Referee Report

Fractional Decomposition Tree Algorithm: A tool for studying the integrality gap of Integer Programs

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Summary

This paper presents a new polytime algorithm for finding a feasible solution for an integer program where all variables are binary, and presents computational experiments with this algorithm. The idea looks simple and nice, but I have not been able to verify its correctness.

Recommendation

The paper is not very precise in its writing, and I therefore cannot verify the claims they make. In particular, their main contributions use a Theorem (Theorem 3) incorrectly as far as I can tell, and it is unclear to me whether this is fatal or easily fixable. I recommend a thorough revision by the authors, both in presentation of their results, claims of theorems of others they use, and proofs of their own results. I withhold judgement on a recommendation until after this revision.

Detailed comments

- Introduction: Introduction jumps between describing methods and problems the methods are applied to. The there is a subsection 1.2 Experiments with more problem descriptions. Please restructure the introduction so that the flow is more logical.
- p. 2, last paragraph: "Integrality gap" is defined with respect to an IP instance, instead of a family of IP instances (IP formulation) for a problem. This makes the paragraph rather awkward to read, because the statements are not correct as stated.
- p. 3, below equation (1): Why is A defined as a matrix of rational numbers but b a vector or real numbers?
- p. 3: Is there a reason why you choose to not include the coefficients of the objective function in an instance?
- p. 4, Definition 1: Now the integrality gap is defined with respect to a matrix A and vector b, taking the supremum (not maximum, please correct) over all coefficients c, that are nonnegative. This is inconsistent with the previous statement, and also nonstandard as far as I know. Please add some remarks for the reader about this choice. Also, the importance

of considering $c \geq 0$ only should be emphasized more in the text (it is of vital importance when using Carr-Vempala).

- p. 4, equation (4): supremum instead of maximum
- p. 5, Proposition 2: Please specify where this is stated in [GLS93].
- p. 5, below Proposition 2: I don't see that Proposition 2 and Theorem 3 are equivalent as stated. (Also in light of the remark about one result having an algorithm, and the other not.)
- p. 6, Theorem 4: "Moreover, [...]" — C is not specified otherwise; replace "Moreover" by "where".
- p. 6, Theorem 5: Why "$1 \leq$"? Is that not automatic?
- p. 6, below Theorem 5: "in iteration i the algorithm maintains a convex combination of vecotrs" — please phrase this more precisely.
- p. 6, near bottom: "It's subtour-elimination..." — Its
- p. 7, equation (5): sup instead of max
- p. 7, Theorem 6: "Moreover, [...]" — C is not specified otherwise; replace "Moreover" by "where".
- p. 9, last bullet point of Contribution summary: "can support theoretical work" — be more specific about what you did, and what it showed.
- p. 9, Lemma 7: add $\tilde{x}$ right after "Given an integral vector"
- p. 9, bottom: Isn't it even easier? Is $\{1\}^n$ not a point in $\mathcal{D}(P)$ (provided $P \neq \emptyset$)?
- p. 11, 3rd line below definitions of $\mathrm{LP}^{(\ell)}$ and $\mathrm{IP}^{(\ell)}$: "By Theorem 3, there exist [not exists] $\tilde{z}_i \in S$" In the statement of Theorem 3 this is $\mathcal{D}(S)$. Is S the same as $\mathcal{D}(S)$ here? If so explain.

  If not, can you still draw the conclusion that $(IP)^{(\ell)} \neq \emptyset$?
- p. 11, third paragraph: Same problem as previous bullet point.
- p. 13, top of page: Same problem.
- p. 24 and following, references: missing capitalization in a few entries