LECTURE 15: MRI SIGNAL AND CONTRAST

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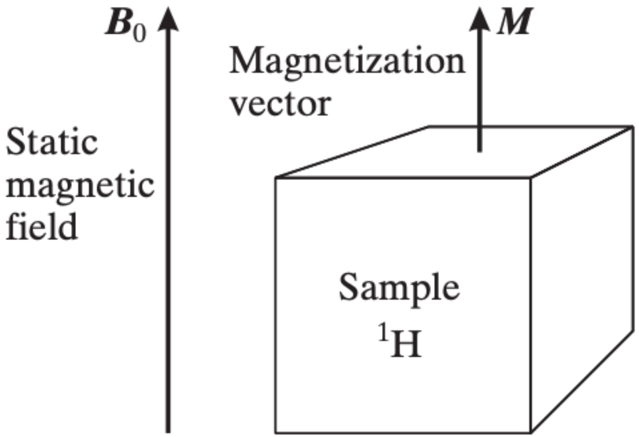
Bulk Magnetization

- The Magnetization M0, can be described by

$$M_0 = \frac{B_0 \gamma^2 \hbar^2}{4kT} P_D$$

- k is Boltzmann’s Distribution



There is a lot of a water!!!

- 18 g of water is approximately 18 ml and has approximately 2 moles of hydrogen protons
- Consider the protons in 1mm x 1 mm x 1 mm cube.
- $2*6.62*10^{23}*1/1000*1/18 = 7.73 \times 10^{19}$ protons/mm^3
- If we have 7 excesses protons per 2 million protons, we get .25 million billion protons per cubic millimeter!!!!

Precession

- Because **M**(t) is a magnetic moment, it experiences a torque when an external, time-varying magnetic field **B**(t) is applied

$$\frac{d\mathbf{M}(t)}{dt} = \gamma \mathbf{M}(t) \times \mathbf{B}(t)$$

- If the initial magnetization vector **M**(0) was oriented at an angle α relative to the *z*-axis

$$M_x(t) = M_0 \sin\alpha \cos(-\gamma B_0 t + \phi),$$

$$M_y(t) = M_0 \sin\alpha \sin(-\gamma B_0 t + \phi),$$

$$M_z(t) = M_0 \cos\alpha,$$

Transverse Magnetization & Signal

- The *transverse magnetization* is defined by

$$M_{xy}(t) = M_x(t) + jM_y(t)$$

- Phase (of the transverse magnetization) is the angle of the complex number M_{xy}

$$\phi = \tan^{-1} \frac{M_y}{M_x}$$

- Inserting the initial magnetization vectors into the transverse magnetization, we get that the transverse magnetization can be written as

$$M_{xy}(t) = M_0 \sin\alpha \, e^{-j(2\pi \nu_0 t - \phi)}$$

Rotating Frame

It is very convenient to express and visualize the evolution of the magnetization vector in a frame of reference, called the *rotating frame*, that is rotating at the Larmor frequency v0

$$x' = x\cos(2\pi\nu_0 t) - y\sin(2\pi\nu_0 t)\,,$$
$$y' = x\sin(2\pi\nu_0 t) + y\cos(2\pi\nu_0 t)\,,$$
$$z' = z\,.$$

In the rotating frame, the transverse magnetization becomes

$$M_{x'y'}(t) = M_0 \sin\alpha e^{j\phi}$$

The Bloch Equation

- In the rotating frame: $\vec{r} = [x,\ y,\ z]^T$

$$\vec{B} = \gamma \vec{G} \cdot \vec{r}\hat{k} + B_{1x}\hat{i} + B_{1y}\hat{j}$$

$$\frac{d\vec{M}}{dt} = \underbrace{-\gamma \vec{B} \times \vec{M}}_{\text{precession}} - \underbrace{\frac{M_x\hat{i} + M_y\hat{j}}{T_2}}_{\text{transverse decay}} - \underbrace{\frac{M_z - M_0}{T_1}\hat{k}}_{\text{long. recovery}}$$

