Portfolio
MATH 476

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Exercise 1

Forward Contract Payoff

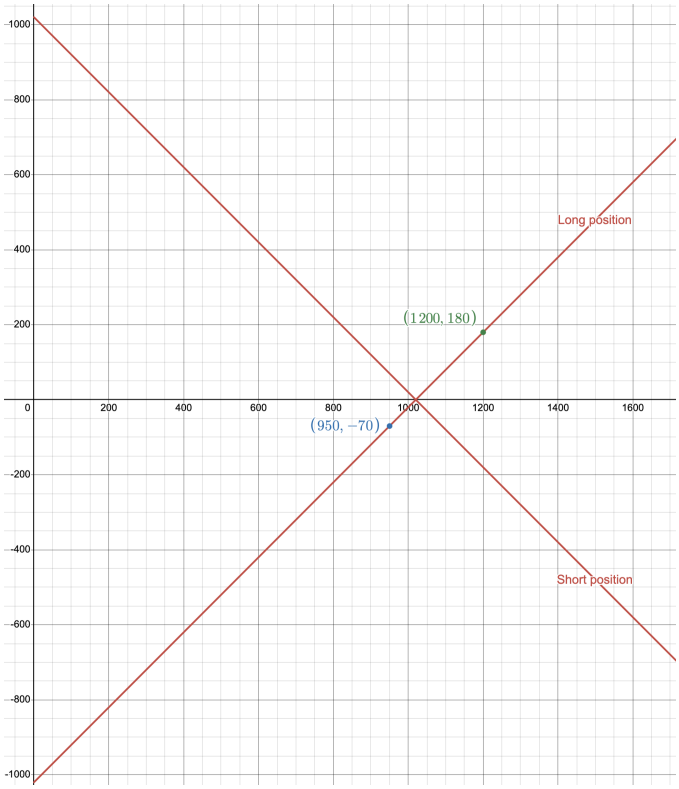
1. The payoff from a long position (buying the asset) in a forward contract is $S_T - K$.
2. The payoff from a short position (selling the asset) in a forward contract is $K - S_T$.

Exercise 2

Forward Contract on Stock Index
We know the current price is \$1000 and the 6-month forward price is \$1020.

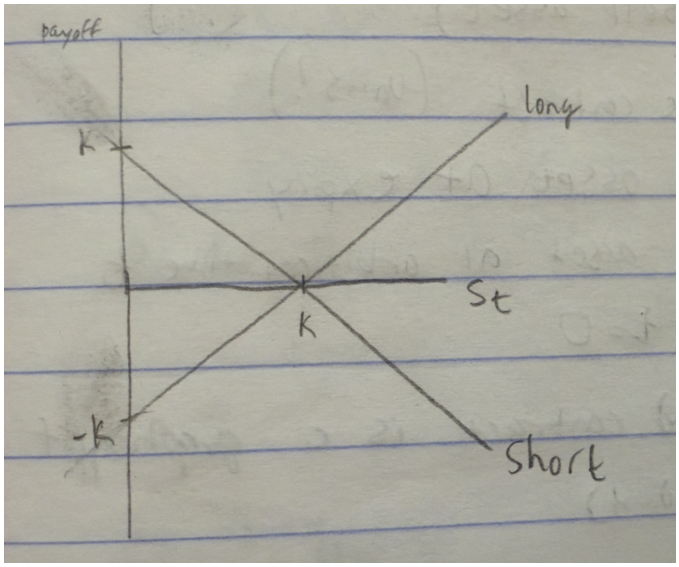
1. If the price is \$950 in 6 months, the long position will lose \$70 (950 - 1020).
2. If the price is \$1200 in 6 months, the long position will gain \$180 (1200 - 1020).

The forward contract allows for a profit if the value of the asset increases after 6 months, without having to actually own the asset.
Payoff Diagram:



Exercise 3

Payoff Diagrams for Forward Contract



Exercise 4

Forward Contract on Foreign Exchange
The bank agrees to a 6-month forward contract to purchase 1 million GBP in 6 months.

1. If the spot price is 1.3000 in 6 months, the bank will make $(1.3000 - 1.2230) \cdot 1000000 = \$77000$.
2. If the spot price is 1.2000 in 6 months, the bank will lose $(1.2000 - 1.2230) \cdot 1000000 = \$23000$.

Exercise 5

Forward Contract on Foreign Exchange
An investor enters into a short forward contract to sell 100,000 GBP for USD at 1.3000 USD per pound.

1. If the spot price is 1.2900 at the end of the contract, the short position gains $(1.3000 - 1.2900) \cdot 100000 = \$1000$.
2. If the spot price is 1.3200 at the end of the contract, the short position loses $(1.3000 - 1.3200) \cdot 100000 = \$2000$.

