Wavefield migration by PSPI on HPC architectures

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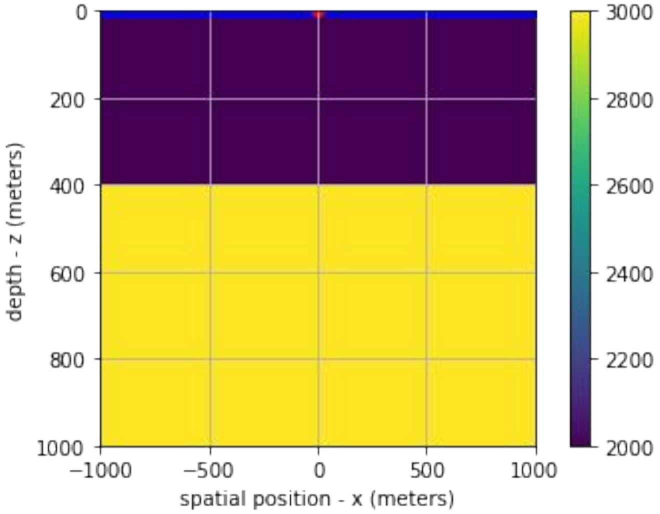
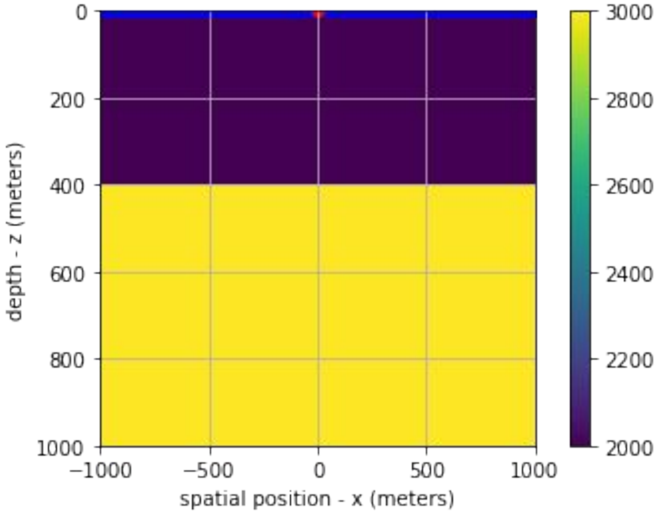
Wavefield Migration

The process of seismic migration involves the application of mathematical algorithms to **position the seismic data** accurately in **space** and **time**, so that the interpreted images align with the true subsurface features. It attempts to **reconstruct the subsurface structures and boundaries**, which can provide valuable insights into the presence and location of geological formations.

Wavefield Migration



Wavefield Migration

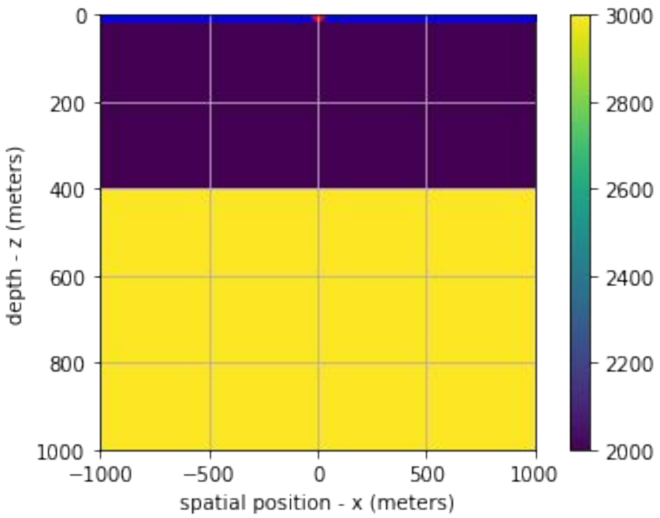


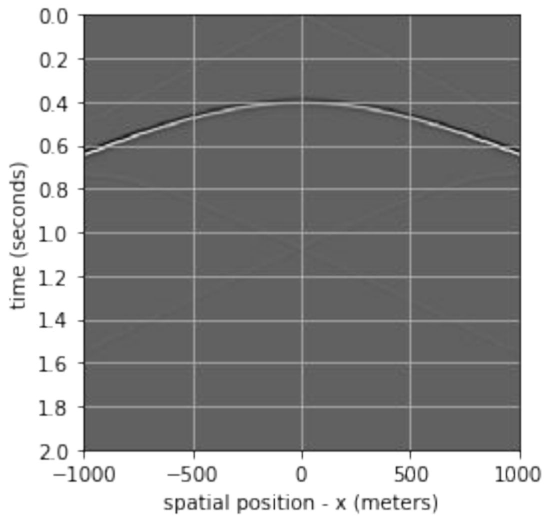
Let us assume we have the model on the left.

