

# Homework #1

(due Friday, January 30, by 3:00 p.m.)

Please include your name (with your last name underlined>), your NetID, and your discussion section number at the top of the first page.



**1 – 4.** Do NOT use a computer. You may only use  $+$ ,  $-$ ,  $\times$ ,  $\div$ , and  $\sqrt{\quad}$  on a calculator. Show all work.

**1.** Evaluate the following integrals:

a)  $\int_0^{\infty} x^2 e^{-3x} dx;$       b)  $\int_0^{\sqrt{\pi}} x \sin(x^2) dx;$       c)  $\int_1^{\infty} \frac{x}{(1+x)^4} dx.$

**2.** Let  $c > 0$ . Consider  $f(x, y) = x + cy$  and  $A = \{(x, y): 0 < y < x < 1\}$ .

- Sketch  $A$ . That is, sketch  $\{(x, y): 0 < y < x < 1\}$ .
- Set up the double integral(s) of  $f(x, y)$  over  $A$  with the outside integral w.r.t.  $x$  and the inside integral w.r.t.  $y$ .
- Set up the double integral(s) of  $f(x, y)$  over  $A$  with the outside integral w.r.t.  $y$  and the inside integral w.r.t.  $x$ .
- Find the value of  $c$  such that  $\iint_A f(x, y) dx dy = 1$ .

**3.** Consider  $f(x, y) = x y^3$  and  $A = \{ (x, y): 0 < y < 1, y < x < 2 \}$ .

- a) Sketch  $A$ . That is, sketch  $\{ (x, y): 0 < y < 1, y < x < 2 \}$ .
- b) Set up the double integral(s) of  $f(x, y)$  over  $A$  with the outside integral w.r.t.  $x$  and the inside integral w.r.t.  $y$ .
- c) Set up the double integral(s) of  $f(x, y)$  over  $A$  with the outside integral w.r.t.  $y$  and the inside integral w.r.t.  $x$ .
- d) Use either (b) or (c) to evaluate  $\iint_A f(x, y) dx dy$ .

**4.** Evaluate the following sums (do NOT use a calculator):

- a)  $\sum_{k=3}^{\infty} \frac{2}{5^k}$ ;
- b)  $\sum_{k=1}^{\infty} 0.6^{2k+1}$ ;
- c)  $\sum_{k=5}^{\infty} \frac{2^k}{k!}$ .

**5.** Suppose  $P(A) = 0.70$ ,  $P(B) = 0.50$ ,  $P(A \cup B') = 0.80$ .

- a) Find  $P(A \cap B)$ .
- b) Find  $P(A \cup B)$ .
- c) Find  $P(A|B)$ .

**6.** Suppose  $P(A) = 0.40$ ,  $P(B) = 0.34$ ,  $P(C) = 0.55$ ,  
 $P(A \cap B) = 0.19$ ,  $P(A \cap C) = 0.25$ ,  $P(B \cap C) = 0.17$ ,  
 $P(A \cap B \cap C) = 0.07$ .

- a) Find  $P(A \cup B \cup C)$ .
- b) Find  $P((A \cap B) \cup C)$ .
- c) Find  $P(A \cap (B \cup C))$ .

**7.** Let  $a > 1$ . Suppose  $S = \{ 2, 3, 4, 5, 6, \dots \}$  and  $P(k) = \frac{c}{a^k}$ ,  $k = 2, 3, 4, 5, 6, \dots$ .

- a) Find the value of  $c$  that makes this a valid probability distribution.
- b) Find  $P(\text{outcome is odd})$ .
- c) Find  $P(\text{outcome is less than or equal to } 5)$ .

8. Suppose  $S = \{ 2, 3, 4, 5, 6, \dots \}$  and  $P(k) = c \frac{2^k}{k!}$ ,  $k = 2, 3, 4, 5, 6, \dots$ .

- a) Find the value of  $c$  that makes this a valid probability distribution.
- b) Find  $P(\text{outcome is greater than or equal to } 5)$ .

9. A family that owns two automobiles is selected at random. Suppose that the probability that the older car is American is 0.80, the probability that the newer car is American is 0.70, and the probability that both the older and the newer cars are American is 0.60.

- a) Find the probability that at least one car is American (i.e. that either the older car or the newer car, or both cars are American).
- b) What is the probability that the newer car is American, given that the older car is American?
- c) What is the probability that the newer car is American, given that the older car is not American?

10. At Anytownville College, 25% of all students are enrolled in an introductory level statistics class, and 20% of all students are seniors. It is also known that 15% of the seniors are enrolled in an introductory level statistics class.

- a) *Initech* is offering an internship to a Anytownville College student who is either a senior or is enrolled in an introductory level statistics class, or both. What proportion of Anytownville College students are eligible for this internship?
- b) What proportion of students enrolled in an introductory level statistics class at Anytownville College are seniors?
- c) Given that a student selected at random is not a senior, what is the probability that he/she is enrolled in an introductory level statistics class?