Thesis

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Abstract

(what is accretion) When matter falls into black hole.. Accretion is efficient source of energy...would release XXX ergs of energy...so why don't we see this from black hole at center of galaxy? Such flows are called LLAGNs, ongoing theory. Typical calculations assume plasma is fluid model, But this type of accretion is hot, plasma is almost collisionless, so kinetic theory more appropriate. can we approximate the computationally-expensive kinetic theory with a fluid closure? We simulate using shearing box and find effective viscosity/resistivity....and find that [results]

Contents

Chapter 1

Introduction

Black holes are now a popular fixture in science fiction books and movies. From tiny primordial black holes (XXXXXX) to

but things rarely fall radially inward. Usually the matter has some angular momentum. Since angular momentum is conserved, a disk usually forms around the black hole as a repository for rotating matter. Processes within the disk then lead to turbulence and an effective friction that results in an outward transport of angular momentum and hence the falling in, or "accretion", of matter. For reasons discussed in Chapter **??**, the infalling matter (mostly hydrogen) heats up past the ionization threshold. We shall thus refer to the matter as a plasma; different types of plasma and the effect on accretion flows will be discussed in Chapter **??**. For now, we motivate this paper by noting that gravitational binding energy is released as matter falls into a black hole, a process that is among the most efficient energy sources in the universe (several hundred times more efficient than fusion [**?**]). This energy presumably goes into radiation and thus

These processes and the debate surrounding them are discusssed throughout the paper, particularly

accretion=falling in. But how does accretion work?

Yay introduction.

1.1 AGN and Accretion Disk Observations

How explain low luminosity from AGN (Sgr A*), x-ray binaries in quiescence
rotation measure stability in Sgr A (Sharma, Quataert, Stone 2008)

Chapter 2

Classification and Basic Physics of Accretion Disks

Accretion disks occur in many different types, from those are protostars to those in binary star systems and those around black holes. Depending on their context, these accretion flows have different properties. While much of the physics of accretion disks involves magnetohydrodynamics (including the mechanism behind accretion itself) as will be discussed in subsequent chapters, the classification of accretion flows is possible even at a more basic level: for example, hot vs. cold accretion flows.

This chapter clarifies some of the jargon in the extensive literature and characterizes the different types of accretion flows in Section 2.1, focusing on hot accretion flows (the subject of the original research in Chapter **??**) and their properties in Section 2.2. Some fundamental physics are presented in Sections **??** and **??**: respectively radiation processes and mechanics. These sections provide background for more complex calculations such as those in Chapter **??** as well as a connection back to observations.

2.1 Types of Accretion Flows

Accretion flows are generally characterized by their temperature, their radiative efficiency, and/or their thickness. These types and the relationships among them will be clarified and discussed soon enough: we first note that accretion does not have to lead to the formation of a disk. Indeed, some of the earliest studies on accretion concerned matter falling in radially and uniformly from all directions onto a compact object: spherical accretion [**?**].

However, such accretion flows are unlikely to occur in nature because matter will almost always be rotating with respect to the compact object and hence have angular momentum.

Another general type of accretion flows

2.2 Properties of Hot Accretion Flows

2.3 Radiation Processes

observations...

2.4 Mechanics

This section presents a series of calculations pertaining to important elements of accretion disks.

2.4.1 Eddington Luminosity

Radiation from a compact object generates a radiation pressure that begins to counter the gravitational force from the central object. When the luminosity is high enough, the object's gravity is no longer enough to keep it together. When the radiation pressure and gravity exactly cancel, the system is in equilibrium. We can derive the value of the luminosity at this equilibrium, the "Eddington luminosity" or "Eddington limit" as follows (following [**?**]xxxformatxx):

