Homework 4
MATH 476

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Exercise 45

$S_0 = 19$, $c = 1$, $K = 20$, $r = 0.04$, expiry in 3 months means $T = 1/4$. We can use put-call parity to calculate the price of a put option with same strike price and expiry.

$$\begin{gathered} c + Ke^{-rT} = p + S_0 \\ 1 + 20e^{-0.04 \cdot 0.25} = p + 19 \\ p = 1 + 20e^{-0.04 * 0.25} - 19 \\ p = 1.80 \end{gathered}$$

Exercise 46

$S_0 = 130$, expiry in one year means $T = 1$, $c = 20$, $p = 5$, $K = 120$. We can use put-call parity to calculate the risk-free interest rate.

$$\begin{gathered} c + Ke^{-rT} = p + S_0 \\ 20 + 120e^{-r \cdot 1} = 5 + 130 \\ e^{-r} = \frac{5+130-20}{120} \\ e^{r} = \frac{120}{115} \\ r = \ln(\frac{120}{115}) = 0.0426 \end{gathered}$$

Thus the risk-free interest rate is 4.26%.

Exercise 47

$S_0 = 31$, $c = 3$, $p = 2.25$, $K = 30$, $T = 0.25$, $r = 0.1$. Note that put-call parity does not hold here:

$$\begin{gathered} c + Ke^{-rT} = p + S_0 \\ 3 + 30e^{-0.1 \cdot 0.25} = 2.25 + 31 \\ 32.26 \neq 33.25 \end{gathered}$$

Thus we should be able to construct an arbitrage opportunity. Consider a portfolio where we buy the call option and short-sell the put option and the stock. At $t = 0$, the cash flow is

$$-c + p + S_0 = \$30.25$$

Thus we have positive cash flow at $t = 0$. We can then invest this at the risk-free interest rate. At expiry, this will be worth \$31.02. At expiry, we have two cases.

1. $S_T \leq 30$. Then we let the ECO expire, the EPO will be exercised, and we return the stock we shorted. The payoff will be $0 - 30 = -30$, and since we started with \$31.02, we have \$1.02 profit.
2. $S_T > 30$. Then we exercise the ECO, the EPO expires worthless, and we return the stock we shorted. The payoff is $-K + S_T - S_T = -30$. Since we started with \$31.02 we have \$1.02 profit.

Thus in all cases, we make a profit with positive initial cash flow, and this was an arbitrage opportunity.

Exercise 54