Exercises III

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Exercise 2

We first need to calculate the 12 equations that we need to solve. For a systems of 3-bodies with masses m_1, m_2, and m_3 and initial positions $\vec{x}_1$, $\vec{x}_2$, and $\vec{x}_3$, this is straightforward. Note that we restrict the motion to a plane so that $\vec{x}_i = x_{ix}\hat{x} + x_{iy}\hat{y}$.

Defining separation vectors

$$\begin{aligned}
\vec{r}_{12} &= \vec{x}_2 - \vec{x}_1 \\
\vec{r}_{23} &= \vec{x}_3 - \vec{x}_2 \\
\vec{r}_{31} &= \vec{x}_1 - \vec{x}_3
\end{aligned}$$

the equations of motion are (setting Newton's gravitational constant $G = 1$ and using dot for time derivative):

$$\begin{aligned}
\ddot{\vec{x}}_1 &= \frac{m_2}{|r_{12}|^3}\vec{r}_{12} - \frac{m_3}{|r_{31}|^3}\vec{r}_{31} \\
\ddot{\vec{x}}_2 &= \frac{m_3}{|r_{23}|^3}\vec{r}_{23} - \frac{m_1}{|r_{12}|^3}\vec{r}_{12} \\
\ddot{\vec{x}}_3 &= \frac{m_1}{|r_{31}|^3}\vec{r}_{31} - \frac{m_2}{|r_{23}|^3}\vec{r}_{23}
\end{aligned}$$

where $|r_{ij}|$ is the magnitude of the vector $\vec{r}_{ij}$.

By introducing the variables $\vec{v}_i = \dot{\vec{x}}_i$, we have the full set of 12 equations (six times two components x/y each):

$$
\begin{aligned}
\dot{\vec{v}}_1 &= \frac{m_2}{|r_{12}|^3}\vec{r}_{12} - \frac{m_3}{|r_{31}|^3}\vec{r}_{31} \\
\dot{\vec{v}}_2 &= \frac{m_3}{|r_{23}|^3}\vec{r}_{23} - \frac{m_1}{|r_{12}|^3}\vec{r}_{12} \\
\dot{\vec{v}}_3 &= \frac{m_1}{|r_{31}|^3}\vec{r}_{31} - \frac{m_2}{|r_{23}|^3}\vec{r}_{23} \\
\dot{\vec{x}}_1 &= \vec{v}_1 \\
\dot{\vec{x}}_2 &= \vec{v}_2 \\
\dot{\vec{x}}_3 &= \vec{v}_3
\end{aligned}
$$

Now we can use the fourth-order Runge-Kutta integrator to plot orbits of the three bodies. As a test, we set the third body to be at the origin and have zero velocity so that the system acts as a two-body problem. The proof-of-concept is shown in Figure 1.

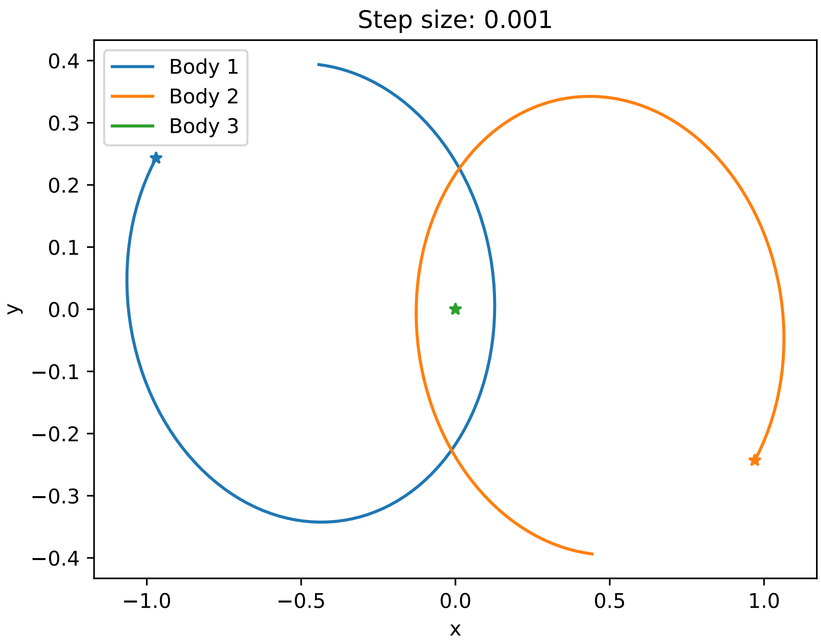

Figure 1: Fourth-order Runge-Kutta proof-of-concept. Stars indicate starting location. Results are similar for other step sizes.

Part a

Simply plugging in the specificied initial conditions yields the plot in Figure 2. Results are indistinguishable for step sizes between 0.01 and 0.001.

The code for the exercises is as follows:

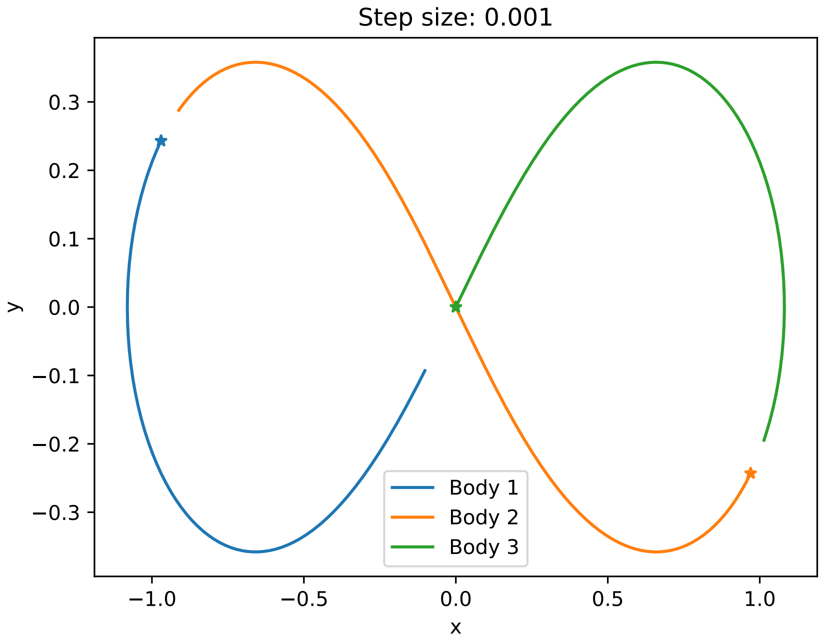

Figure 2: Fourth-order Runge-Kutta three-body integration. Stars indicate starting location. Results are similar for other step sizes.