Gauss Jordan method

1. Core Concepts and Formula

The Gauss-Jordan Method is an extension of the Gauss elimination method for solving systems of equations

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$$Ax=b$$

. The aim is to reduce the system to a form, where the solution vector can be read off directly. This will be accomplished by transforming the matrix A into the identity matrix,I, and the right-hand side b into a new vector b' ,Once this is achieved, the solution is simply x=b'.

It entails the same process as in the Gauss elimination method but with one main difference: Gauss-Jordan does the elimination of off-diagonal elements both below and above the diagonal. The resultant system is simpler, and no back-substitution is required.

In addition to solving for the vector x, the Gauss-Jordan method also yields the inverse of the coefficient matrix A, thus being a powerful tool. However, it involves more computational work compared to the Gauss elimination method. To summarize, the Gauss-Jordan method is a modification of Gauss elimination in which the aim is to obtain an identity matrix, rather than just an upper triangular form. It avoids back-substitution and gives a direct solution but at the expense of more computation.

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2. Example 1 :

Problem : Solve the following equations by Gauss-Jordan method.

$$x + 3y + 2z = 17$$
$$x + 2y + 3z = 16$$
$$2x - y + 4z = 13$$

Solution:

Consider $x + 3y + 2z = 17$ (E.1)
$x + 2y + 3z = 16$ (E.2)
$2x - y + 4z = 13$ (E.3)

$x + 3y + 2z = 17$ (E.1) (–2) + (E.3)
$x + 2y + 3z = 16$ (E.2) (–1) + (E.1)
$2x - y + 4z = 13$

$x + 3y + 2z = 17$
$y - z = 1$ x(2)+(E.1)
$7y = 21$ $\Rightarrow y = 3$

$x + 5y = 19$
$y - z = 1$ $\Rightarrow z = y - 1 = 3 - 1$ $\Rightarrow$ $\boxed{z = 2}$

$x + 5y = 19$ $\Rightarrow x = 19 - 5 \times 3$ $\Rightarrow$ $\boxed{x = 4}$

2. Example 2 :

Problem : Solve the following set of equations by Gauss-Jordan method.

$$2x_1 + x_2 - 3x_3 = 11$$
$$4x_1 - 2x_2 + 3x_3 = 8$$
$$-2x_1 + 2x_2 - x_3 = -6$$

Solution:

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The augmented matrix is

$$\left[\begin{array}{ccc|c} 1 & -2 & 0 & -4 \\ 0 & -5 & 1 & -9 \\ 4 & 0 & -3 & -10 \end{array}\right]$$

Multiplying 1st row by –4 and adding the result to the 3rd row, we obtain

$$-4R_1 + R_3 \rightarrow \left[\begin{array}{ccc|c} 1 & -2 & 0 & -4 \\ 0 & -5 & 1 & -9 \\ 0 & 8 & -3 & 6 \end{array}\right]$$

Now, multiply the 2nd row by –1/5

$$-\frac{1}{5}R_2 \rightarrow \left[\begin{array}{ccc|c} 1 & -2 & 0 & -4 \\ 0 & 1 & -1/5 & 9/5 \\ 0 & 8 & -3 & 6 \end{array}\right]$$

Multiply the 2nd row and add the result to the 1st row. Then multiply the 2nd row by –8 and add the result to the 3rd row.

$$2R_2 + R_1 \rightarrow \left[\begin{array}{ccc|c} 1 & 0 & -2/5 & -2/5 \\ 0 & 1 & -1/5 & 9/5 \\ 0 & 0 & -7/5 & -42/5 \end{array}\right]$$

Multiply 3rd row by –5/7

$$-\frac{5}{7}R_3 \rightarrow \left[\begin{array}{ccc|c} 1 & 0 & -2/5 & -2/5 \\ 0 & 1 & -1/5 & 9/5 \\ 0 & 0 & 1 & 6 \end{array}\right]$$

Multiply 3rd row by 2/5 and add the result to 1st row. Then multiply 3rd row by 1/5 and add the result to 2nd row.

$$\begin{array}{r} \frac{2}{5}R_3 + R_1 \rightarrow \\ \\ \frac{1}{5}R_3 + R_2 \rightarrow \end{array} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 6 \end{array}\right]$$

Hence, the last matrix above represents the system with $x = 2$, $y = 3$ and $z = 6$.