

1. M1

(a) Consider the arithmetic computation below.

$$3 + 4[5 - 12] - 6(3) + (4 + 0) = 3 + 4[5 - 12] - 6(3) + 4 \quad (1)$$

$$= 4[5 - 12] - 6(3) + 4 + 3 \quad (2)$$

$$= 20 - 48 - 18 + 4 + 3 \quad (3)$$

$$= -39.$$

For each of the steps (1), (2), and (3) identify which of the Axioms of Integer Arithmetic are used in the simplification step.

Solution: $3 + 4[5 - 12] - 6(3) + 4 \dots (1)$ additive identity

$4[5 - 12] - 6(3) + 4 + 3 \dots (2)$ commutativity of addition

$20 - 48 - 18 + 4 + 3 \dots (3)$ distributive

(b) Create and simplify an expression that uses associativity of addition, multiplicative identity, and the distributive law.

Solution:

(1) Associativity of addition

$$= a + (b + c) = (a + b) + c$$

(2) Multiplication identity

$$1 * a = a$$

(3) Distributive law

$$a(b + c) = ab + ac$$

Example

$$3 + 4(6 + 4) + (7(3) + 5(1))$$

$$= (3 + 4(6 + 4)) + 7(3) + 5(1) \dots \text{used (1)}$$

$$= (3 + (24 + 14)) + 7(3) + 5(1) \dots \text{used (3)}$$

$$= (3 + 24 + 14) + 21 + 5 \dots \text{used (2)}$$

$$= 67$$

2. M2

For each statement below determine whether each statement is correct for integers a , b , and c . If the statement is correct, then prove it. If the statement is incorrect, then modify it so that it is correct. Be sure to state which Order Axiom(s) you have applied.

- (a) If $a < b$, then $c \cdot a < c \cdot b$.

Solution: Incorrect.

If $a < b$, then $c \cdot a < c \cdot b$ when $c > 0$

- (b) If $a < b$, then $a + c < b + c$.

Solution: Correct

If $a < b$, then we can assume that $a + r = b$, where r is in \mathbb{Z}

Hence, $a + c + r = b + c$, when c is in \mathbb{Z}

Therefore, then $a + c < b + c$.

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If $a < b$, then we can assume that $a + r = b$, where r is in \mathbb{Z}

Hence, $a + c + r = b + c$, when c is in \mathbb{Z}

Therefore, then $a + c < b + c$.

- (c) If $a < b$, $b < c$, and $c < d$, then $a < d$.

Solution: With the same way part (b) Since If $a < b$, then If $a + r = b$, when r is in \mathbb{Z}

and $b < c$, so it follows that $b + r = c$, it is also same with $a + r + r = b + r = c$

also, if $c < d$, then $c + r = d$, it is same with $a + r + r + r = b + r + r = c + r = d$

Therefore, $a + 3r = d$

Hence, $a < d$

- (d) If $a \not\leq b$ and $a \not\leq b$, then $a = b$.

Solution: Given that $a \not\leq b$ and $a \not\leq b$

For $a \not\leq b$

then, it can be either $a < b$ or $a = b$, but $a < b$ is a contradiction by given $a \not\leq b$.

For $a \not\leq b$,

then, it can be either $a > b$ or $a = b$, but $a > b$ is a contradiction by given $a \not\leq b$.

Therefore, $a = b$.