Zadaća 4

iz predmeta Matematička logika i teorija izračunljivosti

Prezime i ime: Mašović Haris

Br. indexa: 17993

Zadatak	Bodovi
1	
2	
3	
4	
5	
6	
7	

Elektrotehnički fakultet Sarajevo

1. Rješenje zadatka

   * Ukoliko označimo date jezike sa:
   S - Španski
   I - Italijanski
   P - Portugalski
   J - Japanski

   * Iz postavke imamo sljedeće uslove:
   $\#(S \cup I \cup P \cup J) = 277$
   $\#(S \cap I) = 23$
   $\#(S \cap P) = 23$
   $\#(S \cap J) = 23$
   $\#(I \cap P) = 23$
   $\#(I \cap J) = 23$
   $\#(P \cap J) = 23$
   $\#(S \cap I \cap P) = 11$
   $\#(S \cap P \cap J) = 11$
   $\#(S \cap I \cap J) = 11$
   $\#(I \cap P \cap J) = 11$
   $\#(S \cap I \cap P \cap J) = 5$
   $\#S = \#I + \#P \quad (1)$
   $\#I = \#J + 4 \quad (2)$
   $\#P = \#J - 6 \quad (3)$

   * Dalje možemo izračunati kardinalni broj svakog skupa na sljedeći način i korštenje jednačina (1),(2),(3):

   $\#(S \cup I \cup P \cup J) = 277 = \#S + \#I + \#P + \#J - (\#(S \cap I) + \#(S \cap P) + \#(S \cap J) + \#(I \cap P) + \#(I \cap J) + \#(P \cap J)) + \#(S \cap I \cap P) + \#(S \cap P \cap J) + \#(S \cap I \cap J) + \#(I \cap P \cap J) - \#(S \cap I \cap P \cap J) =$
   $384 = 5\#J - 4 \Rightarrow \#J = 76$

   ** Uvrštavajući nazad u uslove (1),(2),(3) imamo:
   $\#P = 70, \#I = 80, \#S = 150$

2. Rješenje zadatka
   a)

   $(A \setminus B) \setminus C = A \setminus (B \cup C)$
   $(A \cap C(B)) \setminus C = A \cap C(B \cup C)$
   $A \cap C(B) \cap C(C) = A \cap C(B) \cap C(C)$

b)

$(A \setminus B)\ (B \setminus C) = A \setminus B$
$(A \cap C(B)) \setminus (B \cap C(C)) = A \cap C(B)$
$A \cap C(B) \cap (C(B) \cup C) = A \cap C(B)$
$A \cap (C(B) \cup (C(B) \cap C)) = A \cap C(B)$
$A \cap (C(B) \cap (C \cup U)) = A \cap C(B)$
$A \cap C(B) = A \cap C(B)$

3. Rješenje zadatka

a)

* Ovo ćemo dokazati pomoću zakona idempotentnosti koji glasi:
$A \cup A \cup A \cup ... \cup A = A$

* Dalje ukoliko raspišemo ovih n skupova imamo:
$A \cap (B_1 \cup B_2 \cup B_3 \cup ... \cup B_n) = (A \cap B_1) \cup (A \cap B_2) \cup (A \cap B_3) \cup ... \cup (A \cap B_n)$

** Isključenjem trećeg imamo:

$A \cap (B_1 \cup B_2 \cup B_3 \cup ... \cup B_n) = A \cap (B_1 \cup B_2 \cup B_3 \cup ... \cup B_n)$

*** Samim tim dokaz vrijedi uz početne uslove.

b)

* Ovo ćemo dokazati pomoću zakona idempotentnosti koji glasi:
$A \cap A \cap A \cap ... \cap A = A$

* Dalje ukoliko raspišemo ovih n skupova imamo:
$(A \text{ x } B_1) \cap (A \text{ x } B_2) \cap (A \text{ x } B_3) \cap ... \cap (A \text{ x } B_n) = A \text{ x } (B_1 \cap B_1 \cap B_1 \cap ... \cap B_n)$

** Isključenjem trećeg imamo:

$A \text{ x } (B_1 \cap B_1 \cap B_1 \cap ... \cap B_n) = A \text{ x } (B_1 \cap B_1 \cap B_1 \cap ... \cap B_n)$

*** Samim tim dokaz vrijedi uz početne uslove.

4. Rješenje zadatka

* Funkcija očigledno nije inverzna jer nije zadovoljeno pravilo '1-1' tj. injektivnost , zatim funkcija nije surjektivna jer kodomen nije čitav Z, što znači da funkcija nije ni bijektivna samim tim nema inverznu funkciju.

