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1. Consider the probability mass function (pmf):

$$P(X = x) = \begin{cases} \frac{1}{4}, & x = -1 \\ \frac{1}{2}, & x = 0 \\ \frac{1}{4}, & x = 1 \end{cases}$$

- Find the cumulative distribution function (cdf), $F(x)$, of X over all values of x .
- Sketch a graph of $F(x)$.
- Find the mean and variance of X .
- Find the moment generating function (MGF), $M_X(t)$.

2. Consider the following mixed discrete and continuous CDF,

$$F(x) = \begin{cases} 0, & x < 0 \\ \frac{x+1}{4}, & 0 \leq x < 1 \\ 1, & x \geq 1 \end{cases}$$

- For which values of x is $P(X = x) > 0$?
- Find $P\left(-\frac{1}{2} < x \leq \frac{1}{2}\right)$.
- Find $P\left(x > \frac{1}{2}\right)$.
- Find $E(X)$. (Hint: Consider the discrete and continuous parts separately and add the results.)

3. Let X be the number of rolls of a six-sided dice that is needed before the first 6 is observed.

- What is the pmf for X ?
- Confirm $\sum_{x=1}^{\infty} p(x) = 1$.
- Find the cdf $F(x)$.
- Derive the mgf and use it to find $\mu = E(X)$.

4. Let $Y = X^2$, where X is the random variable from problem 3 above.

- What is the support of Y and its pmf?
- Find $E(Y)$.

5. Let $U \sim U(0,1)$. Find the cdf and pdf for the following transformations of U .

- a. $X = 1 - U$
- b. $Y = \frac{U}{1-U}$
- c. $W = \ln(Y)$

6. Reconsider the random variables X , Y , and W in problem 5.

- a. Find $E(X)$ if possible.
- b. Find $E(Y)$ if possible.
- c. Find $E(W)$ if possible.

7. Suppose the random variable X has a cumulative distribution function (cdf) given by:

$$F(x) = \frac{e^{x/2}}{1 + e^{x/2}}, \quad -\infty < x < \infty.$$

Find formulas for the median (50th percentile) and interquartile range (IQR = 75th percentile – 25th percentile) of the distribution of X .

8. Consider a random variable X with the pdf

$$f_X(x) = 5x^4, \quad 0 < x < 1.$$

- a. Find $E(X)$
- b. Find $E\left(\frac{1}{X}\right)$. Does it equal $\frac{1}{E(X)}$?
- c. Find the pdf for $Y = \frac{1}{X}$ being sure to specify the range for the support of Y .

9. Consider a discrete random variable X with the probability mass function (pmf)

$$p(x) = e^{-5} \frac{5^x}{x!}, \quad x = 0, 1, 2, 3, \dots$$

- a. Show that this is a valid pmf.
- b. Find $E(X)$
- c. Find $E(X(X - 1))$
- d. Find $E((X - 5)^2)$

10. Consider a continuous random variable X with the pdf $f(x) = \frac{1}{5}e^{-x/5}$, $0 < x < \infty$.
- Show that this is a valid pdf.
 - Find $E(X)$.
 - Obtain a formula for $P(X > x)$ for all x .
 - Obtain a formula for the conditional probability $P(X > 10 + x \mid X > 10)$ for all x .