Exercises III

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Exercise 2

We first need to calculate the 12 equations that we need to solve. For a systems of 3-bodies with masses m_1, m_2, and m_3 and initial positions $\vec{x}_1$, $\vec{x}_2$, and $\vec{x}_3$, this is straightforward. Note that we restrict the motion to a plane so that $\vec{x}_i = x_{ix}\hat{x} + x_{iy}\hat{y}$.

Defining separation vectors

$$\begin{aligned}
\vec{r}_{12} &= \vec{x}_2 - \vec{x}_1 \\
\vec{r}_{23} &= \vec{x}_3 - \vec{x}_2 \\
\vec{r}_{31} &= \vec{x}_1 - \vec{x}_3
\end{aligned}$$

the equations of motion are (setting Newton's gravitational constant $G = 1$ and using dot for time derivative):

$$\begin{aligned}
\ddot{\vec{x}}_1 &= \frac{m_2}{|r_{12}|^3}\vec{r}_{12} - \frac{m_3}{|r_{31}|^3}\vec{r}_{31} \\
\ddot{\vec{x}}_2 &= \frac{m_3}{|r_{23}|^3}\vec{r}_{23} - \frac{m_1}{|r_{12}|^3}\vec{r}_{12} \\
\ddot{\vec{x}}_3 &= \frac{m_1}{|r_{31}|^3}\vec{r}_{31} - \frac{m_2}{|r_{23}|^3}\vec{r}_{23}
\end{aligned}$$

where $|r_{ij}|$ is the magnitude of the vector $\vec{r}_{ij}$.

By introducing the variables $\vec{v}_i = \dot{\vec{x}}_i$, we have the full set of 12 equations (six times two components x/y each):

$$\dot{\vec{v}}_1 = \frac{m_2}{|r_{12}|^3}\vec{r}_{12} - \frac{m_3}{|r_{31}|^3}\vec{r}_{31}$$
$$\dot{\vec{v}}_2 = \frac{m_3}{|r_{23}|^3}\vec{r}_{23} - \frac{m_1}{|r_{12}|^3}\vec{r}_{12}$$
$$\dot{\vec{v}}_3 = \frac{m_1}{|r_{31}|^3}\vec{r}_{31} - \frac{m_2}{|r_{23}|^3}\vec{r}_{23}$$
$$\dot{\vec{x}}_1 = \vec{v}_1$$
$$\dot{\vec{x}}_2 = \vec{v}_2$$
$$\dot{\vec{x}}_3 = \vec{v}_3$$

Now we can use the fourth-order Runge-Kutta integrator to plot orbits of the three bodies. As a test, we set the third body to be at the origin and have zero velocity so that the system acts as a two-body problem. The proof-of-concept is shown in Figure 1.

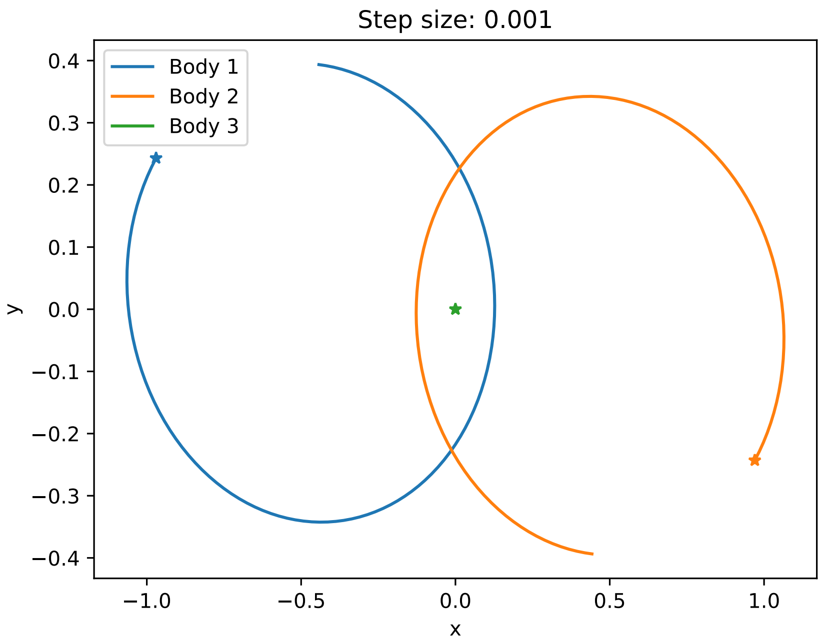

Figure 1: Fourth-order Runge-Kutta proof-of-concept. Stars indicate starting location. Results are similar for other step sizes.