The Bloch Equation

- **In Matrix Form :**

$$\begin{bmatrix} \dot{M}_x \\ \dot{M}_y \\ \dot{M}_z \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & \gamma \vec{G}\cdot\vec{r} & -\gamma B_{1y} \\ -\gamma \vec{G}\cdot\vec{r} & 0 & \gamma B_{1x} \\ \gamma B_{1y} & -\gamma B_{1x} & 0 \end{bmatrix} \begin{bmatrix} M_x \\ M_y \\ M_z \end{bmatrix}}_{\text{Precession}} + \underbrace{\begin{bmatrix} -\frac{1}{T_2} & 0 & 0 \\ 0 & -\frac{1}{T_2} & 0 \\ 0 & 0 & -\frac{1}{T_1} \end{bmatrix} \begin{bmatrix} M_x \\ M_y \\ M_z \end{bmatrix}}_{\text{relaxation}} + \underbrace{\begin{bmatrix} 0 \\ 0 \\ \frac{1}{T_1} \end{bmatrix} M_0}_{\text{recovery}}$$

The Bloch Equation

- Combined (rotating frame)

$$\begin{bmatrix} \dot{M}_x \\ \dot{M}_y \\ \dot{M}_z \end{bmatrix} = \underbrace{\begin{bmatrix} -\frac{1}{T_2} & \gamma\vec{G}\cdot\vec{r} & -\gamma B_{1y} \\ -\gamma\vec{G}\cdot\vec{r} & -\frac{1}{T_2} & \gamma B_{1x} \\ \gamma B_{1y} & -\gamma B_{1x} & -\frac{1}{T_1} \end{bmatrix}}_{\text{Rotation + relaxation}} \begin{bmatrix} M_x \\ M_y \\ M_z \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \\ 0 \\ \frac{1}{T_1} \end{bmatrix} M_0}_{\text{Recovery}}$$

Rotation +
relaxation

Recovery

- T1 is <u>the</u> source of all signals!
- Magnetization distribution unknown
- Can probe by changing B1 and G

Excitation

- **Excitation:** an RF pulse generates a magnetic field B1, which rotates M into the xy plane.
- B_1 is a
  - Radiofrequency (RF)
  - Short Duration Pulse (~0.1 to 5 ms)
  - Small Amplitude (<30 μT)
  - Circularly Polarized (@Larmor Frequency)
  - Perpendicular to B_0

Chosen to excite specific range of frequencies (kHz).

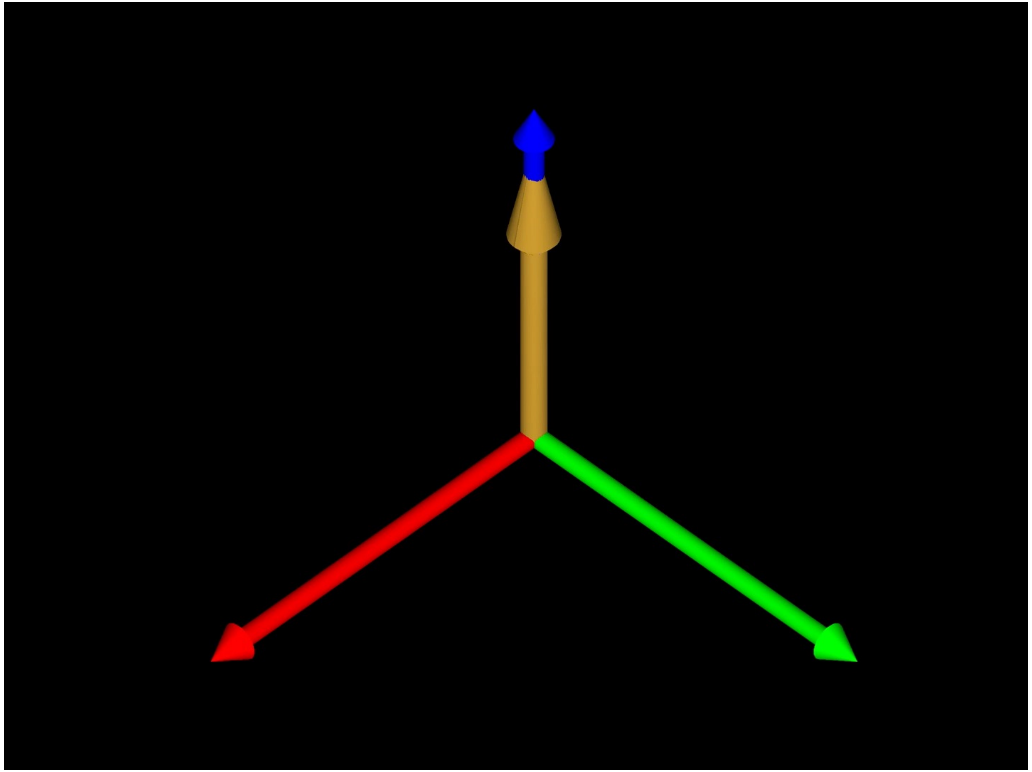
Matched to Larmor Frequency (MHz)

