FUNUGA

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21/5/24

§1 P1

Let n be a positive integer and S be a set of 2^n elements. Let $A_1, A_2, ...A_n$ be randomly and independent chosen subsets of S where each possible subset of S is chosen with equal probability. Let P_n be the probability that

$$A_1 \cup A_2 \cup A_3 \cup ... \cup A_n = S \text{ and } A_1 \cap A_2 \cap ... \cap A_n = \phi$$

Find the value of $\lim_{n\to\infty} P_n$.

(A) $\frac{1}{2}$ **(B)** $\frac{1}{e}$ **(C)** $\frac{2}{3}$ **(D)** $\frac{1}{e^2}$

§2 P2

There are $n \geq 4$ sectors in which a circular disc is divided. There are k colours. Calculate the number of ways the sectors can be coloured such that no two adjacent sectors are coloured with the same colour.

(A) k^n **(B)** $\frac{k!}{n!}$ **(C)** $k^2(k-1)^{n-2}$ **(D)** $k(k-1)^{n-1}$

§3 P3

Let F_1, F_2 be the feet of the perpendiculars from foci S_1 and S_2 of the ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$ on the tangent at any point P of the ellipse. Then the minimum value of $(S_1F_1 + S_2F_2)$ is :

(A) 2 **(B)** 3 **(C)** 6 **(D)** 8

§4 P4

The value of the greatest integer less or equal to $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}(n+1)}$ is :

(A) 1 **(B)** 2 **(C)** 0 **(D)** 5

§5 P5

Say a number to be algebraic if it is the root of an integer polynomial. Consider these two statements:

(1)$\sin 1°$ is algebraic

and

(2)$\cos 1°$ is algebraic. Then the correct statement is/are:

(A) only 1 **(B)** only 2 **(C)** both 1, 2 **(D)** neither of 1, 2

§6 P6

If α is a real root of the equation $x^{11} - x^{10} + x^8 - x^7 + x^5 - x^4 + x^2 - x - 20 = 0$ then α^{12} is:
(A) $= 61$ **(B)** > 61 **(C)** < 61 **(D)** can't say

§7 P7

The value of the limit $\lim_{x\to 0}\left(\frac{(1+x)^{\frac{2}{x}}}{e^2}\right)^{\frac{4}{\sin x}}$:
(A) e^2 **(B)** e^4 **(C)** e^{-4} **(D)** e^8

§8 P8

If x_1, x_2, x_3 be real roots of the equation $x^3 - 3x + 1 = 0$ and $[x]$ denotes the greatest integer less or equal to x and $[x_1] + [x_2] + [x_3] = -k$ then k is equal to :
(A) 1 **(B)** 2 **(C)** 0 **(D)** 3

§9 P9

Say the number of onto functions(or, permutations) $f : \{1, 2, 3, ..., n\} \to \{1, 2, 3, ...n\}$ satisfying

$$|f(1) - 1| = |f(2) - 2| = |f(3) - 3| = ... = |f(n) - n|$$

is $P(n)$. Then the value of $P(2024) + P(2025)$ is:
(A) 11 **(B)** 12 **(C)** 13 **(D)** 14

§10 P10

Area of the region enclosed by the conic $5x^2 + 6xy + 2y^2 + 7x + 6y + 6 = 0$ is $\frac{a\pi}{b}$ where a, b are natural number with $\gcd(a, b) = 1$. The value of $(a^2 + b^2)$ is :
(A) 13 **(B)** 5 **(C)** 2 **(D)** 17