Preparation problems for the discussion sections on August 25th and 27th

1. *For the following systems determine:*

(1) *the augmented matrix,*
(2) *an echelon form of the matrix,*
(3) *the reduced echelon form of the matrix,*
(4) *whether the system is consistent,*
(5) *a parametric description of the set of solutions,*
(6) *how many solutions the system has, and*
(7) *the geometric interpretation of the set of solutions.*

System A:

$$\begin{aligned} x_2 &= 3 \\ x_1 + 2x_2 &= 4 \end{aligned}$$

System B:

$$\begin{aligned} x_1 + x_2 &= 3 \\ 2x_1 + 2x_2 &= 6 \end{aligned}$$

System C:

$$\begin{aligned} x_1 + x_2 &= 3 \\ 2x_1 + 2x_2 &= 7 \end{aligned}$$

2. *Find a parametric description of the set of solutions of*

$$\begin{aligned} x_1 + 3x_2 - 5x_3 &= 4 \\ x_1 + 4x_2 - 8x_3 &= 7 \\ -3x_1 - 7x_2 + 9x_3 &= -6 \end{aligned}$$

3. *For which values of h_1 and h_2 is the following system consistent?*

$$\begin{aligned} x_1 &= h_1 \\ x_2 &= 5 \\ x_1 + 2x_2 &= h_2 \end{aligned}$$

4. *According to the* New York Times, *Sony's 2014 film* The Interview *grossed* \$15 *million in its first four days through* \$15 *purchases and* \$6 *rentals. Sony did not reveal what the numbers of each of these were, but they did reveal that there were roughly* 2 *million transactions in all. While Sony may not have told the number of each type of transactions, what can Math 415 tell the reporter?*

Tutoring Room : Time and Place TBA

Midterm Dates: September 29th, October 22nd, November 19th (All Midterms 7-8:15 PM, see `learn.illinois.edu` **for locations)**

5. *Let $A = [a_{ij}]_{3\times 4}$, and let $B = [b_{ij}]_{3\times 4}$ be an echelon form of A.*

(1) *Is it true that, if $a_{11} = 0$, then $b_{11} = 0$?*
(2) *Is it true that, if A has a column of zeros, then B also has a column of zeros?*
(3) *Suppose B has a row of zeros. What can you say about rows of A? (Explain.)*
(4) *Suppose we form a new matrix using some columns of A, let's say the first and the third column. What is an echelon form corresponding to this new matrix?*

6. *Show that the interchange of two rows of a matrix can be accomplished by a finite sequence of elementary row operations of the other two types.*

The following may be useful in the above problems:

Definition. A system of linear equations is **consistent** if there **exists** a solution.

Definition. A matrix is in **(row) echelon form** if

(1) All nonzero rows are above all zero rows.
(2) Each *leading entry* (i.e., leftmost nonzero entry) of a row is in a column to the right of the leading entry of the row above it.
(3) All entries in a column below a leading entry are zero.

A matrix is in **reduced (row) echelon form** if in addition to (1), (2) and (3) above it also satisfies:

(4) The leading entry in each nonzero row is 1.
(5) Each leading 1 is the only nonzero entry in its column.

Definition. A **pivot position** is the position of a leading entry in an echelon form of the matrix. A **pivot** is a nonzero number that either is used in a pivot position to create zeros or is changed into a leading 1, which in turn is used to create zeros. A **pivot column** is a column that contains a pivot position.

Definition. An **elementary row operation** is one of the following:

- **Replacement:** Add a multiple of one row to another row (denoted $R_i \to R_i + cR_j$),
- **Interchange:** Interchange two rows (denoted $R_i \leftrightarrow R_j$), or
- **Scaling:** Multiply all entries in a row by a nonzero constant (denoted $R_i \to cR_i$).