Part a

Simply plugging in the specificied initial conditions yields the plot in Figure 2. Results are indistinguishable for step sizes between 0.01 and 0.001.

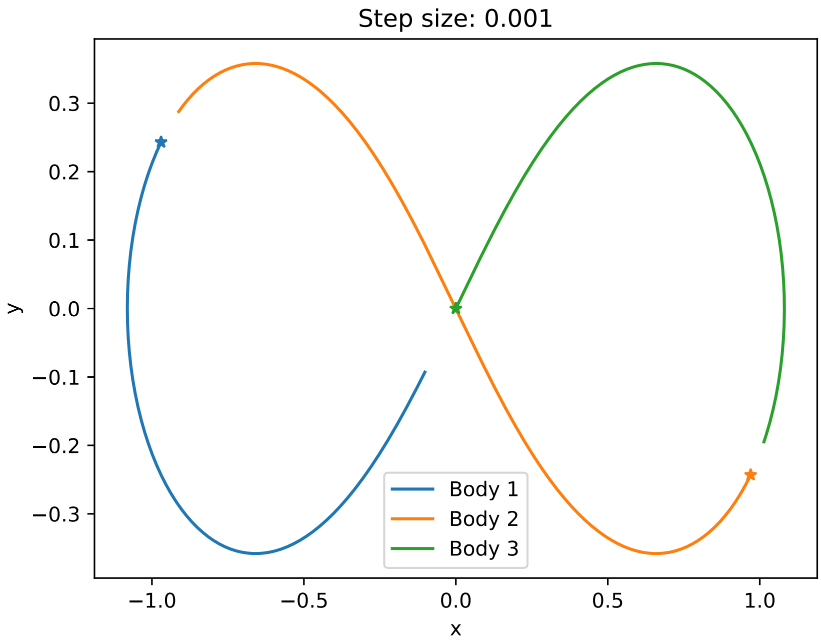

Figure 2: Fourth-order Runge-Kutta three-body integration. Stars indicate starting location. Results are similar for other step sizes.

Part b: Trajectories

Meissel-Burrau Problem using RK4
[h!]

Part b: Distances

[h!]

Part b: Total Energy Error

[h!]

