

CS 373: Intro to Theory of Computation
Spring 2010 MockMidterm II, March, 2010

INSTRUCTIONS (read carefully)

- Print your name and netID here and netID at the top of each other page.

NAME:

NETID:

- It is wise to skim all problems and point values first, to best plan your time. If you get stuck on a problem, move on and come back to it later.
- Points may be deducted for solutions which are correct but excessively complicated, hard to understand, hard to read, or poorly explained.
- This is a closed book exam. No notes of any kind are allowed. Do all work in the space provided, using the backs of sheets if necessary. See the proctor if you need more paper.
- Please bring apparent bugs or unclear questions to the attention of the proctors.

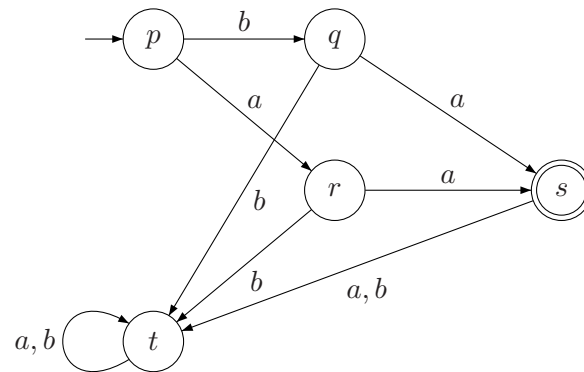
Problem 1: True/False (10 points)

Completely write out “True” if the statement is necessarily true. Otherwise, completely write “False”. Other answers (e.g. “T”) will receive credit only if your intent is unambiguous. For example, “ $x + y > x$ ” has answer “False” assuming that y could be 0 or negative. But “If x and y are natural numbers, then $x + y \geq x$ ” has answer “True”. You do not need to explain or prove your answers.

1. Let M be a DFA with n states such that $L(M)$ is infinite. Then $L(M)$ contains a string of length at most $2n - 1$.
2. Let $L_w = \{\langle M \rangle \mid M \text{ is a TM and } M \text{ accepts } w\}$, where w is some fixed string. Then there is an enumerator for L_w .
3. The set of undecidable languages is countable.
4. There is a bijection between the set of Turing-recognizable languages and the set of decidable languages.
5. If L is a non-regular language over Σ^* , and h is a homomorphism, then $h(L)$ must also be non-regular. Is this statement correct?
6. Suppose all the words in language L are no more than 1024 characters long. Then L must be regular. Is this statement correct?
7. A minimum size NFA for a regular language L , always has strictly fewer states than the minimum size DFA for the language L . True or False?
8. Let L_i be a regular language, for $i = 1, \dots, \infty$. Is the language $\bigcup_{i=1}^{\infty} L_i$ always regular? True or false?

Problem 2: Minimization (20 points)

Minimize this DFA:



Problem 3: TM design (10 points)

Give the state diagram of a TM M that does the following on input $\#w$ where $w \in \{0,1\}^*$. Let $n = |w|$. If n is even, then M converts $\#w$ to $\#0^n$. If n is odd, then M converts $\#w$ to $\#1^n$. Assume that ϵ is an even length string.

The TM should enter the accept state after the conversion. We don't care where you leave the head at the end of the conversion. The TM should enter the reject state if the input string is not in the right format. However, your state diagram does not need to explicitly show the reject state or the transitions into it.

Problem 4: UTM (10 points)

Show L is TM-recognizable:

$$L = \{\langle M \rangle \mid M \text{ accepts some string } w \text{ in at most } |w| \text{ steps.}\}$$

Problem 5: Decidability (10 points)

Show that

$EQINT_{DFA} = \{\langle A, B, C \rangle \mid A, B, C \text{ are DFAs over the same alphabet } \Sigma, \text{ and } L(A) = L(B) \cap L(C)\}$
is decidable.

This question does not require detail at the level of tuple notation. Rather, keep your proof short by exploiting theorems and constructions we've seen in class.

Problem 6: Reduction (20 points)

Prove that $L = \{\langle M, w \rangle \mid M \text{ accepts } w \text{ in more than 3 steps}\}$ is undecidable.
(You may use the fact that $A_{TM} = \{\langle M, w \rangle \mid M \text{ accepts } w\}$ is undecidable.)

Problem 7: Non-regularity (20 points)

(a) Prove that the following language L is not regular ($\Sigma = \{0, 1\}$):

$$L = \{0^n 10^m \mid n < m\}$$

Your proof should use MNT (or the pumping lemma).

(b) Assume $A = \{0^n 10^n \mid n \geq 0\}$ is non-regular. Use this together with closure properties to give another proof for non-regularity of L .