The Secant method

1. Core Concepts and Formula

The **Secant method** is similar to the **Newton-Raphson method**, but it removes the need for calculating derivatives. The main drawback of Newton-Raphson is that it requires the derivative of the function at several points, which can be time-consuming. In some cases, calculating the derivative is difficult or not possible.

To address this, the **Secant method** approximates the derivative using finite differences. Instead of calculating f′(x) analytically, it uses the **backward difference** formula:

$$f'(x_i) = (f(x_i) - f(x_{i-1}))/(x_i - x_{i-1})$$

This method does not require $f(x_i)*f(x_{i-1})<0$, unlike Newton-Raphson.

From the Newton-Raphson method, we can derive the Secant update formula:

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)} = x_i - \frac{f(x_i)(x_i - x_{i-1})}{f(x_i) - f(x_{i-1})}$$

The Secant method requires **two initial guesses**, x_0 and x_1, for the root. Geometrically, a secant is drawn between $f(x_{i-1})$ and $f(x_i)$, and the intersection with the x-axis gives x_{i+1}. The process repeats with new secants to find the root.

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2. Graphical Representation and Convergence

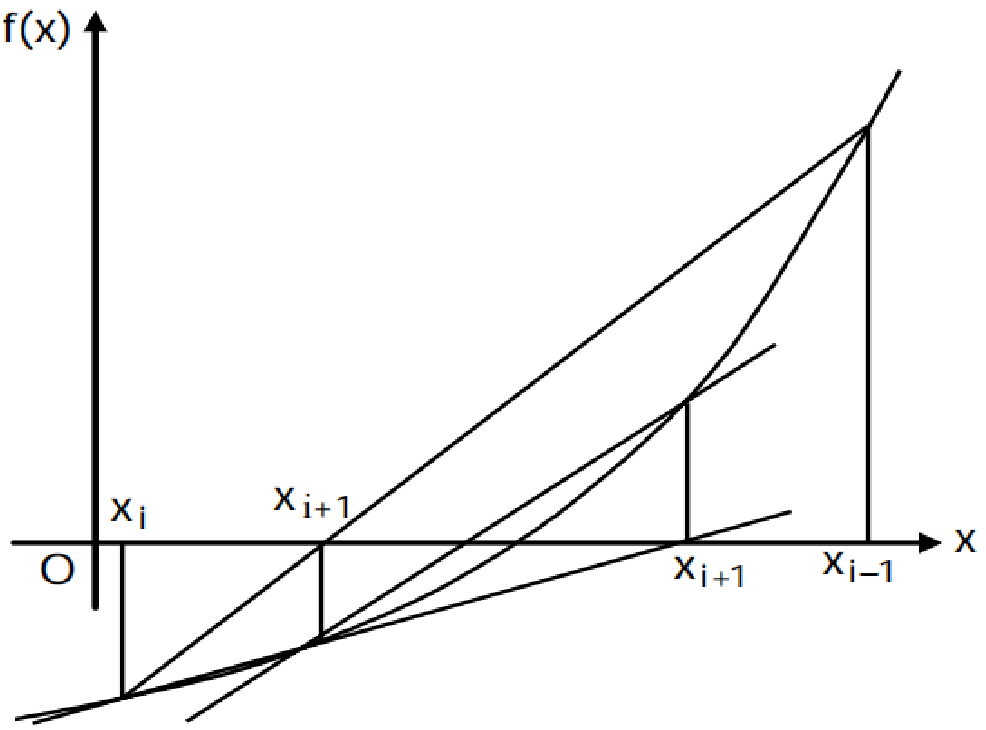

Figure : Secant Method.

3. Step-by-Step Algorithm for Secant method

Algorithm for Secant Method:

1. Start with two initial guesses: x_0 and x_1.

2. Repeat until convergence (i.e., the difference between successive approximations is less than the desired tolerance ϵ\epsilonϵ):

   a. Compute the function values at the two current guesses: $f(x_0), f(x_1)$.
   b. Calculate the new approximation using the Secant formula:

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)} = x_i - \frac{f(x_i)(x_i - x_{i-1})}{f(x_i) - f(x_{i-1})}$$

   c. Check if the absolute difference between the new approximation x_2 and the

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previous guess x_1 is less than the specified tolerance ϵ :

$$|x2-x1|<\epsilon$$

If true, stop the iteration, and x_2 is the root of the equation.

3. Update the values:
   - Set $x_0=x_1$
   - Set $x_1=x_2$
4. Repeat the process from step 2 until convergence is reached.
5. End when the root is found.

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4. Example 1 : $f(x)=x^3-8x-5=0$

Problem : Find a root of the equation $\mathbf{x^3-8x-5=0}$ using the Secant method.

Solution:

Here,

f(x)= x^3−8x-5=0
f(3)=3^3−8*(3)−5=−2
f(4)=4^3−8*(4)−5=−27

Therefore, one root lies between 3 and 4. Let the initial approximations be $x_0=3$ and $x_1=3.5$. Then, x_2 is given by:

$$x_2 = \frac{x_0 f(x_1) - x_1 f(x_0)}{f(x_1) - f(x_0)}$$

x_0	$f(x_0)$	x_1	$f(x_1)$	x_2	$f(x_2)$
3	-2	3.5	9.875	3.08421	-0.33558
3..5	9.875	3.08421	-0.33558	3.09788	-0.05320
3.08421	-0.33558	3.09788	-0.05320	3.10045	0.00039
3.08788	-0.05320	3.10045	0.00039	3.10043	0
3.10045	0.00039	3.10043	0	3.10043	0

Hence, the root of the equation is 3.1004.