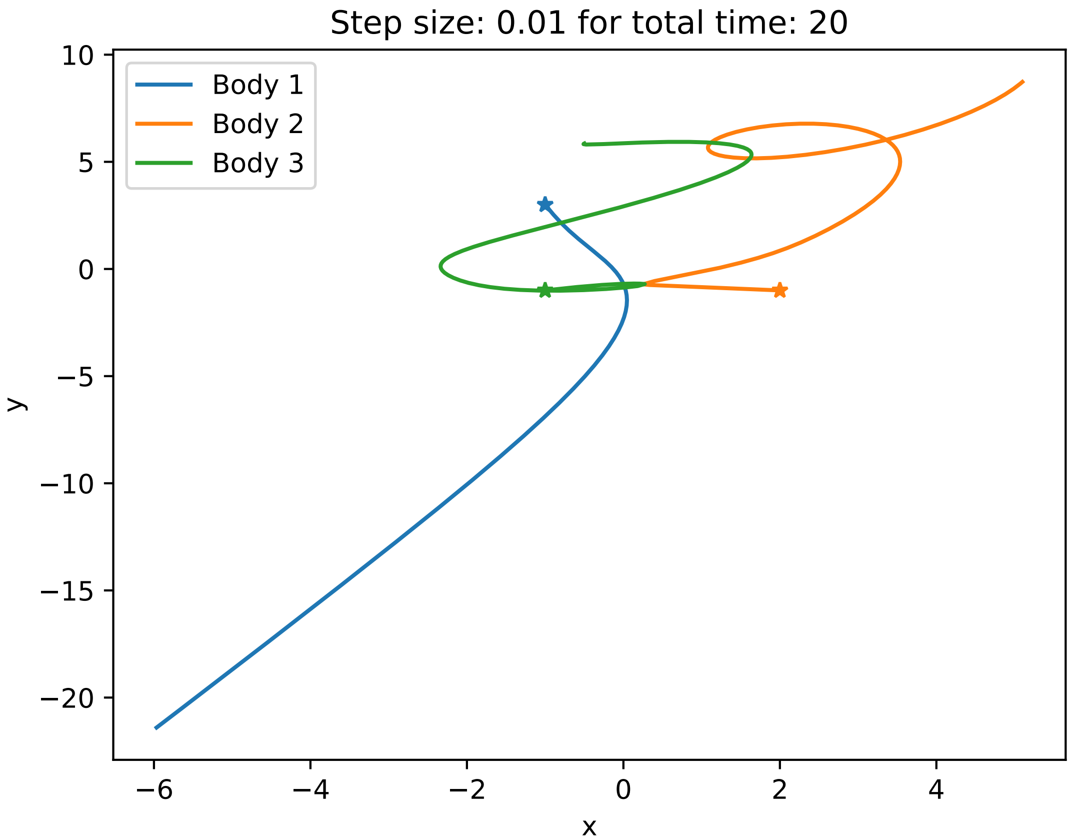

Figure 3: Trajectories h = 0.001

The code for the exercises is as follows:

