The Method of False Position (Regula Falsi)

1. Core Concepts and Formula

The Method of False Position (also called Regula Falsi or the linear interpolation method) is a well-known bracketing method, used to find the root of an equation f(x)=0.

It requires an initial interval (a, b) where the root is bracketed. This means the function f(x) must be continuous and f(a) and f(b) must have opposite signs.

The key difference is that instead of bisecting the interval, it approximates the function f(x) between a and b using a straight line (a chord). The intersection of this line with the x-axis represents an improved estimate of the root. This characteristic is why one endpoint often remains fixed, leading to fast, single-sided convergence.

The Formula :

$$x_s = \{a^* f(b) - b^* f(a)\}/\{f(b) - f(a)\}$$

2. Graphical Representation and Convergence

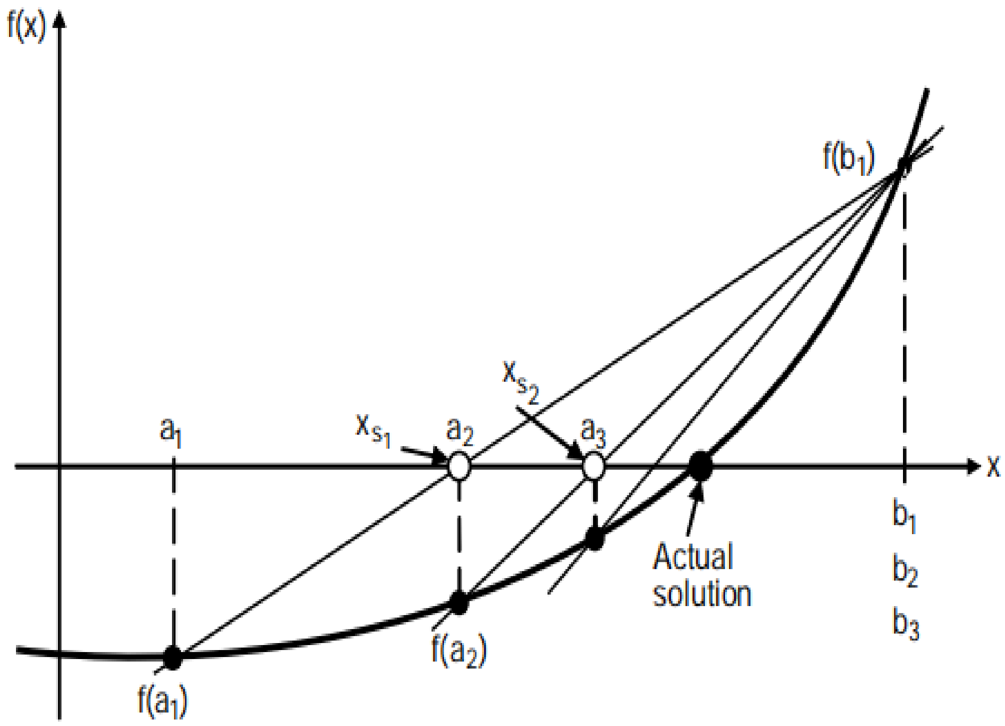

Figure : Method of False position.

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We assume that f(x) is continuous in the interval [a,b] and has a solution. The method starts by choosing an interval $[a_1,b_1]$ where the root lies. The function values at these points, $f(a_1)$ and $f(b_1)$, are connected by a straight line. The first estimate of the root,x_s, is where this line crosses the x-axis.In the second iteration, a new interval $[a_2,b_2]$ is chosen. This interval is either $[a_1,x_{s1}]$ or $[x_s,b_1]$ and the new line connecting the endpoints gives the second estimate, x_{s2}. This process continues with new intervals, and the method repeats until the solution is accurate Enough.

3. Step-by-Step Algorithm for the method of False Position

1. Define the first interval (a,b) such that a solution exists between them. Check f(a)*f(b) < 0.

2. Compute the first estimate of the numerical solution x_s using the formula:

$$x_s = \{a^* f(b) - b^* f(a)\}/\{f(b) - f(a)\}$$

3. Determine the New Interval: This interval is either $[a_1,x_{s1}]$ or $[x_s,b_1]$ and check the sign of the product
   $f(a)*f(x_s)$:
   * If $f(a)*f(x_s) < 0$, the solution is between a and x_s. Set the new interval as (a, x_s).
   * If $f(a)*f(x_s) > 0$, the solution is between x_s and b. Set the new interval as (x_s, b).

4. Repeat: Go back to step 2 with the new interval until the desired accuracy is attained.

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4. Example 1 : f(x)=e^x - 3*x^2

Problem: Use the False Position method to find a root of the function $f(x)=e^x-3x^2 to an accuracy of 5 digits. The root is known to lie between a=0.5 and b=1.0.

Initial Check:
f(0.5) =0.89872 (Positive)
f(1.0) =-0.28172 (Negative)
A root is bracketed. The convergence table below shows that the endpoint $b=1.0$ remains fixed, as illustrated in the graph.

n	a	b	f(a)	f(b)	Xs1	f(Xs1)	R lative Error (ξ
1	0.5	1	0.89872	-0.28172	0.88067	0.08577	
2	0.88067	1	0.08577	-0.28172	0.90852	0.00441	0.03065
3	0.90852	1	0.00441	-0.28172	0.90993	0.00022	0.00155
4	0.90993	1	0.00022	-0.28172	0.91	1e-05	0.00008
5	0.91	1	1e-05	-0.28172	0.91001	0	3.7952×10^{-6}

Result: The root is 0.91 accurate to five digits.

5. Example 2 : cos(x) - 3*x + 5 = 0

Problem: Find the real root of the transcendental equation cos(x) - 3*x + 5 = 0 correct to four decimal places.

Here, f(x)=cosx−3x+5=0
f(0)=cos0−3(0)+5=5>0
f(π2)=cos(π2)−3(π2)+5=−3π/2+5<0

Therefore, a root of f(x)=0 lies between 0 and π/2. We apply the method of False Position with a=0

and b=π/2.

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n	a	b	f(a)	f(b)	X_{s1}	f(X_{s1})
1	0	1.5708	6	0.28761	1.64988	-0.02866
2	1.64988	1.5708	-0.02866	0.28761	1.64272	-0.00001
3	1.64272	1.5708	-0.00001	0.28761	1.64271	0

Result: The root is 1.6427 accurate to four decimal places.