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1. Description

This repository contains the Matlab code for performing the slope homogeneity tests in panel data models, including the Swamy's test by Swamy (1970) and the Delta tests by Pesaran and Yamagata (2008).

2. Citation

This code is provided as supplementary material for our paper:
Haghnejad, A., Samadi, S., Nasrollahi, K., Azarbayjani, K., & Kazemi, I. (2020). Market power and efficiency in the Iranian banking industry. *Emerging Markets Finance and Trade*, 56(13), 3217-3234.
Please cite this paper if you are using the codes in your research.

3. Slope homogeneity tests

The procedure for testing slope homogeneity in the panels is based on the following model:

$$y_{it} = \alpha_i + \boldsymbol{\beta}_i'\mathbf{x}_{it} + u_{it}, \quad i = 1, 2, \dots, N, \qquad t = 1, 2, \dots, T, \tag{1}$$

where i is the cross-sectional dimension, t is the time dimension, y_{it} is the dependent variable, $\mathbf{x}_{it}$ is a $p \times 1$ vector of regressors, and α_i and $\boldsymbol{\beta}_i$ are intercept and a $p \times 1$ vector of unknown slope coefficients, respectively, which are allowed to vary across individuals. The usual procedure for testing slope homogeneity is the standard F test. Considering Equation (1), it tests the null hypothesis of slope homogeneity - $H_0: \boldsymbol{\beta}_i = \boldsymbol{\beta}$, for all i - against the alternative hypothesis of slope heterogeneity - $H_0: \boldsymbol{\beta}_i = \boldsymbol{\beta}_j$, for a non-zero fraction of pair-wise slopes for $i \neq j$. This test is appropriate in the context of panel data models with strictly exogenous regressors, homoscedastic error variances, and N being small relative to T (Pesaran and Yamagata, 2008).

By relaxing the homoscedasticity assumption in the standard F test, Swamy (1970) develops a slope homogeneity test based on the dispersion of individual slope estimates from an appropriate pooled estimator. The Swamy's test statistic is given by

$$\hat{S} = \sum_{i=1}^{N} (\widehat{\boldsymbol{\beta}}_i - \widehat{\boldsymbol{\beta}}_{WFE})' \frac{\mathbf{X}_i' \mathbf{M}_\tau \mathbf{X}_i}{\hat{\sigma}_i^2} (\widehat{\boldsymbol{\beta}}_i - \widehat{\boldsymbol{\beta}}_{WFE}), \tag{2}$$

where $\mathbf{X}_i = (\mathbf{x}_{i1}, \dots, \mathbf{x}_{iT})'$, $\widehat{\boldsymbol{\beta}}_i$ is the OLS estimate of slope coefficients for unit i, $\mathbf{M}_\tau = \mathbf{I}_T - \boldsymbol{\tau}_T(\boldsymbol{\tau}'_T\boldsymbol{\tau}_T)^{-1}\boldsymbol{\tau}'_T = \mathbf{I}_T - \frac{\boldsymbol{\tau}_T\boldsymbol{\tau}'_T}{T}$ where $\mathbf{I}_T$ is an identity matrix of order T and $\boldsymbol{\tau}_T$ is a $T \times 1$ vector of ones, and $\hat{\sigma}_i^2$ is the estimate of error variance, σ_i^2, which can be written as follows:

$$\hat{\sigma}_i^2 = \frac{(\mathbf{y}_i - \mathbf{X}_i\widehat{\boldsymbol{\beta}}_i)' \mathbf{M}_\tau (\mathbf{y}_i - \mathbf{X}_i\widehat{\boldsymbol{\beta}}_i)}{T - p - 1}, \tag{3}$$

where $\mathbf{y}_i = (y_{i1}, \dots, y_{iT})'$, and $\widehat{\boldsymbol{\beta}}_{WFE}$ is the weighted fixed effects (WFE) estimator of slope coefficients, as given by

$$\widehat{\boldsymbol{\beta}}_{WFE} = \left(\sum_{i=1}^{N} \frac{\mathbf{X}_i' \mathbf{M}_\tau \mathbf{X}_i}{\hat{\sigma}_i^2} \right)^{-1} \sum_{i=1}^{N} \frac{\mathbf{X}_i' \mathbf{M}_\tau \mathbf{y}_i}{\hat{\sigma}_i^2}. \tag{4}$$

Under the null hypothesis of slope homogeneity, in the case where N is fixed and $T \to \infty$, the $\hat{S}$ statistic is asymptotically distributed as χ^2 with $p(N-1)$ degrees of freedom.