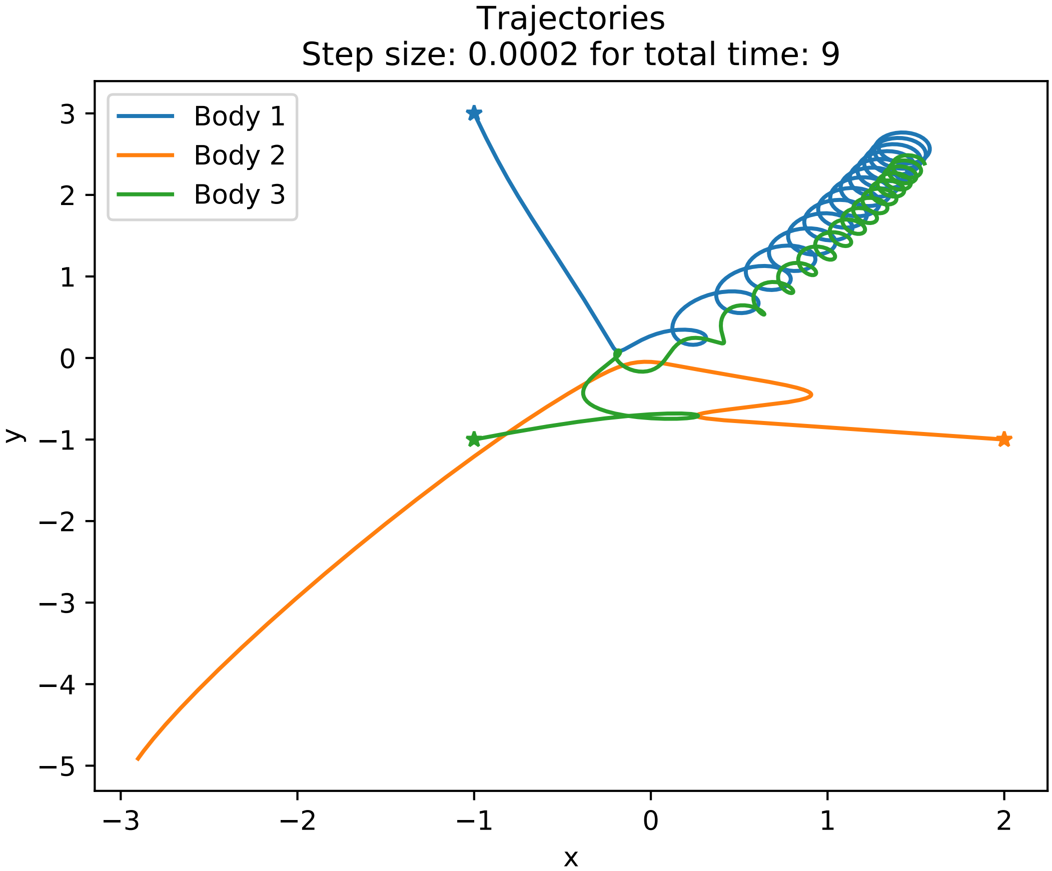

Figure 4: Trajectories h = 0.0002

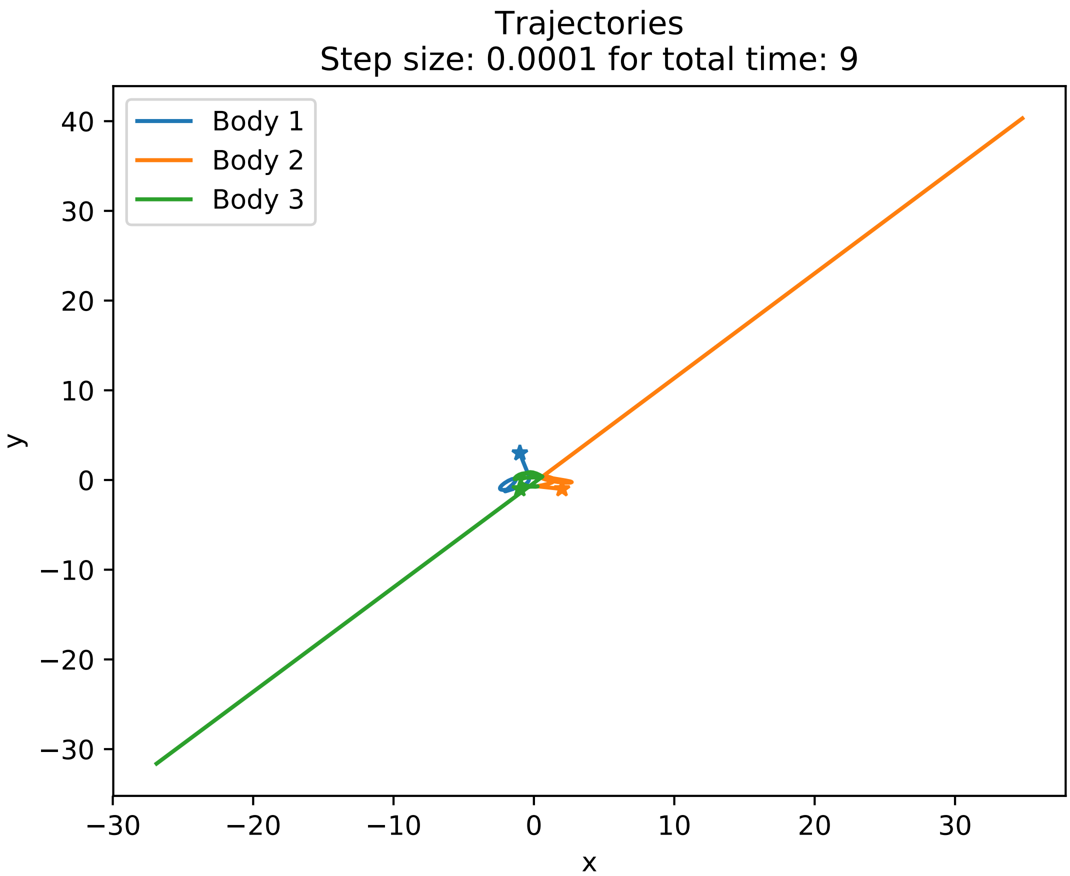