Perpendicular to B_0

$$\vec{B}_1(t) = 2B_1^e(t)\cos(\omega_{RF}t + \theta)\vec{i}$$

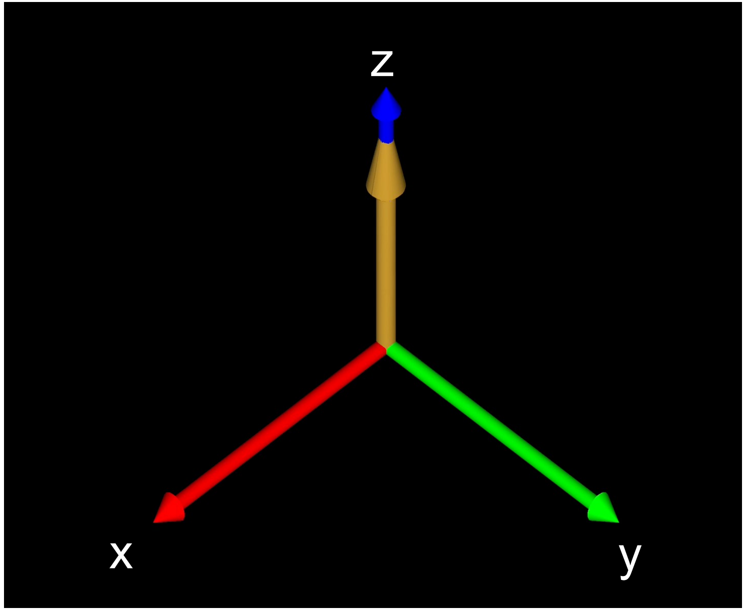
RF Excitation – Laboratory Frame

B_0 causes ***precession*** about z-axis. B_1 causes ***nutation*** *(Forced Precession)*.