We place a source at the center x = 0 m, and place receivers on the surface level z = 0.

The medium presents a horizontal reflector at depth 400 m due to the velocity contrast from 2000 to 3000 m/s.

Wavefield Migration





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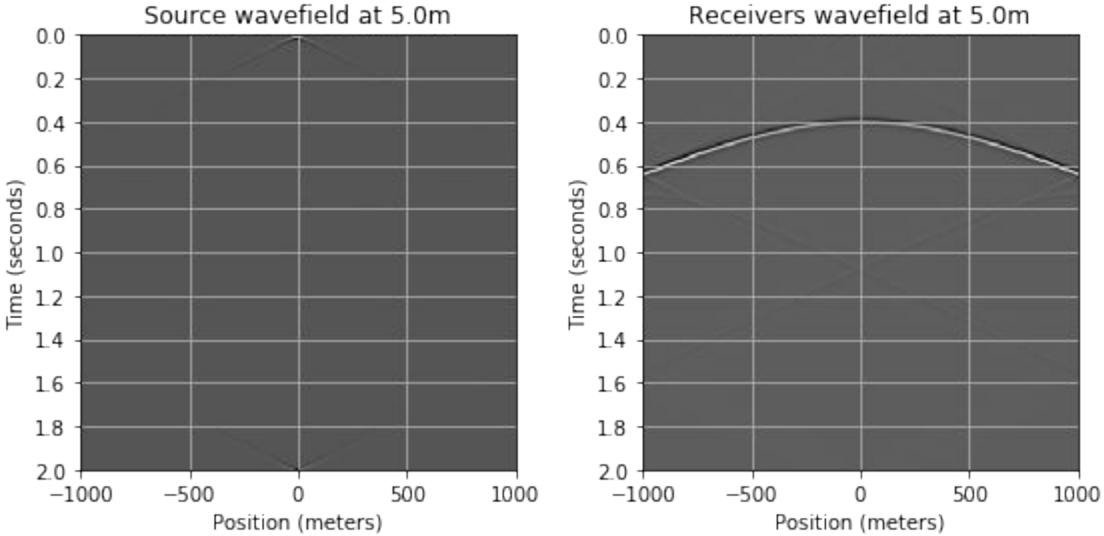

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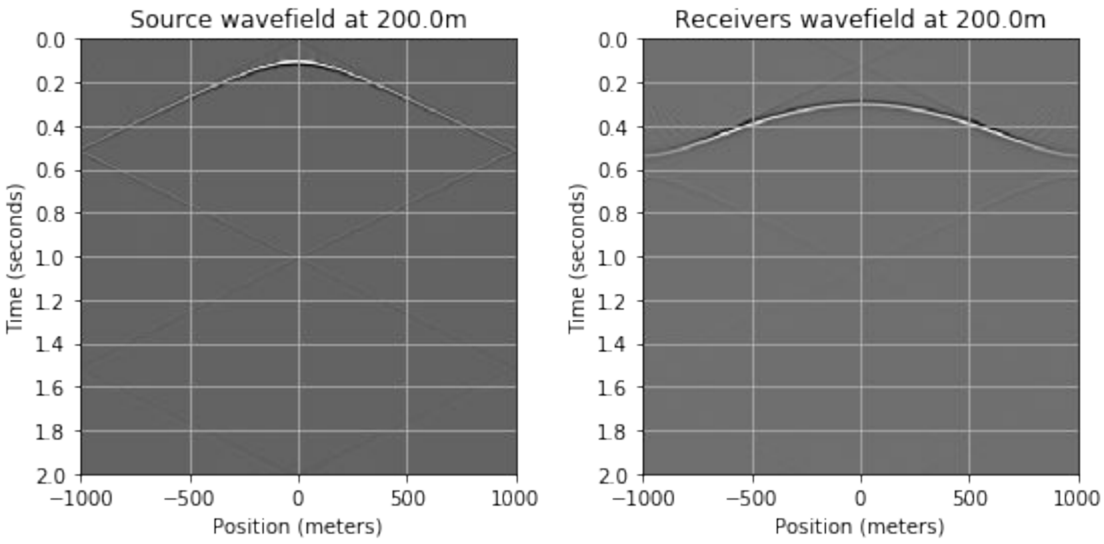

