CS203 (2023) – Second assignment

Total marks: 40

- **Note.** Answers without clear and concise explanations will not be taken into account. Use of immoral means will get severe punishment.

Name:_____

Roll No:_____

Questions

1. **(15 marks)** Write a python program which takes a random variable X with range having 5 elements (each element can have probability as a multiple of $1/1000$). Draw n random samples from X and compute the sample mean. Repeat this experiment multiple times and plot these sample means.

   You should get a plot closer and closer to bell shape, as n increases.

   Solution: □

2. **(5+3+7 marks)** In an undirected graph $G = (V, E)$, a cut is specified by an $S \subseteq V$, where the size of the cut is the number of edges between S and $\bar{S}$. A random cut is obtained by keeping each vertex of V in S with probability half.

   (a) Show that a random cut will have expected size $|E|/2$. Justify that each random bit (whether the vertex belongs to S or not) can be pairwise independent.

   (b) Using pairwise independent bits (the generation was shown in class), construct an efficient algorithm to find such a cut.

   Solution:

   □

3. **(10 marks)** Let U be a set of n elements and $S_1, S_2, \cdots, S_m$ be subsets of U. For a function $f : U \to \{-1, 1\}$, define $f(S_i) := \sum_{x \in S_i} f(x)$. Show that there exist a function f such that for all i, $f(S_i) \leq 100\sqrt{n \ln m}$.

   Hint: Assign $1, -1$ randomly to U. Find the probability that a particular S_i is bad. What is the expected number of S_i which are bad?

   Solution: □