** Generalizirana inverzna funkcija glasi:

$$F^{-1}(x) = \begin{cases} \{\emptyset\} & , \quad x = 0 \\ \{\{a\}, \{b\}, \{c\}\} & , \quad x = 1 \\ \{\{a,b\}, \{a,c\}, \{b,c\}\} & , \quad x = 2 \\ \{a,b,c\} & , \quad x = 3 \\ \emptyset & , \quad \text{x} \in \mathbb{Z} \setminus \ \text{\{0,1,2,3\}} \end{cases}$$

5. Rješenje zadatka

$$((\lambda f.\lambda x.f(f(x)))((\lambda y.2y+1)(\lambda t.t^3+2t^2-t+4)))(2) =$$
$$= ((\lambda f.\lambda x.f(f(x)))(2(\lambda t.t^3+2t^2-t+4)+1))(2) =$$
$$\triangleright f = 2(\lambda t.t^3+2t^2-t+4)+1$$
$$= (\lambda x.f(f(x)))(2) =$$
$$= f(f(2)) =$$
$$= f((2(\lambda t.t^3+2t^2-t+4)+1)(2)) =$$
$$= f(2(2^3+2\cdot 2^2-2+4)+1) =$$
$$= f(37) =$$
$$= (2(\lambda t.t^3+2t^2-t+4)+1)(37) =$$
$$= 2(37^3+2\cdot 37^2-37+4)+1 =$$
$$= 2(50653+2738-37+4)+1 =$$
$$= 106717$$

6. Rješenje zadatka

   a)

   Vrijedi $xR^{-1}y$ ako i samo ako vrijedi yRx.

$$R^{-1} = \{(6,1),(5,2),(5,3),(5,4),(4,5),(5,5)\}$$

   Relaciju R možemo predstaviti relacionom matricom:

$$M = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

   Sada relaciju R^2 i R^3 možemo naci pomocu Booleovog proizvoda matrica:

$$M^2 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \circ \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$R^2 = \{(2,4),(2,5),(3,4),(3,5),(4,4),(4,5),(5,4),(5,5)\}$$

$$M^3 = M^2 \circ M = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \circ \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$R^3 = R^2 = \{(2,4),(2,5),(3,4),(3,5),(4,4),(4,5),(5,4),(5,5)\}$$

   Simetricno zatvorenje jeste unija relacije i njene inverzne relacije.

$$R^s = R \cup R^{-1} = \{(1,6),(2,5),(3,5),(4,5),(5,2),(5,3),(5,4),(5,5),(6,1)\}$$

   Vrijedi R^+ ako i samo ako je ikako moguce iz x doci u y na strelicastom dijagramu tj. sve vrijednosti $\top$ bez glavne dijagonale.

$$R^+ = \{(1,6),(2,4),(2,5),(3,4),(3,5),(4,4),(4,5),(5,4),(5,5)\}$$

   Da bismo našli R^*, zatvorenje R^+ dopunjujemo sa refleksivnošću.

$$R^* = \{(1,1),(1,6),(2,2),(2,4),(2,5),(3,3),(3,4),(3,5),(4,4),(4,5),(5,4),(5,5),(6,6),(7,7)\}$$

b)

$$M \circ M^{-1} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \circ \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$R \circ R^{-1} = \{(1,1), (2,2), (2,3), (2,4), (2,5), (3,2), (3,3), (3,4), (3,5), (4,2), (4,3), (4,4), (4,5), (5,2), (5,3), (5,4), (5,5)\}$$

$$M^2 \circ (M^{-1})^2 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \circ \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$R^2 \circ (R^{-1})^2 = \{(2,2), (2,3), (2,4), (2,5), (3,2), (3,3), (3,4), (3,5), (4,2), (4,3), (4,4), (4,5), (5,2), (5,3), (5,4), (5,5)\}$$

c)

$$(M \circ M^2) \circ M^3 = \left(\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \circ \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \right) \circ \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} =$$

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \circ \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$(R \circ R^2) \circ R^3 = \{(2,4), (2,5), (3,4), (3,5), (4,4), (4,5), (5,4), (5,5)\}$$

7. Rješenje zadatka

   * Ukoliko posmatramo našu prvu projekciju relacije R2 imamo:

$$\underset{(\#1,\#2)}{\pi}(R2) = \{(8,6),(7,4),(6,5),(2,6),(4,8)\}$$

   * Dalje ukoliko posmatramo lambda spajanje, vidimo da moramo spojiti petorku sa dvojkom, pošto u narednoj iteraciji su nam potrebne samo dvije dobivene sedmorke samo će one biti napisane, tj. idemo za korak više pa imamo:

$$\text{A} = \underset{(\#1>\#2 \ \wedge \ \#1\neg 1)}{\sigma} (R1 \underset{(\#1<\#1+\#2)}{\bowtie} \underset{(\#1,\#2)}{\pi}(R2)) = \{(5,4,3,2,6,8,6),(5,4,3,2,6,7,4),$$

(5,4,3,2,6,6,5),(5,4,3,2,6,2,6),(5,4,3,2,6,4,8), (4,1,1,5,7,8,6),(4,1,1,5,7,7,4),(4,1,1,5,7,6,5),

(4,1,1,5,7,2,6),(4,1,1,5,7,4,8)}

   ** Na kraju posmatramo projekciju i dobijamo konačnu relaciju R:

$$R = \underset{(\#1,\#2,\#(-1))}{\pi}(A) = \{(5,4,6),(5,4,4),(5,4,5),(5,4,6),(5,4,8),(4,1,6),(4,1,4),$$

(4,1,5),(4,1,8)}