Examples for 1.3

The **conditional probability of A, given B** (the probability of event A, computed on the assumption that event B has happened) is

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)} \qquad (\text{assuming } P(B) \neq 0).$$

Similarly, the **conditional probability of B, given A** is

$$P(B \mid A) = \frac{P(A \cap B)}{P(A)} \qquad (\text{assuming } P(A) \neq 0).$$

3. (continued)

The probability that a randomly selected student at Anytown College owns a bicycle is 0.55, the probability that a student owns a car is 0.30, and the probability that a student owns both is 0.10.

	C	C'	
B	0.10	0.45	0.55
B'	0.20	0.25	0.45
	0.30	0.70	1.00

$P(B) = 0.55$, $P(C) = 0.30$, $P(B \cap C) = 0.10$.

a) What is the probability that a student owns a bicycle, <u>given that he/she owns a car</u>?

$$P(B \mid C) = {}^{0.10}\!/_{0.30} = {}^{1}\!/_{3}.$$

b) <u>Suppose a student does not have a bicycle.</u> What is the probability that he/she has a car?

$$P(C \mid B') = {}^{0.20}\!/_{0.45} = {}^{4}\!/_{9}.$$

5. (continued)

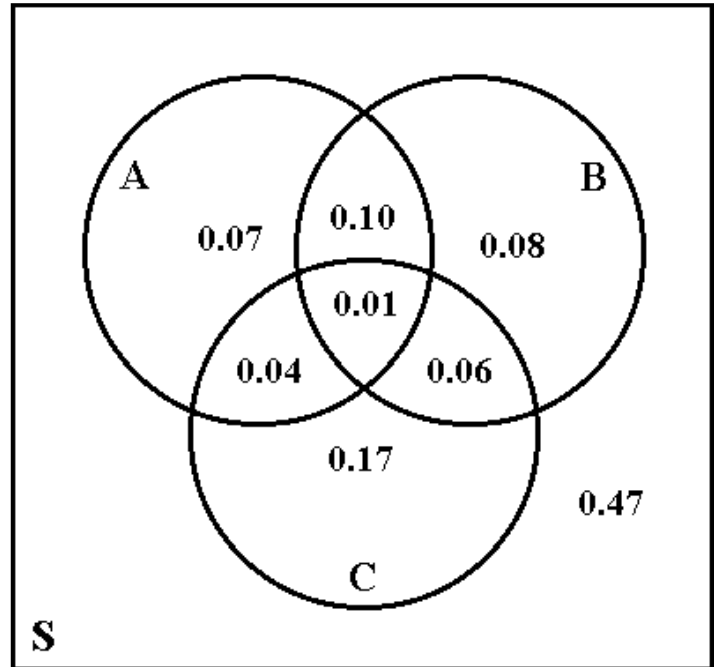Suppose

$P(A) = 0.22,$

$P(B) = 0.25,$

$P(C) = 0.28,$

$P(A \cap B) = 0.11,$

$P(A \cap C) = 0.05,$

$P(B \cap C) = 0.07,$

$P(A \cap B \cap C) = 0.01.$

Find the following:

a) $P(B \mid A)$

$P(B \mid A) = {}^{0.11}/_{0.22} = \mathbf{0.50}.$

b) $P(B \mid C)$

$P(B \mid C) = {}^{0.07}/_{0.28} = \mathbf{0.25}.$

c) $P(B \cap C \mid A)$

$P(B \cap C \mid A) = {}^{0.01}/_{0.22} = {}^{1}/_{22}.$

d) $P(B \cup C \mid A)$

$P(B \cup C \mid A) = {}^{0.15}/_{0.22} = {}^{15}/_{22}.$

e) $P(C \mid A \cup B)$

$P(C \mid A \cup B) = {}^{0.11}/_{0.36} = {}^{11}/_{36}.$

f) $P(C \mid A \cap B)$

$P(C \mid A \cap B) = {}^{0.01}/_{0.11} = {}^{1}/_{11}.$

g) $P(A \cap B \cap C \mid A \cup B \cup C)$

$P(A \cap B \cap C \mid A \cup B \cup C) = {}^{0.01}/_{0.53} = {}^{1}/_{53}.$

Multiplication Law of Probability

If A and B are any two events, then

$$\mathbf{P(A \cap B) = P(A) \cdot P(B \mid A)}$$

$$\mathbf{P(A \cap B) = P(B) \cdot P(A \mid B)}$$

8. It is known that 30% of all the students at Anytown College live off campus. Suppose also that 48% of all the students are females. <u>Of the female students, 25% live off campus.</u>

$$P(\text{ Off }) = 0.30, \qquad P(\text{ F }) = 0.48, \qquad P(\text{ Off } | \text{ F }) = 0.25.$$

a) What is the probability that a randomly selected student is a female <u>and</u> lives off campus?