Exercise 6

Forward Contract on Foreign Exchange
A trader enters into a short forward contract to sell 100 million yen at \$0.0090 per yen.

1. If the spot price is 0.0084 at the end of the contract, the short position gains $(0.0090 - 0.0084) \cdot 100000000 = \$60000$.
2. If the spot price is 0.0101 at the end of the contract, the short position loses $(0.0090 - 0.0101) \cdot 100000000 = \$110000$.

Exercise 7

ECO with T = 10 days, K = \$250

1. If $S_T = \$270$, then the holder of the ECO will exercise the option and the payoff will be $270 - 250 = \$20$
2. If $S_T = \$230$, then the holder of the ECO will let the option expire worthless. The payoff is zero and the holder of the option only loses the option premium.

Exercise 8

Expected Value = $\frac{1}{2} \cdot \$20 + \frac{1}{2} \cdot \$0 = \$10$

Exercise 9

Suppose that an investor did indeed pay c = 10 dollars for an ECO.

1. If $S_T = \$270$, then the payoff is \$20. The net profit is $20 - 10 = \$10$. In this case the net profit is 100% of the initial cost.
2. If $S_T = \$230$, then the payoff is \$0. The net profit is $0 - 10 = -\$10$. In this case, the loss is 100% of the initial cost.

Exercise 10

Suppose that the investor purchases the stock for \$250 outright instead of buying an option.

1. If $S_T = \$270$, then the profit is \$20, which is 8% of the initial cost.
2. If $S_T = \$230$, then the profit is -\$20, which is also 8% of the initial cost.

Compared to buying a call option, purchasing the stock outright has less risk in terms of potential percentage gained or lost. However, the initial cost is much higher.

Exercise 11

EPO with 100 shares of underlying stock, $K = \$70$, current price is \$65. If $S_T = \$55$, then the holder will exercise the option. The payoff will be $100 \cdot (70 - 55) = \$1500$.

Exercise 12

ECO with $K = \$100$, 100 underlying shares, $c = \$500$, $S_T = \%102$.

1. Option 1: Exercise option. The per-share gain is \$2, so the total gain is \$200. Subtracting the \$500 initial cost, the investor would lose \$300.
2. Option 2: Let option expire worthless. The total gain is \$0 and the option cost \$500, so the total loss is \$500.

In this case, exercising the option would let the investor reduce their losses.

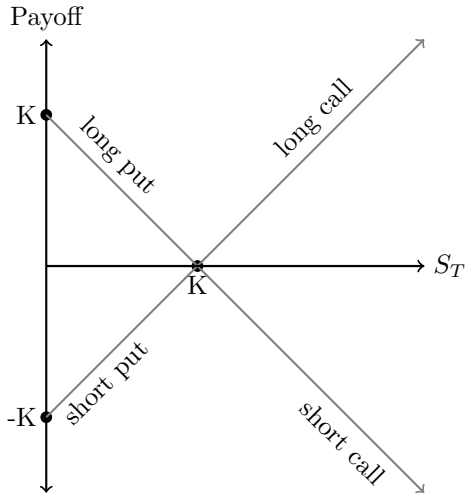
Exercise 13

1. Long position in ECO: If $S_T \leq K$, the long position lets the option expire worthless. The payoff is 0. If $S_T > K$, the long position exercises the option. The payoff is $S_T - K$. Thus, the payoff is $\max(S_T - K, 0)$.
2. Short position in ECO: If $S_T \leq K$, then the long will let the option expire and the payoff to the short is 0. If $S_T > K$, the long will exercise and the payoff to the short is $K - S_T$. Thus, the payoff is $\min(K - S_T, 0)$.

3. Long position in EPO: If $S_T \leq K$, then the payoff will be $K - S_T$. If $S_T > K$, then the payoff will be 0.

4. Short position in EPO: If $S_T \leq K$, then the payoff is $S_T - K$. If $S_T > K$, then the payoff will be 0. So the payoff is $\min(S_T - K, 0)$.

Exercise 14

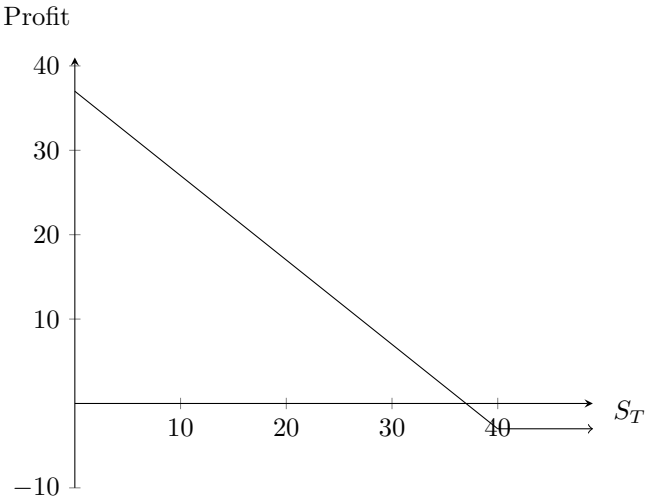


Exercise 15

Investor buys EPO for \$3, current price is \$42, and $K = \$40$. The profit is calculated as

