

# Solution

## 5.10

a.

After deleting the single case in the last row, we have the statistics  $z^2 = 1.078$  and  $p\text{-value} = 0.299$ . The test suggests there is no evidence of a linear trend. However, with this single observation present, the statistics  $z^2 = 6.6$  and  $p\text{-value} = 0.010$ , so the test suggests strong evidence of a positive slope. Without the single observation, there won't be statistical significance of the linear trend between malformation and alcohol consumption.

b.

Using alcohol consumption scores (1, 2, 3, 4, 5), we have the statistics  $z^2 = 1.828$  and  $p\text{-value} = 0.176$ , so the test suggests there is no evidence of a linear trend. However, using scores (0.0, 0.5, 1.5, 4.0, 7.0), the test would suggest strong evidence of a positive slope.

## 5.35

a.

Treating the data as  $n$  Bernoulli observations, we have the kernel of the likelihood function to be  $\prod_{i=1}^N \prod_{j=1}^{n_i} \pi_i^{y_{ij}} (1 - \pi_i)^{(1-y_{ij})} = \prod_{i=1}^N \pi_i^{(\sum_{j=1}^{n_i} y_{ij})} (1 - \pi_i)^{(n_i - \sum_{j=1}^{n_i} y_{ij})}$ .

Treating the data as  $N$  binomial observations, we have the kernel of the likelihood function to be  $\prod_{i=1}^N \pi_i^{(\sum_{j=1}^{n_i} y_{ij})} (1 - \pi_i)^{(n_i - \sum_{j=1}^{n_i} y_{ij})}$ .

They are the same.

b.

For the saturated model of Bernoulli data form, the likelihood function is  $\prod_{i=1}^N \prod_{j=1}^{n_i} y_{ij}^{y_{ij}} (1 - y_{ij})^{(1-y_{ij})}$  with  $n$  parameters.

For the saturated model of binomial data form, the likelihood function is  $\prod_{i=1}^N \binom{n_i}{\sum_{j=1}^{n_i} y_{ij}} [\sum_{j=1}^{n_i} y_{ij} / n_i]^{\sum_{j=1}^{n_i} y_{ij}} [1 - (\sum_{j=1}^{n_i} y_{ij}) / n_i]^{(n_i - \sum_{j=1}^{n_i} y_{ij})}$  with  $N$  parameters.

So they are different.

c.

The difference between deviances is  $-2(L_0 - L_S) - [-2(L_1 - L_S)] = -2(L_0 - L_1)$  which is not related to the likelihood function of the saturated model and only related to the kernel of the likelihood function (constant terms cancelled out after the subtraction between loglikelihoods). As we have shown in part (a) that using either form of data entry, we have the same kernel of the likelihood function, so the difference between deviances does not depend on the form of data entry.