4, … } (positive integers) and

$$\mathrm{P}(1) = p, \qquad \mathrm{P}(k) = \frac{(\ln 3)^k}{k!}, \quad k = 2, 3, 4, \ldots .$$

Must have $\sum_{\text{all } x} p(x) = 1.$ $\Rightarrow$ $p(1) + \sum_{k=2}^{\infty} \frac{(\ln 3)^k}{k!} = 1.$

$$\sum_{k=2}^{\infty} \frac{(\ln 3)^k}{k!} = \sum_{k=0}^{\infty} \frac{(\ln 3)^k}{k!} - 1 - \ln 3 = e^{\ln 3} - 1 - \ln 3 = 2 - \ln 3.$$

$p(1) + 2 - \ln 3 = 1.$ $\Rightarrow$ $p(1) = \mathbf{\ln 3 - 1} \approx 0.0986.$

3. Suppose S = { 3, 4, 5, 6, … } and $\mathrm{P}(k) = \frac{C}{5^k}, \quad k = 3, 4, 5, 6, \ldots .$

a) Find the value of C that would make this a valid probability model.

$$\sum_{k=3}^{\infty} \frac{C}{5^k} = \frac{\textit{first term}}{1 - \textit{base}} = \frac{\frac{C}{5^3}}{1 - \frac{1}{5}} = \frac{\frac{C}{125}}{\frac{4}{5}} = \frac{C}{100} = 1.$$

$\Rightarrow$ $C = \mathbf{100}.$

b) Find P(even outcome).

P(even outcome) $= p(4) + p(6) + p(8) + \ldots$

$$= \frac{100}{5^4} + \frac{100}{5^6} + \frac{100}{5^8} + \ldots = \frac{\textit{first term}}{1 - \textit{base}} = \frac{\frac{100}{5^4}}{1 - \frac{1}{5^2}} = \frac{4}{24} = \mathbf{\frac{1}{6}}.$$

4. At *Initech*, 60% of all employees surf the Internet during work hours. 24% of the employees surf the Internet and play MMORPG during work hours. It is also known that 72% of the employees either surf the Internet or play MMORPG (or both) during work hours.

$$P(\text{Internet}) = 0.60, \qquad P(\text{Internet} \cap \text{MMORPG}) = 0.24,$$

$$P(\text{Internet} \cup \text{MMORPG}) = 0.72.$$

a) What proportion of the employees play MMORPG during work hours?

$$P(\text{Internet} \cup \text{MMORPG}) = P(\text{Internet}) + P(\text{MMORPG}) - P(\text{Internet} \cap \text{MMORPG})$$

$$0.72 = 0.60 + P(\text{MMORPG}) - 0.24$$

$$P(\text{MMORPG}) = \mathbf{0.36}.$$

	MMORPG	MMORPG′	
Internet	**0.24**	0.36	**0.60**
Internet′	0.12	0.28	0.40
	0.36	0.64	1.00

b) If it is known that an employee surfs the Internet during work hours, what is the probability that he/she also plays MMORPG?

$$P(\text{MMORPG} \mid \text{Internet}) = \frac{P(\text{MMORPG} \cap \text{Internet})}{P(\text{Internet})} = \frac{0.24}{0.60} = \mathbf{0.40}.$$

c) Suppose an employee does not play MMORPG during work hours. What is the probability that he/she surfs the Internet?

$$P(\text{Internet} \mid \text{MMORPG}') = \frac{P(\text{Internet} \cap \text{MMORPG}')}{P(\text{MMORPG}')} = \frac{0.36}{0.64} = \frac{\mathbf{9}}{\mathbf{16}} = \mathbf{0.5625}.$$

5. Consider the following experiment:

A letter is chosen at random from the word **STATISTICS**.

a) List all possible outcomes and their probabilities.

10 letters: 1 **A**, 1 **C**, 2 **I**, 3 **S**, 3 **T**.

Possible Outcomes:	**A**,	**C**,	**I**,	**S**,	**T**
Probabilities:	0.10	0.10	0.20	0.30	0.30

b) What is the probability that the letter selected is a vowel?

P(vowel) = P(**A**) + P(**I**) = 0.10 + 0.20 = **0.30**.

6. If $P(A) = 0.7$, $P(B) = 0.5$, and $P(A' \cap B') = 0.1$, find …

a) $P(A \cap B)$; b) $P(A \mid B)$; c) $P(B \mid A)$.

a) $P(A \cup B) = 1 - P(A' \cap B') = 1 - 0.1 = 0.9$;

$0.9 = P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= 0.7 + 0.5 - P(A \cap B)$,

$P(A \cap B) =$ **0.3**.

b) $P(A \mid B) = \dfrac{P(A \cap B)}{P(B)} = \dfrac{0.3}{0.5} = \dfrac{3}{5} =$ **0.6**.

c) $P(B \mid A) = \dfrac{P(A \cap B)}{P(A)} = \dfrac{0.3}{0.7} = \dfrac{\mathbf{3}}{\mathbf{7}}$.

7. 500 people, all of whom drive approximately 10,000 miles per year, were classified according to age and the number of auto accidents each has had during the last three years:

Number of Accidents	*Age (in years)*		
	Under 40	**Over 40**	Total
0	170	80	250
1	80	70	150
More than 1	50	50	100
Total	300	200	500

A person is selected at random from those 500.

a) What is the probability that the person selected is over 40 and has had more than 1accident?

$$^{50}/_{500} = \mathbf{0.10}.$$

b) What is the probability that the person selected is either over 40 or has had more than 1 accident (or both)?

$$^{250}/_{500} = \mathbf{0.50}.$$

c) Find the probability that the person selected is over 40 given that he/she has had more than 1 accident.

$$^{50}/_{100} = \mathbf{0.50}.$$

d) Suppose that the person selected is over 40. What is the probability that he/she has had more than 1 accident?

$$^{50}/_{200} = \mathbf{0.25}.$$

e) Find the probability that the person selected is over 40 given that he/she has had at most 1 accident.

$$^{150}/_{400} = \mathbf{0.375}.$$

f) Find the probability that the person selected has had more than 1 accident given that he/she has had at least one accident.

$$^{100}/_{250} = \mathbf{0.40}.$$