$$P(\text{ F} \cap \text{Off }) = P(\text{ F }) \times P(\text{ Off } | \text{ F }) = 0.48 \times 0.25 = \textbf{0.12}.$$

	Off	On	
F	0.12	0.36	0.48
M	0.18	0.34	0.52
	0.30	0.70	1.00

b) What is the probability that a randomly selected student either is a female <u>or</u> lives off campus, or both?

$$P(\text{ F} \cup \text{Off }) = P(\text{ F }) + P(\text{ Off }) - P(\text{ F} \cap \text{Off }) = 0.48 + 0.30 - 0.12 = \textbf{0.66}.$$

OR

$$P(\text{ F} \cup \text{Off }) = P(\text{ F} \cap \text{Off }) + P(\text{ F}' \cap \text{Off }) + P(\text{ F} \cap \text{Off}')$$

$$= 0.12 + 0.18 + 0.36 = \textbf{0.66}.$$

OR

$$P(\text{ F} \cup \text{Off }) = 1 - P(\text{ F}' \cap \text{Off}') = 1 - 0.34 = \textbf{0.66}.$$

c) What proportion <u>of the off-campus students</u> are females?

$$P(\text{ F} | \text{Off }) = {}^{0.12}\!/_{0.30} = \textbf{0.40}.$$

d) What proportion <u>of the male students</u> live off campus?

$$P(\text{ Off } | \text{ M }) = {}^{0.18}\!/_{0.52} = {}^{9}\!/_{26} \approx \textbf{0.346154}.$$

9. Suppose that Joe's Discount Store has received a shipment of 25 television sets, 5 of which are defective. On the following day, 2 television sets are sold.

a) Find the probability that both of the television sets are defective.

$$P(\text{ both defective }) = P(\text{ 1st D} \cap \text{2nd D}) = P(\text{ 1st D}) \times P(\text{ 2nd D} \mid \text{1st D})$$

$$= \tfrac{5}{25} \times \tfrac{4}{24} = \mathbf{\tfrac{1}{30}}.$$

b) Find the probability that at least one of the two television sets sold is defective.

✓	D	D	$\tfrac{5}{25} \times \tfrac{4}{24} = \tfrac{1}{30}.$
✓	D	D′	$\tfrac{5}{25} \times \tfrac{20}{24} = \tfrac{5}{30}.$
✓	D′	D	$\tfrac{20}{25} \times \tfrac{5}{24} = \tfrac{5}{30}.$
✗	D′	D′	

$$P(\text{ at least one D}) = \tfrac{1}{30} + \tfrac{5}{30} + \tfrac{5}{30} = \mathbf{\tfrac{11}{30}}.$$

OR

$$P(\text{ at least one D}) = 1 - P(\text{ D}'\text{D}') = 1 - \tfrac{20}{25} \times \tfrac{19}{24} = 1 - \tfrac{19}{30} = \mathbf{\tfrac{11}{30}}.$$

10. Cards are drawn one-by-one **without** replacement from a standard 52-card deck. What is the probability that …

a) … both the first and the second card drawn are ♥'s?

$$P(\text{ 1st ♥} \cap \text{2nd ♥}) = P(\text{ 1st ♥}) \times P(\text{ 2nd ♥} \mid \text{1st ♥}) = \tfrac{13}{52} \times \tfrac{12}{51} = \mathbf{\tfrac{1}{17}}.$$

b)   … the first two cards drawn are a ♥ and a ♣ (or a ♣ and a ♥)?

$$P(\text{1st } ♥ \cap \text{2nd } ♣) + P(\text{1st } ♥ \cap \text{2nd } ♣) = {}^{13}\!/_{52} \times {}^{13}\!/_{51} + {}^{13}\!/_{52} \times {}^{13}\!/_{51}.$$

c)   … there are at least two ♥'s among the first three cards drawn?

♥   ♥   ~~♥~~ $${}^{13}\!/_{52} \times {}^{12}\!/_{51} \times {}^{39}\!/_{50}$$

or   +

♥   ~~♥~~   ♥ $${}^{13}\!/_{52} \times {}^{39}\!/_{51} \times {}^{12}\!/_{50}$$

or   +

~~♥~~   ♥   ♥ $${}^{39}\!/_{52} \times {}^{13}\!/_{51} \times {}^{12}\!/_{50}$$

or   +

♥   ♥   ♥ $${}^{13}\!/_{52} \times {}^{12}\!/_{51} \times {}^{11}\!/_{50}$$

$${}^{19968}\!/_{132600} \approx \mathbf{0.150588}.$$