Assignment 01

1. Consider $r(t) = (\sin t - t\cos t)i + (\cos t + t\sin t)j + k$ to find the unit bi-normal.
2. Identify the graph of $r(t) = 4\cos t\,\hat{\imath} + 4\sin t\,\hat{\jmath} + 3t\hat{k}$ and find its unit binormal at $t = \frac{\pi}{4}$.
3. Using the implicit partial differentiation, evaluate $\frac{\partial z}{\partial x}$ for $y^2e^{5x} - \cos(\ln z) = 1$.
4. Find the directional derivative of $f(x, y, z) = x^2y^3 + \ln z + \cos^{-1}\left(\frac{x}{2}\right)$ at the point $(-1, -2, 1)$ in the direction of the vector $a = 2i + j - 2k$.
5. Find the directional derivative of $z = x^2 + y^2$ at the point $P(-1, -2, 5)$ in the direction of $\vec{a} = -\hat{\imath} + 6\hat{\jmath} + 2\hat{k}$.
6. Find a unit vector in the direction in which $f(x, y, z) = 4e^{xy}\cos z$ decreases most rapidly at $P\left(0, 1, \frac{\pi}{4}\right)$, and find the rate of change of f at P in that direction.
7. Consider the surface $x^3 - y^2z - yz^2 = 6$ to find the parametric equations of the line that is normal to the surface at the point $(2,1,1)$.
8. Find the parametric equation of tangent plane and normal line to the surface $F(x, y, z) = x^3z + 2xy^3z^2$ at the point$(-2,0,3)$.
9. Find the equation of tangent line of the intersection of the surfaces $z = x^2 + y^2$ and $xyz = 2$ at $(1,1,2)$.
10. Find the maximum rate of change of $F(x, y, z) = z - \sqrt{x^2 + y^2}$ at $(-4, -3, 5)$ and the unit vector at that direction.
11. Find parametric equation of the tangent line to the curve of intersection of the cone $z = \sqrt{x^2 + y^2}$ and the plane $x - 2y - z = 5$ at $(4, -3, 5)$.
12. Find the force field corresponding to the potential function $\emptyset(x, y, z) = \frac{yz}{x^2} + \frac{xz}{y^2} + \frac{xy}{z^2}$. Consider $\nabla\emptyset(1,1,1)$ and write the physical explanation.
13. Find the divergence and curl of $f(x, y, z) = y\tan^{-1}(xz)\,i + ze^{xy}j + \ln\left(\frac{z}{x}\right)k$.
14. Using the proper parameterization evaluate the line integral $\oint_C (x^2 + y^2)dx - y\,dy$, where C is the portion of the circle $x^2 + y^2 = 16$ in the third quadrant.
15. Evaluate $\int_C xy\,dx + xy\,dy + z^2\,dz$ along the curve $C: x = \sin t, y = \cos t, z = t^2$ within the interval $0 \le t \le \frac{\pi}{2}$.
16. If $z_1 = -3i$ and $z_2 = 7$, show that z_1, $z_1 + z_2$, and $z_1 - z_2$ lie in the same line.

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17. Write $\frac{3e^{1+\pi i}}{e^{\frac{\pi}{2}i}}$ in the $a+ib$ form.

18. Find all the values of $\left(\frac{\sqrt{3}-i}{i}\right)^{\frac{1}{3}}$ and $\left(\frac{\sqrt{2}-i}{2}\right)^{\frac{1}{5}}$.

19. Write $\frac{-3(1-i)^2}{2+2\sqrt{3}i}$ in the CiS form.

20. Evaluate $\left(3e^{\frac{\pi i}{3}}\right)\left(5e^{-\frac{5\pi i}{2}}\right)^{-3}\left(2e^{\frac{\pi i}{6}}\right)\left(4e^{\frac{2\pi i}{3}}\right)^{-2}$.

21. Find the modulus and argument of $\frac{-3(-1-i)^3}{i+i^4}$.

22. Describe the set of points z in the complex plane satisfying (i) $|2z-i|=4$ (ii) Re $(z^2)\geq 4$ (iii) $|z-1|=|z-i|$ (iv) $|z-i|=\text{Im}(z)+1$ (v) $|z+1-i|=5$.

23. Find and locate (graphically) all the roots of $\left(\sqrt{3}-i\right)^{\frac{1}{4}}$ and $(-i)^{\frac{1}{2}}$.

24. Evaluate $\lim_{z\to 2}\frac{z-2}{z^3-8}$ and $\lim_{z\to 0}\left(\frac{\cos z}{z}\right)^{\frac{1}{z}}$.

25. Show that $\frac{\log(z)}{z^2-3z+2}$ is continuous for all z outside $|z+2|=1$.

26. Show that $u(x,y)=x^2-y^2-2x$ is harmonic and hence find its harmonic conjugate $v(x,y)$. Also, confirm $w=u(x,y)+iv(x,y)=f(z)$.

27. Show that $u(x,y)=3x^2y+2x^2-y^3-2y^2$ is a harmonic function. Find the harmonic conjugate $v(x,y)$ such that $f(z)=u+iv$ is analytic.

28. Evaluate $\log(-1-i)$.

29. Solve $e^z+i=0$ for z.

30. Why $e^{\frac{1}{z}}$ is analytic everywhere except the origin? Explain.