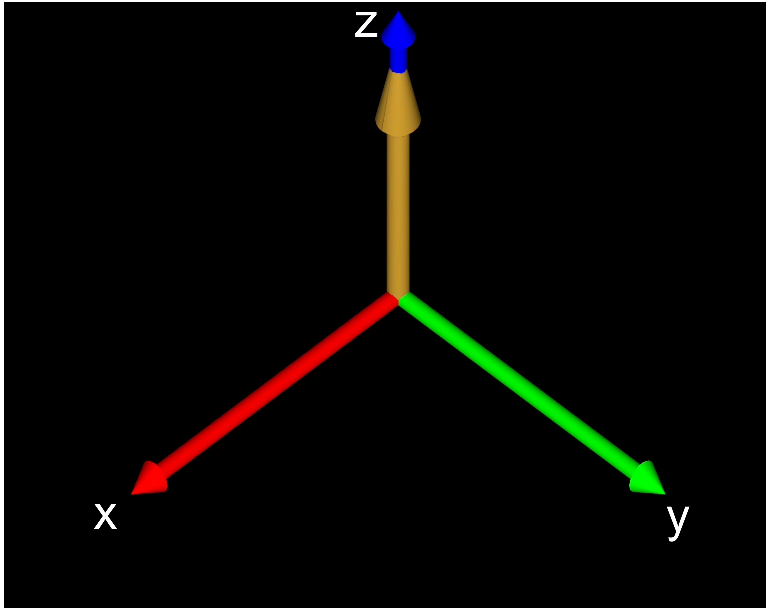


RF Excitation – Rotating Frame

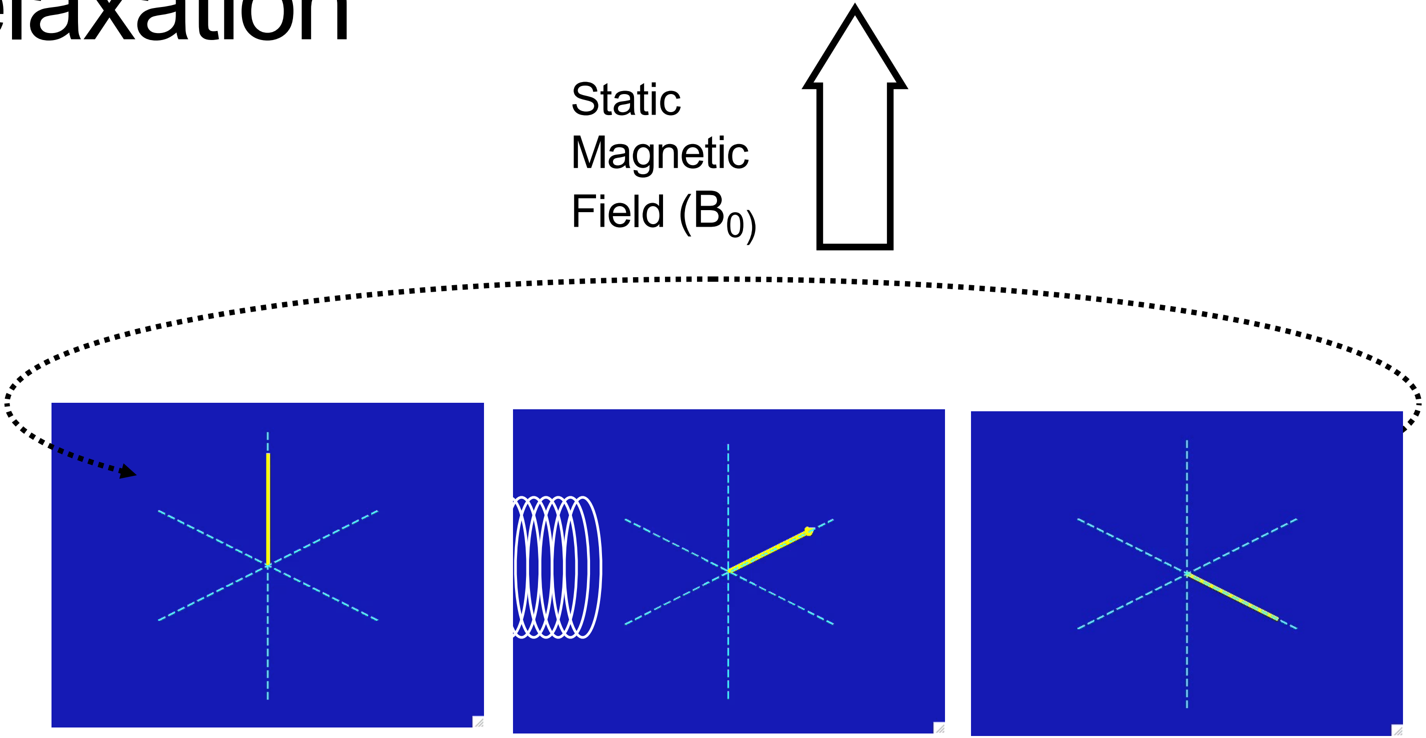
Laboratory Frame



Rotating Frame



Relaxation



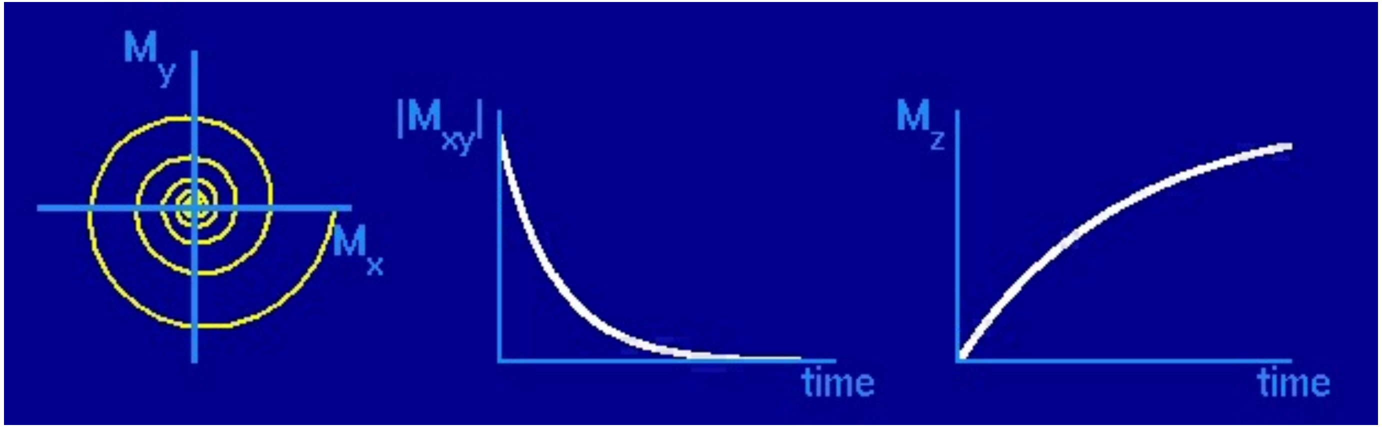
Excitation

Precession (Reception)

Relaxation (Recovery)

Relaxation and Precession

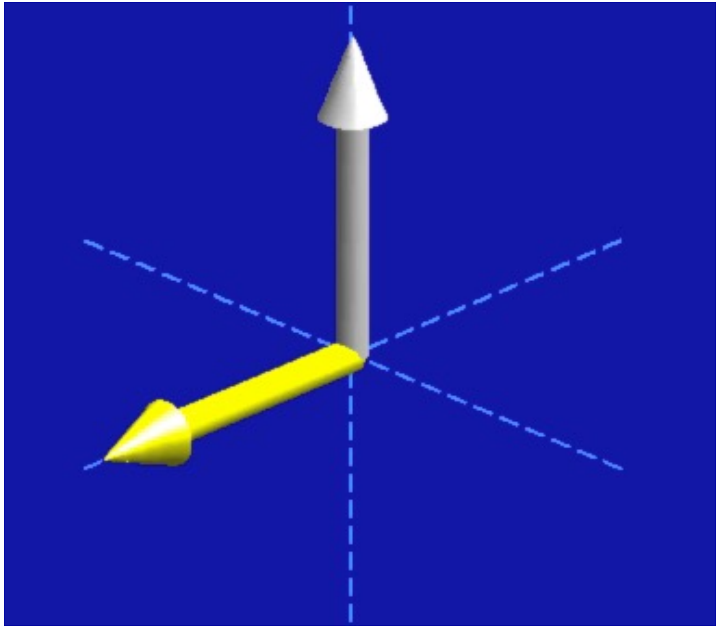