Figure 5: Trajectories h = 0.0001

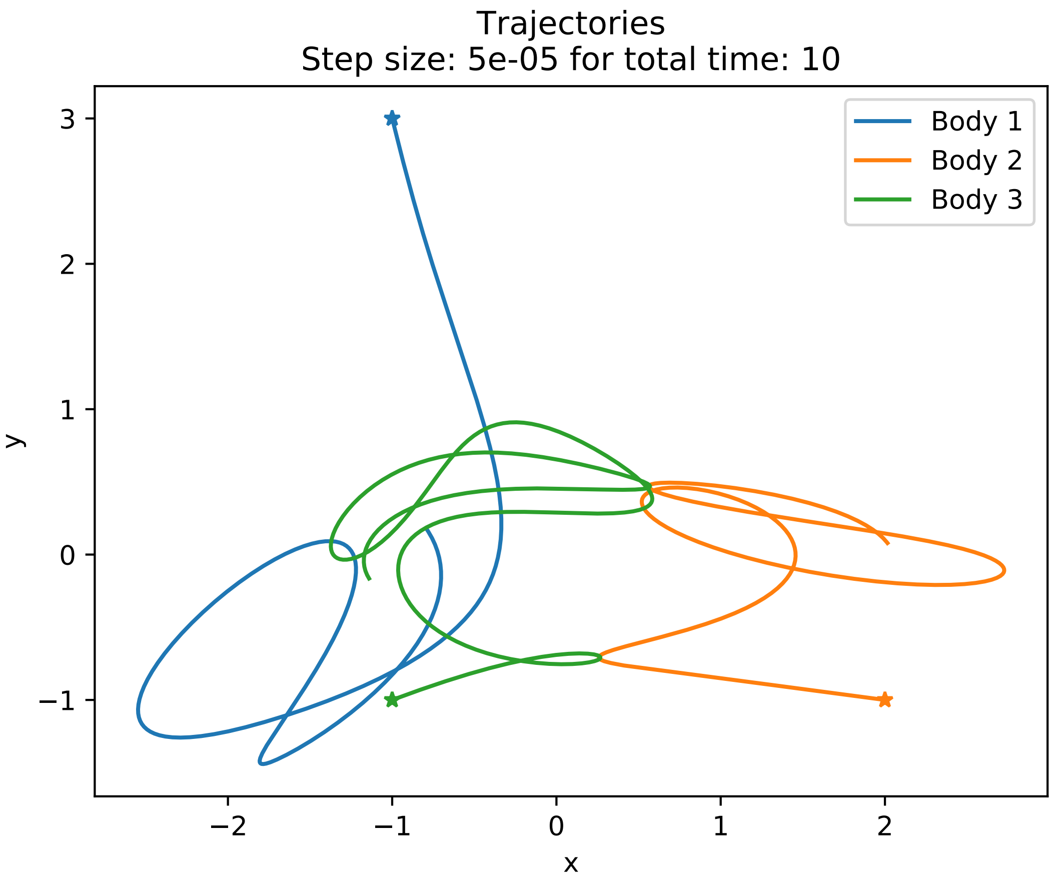

Figure 6: Trajectories h = 0.00005

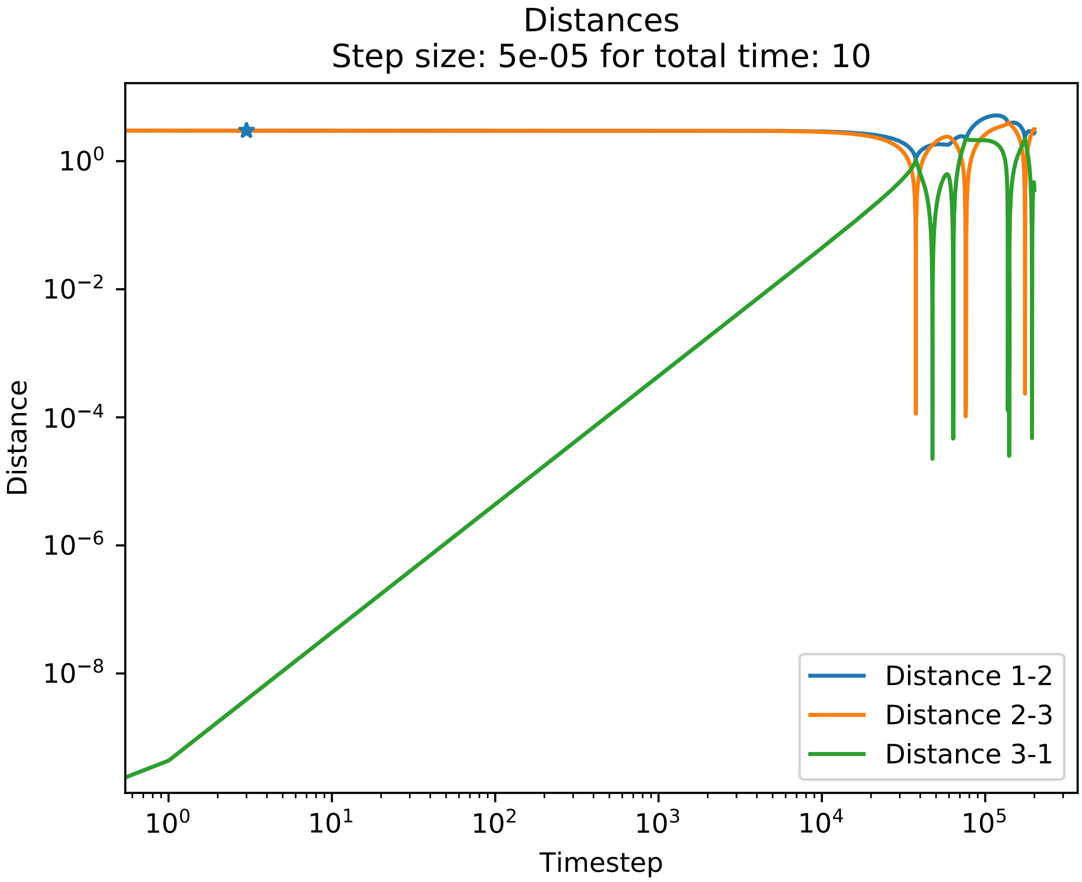