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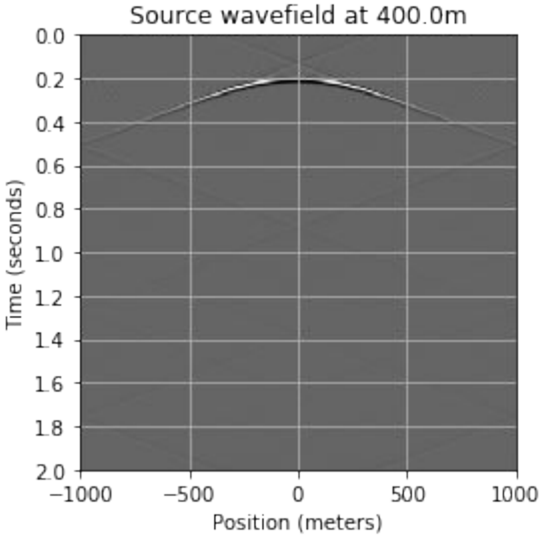

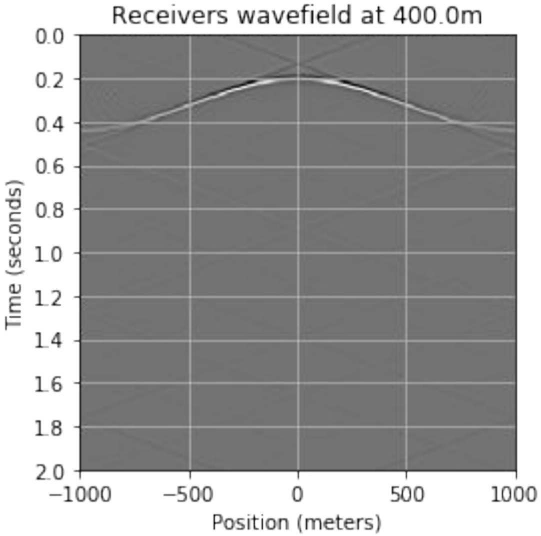

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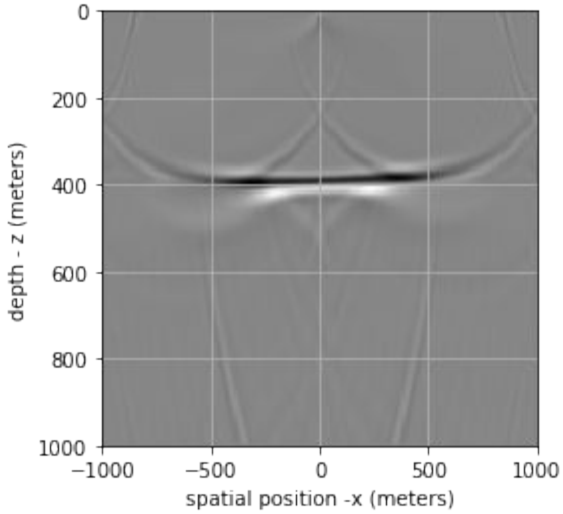