As pointed out by Pesaran and Yamagata (2008), like the standard F test, the Swamy's test is applicable for panels where N is small relative to T. To address this problem, they propose a standardized version of the Swamy's test that is appropriate for panels where N

could be large relative to T. The test statistic proposed, denoted by $\tilde{\Delta}$, is based on a modified version of the Swamy's statistic as

$$\tilde{S} = \sum_{i=1}^{N} (\widehat{\boldsymbol{\beta}}_i - \widetilde{\boldsymbol{\beta}}_{WFE})' \frac{\mathbf{X}_i' \mathbf{M}_\tau \mathbf{X}_i}{\tilde{\sigma}_i^2} (\widehat{\boldsymbol{\beta}}_i - \widetilde{\boldsymbol{\beta}}_{WFE}), \tag{5}$$

where the error variances for the individual units (σ_i^2) are estimated using the standard fixed effects (FE) estimator ($\widehat{\boldsymbol{\beta}}_{FE}$), rather than the OLS estimator ($\widehat{\boldsymbol{\beta}}_i$), namely

$$\tilde{\sigma}_i^2 = \frac{\left(\mathbf{y}_i - \mathbf{X}_i \widehat{\boldsymbol{\beta}}_{FE}\right)' \mathbf{M}_\tau \left(\mathbf{y}_i - \mathbf{X}_i \widehat{\boldsymbol{\beta}}_{FE}\right)}{T-1}, \tag{6}$$

and instead of $\widehat{\boldsymbol{\beta}}_{WFE}$, $\widetilde{\boldsymbol{\beta}}_{WFE}$ is applied which is the WFE estimator computed using $\tilde{\sigma}_i^2$ (instead of $\hat{\sigma}_i^2$), namely

$$\widetilde{\boldsymbol{\beta}}_{WFE} = \left(\sum_{i=1}^{N} \frac{\mathbf{X}_i' \mathbf{M}_\tau \mathbf{X}_i}{\tilde{\sigma}_i^2} \right)^{-1} \sum_{i=1}^{N} \frac{\mathbf{X}_i' \mathbf{M}_\tau \mathbf{y}_i}{\tilde{\sigma}_i^2}. \tag{7}$$

Then the standardized dispersion statistic is developed as

$$\tilde{\Delta} = \sqrt{N} \left(\frac{N^{-1} \tilde{S} - p}{\sqrt{2p}} \right). \tag{8}$$

Under the normally distributed errors, Pesaran and Yamagata (2008) improve the small sample properties of the statistic $\tilde{\Delta}$ by considering the following mean and variance bias adjusted version of $\tilde{\Delta}$,

$$\tilde{\Delta}_{adj} = \sqrt{N} \left(\frac{N^{-1} \tilde{S} - E(\tilde{z})}{\sqrt{V(\tilde{z})}} \right), \tag{9}$$

where $E(\tilde{z}) = p$ and $V(\tilde{z}) = 2p(T-p-1)/(T+1)$. Monte Carlo experiments carried out in Pesaran and Yamagata (2008) show that in the case of models with strictly exogenous covariates and non-normally distributed errors, the $\tilde{\Delta}$ statistic is asymptotically distributed as $N(0,1)$ when $N \to \infty$ and $T \to \infty$ such that $\sqrt{N}/T^2 \to 0$. When the errors are normally distributed, the $\tilde{\Delta}_{adj}$ statistic is shown to be distributed as $N(0,1)$ if $N \to \infty$ and $T \to \infty$ without any restrictions on the relative expansion rates of N and T.

References

Haghnejad, A., Samadi, S., Nasrollahi, K., Azarbayjani, K., & Kazemi, I. (2020). Market power and efficiency in the Iranian banking industry. *Emerging Markets Finance and Trade*, 56(13), 3217-3234.

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