Problem statement

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1 Background

My group is working on primary migration of oil and gas in source rock. We want to understand the coupling between fluid production, fracturing and drainage, and for this purpose we have devised an analog 2D experiment which we hope can give us some insight: Gelatine containing yeast and sugar is confined between two glass plates (Hele-Shaw cell). When the yeast consumes sugar it produces CO_2. We observe nucleation of bubbles that later may propagate as long, relatively straight fractures. There is no fracture branching, and the resulting fractures open and close as gas is expelled through the fracture network.

2 Suggested problem for this project

Eventually I want to model this complex problem in detail, but for the purpose of this project I suggest focusing on a smaller sub-problem:

- Gas flow through a fracture which is being held open by the gas pressure.

More precisely I suggest modelling the following coupled problem (The model setup is shown in figure 1):

- Flow of gas in the fracture (Navie-Stokes simplified to Pouiseuille flow (cubic law)) Solve by finite difference or element in 1D.
- Elastic response of gel to pressure in fracture, using the solution of the flow problem as boundary condition. Solve by finite element in 2D.

For simplicity I would assume symmetry around the x-axis and only model one half plane for the elasticity problem. I will assume that the facture has some small opening h_0 (i.e capacity to hold gas) even when it is closed, because the surfaces not fit perfectly together. The fracture opening is calculated from $h(x) = 2u(x, 0) + h_0$.

Can I assume that the flow happens much slower than the speed of sound in the gel so that the elastic problem can be considered static?

2.1 Flow equation

One can derive the flow equation from the mass conservation equation,

$$\frac{d\rho}{dt} + \frac{d}{dx}(\rho v) = 0, \tag{1}$$

where $\rho = N/V$ is the particle density and v is the flow velocity: The number of particles per unit length of fracture is $\eta(x,t) = \int_0^{h(x,t)} \rho(x,y,t)dy$, and the time derivative of this is

$$\frac{d\eta}{dt} = \int_0^{h(x,t)} \frac{d}{dt}\rho(x,y,t)dy + \rho(x,h,t)\frac{dh}{dt} \tag{2}$$

If one now assume that the density is constant in y and that the velocity profile is $u(x,y) = \frac{y(y-h)}{2\mu}\frac{dp}{dx}$ (Poiseuille flow) we get

$$\begin{aligned}
\frac{d\eta}{dt} &= \int_0^{h(x,t)} -\frac{d}{dx}(\rho v)\, dy + \rho(x,t)\frac{dh}{dt} \\
&= \int_0^{h(x,t)} \frac{d}{dx}\left(\rho \frac{y(y-h)}{2\mu}\frac{dp}{dx}\right) dy + \rho(x,t)\frac{dh}{dt} \\
&= \frac{d}{dx}\left(\rho \frac{h^3}{12\mu}\frac{dp}{dx}\right) + \rho(x,h,t)\frac{dh}{dt},
\end{aligned} \tag{3}$$

and, using $\eta = h\rho$ and $\rho = \frac{Nm_a}{V} = \frac{pm_a}{kT}$ (ideal gas law, m_a = molecular mass),

$$h\frac{dp}{dt} = \frac{d}{dx}\left(\frac{ph^3}{12\mu}\frac{dp}{dx}\right) \tag{4}$$

I'm not sure if we can solve the problem in this form, because the change in h does not appear explicitly and the p changes with the opening? It might be better to keep the problem in terms of the variable η, because it remains constant when the opening changes. The equation then becomes

$$\frac{d\eta}{dt} = \frac{kT}{12\mu m_a}\frac{d}{dx}\left(\eta h\frac{d\eta}{dx} - \eta^2\frac{dh}{dx}\right) + \frac{\eta}{h}\frac{dh}{dt}, \tag{5}$$

and at every time step one can find the pressure using $p = \frac{kT}{m_a h}\eta$.

Maybe we cannot use (4) or (5), but rather have to solve the problem in several steps, like this (ensures conservation of the gas):

1. Solve $h\frac{dp}{dt} = \frac{d}{dx}\left(\frac{ph^3}{12\mu}\frac{dp}{dx}\right)$ with static h.
2. Iterate until convergence:
   (a) Find new h by solving the elastic problem with new p.
   (b) Adjust p using $p_{i+1} = \frac{h_{i+1}}{h_i}p_i$

I would appreciate any suggestions or ideas!

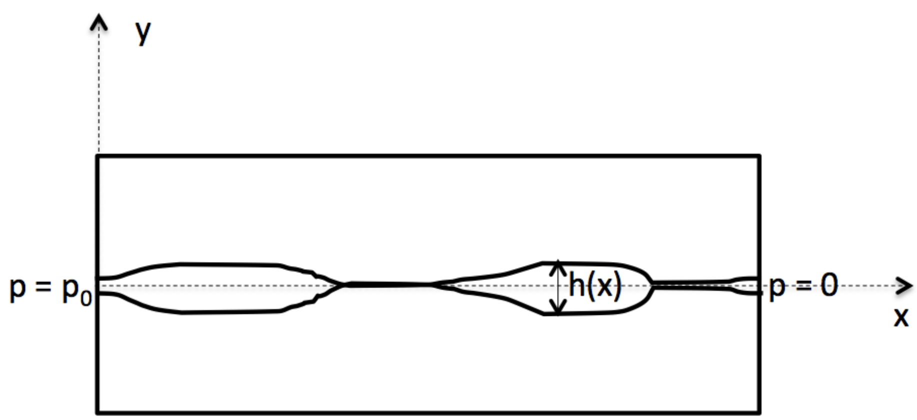

Figure 1: Model setup