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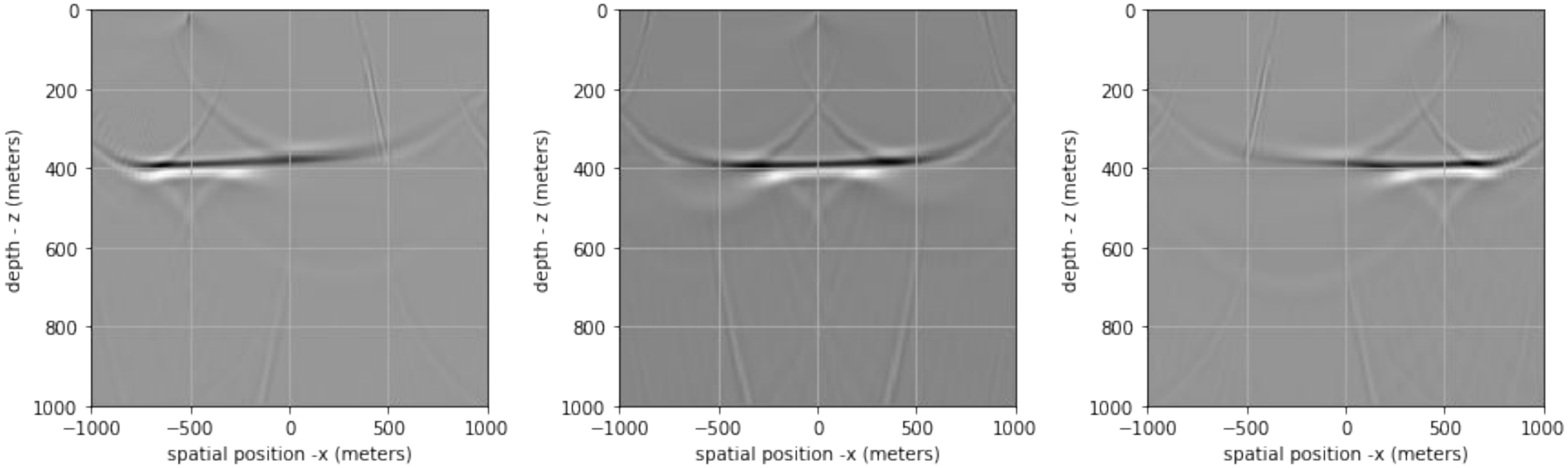
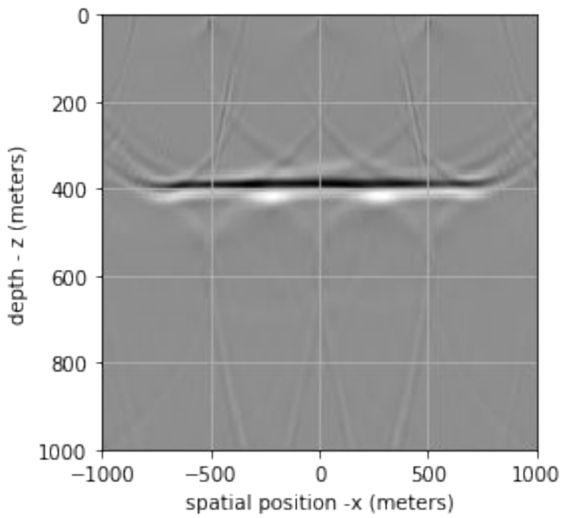


Wavefield Migration



Phase-Shift propagation

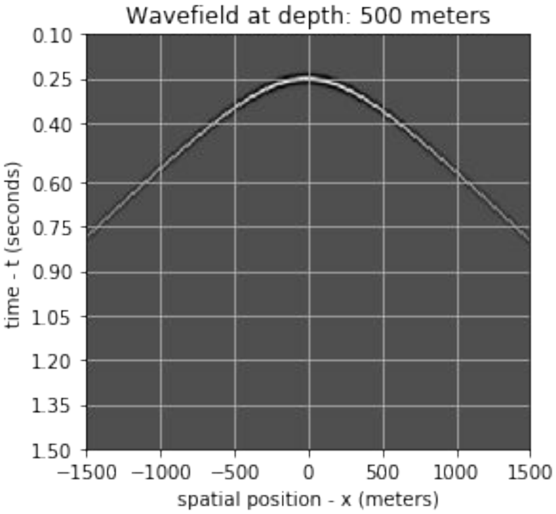
When the velocity model is homogeneous, we transform the wave-equation to the *frequency-wavenumber* domain with solutions of the form:

$$\tilde{P}_{z+\Delta z}(k_x;\omega) = e^{-ik_z\Delta z}P_z(k_x;\omega)$$

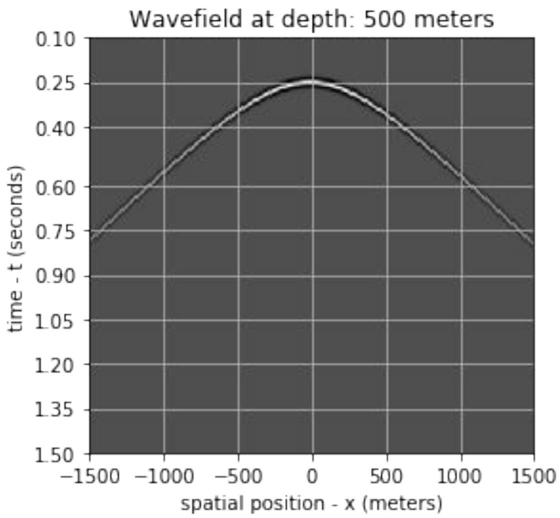
$$k_z = \sqrt{\frac{\omega^2}{c^2} - k_x^2} \quad , \quad \omega > 0$$

