Quiz 9

1. Recall that regular languages are closed under homomorphisms, i.e., if L is regular and h is a homomorphism then $h(L)$ is regular. Based on this observation, which of the following statements must be necessarily true?

   (A) If $h(L)$ is regular then L is regular.

   (B) If $h(L)$ is not regular then L is not regular.

   (C) If L is not regular then $h(L)$ is not regular.

   (D) All of the above.

   Correct answer is (B).

2. Let $h : \{0,1\}^* \to \{a\}^*$ be a homomorphism defined as follows: $h(0) = a$ and $h(1) = \epsilon$. Let $L_{0n1n} = \{0^n 1^n \mid n \geq 0\}$. Taking $A \subset B$ to mean A is a proper subset of B, which of the following is true?

   (A) $h^{-1}(h(L_{0n1n})) = L_{0n1n}$

   (B) $h^{-1}(h(L_{0n1n})) \subset L_{0n1n}$

   (C) $L_{0n1n} \subset h^{-1}(h(L_{0n1n}))$

   (D) $h^{-1}(h(L_{0n1n})) \cap L_{0n1n} = \emptyset$

   Correct answer is (C).

3. Let $h : \{a,b\}^* \to \{0,1\}^*$ be a homomorphism defined as follows: $h(a) = 01$ and $h(b) = 10$. Let $L = \mathbf{L}((00 \cup 1)^*)$. Taking $A \subset B$ to mean A is a proper subset of B, which of the following is true?

   (A) $h(h^{-1}(L)) = L$

   (B) $h(h^{-1}(L)) \subset L$

   (C) $L \subset h(h^{-1}(L))$

   (D) $h(h^{-1}(L)) \cap L = \emptyset$

   Correct answer is (B).

4. In the notes for lecture 9 posted on the website, it is shown that the language $L_{0n1n} = \{0^n 1^n \mid n \geq 0\}$ is not regular. Consider the homomorphism $h : \{0,1\}^* \to \{a\}^*$ defined as: $h(0) = a$, and $h(1) = \epsilon$. What can we conclude on the basis of the languages L_{0n1n} and $h(L_{0n1n})$?

   (A) Non-regular languages are not closed under homomorphisms.

   (B) Non-regular languages are closed under homomorphisms.

   (C) Non-regular languages are closed under inverse homomorphisms.

   (D) Non-regular languages are not closed under inverse homomorphisms.

   Correct answer is (A).

5. In the notes for lecture 9 posted on the website, it is shown that the language $L_{0n1n} = \{0^n 1^n \mid n \geq 0\}$ is not regular. Consider the homomorphism $h : \{a\}^* \to \{0,1\}^*$ defined as: $h(a) = 01$. What can we conclude on the basis of the languages L_{0n1n} and $h^{-1}(L_{0n1n})$?

(A) Non-regular languages are not closed under homomorphisms.

(B) Non-regular languages are closed under homomorphisms.

(C) Non-regular languages are closed under inverse homomorphisms.

(D) Non-regular languages are not closed under inverse homomorphisms.

Correct answer is (D).