

# IOQM MOCK

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**P1.** In ISI canteen, the only cold drink you can get is Campa Cola, worth 10 rupees. Arnab is tossing a coin, with probability of heads appearing  $\frac{1}{4}$  and tails  $\frac{3}{4}$ . Now if heads appears, Arnab gives Shounak 1 rupee and if tails, Shounak gives Arnab one rupee. Whoever has 10 rupees at hand, can buy a Campa Cola once. If the probability that Shounak can buy a Campa Cola at least 10 times is  $\frac{a}{b}$  with  $a, b$  are coprime integers, find largest prime divisor of  $b$ . [2]

**P2.** The degree-6 polynomial  $f$  satisfies

$$f(7) - f(1) = 1, \quad f(8) - f(2) = 16, \quad f(9) - f(3) = 81, \quad f(10) - f(4) = 256, \quad f(11) - f(5) = 625.$$

Compute sum of digits of  $f(15) - f(-3)$ . [2]

**P3.** Let  $r, s, t$  be the distinct roots of

$$x^3 - 2022x^2 + 2022x + 2022.$$

Compute

$$\frac{1}{1-r^2} + \frac{1}{1-s^2} + \frac{1}{1-t^2}.$$

. If the answer you got is  $\frac{m}{n}$  with  $m, n$  coprime integers, find last two digits of  $m + n$  [2]

**P4.** Triangle  $\triangle RSM$  has  $RS = 4$ ,  $RM = 6$ , and  $SM = 8$ . Point  $A$  lies on line  $\overleftrightarrow{RS}$  and point  $C$  lies on line  $\overleftrightarrow{RM}$  such that  $AC$  is parallel to  $SM$  and the center of the circle inscribed in triangle  $\triangle RAC$  lies on  $SM$ . Compute  $5 \cdot AC$ . [2]

**P5.** How many ways can Ted Mosby say the four words "Robin", "Scherbatsky", "Barney", "Stinson" (once each) such that he never says "Robin Stinson" or "Barney Scherbatsky"? [2]

**P6.** In a T20 cricket tournament, in each match, the winner gets 3 points and the loser gets none. There are no ties (Super Over decides, and if it fails too, winner is decided like CWC 2019). In case of a match washed out, both teams get one point each. Midway through the tournament, the points table looked as follows. Can you guess how many matches were washed out?

SL No.	Team	Number of matches played	Points
1	India	6	9
2	Australia	6	11
3	New Zealand	7	11
4	England	6	6
5	Pakistan	7	9
6	Bangladesh	6	9
7	South Africa	5	7
8	Sri Lanka	5	4

[2]

**P7.**  $AB$  is a diameter of a circle, and  $C, D$  are points on the circle (same side of  $AB$ ) such that  $\angle CAB = 30^\circ$ ,  $\angle DAB = 45^\circ$ .  $AC, BD$  intersect at  $I$ . What is the value of

$$\left( \frac{[\triangle AID]}{[\triangle BIC]} \right)^2 ?$$

[2]

**P8.** Find the sum of all  $n \in \mathbb{N}$  such that  $n^n + 1$  is prime, and has at most 19 digits.

[2]

**P9.** Let  $ABCD$  be a cyclic quadrilateral, with  $AB = 7$ ,  $BC = 11$ ,  $CD = 13$ , and  $DA = 17$ . Let the incircle of  $ABD$  hit  $BD$  at  $R$  and the incircle of  $CBD$  hit  $BD$  at  $S$ . What is the length of  $RS$ ?

[2]

**P10.** The radius of the circumscribing circle of an Octagon having sides  $a, a, a, a, b, b, b, b$  is  $\frac{m}{\sqrt{n}} \sqrt{a^2 + \sqrt{2}ab + b^2}$ . Find  $m + n$ .

[2]

**P11.** Being bored by their Statistics assignment, Ahan and Sayan are playing a game. They are very jobless and can make a heap of upto 2025 stones, stolen from their geology classroom. Now, they are allowed to remove any number of stones they want, from the set  $\{1, 3, 4\}$ .

e.g. If they start with 2 stones, Ahan has to play the current move then he can only remove 1 stone and Sayan has to remove the remaining stone in his move.

**The guy who removes the last stone wins.** Say, they are starting with a heap of  $N$  stones. Sayan is starting the game. Given that they play optimally, find number of values of  $N \leq 2025$ , for which Ahan has a winning strategy. Report the last two digits of your answer.

[3]

**P12.** Let  $AEF$  be a triangle with  $EF = 20$  and  $AE = AF = 21$ . Let  $B$  and  $D$  be points chosen on segments  $AE$  and  $AF$ , respectively, such that  $BD$  is parallel to  $EF$ . Point  $C$  is chosen in the interior of triangle  $AEF$  such that  $ABCD$  is cyclic. If  $BC = 3$  and  $CD = 4$ , then the ratio of areas  $\frac{[ABCD]}{[AEF]}$  can be written as  $\frac{a}{b}$  for relatively prime positive integers  $a, b$ . Compute  $a + b$ .

[3]

**P13.** There are two distinguishable flagpoles, and there are 19 flags, of which 10 are identical blue flags, and 9 are identical green flags. Let  $N$  be the number of distinguishable arrangements using all of the flags in which each flagpole has at least one flag and no two green flags on either pole are adjacent. Find the sum of distinct prime divisors of  $N$ .

[3]

**P14.** How many ways are there to shade a total of 8 unit squares in a  $4 \times 4$  grid such that there are exactly 2 shaded squares in each row and each column?

[3]

**P15.** Triangle  $ABC$  is isosceles with  $AC = BC$  and  $\angle ACB = 106^\circ$ . Point  $M$  is in the interior of the triangle so that  $\angle MAC = 7^\circ$  and  $\angle MCA = 23^\circ$ . Find  $\angle CMB$  in degrees.

[3]

**P16.** Find  $a + b + c$  if  $a, b, c$  are digits and  $(\overline{abc})^3 = (3(a + b + c))^5$ .

[3]

**P17.** A positive integer is called "ISISH" if the number of 1's in its binary representation is even. For example,  $6 = 110_2$  is a "ISISH" number. What is the sum of square of digits of the 2025<sup>th</sup> smallest "ISISH" number?

[3]

**P18.** Let  $P$  be a 10-degree monic polynomial with roots  $r_1, r_2, \dots, r_{10} \neq 0$  and let  $Q$  be a 45-degree monic polynomial with roots  $\frac{1}{r_i} + \frac{1}{r_j} - \frac{1}{r_i r_j}$  where  $i < j$  and  $i, j \in \{1, \dots, 10\}$ . If  $P(0) = Q(1) = 2$ , then  $\log_2(|P(1)|)$  can be written as  $\frac{a}{b}$  for relatively prime integers  $a, b$ . Find  $a + b$ .

[3]

**P19.** Circle  $\Omega$  has radius 13. Circle  $\omega$  has radius 14 and its center  $P$  lies on the boundary of circle  $\Omega$ . Points  $A$  and  $B$  lie on  $\Omega$  such that chord  $AB$  has length 24 and is tangent to  $\omega$  at point  $T$ . Find  $AT \cdot BT$ .

[3]

**P20.** A fenced, rectangular field measures 24 meters by 52 meters. Inesh, for his Agriculture project has to partition this plot, he has 1994 meters of fence that can be used for internal fencing to partition the field into congruent, square test plots. The entire field must be partitioned, and the sides of the

squares must be parallel to the edges of the field. What is the largest number of square test plots into which the field can be partitioned using all or some of the 1994 meters of fence? report the product of the non-zero digits of your answer. [3]

**P21.** Let the number of coprime natural numbers less or equal to  $n$  be denoted by  $\phi(n)$ . Find the sum  $\sum_{n=1}^{64} ((-1)^n \phi(n) \lfloor \frac{64}{n} \rfloor)$ . Report the last two digits of the absolute value of your answer. [5]

**P22.** A positive integer  $n$  is *Ahan's favourite number* if it is possible to partition the set  $\{1, 2, \dots, n\}$  into disjoint pairs such that the sum of the two elements in each pair is a power of 3.

For example, 6 is Ahan's favourite number because

$$\{1, 2, 3, 4, 5, 6\} = \{1, 2\} \cup \{3, 6\} \cup \{4, 5\}$$

and

$$1 + 2 = 3^1, \quad 3 + 6 = 3^2, \quad 4 + 5 = 3^2$$

are all powers of 3.

How many positive integers less than or equal to 2024 are Ahan's favourite numbers? Report the last two digits of your answer. [5]

**P23.** If

$$\frac{\binom{1013}{0}}{\binom{2025}{1011}} + \frac{\binom{1013}{1}}{\binom{2025}{1012}} + \dots + \frac{\binom{1013}{1013}}{\binom{2025}{2024}} = \frac{a}{b}.$$

where  $a, b$  are coprime ( $\gcd(a, b) = 1$ ) natural numbers (positive integers), find largest prime factor of  $b$ . [5]

**P24.** Let  $f$  be a monic cubic polynomial satisfying  $f(x) + f(-x) = 0$  for all real numbers  $x$ . For all real numbers  $y$ , define  $g(y)$  to be the number of distinct real solutions  $x$  to the equation  $f(f(x)) = y$ . Suppose that the set of possible values of  $g(y)$  over all real numbers  $y$  is exactly  $\{1, 5, 9\}$ . Compute the sum of all possible values of  $f(10)$ . Report the last two digits as your answer. [5]

**P25.** Let  $ABCD$  be a rectangle and  $E$  be a point on segment  $AD$ . We are given that quadrilateral  $BCDE$  has an inscribed circle  $\omega_1$  that is tangent to  $BE$  at  $T$ . The incircle  $\omega_2$  of  $\triangle ABE$  is also tangent to  $BE$  at  $T$ . If the ratio of the radius of  $\omega_1$  to the radius of  $\omega_2$  is

$$\frac{a + \sqrt{b}}{c},$$

where  $a, b, c$  are mutually co-prime positive integers, find  $a + b + c$ . [5]

**P26.** Let  $F_0 = 0, F_1 = 1$ , and  $F_n = F_{n-1} + F_{n-2}$ . How many subsets  $S$  of  $\{1, 2, \dots, 99\}$  are there such that

$$F_{100} - 1 = \sum_{i \in S} F_i?$$

[5]

**P27.** Find the sum (in base 10) of the three greatest numbers less than  $1000_{10}$  that are palindromes in both base 10 and base 5. Report the sum of digits of your answer [5]

**P28.** For each even positive integer  $x$ , let  $g(x)$  denote the greatest power of 2 that divides  $x$ . For example,  $g(20) = 4$  and  $g(16) = 16$ . For each positive integer  $n$ , let  $S_n = \sum_{k=1}^{2^{n-1}} g(2k)$ . Find the greatest integer  $n$  less than 1000 such that  $S_n$  is a perfect square. [5]

**P29.** Drohan has bought a tripod to record a song. The tripod has three legs each of length 5 feet. When the tripod is set up, the angle between any pair of legs is equal to the angle between any other pair, and the top of the tripod is 4 feet from the ground. In setting up the tripod, Drohan, by mistake, breaks off the lower 1 foot of one leg. Let  $h$  be the height in feet of the top of the tripod from the ground when the broken tripod is set up. Then  $h$  can be written in the form  $\frac{m}{\sqrt{n}}$ , where  $m$  and  $n$  are positive integers and  $n$  is not divisible by the square of any prime. Find  $\lfloor m + \sqrt{n} \rfloor$ . (The notation  $\lfloor x \rfloor$  denotes the greatest integer that is less than or equal to  $x$ .) Report the sum of digits of your answer. [5]

**P30.** Circles  $\omega_1$  and  $\omega_2$  intersect at points  $X$  and  $Y$ . Line  $\ell$  is tangent to  $\omega_1$  and  $\omega_2$  at  $A$  and  $B$ , respectively, with line  $AB$  closer to point  $X$  than to  $Y$ . Circle  $\omega$  passes through  $A$  and  $B$  intersecting  $\omega_1$  again at  $D \neq A$  and intersecting  $\omega_2$  again at  $C \neq B$ . The three points  $C, Y, D$  are collinear,  $XC = 67$ ,  $XY = 47$ , and  $XD = 37$ . Find  $AB^2/10$ . [5]

In life, you will be challenged in each and every step. This mock is nothing in comparison. So, better suit up! For boosting your confidence here is a legendary specimen of what your attitude should be :