Figure 7: Distances, h = 0.00005

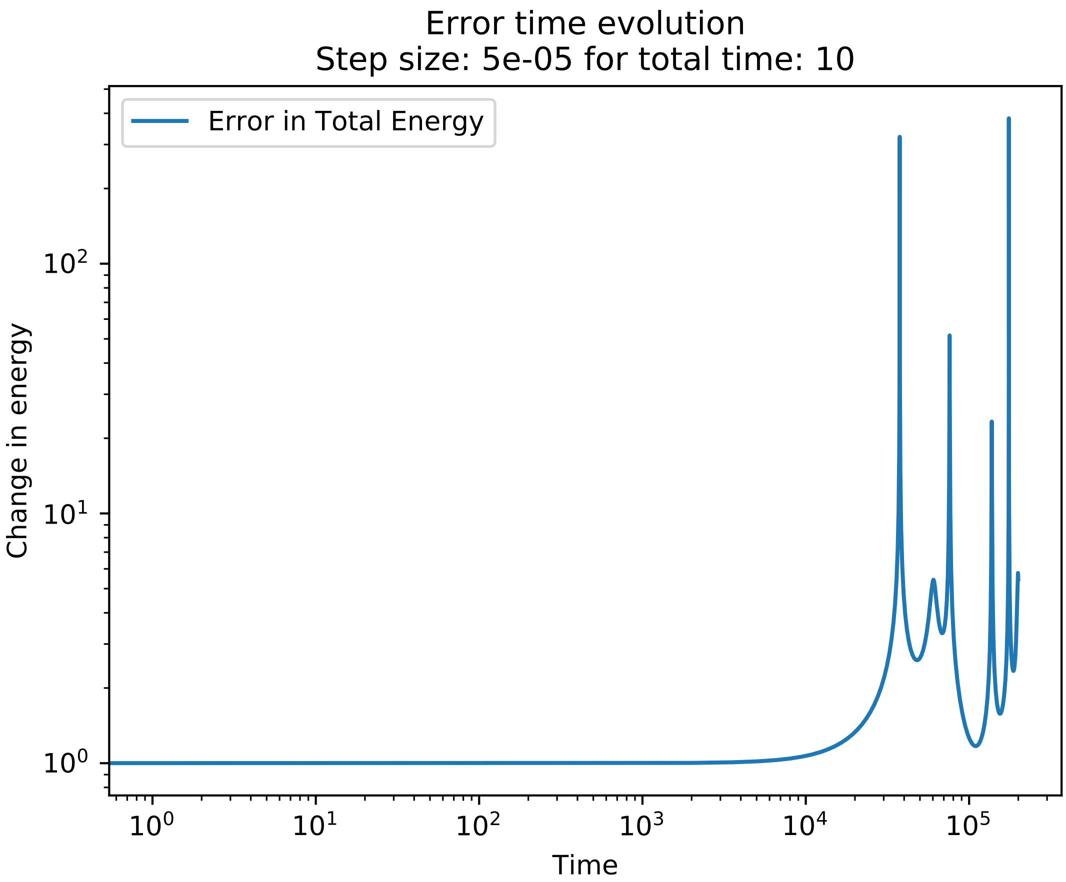

Figure 8: Error in Total Energy