Assume the radiation has a flux F_{rad} and an opacity, or scattering cross-section times mass, of κ (for ionized hydrogen, $\kappa = \sigma_T/m_p$ where m_p is the mass of the proton and σ_T is the Thomson scattering cross-section of the electron). Then the force balance is given by:

$$\nabla\Phi = \frac{\kappa}{c}F_{\mathrm{rad}} \tag{2.1}$$

The luminosity is defined as the total energy output per time, so we can get it from the flux (amount of light per area per time) by integrating over a surface, which we assume to be spherical:

$$L = \int_S F_{\rm rad} \cdot dS = \frac{c}{\kappa} \int_S \nabla\Phi \tag{2.2}$$

$$= \frac{c}{\kappa} \int_V \nabla^2\Phi = \frac{4\pi Gc}{\kappa} \int_V \rho dV \tag{2.3}$$

$$= \frac{4\pi GMc}{\kappa} \tag{2.4}$$

Note that the force balance equation does not include any forces other than pressure-gravity equilibrium and the radiation pressure; magnetic forces, for example, are excluded.

Black holes can exceed the Eddington luminosity because the energy contained in the infalling matter does not have to be radiated; it can simply be swallowed by the black hole and increase its mass. At high accretion rates, radiation could become trapped by the flow and advect into the black hole. This would lead to low observed luminosity.

2.4.2 Disk Scale Height

The scale height of a disk is a useful scale for defining distances. It comes from considering a stratified disk; that is, one that has a component of gravity in the vertical direction. As Figure **??** illustrates, the gravitational force in the vertical direction is proportional to the force along the purely horizontal ("R") direction and the ratio of the distances:

$$\frac{F_{gR}}{F_{gz}} = \frac{R}{z} \tag{2.5}$$

Introducing the Keplerian rotation speed $\Omega^2 = GM/R^3$, we have

$$F_{gz} = F_{gR}\frac{z}{R} = \frac{GM}{R^2}\frac{z}{R} \qquad\qquad = \Omega^2 z$$

Now relate F_{gz} to the disk density by considering the force balance equation for equilibrium in the vertical direction:

$$-\frac{1}{\rho}\frac{dP}{dz} = F_{gz}$$

Assuming an isothermal equation of state $P = \rho c^2$, this equation can be integrated:

$$-\frac{1}{\rho}c^2\frac{d\rho}{dz} = F_{gz} = \Omega^2 z$$
$$\rho(z) = \rho_0 e^{-\frac{\Omega^2}{c^2}\frac{z^2}{2}} \qquad = \rho_0 e^{-\frac{z^2}{2H^2}}$$

where $H^2 \equiv \frac{c^2}{\Omega^2}$. H can thus be interpreted as the distance a sound wave travels in a shear time (i.e., one rotation). Generally the thickness of a disk is taken to be $2H$. As will be discussed in Section **??**, it is often convenient to choose units such that $H = 1$, resulting in numerical domains with dimensions of $(H, 4H, H)$.

2.4.3 Rayleigh Stability Criterion

In hydrodynamics, disks with a Keplerian velocity profile are stable against perturbations. To see this, simply consider perturbations about a circular orbit in an azimuthally symmetric potential $\Phi = \Phi(r, z)$. In cylindrical coordinates we have

$$\hat{r} = \cos\theta\hat{x} + \sin\theta\hat{y} \tag{2.6}$$
$$\hat{\theta} = -\sin\theta\hat{x} + \cos\theta\hat{y} \tag{2.7}$$

Upon taking the time derivatives, we find that

$$\begin{aligned}\ddot{r} &= \frac{d}{dt}\dot{r} = \frac{d}{dt}\left(\dot{r}\hat{r} + r\frac{d\hat{r}}{dt} + \dot{z}\hat{z}\right) \\ &= \ddot{r}\hat{r} + 2\dot{r}\frac{d\hat{r}}{dt} + r\frac{d^2\hat{r}}{dt^2} + \ddot{z}\hat{z} \\ &= \ddot{r}\hat{r} + 2\dot{r}\dot{\theta}\hat{\theta} + r\ddot{\theta}\hat{\theta} - r\dot{\theta}^2\hat{r} + \ddot{z}\hat{z} \\ &= \left(\ddot{r} - r\dot{\theta}^2\right)\hat{r} + \left(\frac{1}{r}\frac{d}{dt}r^2\dot{\theta}\right)\hat{\theta} + \ddot{z}\hat{z}\end{aligned}$$

From Newton's laws we have

$$F = -m\nabla\Phi = m\ddot{r} \tag{2.8}$$

leading to component-wise equilibrium equations:

$$-\frac{\partial \Phi}{\partial r} = \ddot{r} - r\dot{\theta}^2 \tag{2.9}$$

$$-\frac{1}{r}\frac{\partial \Phi}{\partial \theta} = 0 = \frac{1}{r}\frac{d}{dt}\left(r^2\dot{\theta}\right) \tag{2.10}$$

$$-\frac{\partial \Phi}{\partial z} = \ddot{z} \tag{2.11}$$

Note that Eq. 2.10 gives conservation of specific momentum h_z: i.e., $h_z = r^2\dot{\theta}$ is a constant. For a circular orbit, $\ddot{r} = 0$ and we have $\frac{1}{r_0}\frac{\partial \Phi}{\partial r} = \dot{\theta}^2 = \Omega_0^2$, where we define $\Omega_0^2 \equiv \frac{1}{r_0}\frac{\partial \Phi}{\partial r}|_{r_0}$. Now perturbing this circular orbit, we have

$$r = r_0 + \delta r \tag{2.12}$$

$$\theta = \Omega_0 t + \delta\theta \tag{2.13}$$

$$z = 0 + \delta z \tag{2.14}$$

We obtain equations of motion by linearizing Eqns. 2.9-2.11 and using the perturbed variables:

$$\begin{aligned}
-\frac{\partial \Phi}{\partial r} &\approx -\left(\frac{\partial \Phi}{\partial r}\Big|_{r_0} + \delta r\frac{\partial^2 \Phi}{\partial r^2}\Big|_{r_0}\right) = -\left(\Omega_0^2 r_0 + \delta r\frac{\partial^2 \Phi}{\partial r^2}\Big|_{r_0}\right) \\
&\approx \delta\ddot{r} - r_0\Omega_0^2 - 2\Omega_0 r_0\delta\dot{\theta} - \delta r\Omega_0^2 && (2.15) \\
h_z &\approx r_0^2\Omega + r_0^2\delta\dot{\theta} + 2r_0\Omega_0\delta r \\
&= r_0^2\Omega_0 && (2.16) \\
-\frac{\partial \Phi}{\partial z} &\approx -\left(\delta z\frac{\partial \Phi}{\partial z}\Big|_{r_0,z_0}\right) \\
&\approx \delta\ddot{z} && (2.17)
\end{aligned}$$

From Eq. 2.16, we find that $r_0^2\delta\dot{\theta} = -2r_0\Omega_0\delta r$, and thus $\delta\dot{\theta} = -\frac{2}{r_0}\Omega_0\delta r = -\frac{2}{r_0^3}h_z\delta r$. Using this in the radial equation 2.15 yields

$$\begin{aligned}
\ddot{r} &= 2\Omega_0 r_0\left(-\frac{2}{r_0^3}h_z\delta r\right) + \delta r\left(\Omega_0^2 - \frac{\partial^2 \Phi}{\partial r^2}\Big|_{r_0}\right) && (2.18) \\
&= -\left(3\Omega_0^2 + \frac{\partial^2 \Phi}{\partial r^2}\Big|_{r_0}\right)\delta r && (2.19) \\
&= -\kappa_0^2\delta r && (2.20)
\end{aligned}$$

where $\kappa_0^2 \equiv 3\Omega_0^2 + \frac{\partial^2 \Phi}{\partial r^2}|_{r_0}$ is the epicyclic frequency. The physical significance of this becomes clear upon writing the θ-equation of motion as a coupled harmonic oscillator with the radial equation: a particle will simply execute small circular orbits about its zeroth-order circular path as it travels. This will become an important mechanism distinguishing disk flow from shear flow, as energy can go into epicycles instead of turbulence. Section **??** illustrates this point in the context of accretion.

The epicyclic frequency appears often in astrophysical situations, although under many different guises. For example, remember that $\Omega^2 = \frac{1}{r}\frac{\partial \Phi}{\partial r}$. Then we have

$$\frac{d\Omega^2}{dr} = -\frac{1}{r^2}\frac{\partial \Phi}{\partial r} + \frac{1}{r}\frac{\partial^2 \Phi}{\partial r}$$

such that

$$\frac{\partial^2 \Phi}{\partial r^2} = r\frac{d\Omega^2}{dr} + \frac{1}{r}\frac{\partial \Phi}{\partial r} = r\frac{d\Omega^2}{dr} + \Omega^2$$

With this the epicyclic frequency becomes

$$\begin{aligned} \kappa^2 &= 3\Omega^2 + \frac{\partial^2 \Phi}{\partial r^2} && (2.21) \\ &= 4\Omega^2 \left(1 + \frac{A}{\Omega}\right) && (2.22) \end{aligned}$$

where $A \equiv \frac{r}{4\Omega^2}\frac{d\Omega^2}{dr}$ is the Oort A-value, also expressed $A = -\frac{1}{2}d\Omega/d\ln r$. This can be further re-written:

$$\begin{aligned} \kappa^2 &= 4\Omega^2 \left(1 + \frac{r}{4\Omega^2}\frac{d\Omega^2}{dr}\right) \\ &= \frac{1}{r^2}\left(4\Omega^2 r^3 + r^4 \frac{d\Omega^2}{dr}\right) \\ &= \frac{1}{r^3}\frac{d}{dr}\left(\Omega^2 R^4\right) \end{aligned}$$

which is how it is written in, for example, [**?**]. For Keplerian rotation profiles, $A = -\frac{3}{4}\Omega$, and $\kappa^2 = \Omega^2$.

The stability of hydrodynamical disks can be done in multiple ways. Here, an intuitive argument is first presented showing that angular momentum must decrease outward for stability. This condition is put on firmer theoretical ground in the second method, which also introduces tools that will be used later in performing linear analysis of the magnetohydrodynamic equations (see Section **??**).

Equations for disk...continuity, momentum balance (will add B fields later) Du/Dt=d2u/dt2+du/dx dx/dt

2.4.4 Keplerian vs. Shear Flow

2.4.5 Angular Momentum Conservation in Ideal MHD

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{v}) = 0 \tag{2.23}$$

$$\rho \frac{\partial \vec{v}}{\partial t} + (\rho \vec{v} \cdot \nabla) \vec{v} = -\nabla \left(P + \frac{B^2}{8\pi} \right) - \rho \nabla \Phi + \left(\frac{\vec{B}}{4\pi} \cdot \nabla \right) \vec{B} \tag{2.24}$$

$$\frac{\partial \vec{B}}{\partial t} = \nabla \times \left(\vec{v} \times \vec{B} \right) \tag{2.25}$$

Although it can be re-distributed, angular momentum is ultimately conserved in ideal MHD systems. Since conservation of angular momentum is a central idea to the rest of this paper, this section will show how to achieve it from Eq. 2.24 (neglecting gravity). The basic idea is to multiply the ϕ-component by R and re-arrange terms into conservative form, as per [**?**]. Before re-arranging, the full equation is

$$R \left[\rho \frac{\partial \vec{v}}{\partial t} + (\rho \vec{v} \cdot \nabla) \vec{v} = -\nabla \left(P + \frac{B^2}{8\pi} \right) - \rho \nabla \Phi + \left(\frac{\vec{B}}{4\pi} \cdot \nabla \right) \vec{B} \right]_\phi \tag{2.26}$$

where the notation of $[\ ]_\phi$ indicates the ϕ-component of the expression in square brackets. It is best to continue term-by-term.

The density terms are rather straightforward:

$$\begin{aligned} \left[R\rho \frac{\partial \vec{v}}{\partial t} \right]_\phi &= R\rho \frac{\partial v_\phi}{\partial t} \\ &= \frac{\partial}{\partial t} (R\rho v_\phi) - v_\phi R \frac{\partial \rho}{\partial t} \\ &= \frac{\partial}{\partial t} (R\rho v_\phi) + v_\phi R \nabla \cdot (\rho \vec{v}) \\ &= \frac{\partial}{\partial t} (R\rho v_\phi) + \nabla \cdot (R\rho v_\phi \vec{v}) - \rho \vec{v} \cdot \nabla (v_\phi R) \end{aligned} \tag{2.27}$$

where in the second line, the continuity equation 2.23 was used and in the fourth, the identity A.1 was used. Now we look more closely at the last term on the right:

$$\rho \vec{v} \cdot \nabla (v_\phi R) = \rho \left(v_R v_\phi + R v_R \frac{\partial v \phi}{\partial R} + v_\phi \frac{\partial v_\phi}{\partial \phi} + R v_z \frac{\partial v_\phi}{\partial z} \right) \tag{2.28}$$

However, this is exactly the ϕ-component of $(R\rho\vec{v}\cdot\nabla)\,\vec{v}$, as can be seen by Eq. A.3. Adding Eqns. 2.27 and 2.28, we have:

$$R\rho\frac{\partial v_\phi}{\partial t} + \left[R\left(\rho\vec{v}\cdot\nabla\right)\vec{v}\right]_\phi = \frac{\partial}{\partial t}\left(R\rho v_\phi\right) + \nabla\cdot\left(R\rho v_\phi\vec{v}\right) \tag{2.29}$$

which is in conservative form.

The pressure term is straightforward:

The magnetic terms require more insight. Here we introduce the poloidal magnetic field; that is, the components of the magnetic field in the $R-$ and $z-$directions. Thus $B^2 = B_p^2 + B_\phi^2$ and $B_p^2 = B_R^2 + B_z^2$. Beginning with the magnetic pressure term,

$$\begin{aligned}
\left[R\nabla B^2\right]_\phi &= \hat{e}_\phi\cdot\left[R\nabla B^2\right] \\
&= \hat{e}_\phi\cdot\left[R\nabla B_p^2 + R\nabla B_\phi^2\right] \\
&= \hat{e}_\phi\cdot\left[R\nabla B_p^2\right] + \frac{1}{R}\frac{\partial}{\partial\phi}\left(RB_\phi^2\right) \\
&= \hat{e}_\phi\cdot\left[\nabla\left(RB_p^2\right)\right] + 2B_\phi\frac{\partial B_\phi}{\partial\phi}
\end{aligned}$$

where we can move R into the derivative because $\partial R/\partial\phi = 0$. Now, we add zero via the term $\nabla\cdot\hat{e}_\phi$:

$$\begin{aligned}
\left[R\nabla B^2\right]_\phi &= \hat{e}_\phi\cdot\left[RB_p^2\;\nabla\cdot\hat{e}_\phi + \hat{e}_\phi\cdot\nabla\left(RB_p^2\right) + 2B_\phi\frac{\partial B_\phi}{\partial\phi}\right] \\
&= \nabla\cdot\left(RB_p^2\hat{e}_\phi\right) + 2B_\phi\frac{\partial B_\phi}{\partial\phi}
\end{aligned} \tag{2.30}$$

where Eq. A.1 was used in the last line. We have thus achieved part of the conservative form, but with an extra $\partial B_\phi^2/\partial\phi$ term. We now turn to the magnetic tension term in hopes that it will cancel this extra term. First, however, we note using $\nabla\cdot\vec{B} = 0$ and the definition of $\vec{B}_p$ that

$$\begin{aligned}
\nabla\cdot\vec{B}_p &= \nabla\cdot\left(\vec{B} - B_\phi\hat{e}_\phi\right) \\
&= -\nabla\cdot\left(B_\phi\hat{e}_\phi\right) \\
&= -\frac{1}{R}\frac{\partial B_\phi}{\partial\phi}
\end{aligned} \tag{2.31}$$

The magnetic tension term becomes

$$
\begin{aligned}
\left[R\left(\vec{B}\cdot\nabla\vec{B}\right)\right]_\phi &= RB_R\frac{\partial B_\phi}{\partial R} + B_\phi\frac{\partial B_\phi}{\partial \phi} + RB_z\frac{\partial B_\phi}{\partial z} + B_R B_\phi \\
&= \left[B_R\left(R\frac{\partial B_\phi}{\partial R} + B_\phi\right) + B_z R\frac{\partial B_\phi}{\partial z}\right] + B_\phi\frac{\partial B_\phi}{\partial \phi} \\
&= (B_R, 0, B_z)\cdot\left(\frac{\partial}{\partial R}(RB_\phi), 0, \frac{\partial}{\partial z}(RB_\phi)\right) + B_\phi\frac{\partial B_\phi}{\partial \phi} \\
&= \vec{B}_p\cdot\nabla\left(RB_\phi\right) + B_\phi\frac{\partial B_\phi}{\partial \phi} \qquad (2.32)
\end{aligned}
$$

Using Eq. 2.31, we add zero and draw out a total divergence:

$$
\begin{aligned}
\left[R\left(\vec{B}\cdot\nabla\vec{B}\right)\right]_\phi &= \vec{B}_p\cdot\nabla\left(RB_\phi\right) - RB_\phi\nabla\cdot\vec{B}_p + 2B_\phi\frac{\partial B_\phi}{\partial \phi} \qquad (2.33) \\
&= \nabla\cdot\left(RB_\phi\vec{B}_p\right) + 2B_\phi\frac{\partial B_\phi}{\partial \phi} \qquad (2.34)
\end{aligned}
$$

where Eq. A.1 was used in the last line. Armed with each individual term, we combine them (Eqs. 2.29, 2.30, and 2.34) to find the conservative form of the angular momentum equation:

$$
\begin{aligned}
&\left[R\rho\frac{\partial\vec{v}}{\partial t} + R\left(\rho\vec{v}\cdot\nabla\right)\vec{v} + R\,\nabla P + R\,\nabla\frac{B^2}{8\pi} - R\,\left(\frac{\vec{B}}{4\pi}\cdot\nabla\right)\vec{B}\right]_\phi = \\
&\qquad \frac{\partial}{\partial t}\left(R\rho v_\phi\right) + \nabla\cdot\left(R\rho v_\phi\vec{v}\right) + PRESSURE \\
&\qquad\qquad + \frac{1}{8\pi}\left[\nabla\cdot\left(RB_p^2\hat{e}_\phi\right) + 2B_\phi\frac{\partial B_\phi}{\partial\phi}\right. \\
&\qquad\qquad \left. -2\nabla\cdot\left(RB_\phi\vec{B}_p\right) - 2B_\phi\frac{\partial B_\phi}{\partial\phi}\right] \\
&\quad = \frac{\partial}{\partial t}\left(R\rho v_\phi\right) + \nabla\cdot R\left[\rho v_\phi\vec{v} + PRESSURE + \frac{B_p^2}{8\pi}\hat{e}_\phi - \left(\frac{B_\phi\vec{B}_p}{4\pi}\right)\right] \qquad (2.35)
\end{aligned}
$$

Chapter 3

History and Simulation of Accretion Disks

This chapter examines the development of the theory behind accretion disks, with the ultimate goal of framing the past and current research on accretion disks. Due to the complexity of the equations involved (see Chapter ??), numerical simulations play an integral role in understanding accretion flows. However, said complexity also means that the power of numerical simulations to resolve interesting features has been and is limited by the computational ability and algorithms utilized. The theory and numerical simulations of accretion disks have thus evolved in step, with new insights coming from both sides. As such, it is difficult to disentangle the pure theory from the numerical simulations and the following chapter does not attempt to do so. It is instead an intertwined, mostly chronological account that showcases the thought processes and impetus behind new developments.

We first trace back to some of the early ideas of accretion disks and how they were formed (Section 3.1), leading into Section ??'s discussion of possible sources of turbulence and then Section ??'s modern hypotheses and what this means for the original research of this paper (e.g. Chapter ??).

3.1 Early Thoughts on Accretion

The study of accretion in the modern era emerged from simpler questions than that of AGN luminosity. In 1941, Kuiper [?] recognized the formation of a disk during mass transfer in a binary-star system, while Hoyle and Lyttleton [?] attributed the climatic variation on Earth to the accretion of interstellar matter around the sun. Once compact x-ray sources such as AGNs were actually observed, the role of disks in the production of radiation was explored more thoroughly [?]. At first, however, the basic physics of accretion disks was under discussion.

As mentioned in Section ??, accretion involves the outward transport of angular momentum, which means the slowing down of particles as they drop into closer orbits (Section ??). It seems natural to explain this slowing down via friction; in an accretion disk (or torus), the matter at different radii are not moving at the same velocity (i.e. there is a shear) and hence one might think that there is a sort of coefficient of kinetic friction between particles that slows down their movement and causes them to accrete. The idea that this "molecular" or "shear" viscosity could explain accretion rates was tempting, but not supported by simulations. Early simulations solving the MHD equations for a disk showed accretion rates on the order of XXXXXXXXXX (CITE: XXXXXXXSTILL need some simulations showing molecular rates!!XXXXXXXXXxx), XXXXXXxalso observations!XXXXXXx; however, according to the standard values of molecular viscosity (in, for example, [?]XXXXXXXX make sure cite format right), the standard values of molecular viscosity are around XXXXXXXx. This fantastic difference between theory and simulations resulted in several new ideas for explaining the transport of angular momentum and led to the formulation of one of the most well-known models for thin disks–the α-disk.

The seminal paper of [?] Xmake sure format right!!!XXXXXx explores accretion disks in the context of a binary star system. It essentially characterizes ignorance in the accretion rate via the parameter α, defining the tangential stress $w_{r\phi} = \alpha \rho v_s^2$, where v_s is the sound speed such that $\rho v_s^2/2$ is the disk matter's thermal energy density, although definitions vary to order unity across sources [?]. This formulation temporarily removed the need to explain the source of the viscosity and provides a parameter that is easy to tweak in numerical simulations. Although the original paper takes α as a constant for simplicity, it is generally a function of radius. The relevance of the α parameter is apparent even today, as it is a simple way to gain intuition in accretion problems despite its debated value [?].

Despite its intuitive usefulness, the α prescription offers no mechanism for the transport of angular momentum. XXXXXXXXX proposed that, while pure molecular viscosity could not explain the observed accretion rates, an "effective" viscosity due to eddy interaction could do the job [?]. In other words, turbulence would generate eddies whose interactions would manifest similar to a viscosity. The problem became to find the source of the turbulence that would lead to outward angular momentum transport. This question is the subject of the next section.

3.2 Potential Sources of Turbulence

Supposing that an effective viscosity generated by turbulence can explain observed and simulated accretion rates, the question becomes: what causes this turbulence?

3.2.1 Large Reynolds Number

Some, influenced by laboratory fluid mechanics, believed that the sheer property of having a high Reynolds number (the product of a characteristic velocity and length scale divided by the viscosity; huge in astrophysical flows due to the large length scales involved) satisfactorily accounted for the needed turbulence. The mechanism at hand is called "vortex stretching": due to vortex conservation, the stretching of vortices in a shear flow increases the circulation velocity around the vortex tube. This allows for free energy to be extracted from the shear flow [**?**].

Others, however, suspected that, at least in accretion disks, the flow was fundamentally different than those shear flows explored in the aforementioned fluid mechanics labs. Indeed, as demonstrated nicely in [**?**] and reproduced in Section **??**, Keplerian flows are stable against perturbations (i.e. experience no turbulence) where shear flows are not (given the Rayleight stability criterion is satisfied). The difference is due to epicycles in Keplerian flows, which sink the energy that would otherwise devolve into prominent disturbances. A high Reynolds number is not enough to explain the necessary turbulence.

3.2.2 Convective and Self-gravitating Instabilities

It was long thought that hydrodynamic instabilities could lead to turbulence in accretion disks. One such idea proposed by [**?**]XXXXformatXxx in very flat disks was that gravitational instabilities similar to the Jeans instability could lead to internal heating which would in turn limit the development of the instability, maintaining a somewhat unstable state. While the paper raised important questions concerning heat transport, the effect of the instability is small in hot disks [?].

Convective instabilities have garnered the most interest in terms of generating hydrodynamic turbulence. After all, the Schwarzschild condition...

3.2.3 Nonlocal Effects

If the mechanism for producing turbulence were global, its effect would not be captured by a local viscosity parameter. [?] mentions several possibilities for generating turbulence this way, including waves and shocks created by tidal forces. These effects can produce accretion at rates up to $\alpha = .01$, but only in hot disks. Global disk winds, of the type suggested by [?]XXformat?XX, could also transport angular momentum. These magnetically-driven winds could theoretically sweep matter around in such a way as to account for the high accretion rates without a viscosity while also helping account for AGN jets [?]; however, the presence of these winds in all accretion disks is debated. A more universal and fundamental explanation seems more likely.

3.2.4 Magnetic Fields

Magnetic fields were thought to serve an amplifying role in turbulence transport. That is, with pre-existing turbulence, magnetic fields would tangle and speed along the transportation of magnetic fields [?]. It was thought that the magnetic pressure and pressure due to turbulence were distinct, and that magnetic pressure would be insignificant in disk situations [?], or would require large magnetic fields on the order of $10^7 - 10^8$ G to balance the gravitational pressure of infalling gas [?]. The magnetic field was mainly considered to be important due to consequences of cyclotron radiation as a cooling mechanism [?]. It was not until the early 1990s that the full significance of magnetic fields was appreciated.

In 1991, Balbus and Hawley [?, ?] closed the conceptual circle by showing that turbulence resulted directly from a weak magnetic field. Pre-existing turbulence was not needed; the entire sequence of generating turbulence and transporting turbulence and angular momentum could be derived as a result of a linear instability in the MHD equations (see Section 4.1.1). Numerous numerical simulations have since confirmed the important role of magnetic fields in accretion processes. The next section will give an overview of these numerical simulations and thereby place the simulations in section **??** in context.

3.3 Post-MRI Discovery Developments

shearing box artificial viscosity numerical viscosity/resistivity

why reproduce stone pringle2001...

**lead into accretion theory? then MRI?

3.3.1 Current Research

Although the basic source of turbulence in accretion disks seems to be the MRI, there are still many alterations to GR simulations Foucart et al 2016 Still 2D! Including 2-temperature model, as discussed inXXx. Sadowski et al 2016 Outflows and jets (Bu, Wu, Yuan 2016) still no 3d mti rotating? parrish stone=nonrotating? KINETIC!

Chapter 4

Plasma Astrophysics Theory

Distribution function, MHD equations, kinetic, vlasov eq.

4.1 MRI Basics

The magnetorotational instability (MRI) is a linear instability of the ideal magnetohydrodynamic equations that leads to increased outward angular momentum transport, hence providing the driving mechanism for accretion. In section 4.1.1 the linear analysis itself will be provided; section 4.1.2 will discuss a more physical picture.

4.1.1 MRI Derivation

The MRI arises out of a simple linear analysis of the ideal MHD equations as outlined in section **??**.

4.1.2 MRI Spring Analogy

4.1.3 MRI in Shear and Keplerian Flows

Appendix A

Identities and Derivations?

There is lots of appendices.

A.1 Vector Identities

This section is just for useful identities. A good reference is the NRL Plasma Formulary [**?**].

$$\nabla \left(f\vec{A} \right) = f\nabla \cdot \vec{A} + \vec{A} \cdot \nabla f \tag{A.1}$$

$$\nabla \tag{A.2}$$

A.2 Cylindrical Coordinates

Components of

$$\left[\left(\vec{A} \cdot \nabla \right) \vec{B} \right]_{\phi} = \tag{A.3}$$