## **Problem Set 01**

Ahana Deb

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The code used in this can be found here- https://github.com/ahanadeb/RL\_assignment\_1

## 1 Problem 1: Policy Evaluation

Plotting the values obtained from following  $\pi_{lazy}$  and  $\pi_{aggresive}$  and the difference between them as shown in Fig. 1. The states are represented as continuous rows from 0 to 99.

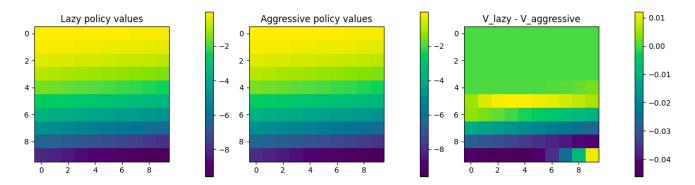


Figure 1: Policy evaluation.

The difference is even more clear when we plot it as a line in Fig. 2. For the given problem, cost of the high service rate action, i.e.,  $c(a_{high}) = 0.01$ . So plotting the difference between the two sets of values obtained, in the left graph, we can see that it doesn't necessarily matter if we choose a lazy or aggressive policy for the first 50 states, i.e., till the length of the queue is 50, but as it increases after that, after state 60, following the lazy policy of a low service rate action is an objectively bad one. If we increase the cost of the high service rate action 10 times, i.e.,  $c(a_{high}) = 0.1$  in the right graph, the cost of this action becomes so great, taking the high rate action is worse than the lazy action even when the queue length approaches its maximum value.

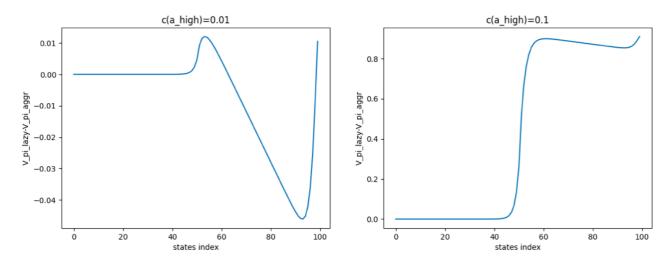


Figure 2: Policy evaluation.

At state 50, for  $c(a_{high}) = 0.01$ , we get the values  $V_{\pi_{lazy}} = -2.396$  and  $V_{\pi_{aggressive}} = -2.399$  and we can say that at this state  $\pi_{lazy}$  policy is slightly better.

At state 80, we get the values  $V_{\pi_{lazy}} = -6.231$  and  $V_{\pi_{aggressive}} = -6.204$  so  $\pi_{aggressive}$  policy is clearly better here, as we can also observe from the left graph in Fig. 2.

## 2 Problem 2: Value Iteration and Policy Iteration

Looking at Fig. 3 obtained from value iteration, we can observe a significant difference in the values occur between the 10th and 20th iterations, but not much occurs afterwards. Now looking at Fig. 4 of policy iteration, practically

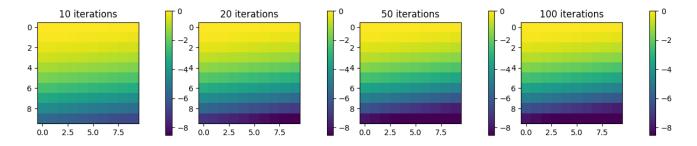


Figure 3: Value Iteration (N=100)

no change can be observed across the iterations, which implies that it is possibly converging very very fast. The time taken for 100 iterations of value iteration is 0.135secs, whereas for 100 iterations of policy iteration, it is 6.22secs (using analytical solution for evaluating policy and not power iteration), roughly 46 times. So, although policy iteration requires a much higher computational cost, it converges in less number of iterations than value iteration. This is further illustrated in Fig. 5. 5 equally spaces states are chosen from the original 100 states, and

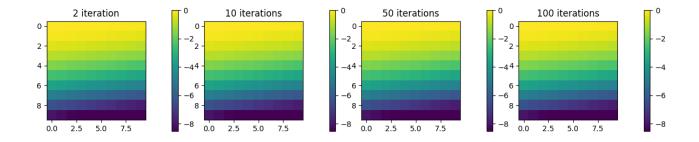


Figure 4: Policy Iteration (N=100)

their values are plotted across the iterations. The blue lines is for the values obtained through value iteration, whereas the red is for policy iteration for the same states. As we can see, the values in policy iteration converges very quickly, almost within the first few iterations, while the value iteration takes more than 30 iterations to achieve stability.

In Fig. 6, the values obtained from the two policies evaluated before are compared with the optimal values. We can see that the optimal values are better than both these cases, since the difference never becomes negative.

The optimal policy obtained from this shows that the best course of action would be to take the lower service rate action  $a_{low}$  from states 0 to state 62, and follow the higher service rate action at every state from state 63 till the queue is full. This is very close to the aggressive policy we evaluated, and can be inferred from the bottom right graph in Fig. 6 as well.

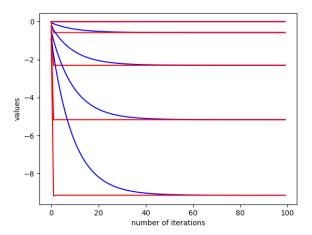


Figure 5: Convergence: Value Iteration vs Policy Iteration

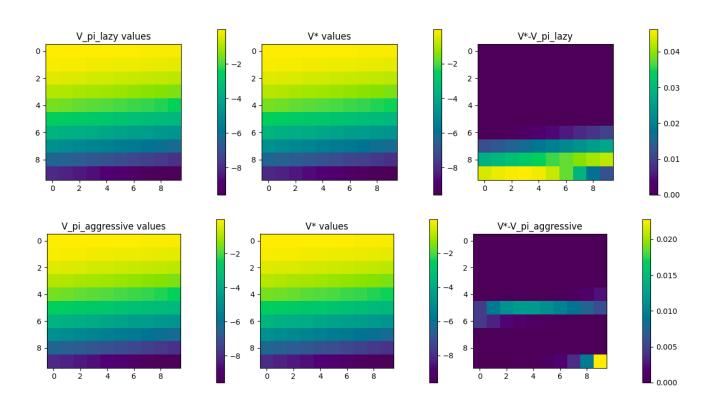


Figure 6: (a) The top 3 plots show the comparison between the values obtained from the  $\pi_{lazy}$  policy and the optimal values. (b) The bottom 3 plots show the comparison between the values obtained from the  $\pi_{aggressive}$  policy and the optimal values. values of M,I.