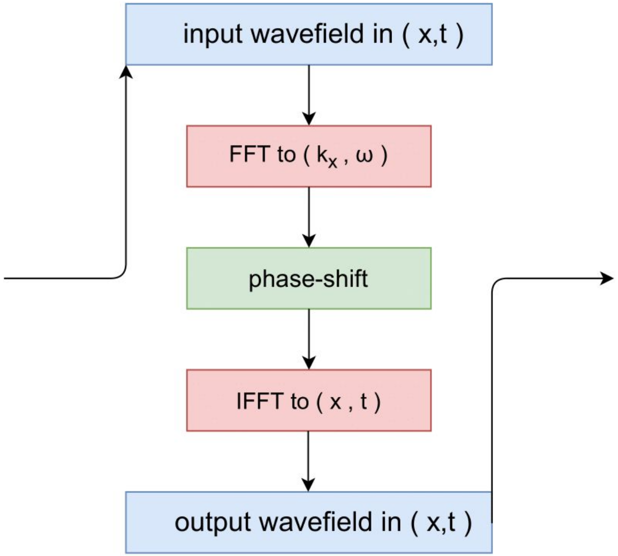
The solution above performs one-way wavefield propagation *forward-in-time*. We can do *backward-in-time* propagation by switching the sign of the exponential term.