$$\begin{cases} 40 - S_T - 3 & S_T < 40 \\ -3 & S_T \geq 40 \end{cases}$$

Since we want the trade to be profitable, we want $40 - S_T - 3 > 0$, or $S_T < 37$. The option will be exercised if $S_T \leq 40$, since that means the payoff will be positive (but not necessarily the profit). The profit diagram is as follows:

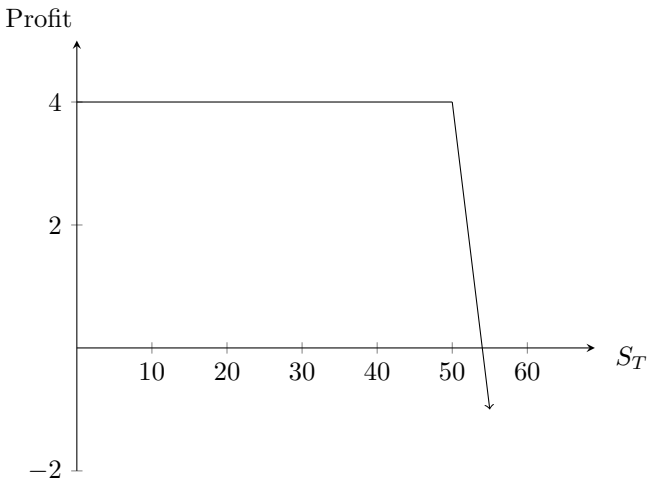


Exercise 16

Investor sells ECO for \$4, $K = \$50$, current price is \$47. The profit is calculated as

$$\begin{cases} 0 + 4 & S_T \leq 0 \\ 50 - S_T + 4 & S_T > 50 \end{cases}$$

Since we want the trade to be profitable, we want $S_T \leq 54$. The option will be exercised when $S_T < 50$, since this is when the payoff is acceptable. The profit diagram is as follows:



Exercise 17

The investor has a short position on an ECO and a long position on an EPO. There are two cases to consider:

1. The long position will exercise if $S_T > K$. Thus, the investor will have to sell to the long position for K. At time $t = T$, the payoff is

   $$\begin{cases} 0 & S_T \leq K \\ K - S_T & S_T > K \end{cases}$$

   or $-\max\{S_T - K, 0\}$.

2. The investor will exercise their long position on the EPO if $S_T < K$. At time $t = T$ the payoff is

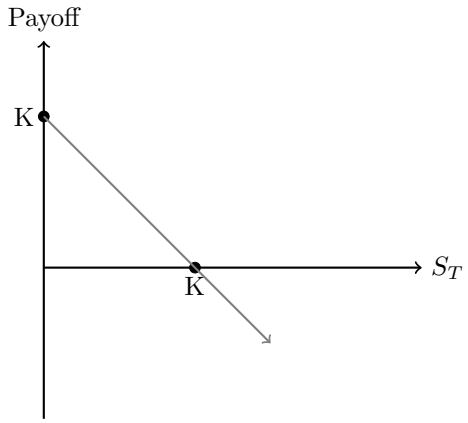
   $$\begin{cases} K - S_T & S_T < K \\ 0 & S_T \geq K \end{cases}$$

   or $-\max\{K - S_T, 0\}$.

Then the overall payoff at expiry will be

$$-\max\{S_T - K, 0\} + \max\{K - S_T, 0\} = \begin{cases} 0 + K - S_T & S_T < K \\ K - S_T + 0 & S_T \geq K \end{cases} = K - S_T.$$

The payoff diagram looks like the following:



Exercise 18

1. Long Position on an ECO: $K = \$45$, $c = \$3$, expiry $t = T$. The profit is represented as

$$\begin{cases} S_T - 45 - 3 & S_T \geq 45 \\ -3 & S_T < 45 \end{cases}$$

2. Long Position on an EPO: $K = \$40$, $c = \$4$, expiry $t = T$. The profit is represented as

$$\begin{cases} 40 - 4 - S_T & S_T < 40 \\ -4 & S_T \geq 40 \end{cases}$$

Then the net profit is

$$\begin{cases} 36 - S_T - 3 & S_T < 40 \\ -3 - 4 & 40 \leq S_T \leq 45 \\ S_T - 48 - 4 & S_T > 45 \end{cases}$$

The profit diagram is as follows: