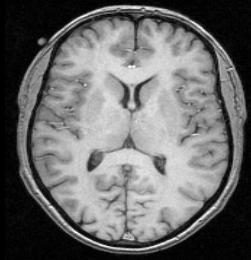


Stanford
MEDICINE

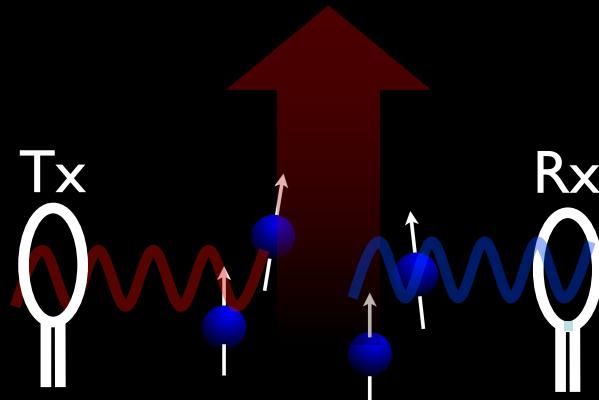
Lecture 14: MRI (chapter 12)

BMP 211



MRI Acquisition

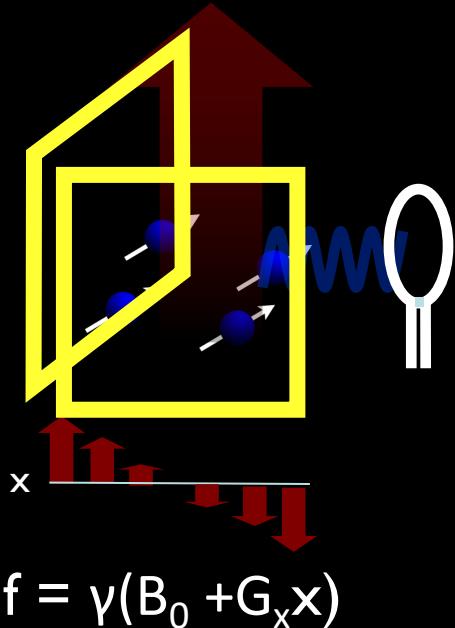
excite and receive



Magnetic dipoles made of water protons precess in plane and flip field = γB_0

- Big magnetic field (B_0)
- RF **excitation** coil sends an RF pulse to excites dipoles
- RF **reception** coil detects signal from excited dipoles

Reception and Image generation



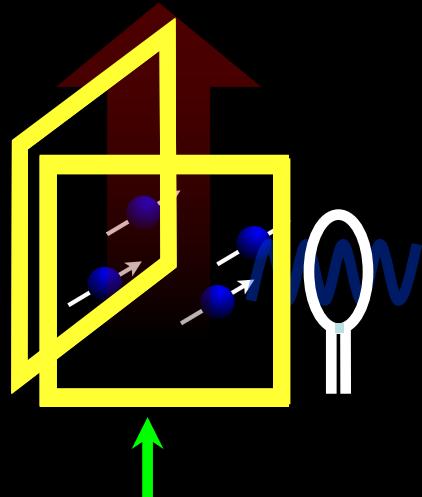
How to differentiate signal from
different spatial location?

→ Spatially encode using
magnetic gradients (G_x & G_y)

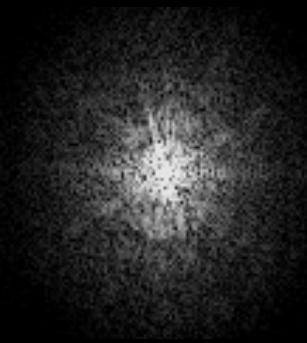
$$f(x,y) = \gamma(B_0 + G_x x + G_y y)$$

Principle of MRI

Acquisition of k-space data



X and Y gradients are applied to spatially encode the location of spins



K-space

Principle of MRI

Image reconstruction

k-space

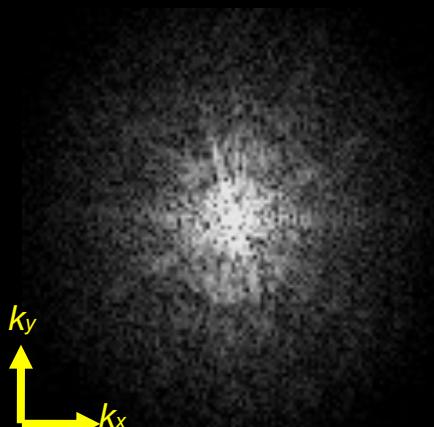
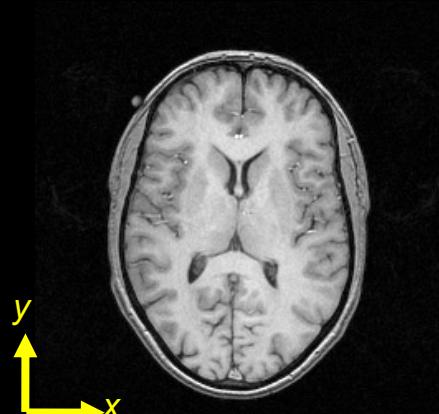
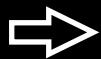


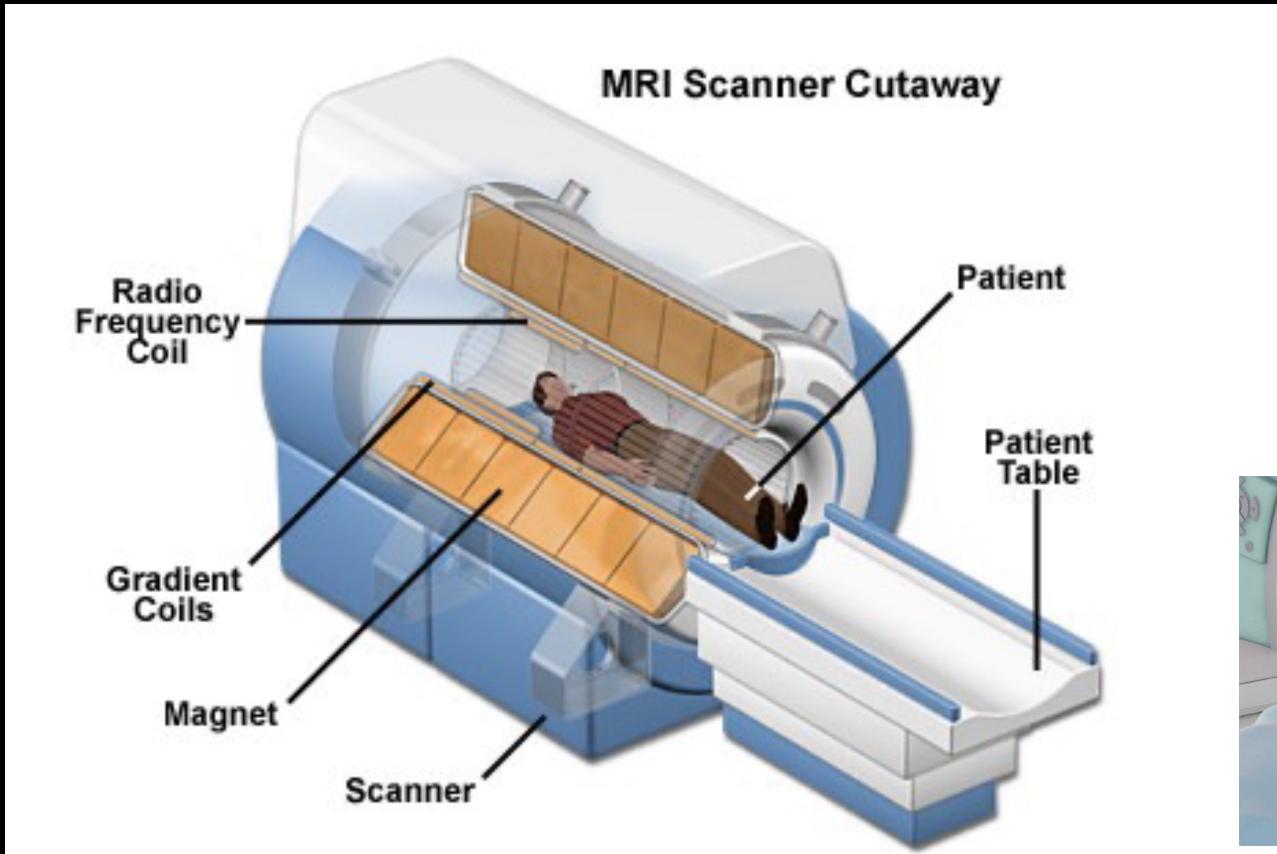
Image space



2DFT⁻¹

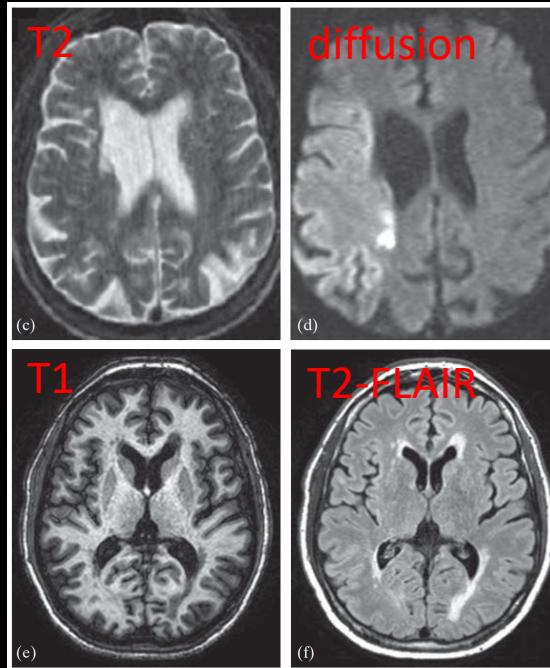
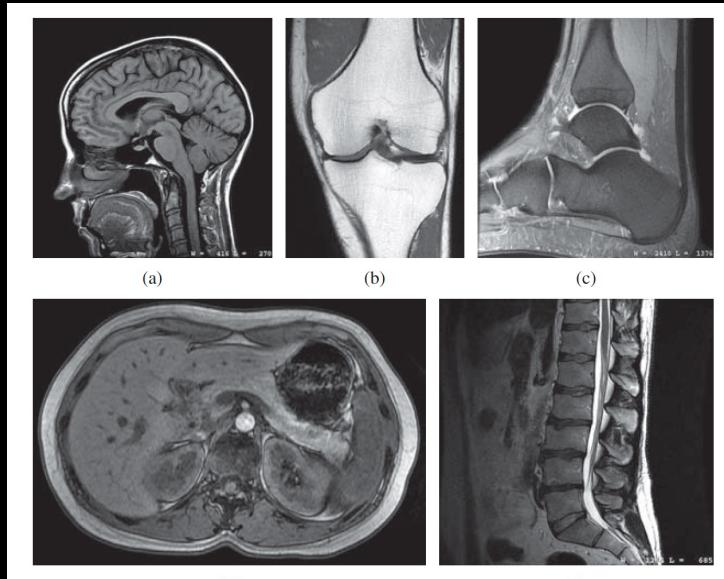


Principle of MRI



Magnetic Resonance Imaging

Pro: High resolution, non-ionizing, excellent soft-tissue contrast w/ many types generated based on tissue properties.



Stroke picked up by diffusion imaging (restricted diffusion)

Multiple Sclerosis Better w/ T2-FLAIR

Con: Expensive equipment, slow imaging speed, e.g. CT exam in minutes, MRI exam 15-40 min slots, bone is done better w/ CT

A Brief History of NMR & MRI

- Based on the principle of Nuclear Magnetic Resonance discovered by Bloch and Purcell *simultaneously* in 1946 (awarded 1952 Nobel Prize in Physics)
- The initial concept for the medical application of NMR originated with Damadian in 1971
- NMR imaging of two tubes of water first demonstrated by Lauterbur in 1973
- Slice selection (and other things) invented by Mansfield in 1973
- First commercial system developed by EMI in 1975
- 2003 Nobel Prize for Physiology or Medicine award to Lauterbur and Mansfield for “their discoveries concerning magnetic resonance imaging”



Edward Purcell
(1912-1997)



Felix Bloch
(1901-1999)

“Magnetic Resonance”



Raymond Damadian
(1936-)



Peter Mansfield
(1933-2017)

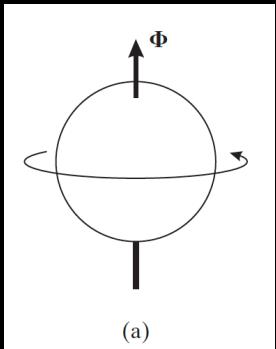


Paul Lauterbur
(1929-2007)

“Imaging”

(Nuclear) Magnetic Resonance (NMR)

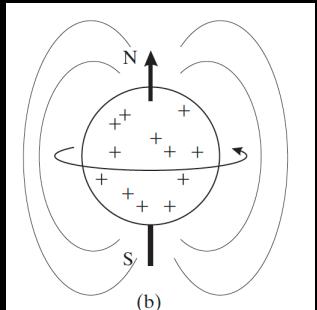
Microscopic Magnetization (w/ Quantum mechanics)



(a)

Nuclear spin system w/ angular momentum (ϕ)

- Nucleus w. odd # of protons, or # of (protons + neutron)
- ^1H , ^{13}C , ^{19}F , ^{31}P
- ^1H main one; nucleus has single proton: 'proton imaging'



(b)

Magnetic Moment (μ)

- A positive spinning charge create magnetic field (like loop of wire)

$$\mu = \gamma \phi$$

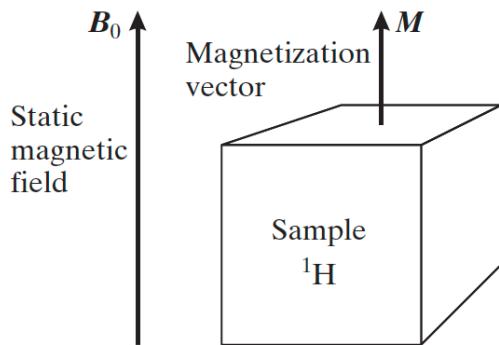
$$\gamma = \frac{\nu}{2\pi},$$

Nucleus	γ [MHz/T]
^1H	42.58
^{13}C	10.71
^{19}F	40.05
^{31}P	11.26

γ is the gyromagnetic ratio (dependent on nuclear system)

- magnetic moment (μ) no prefer orientation until introduced to external field, $B_0 = B_0 \underline{z}$
- then try to align to this field

Macroscopic Magnetization (w/ classical mechanics)



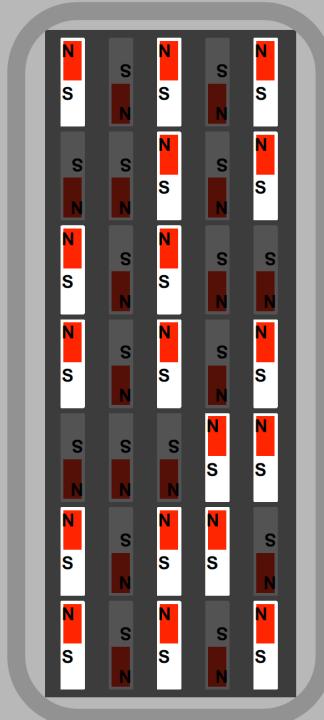
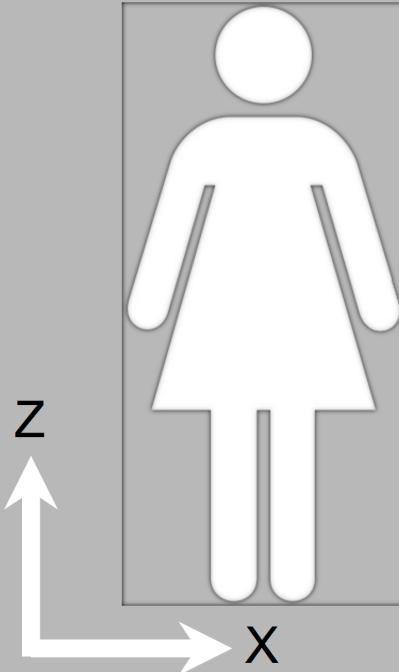
Bulk magnetization

$$\mathbf{M} = \sum_{n=1}^{N_s} \boldsymbol{\mu}_n .$$

$$M_0 = \frac{B_0 \gamma^2 h^2}{4kT} P_D ,$$

- M_0 is signal source in MRI
- P_D : proton density (as in hydrogen nucleus)
- Larger B_0 will give you more M_0
 \rightarrow 1.5T, 3T and now 7T for clinical systems

B₀ Field ON



N
S
Spin-Up

S
N
Spin-Down

$$\frac{N_{\uparrow} - N_{\downarrow}}{N_{total}} \approx \frac{\gamma h B_0}{2K T} \text{ Zeeman Splitting}$$

$$\begin{aligned}\gamma &= 42.58 \times 10^6 \text{ Hz/T} \\ h &= 6.6 \times 10^{-34} \text{ J·s} [\text{Planck' Constant}] \\ T &= 300 \text{ K (room temperature)} \\ K &= 1.38 \times 10^{-23} \text{ J/K} [\text{Boltzmann Constant}] \\ B_0 &= 1.5 \text{ T}\end{aligned}$$

$$\frac{N_{\uparrow} - N_{\downarrow}}{N_{total}} \approx \frac{42.58 \times 10^6 \cdot 6.6 \times 10^{-34} \cdot 1.5}{2 \cdot 1.38 \times 10^{-23} \cdot 300} \approx 4.5 \times 10^{-6}$$

$$\vec{M} = \sum_{n=1}^{N_{total}} \vec{\mu}_n$$

Only a very small number are spin-up relative to spin-down (Zeeman splitting).



- B_0 field always on to create signal source (M_0z)
- Add time_varying B_1 and G fields to excite signal and do image encoding.
- Need ‘equation of motion’ to see how B_0 , B_1 , G fields impact M

Bulk angular momentum (analogous to microscopic pic)

$$\mathbf{M} = \gamma \mathbf{J}.$$

External field exerts torque on M to change its angular momentum

$$\frac{d\mathbf{J}(t)}{dt} = \mathbf{M}(t) \times \mathbf{B}(t),$$

$$\frac{d\mathbf{M}(t)}{dt} = \gamma \mathbf{M}(t) \times \mathbf{B}(t),$$

$$\frac{d\mathbf{M}(t)}{dt} = \gamma \mathbf{M}(t) \times \mathbf{B}(t),$$

If $\mathbf{B} = B_0 \hat{\mathbf{z}}$, then can show that solution is:

$$M_x(t) = M_0 \sin \alpha \cos(-\gamma B_0 t + \phi),$$

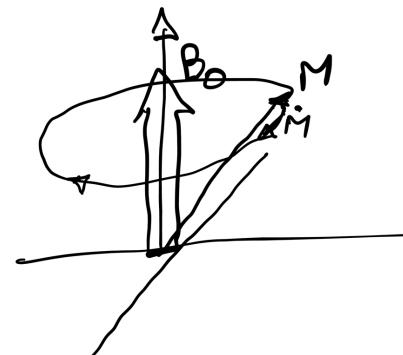
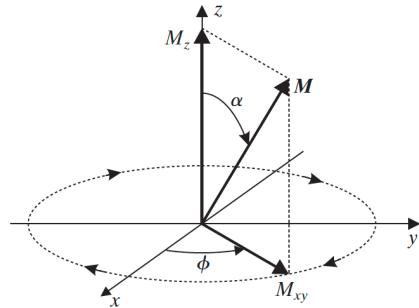
$$M_y(t) = M_0 \sin \alpha \sin(-\gamma B_0 t + \phi),$$

$$M_z(t) = M_0 \cos \alpha,$$

Larmor frequency

$$\omega_0 = \gamma B_0,$$

$$\nu_0 = \gamma B_0,$$



Free-precession

Source of B_0 inhomogeneities

- i) System hardware main field (normally kept v. small, few ppm related to field strength)
- ii) Magnetic susceptibility from material property

- Diamagnetic
- Paramagnetic
- Ferromagnetic



$$\hat{B}_0 = B_0(1 + \chi)$$

where χ is diamagnetic susceptibility

- Body mostly diamagnetic from carbon and O₂ so slightly lower the field
- Iron deposited in brain or removal of oxygen in blood hemoglobin due to neuronal activity can also change χ and be detected.

- iii) Chemical shift

- Local property: molecules to which 1H atom is attached.
- Local electron cloud shield nucleus from full effect of B_0
- ζ is shielding constant
- E.g. Hydrogen nuclei in fat (CH₂) shift by 3.35 ppm from H₂O: -214 Hz at 1.5T ($\gamma\Delta B_0$)

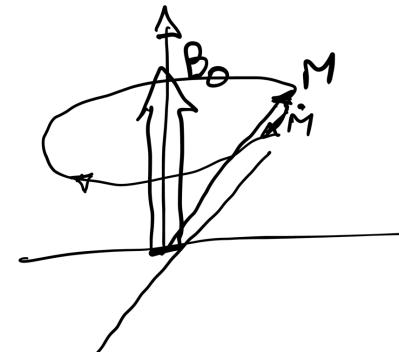
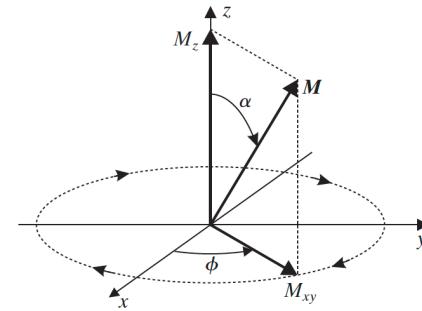
$$\hat{B}_0 = B_0(1 - \zeta),$$

Transverse and Longitudinal Magnetization

$$\vec{M} = M_x \hat{x} + M_y \hat{y} + M_z \hat{z}$$

$\underbrace{M_x \hat{x} + M_y \hat{y}}_{\text{transverse}} + \underbrace{M_z \hat{z}}_{\text{longitudinal}}$

$$M_x + i M_y = |M_{xy}| e^{j\phi}$$
$$= M_0 \sin \alpha e^{-j(z\pi)/y_0 t - \phi}$$

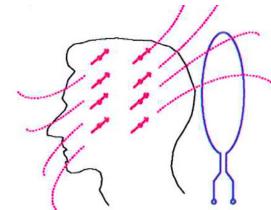


Signal reception

Faraday's law of induction: time varying field across coil will induce voltage.

Principle of reciprocity: look at magnetic field produced from unit current in loop $B^r(r)$

$$V(t) = -\frac{\partial}{\partial t} \int_{\text{object}} \mathbf{M}(\mathbf{r}, t) \cdot \mathbf{B}^r(\mathbf{r}) d\mathbf{r},$$

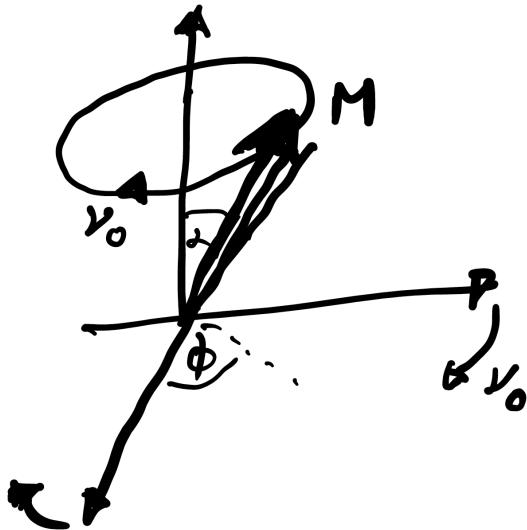


work through a bit of math and assumptions:

$$|V| = 2\pi v_0 V_s M_0 \sin \alpha B^r$$

- voltage proportional to B_0^2 (as both v_0 and M_0)
- α at $\pi/2$ is best.
- V_s is signal per voxel in 'imaging'

Rotating Frame @ Larmor frequency ν_0



$$M_{xy} = M_0 \sin \phi e^{j\phi} e^{-j2\pi\nu_0 t} e^{j2\pi\nu_0 t}$$

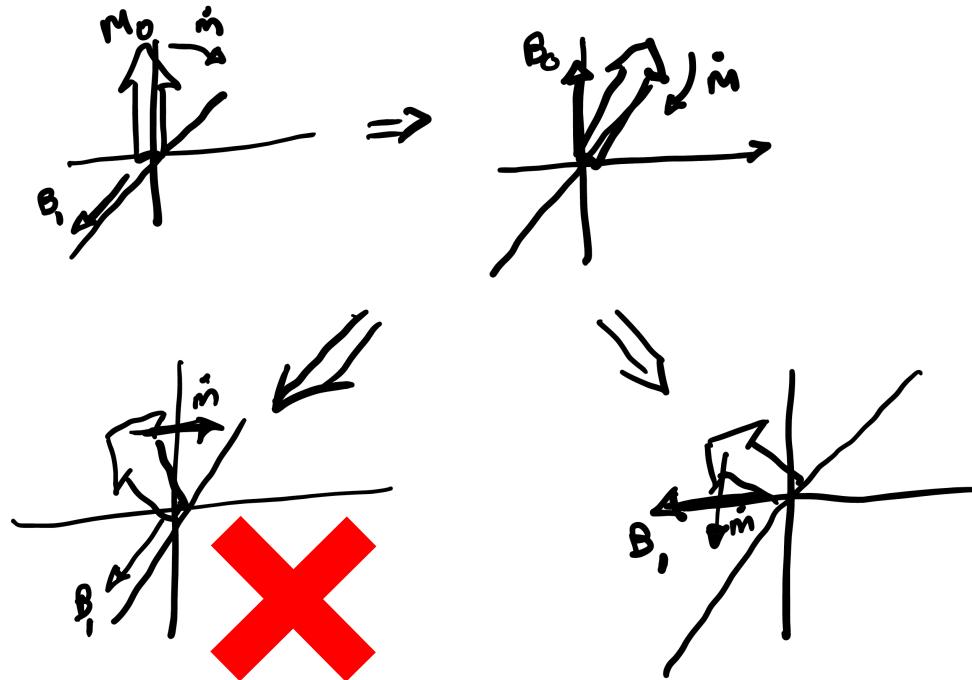
$\underbrace{e^{-j2\pi\nu_0 t} e^{j2\pi\nu_0 t}}_{\cancel{\text{cancel}}}$

$\Rightarrow \text{stationary!}$

Removing a precession will help things be easier to visualize

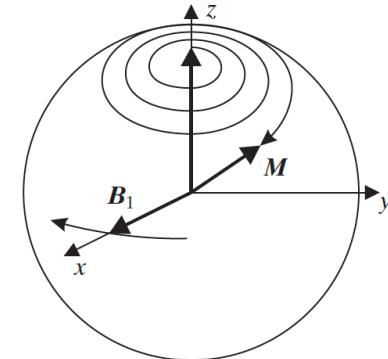
Tipping down M₀ using RF excitation w/ B₁ field

$$\frac{d\mathbf{M}(t)}{dt} = \gamma \mathbf{M}(t) \times \mathbf{B}(t)$$

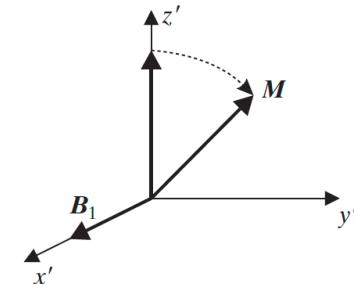


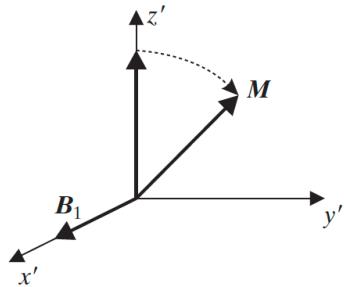
Need B₁ to rotate w/ M @ v₀

Lab frame



Rotating frame





$$B_i = B_i^e(t) e^{-j(2\pi\nu_o t - \phi)}$$

$$B_{i,\text{rot}} = B_i^e(t) e^{j\phi} \quad \boxed{\quad}$$

$$\nu_1 = \gamma B_1$$

$$\begin{aligned} \text{flip} &= 2\pi \int \nu_1 dt \\ &= \gamma \int B_1 dt = \gamma B_1 T_p \end{aligned}$$

Forced-precession

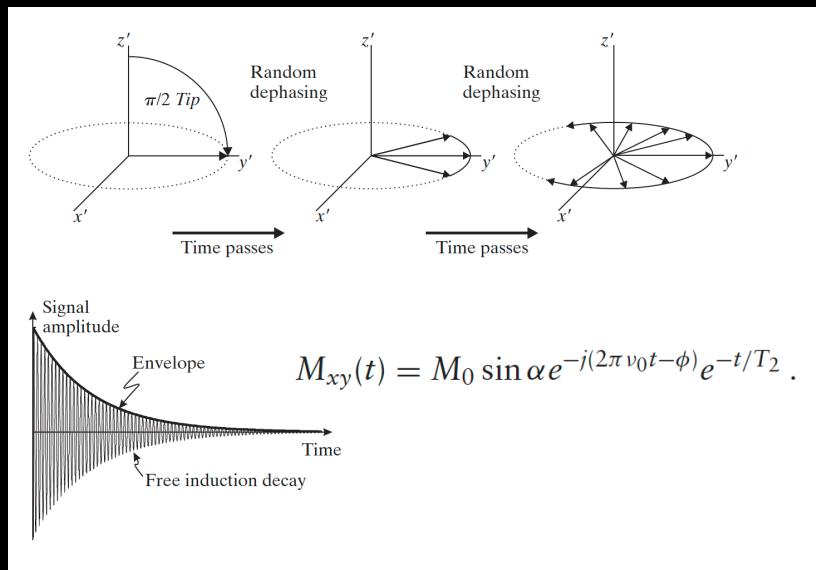
→ for $\frac{\pi}{2}$ pulse of 3 ms

$$B_1 = 1.96 \times 10^{-6} T \quad (\text{v.s. } B_0 = 3 T)$$

Relaxation

Transverse relaxation (M_{xy})

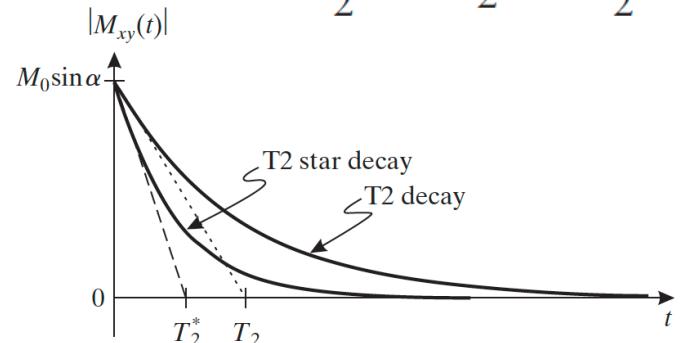
- spin-spin relaxation: perturbation of mag. field from other spins nearby.
- random microscopic motion causes speed up/slow down of precession.



T2 differs for different tissues: create contrast

Additional decay from fixed external field
e.g. susceptibility

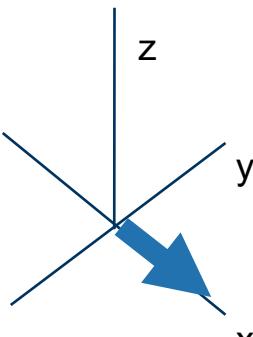
$$\frac{1}{T_2^*} = \frac{1}{T_2} + \frac{1}{T'_2} .$$



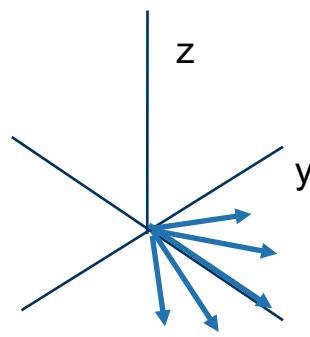
T2*-Dephasing

Wait time TE after excitation before measuring M.

Shorter T2* spins have dephased

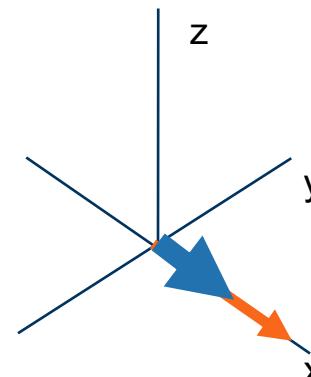


initially



at $t = TE$

vector
sum

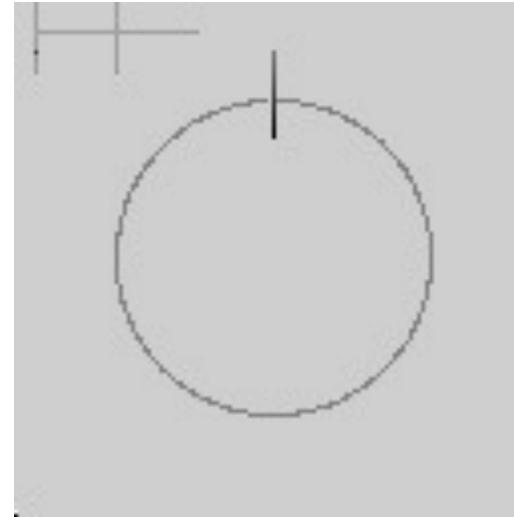
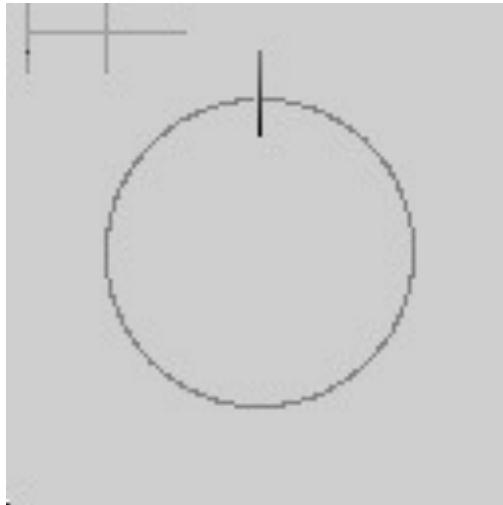


Is smaller...

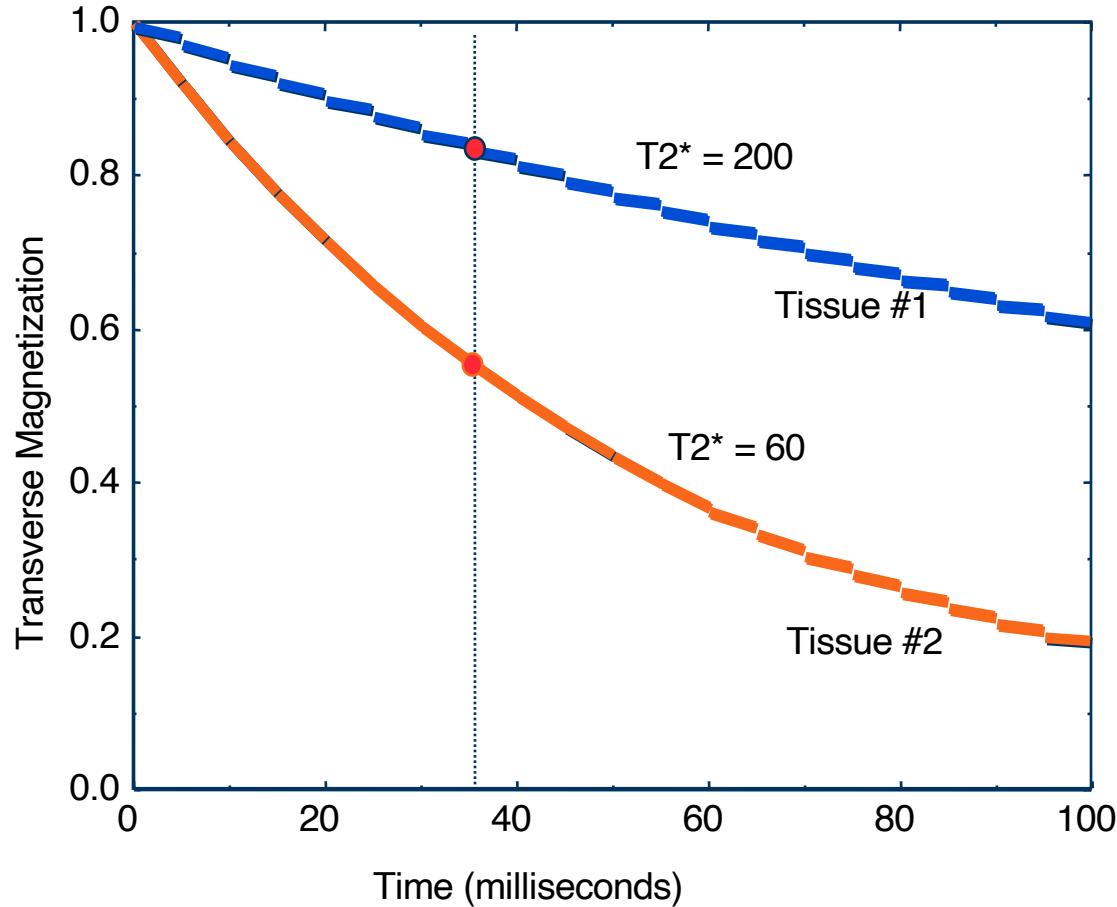
$T2^*$ Dephasing

Just showing the tips of the vectors...

in the laboratory frame and in the rotating frame



$T2^*$ decay graphs

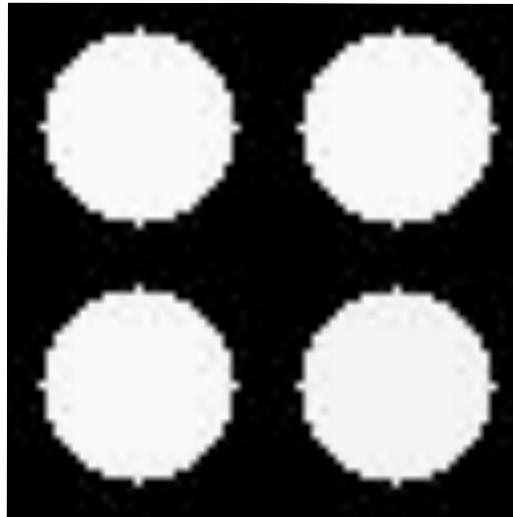


T2* Weighting

Phantoms with
four different T2* decay rates...

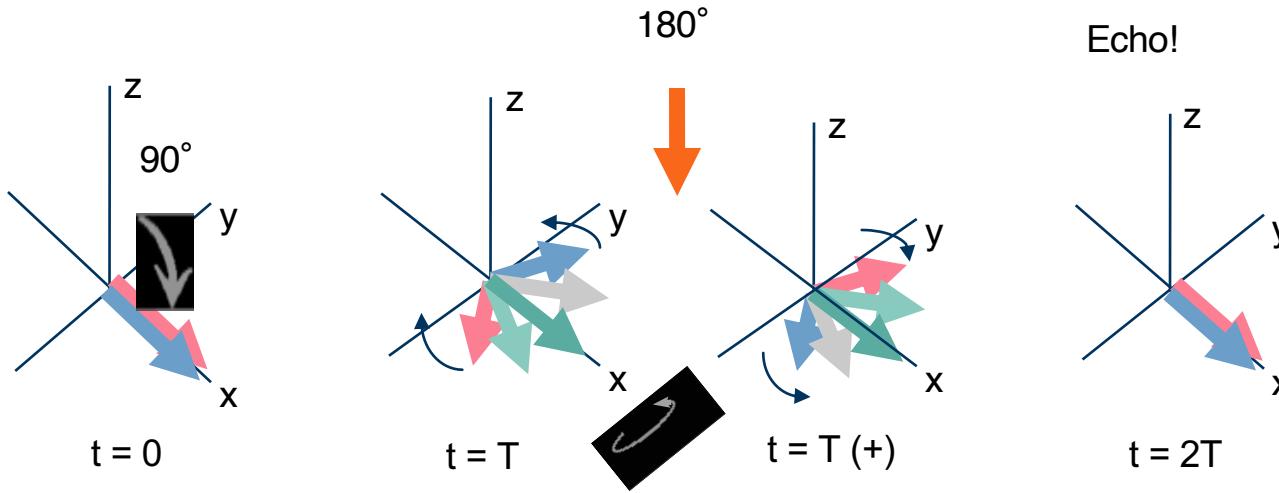
There is no contrast difference
immediately after excitation, must
wait (but not too long!).

Choose TE for maximum intensity
difference.



Spin Echo (T2 contrast)

Some dephasing can be refocused because its due to static fields.



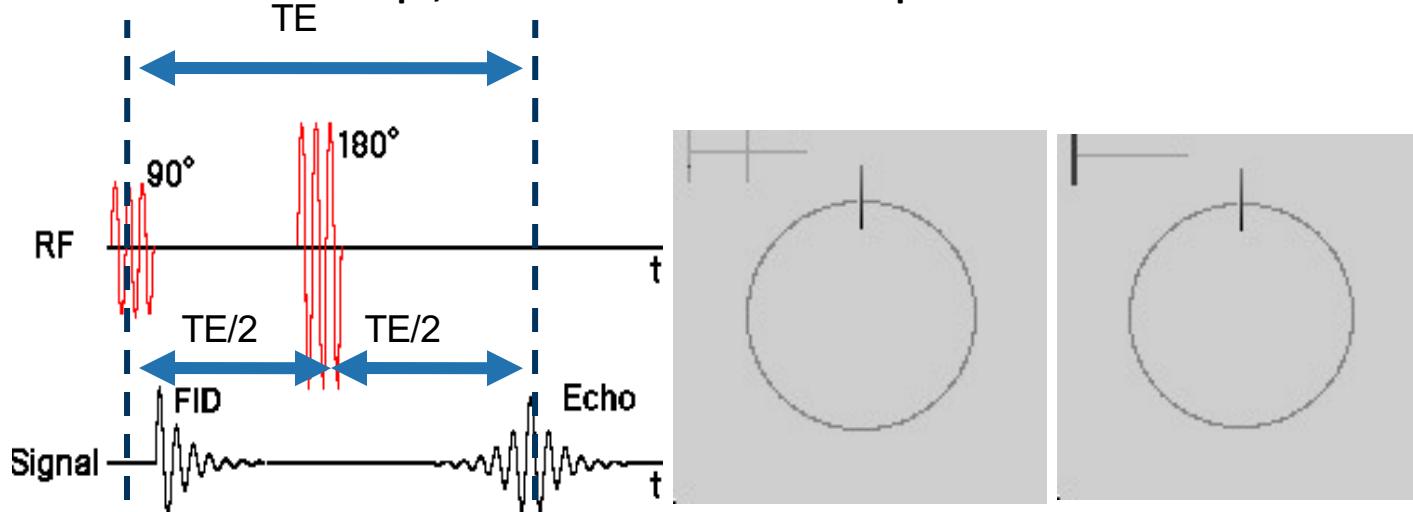
Red arrows precess faster due to local field inhomogeneity than Blue arrow

Spin Echo

180° pulse only helps cancel static inhomogeneity

The “runners” can have static speed distribution.

If a runner trips, he will not make it back in phase with the others.



Shown in

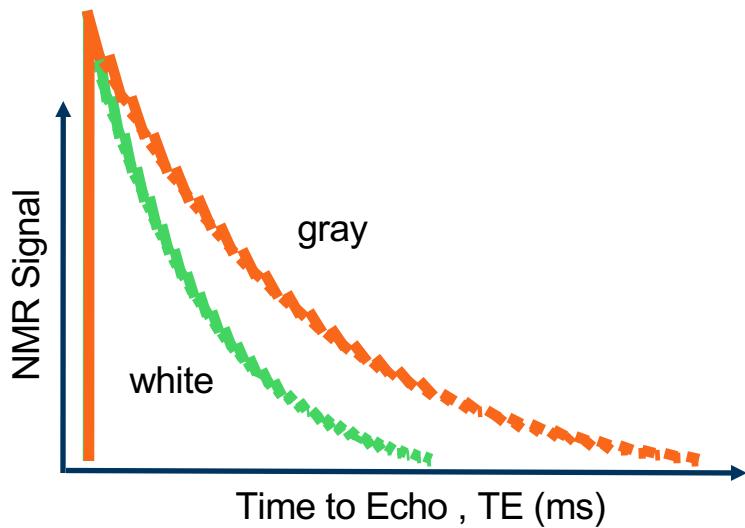
Laboratory Frame

Shown in

Rotating Frame

TE: time to echo

T2 weighted spin echo image

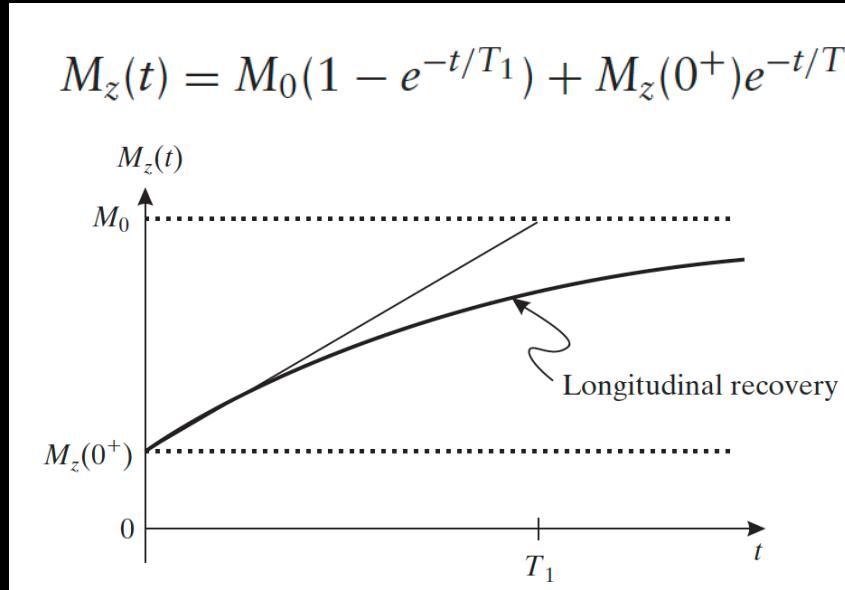


Wald, fMRI MR
Physics

Relaxation

Longitudinal relaxation (M_z)

- Return to equilibrium value M_0



usually: $5T_2 < T_1 < 10T_2$
i.e. quite a bit slower process

Bloch Equation

$$\frac{d\mathbf{M}(t)}{dt} = \gamma \mathbf{M}(t) \times \mathbf{B}(t) - \mathbf{R}\{\mathbf{M}(t) - \mathbf{M}_0\},$$

$$\mathbf{R} = \begin{pmatrix} 1/T_2 & 0 & 0 \\ 0 & 1/T_2 & 0 \\ 0 & 0 & 1/T_1 \end{pmatrix}.$$

$$\mathbf{B}(t) = \mathbf{B}_0 + \mathbf{B}_1(t),$$

Magnetization vector during the MRI experiment