Phase-Shift propagation

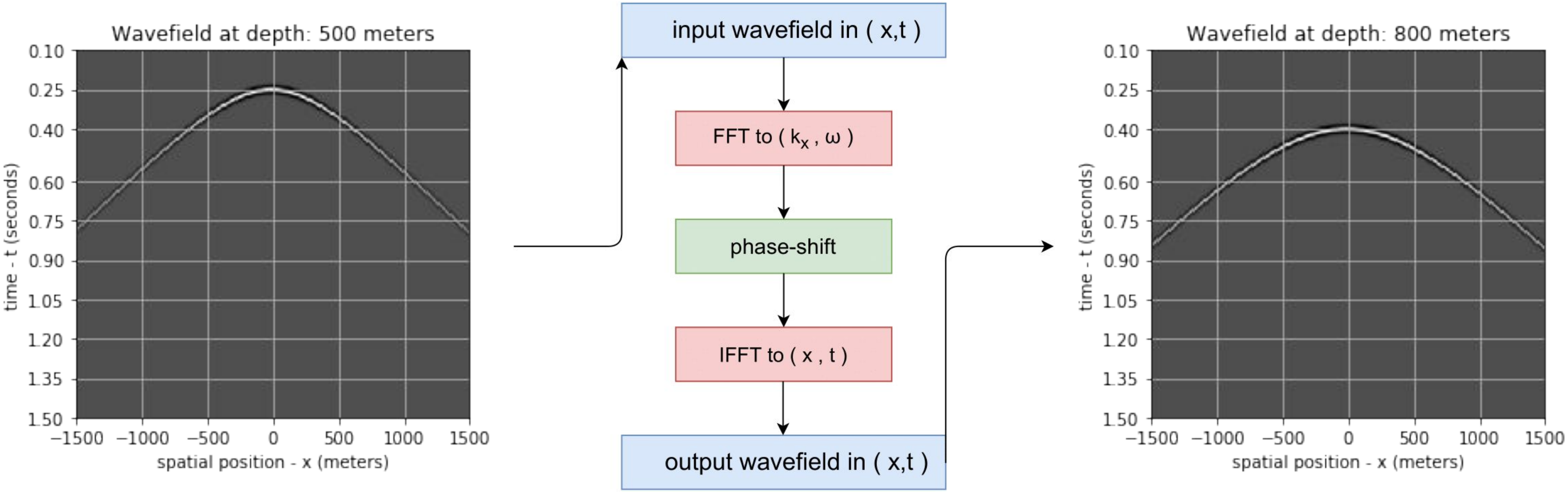


Phase-Shift propagation

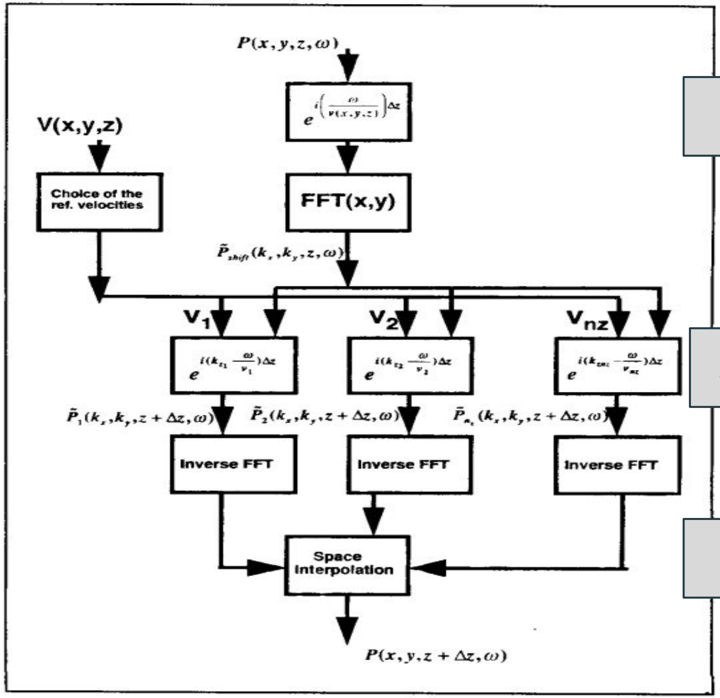




Phase-Shift propagation



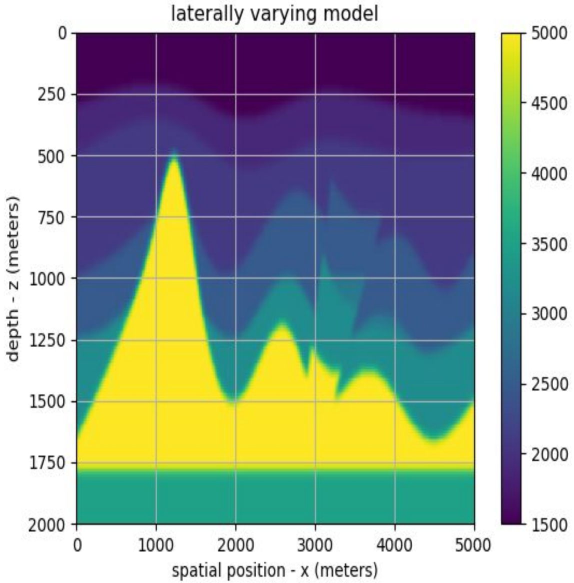
Phase-Shift Plus Interpolation



frequency-space

frequency-wavenumber

frequency-space



Computational complexity

PS_vertical: $N\omega * Nx$
PS_horizontal: $N_{ref} * N\omega * Nx$
FFT_x: $N_{ref} * N\omega * Nx * \text{Log2}(Nx)$
Interpolation: $N\omega * Nx$
Selection: Nx

where,
Nx is the number of spatial points in X direction
$N\omega$ is the number of frequencies
N_{ref} is the number of reference velocities

	6	Bagaini	8	10	12	14	16
FFTs	1.22	1.35	1.59	1.94	2.29	2.66	3.01
PS_h	0.21	0.25	0.27	0.35	0.45	0.54	0.66
Interp.	0.05	0.05	0.05	0.05	0.05	0.05	0.05
PS_v	0.04	0.04	0.04	0.04	0.04	0.04	0.04
Sele.[1]	0.01	0.04	0.01	0.01	0.01	0.01	0.01
Total	1.52	1.73	1.96	2.39	2.84	3.30	3.77

Run-time of the individual kernels for different number of reference velocities. “Bagaini” denotes a statistical approach to the selection of velocities that aims to pick denser close to velocity distribution maxima (<6.7>).

Optimization key-points

- Selection of reference velocities using the statistical entropy of velocity distribution
- Batched FFTs
- Table-driven approach
- Predefine optimal data-layout

Batched FFTs

State-of-the-art HPC libraries for FFTs work based on plans:

1. The plan is created based on the **type** of FFT, the **precision** of the data, the **layout** of data in memory, etc.
2. The plan is executed (actual computation)
3. The plan is destroyed

High-level languages and programming tools usually hide the implementation details; unknown what the underlying implementation does.

We want to have an explicit control over this for performance reasons!

Batched FFTs: Overhead of create/destroy

- **Batched FFTs using cuFFT library**

Algorithm 1:
loop over N
- create plan(1d FFT length NX)
- execute plan
- destroy plan

Algorithm 2:
- plan N x (1d FFTs of length NX)
- execute plan
- destroy plan

- NVIDIA recommends to use batched FFTs for higher performance, so it is expected Algorithm 1 to be slower : **Alg. 1 -> 48s, Alg. 2 -> 8.2s** !!!

- create/destroy plan -> GPU memory allocation/free -> synchronization

Table-driven approach

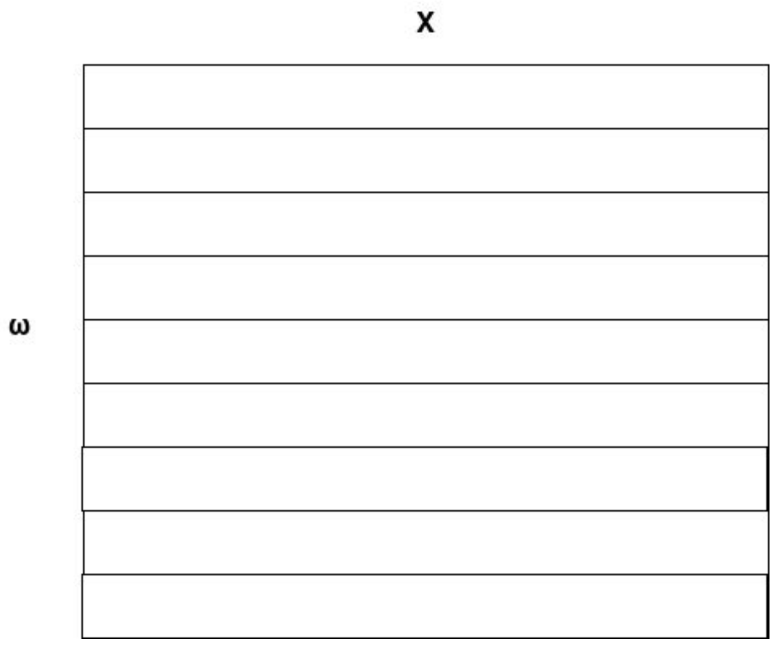
The table-driven approach aims at improving run-time by reducing the number of computations associated with the phase-shift horizontal kernel, which is the second most expensive part of the PSPI algorithm.

$$\tilde{P}_r(x,\omega) = \tilde{P}(x,\omega)\ e^{\pm i\left(k_{z_r} - \frac{\omega}{u_r}\right)\Delta z}$$

$$k_{z_r} = \sqrt{\frac{\omega^2}{u_r^2} - k_x^2}$$

We compute a set of operators that correspond to values $k_r = \frac{\omega}{u_r}$ from 0 to a *kmax*, and during propagation the most suitable is retrieved and applied directly. *--> Up to 40 % speed-up!*

Data-layout of wavefield



Data are:

1. contiguous across X
2. strided by *Nx* points across ω

The layout enables:

1. Batched FFTs using state of the art libraries
2. Develop custom implementations for the other routines optimally

Optimization in the context of HPC starts with defining a suitable data-layout!

Migration by PSPI

For each depth **Z**:

For each source **S**:

- Propagate by PSPI the source wavefield forward in time
- Propagate by PSPI the receivers wavefield backward in time
- Apply imaging condition (IC)

Cross-correlation IC: $$I_{z,s}(x) = Re\left\{ \sum_{\omega} \tilde{P}^{+}(x,\omega)\ \tilde{P}^{-}(x,\omega)^{*} \right\}$$

Assignment

1. Implement the **cross-correlation imaging condition** in:
   a. C-OpenMP,
   b. C-CUDA

2. Remove the direct response from the seismic data
3. Perform imaging using more sources