- Relaxation and precession are independent.
- Magnetization returns exponentially to equilibrium:
  - Longitudinal *recovery* time constant is T_1
  - Transverse *decay* time constant is T_2



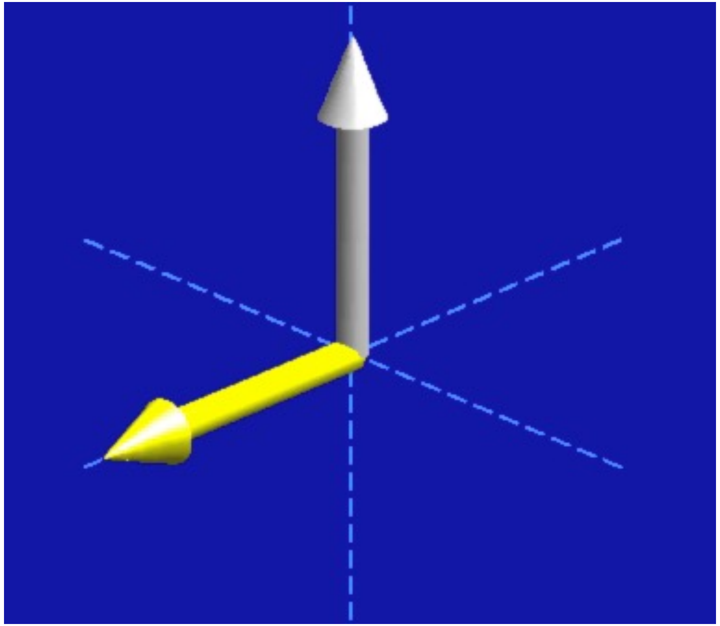
Precession Decay Recovery

Relaxation - Frame of Reference

Laboratory Frame

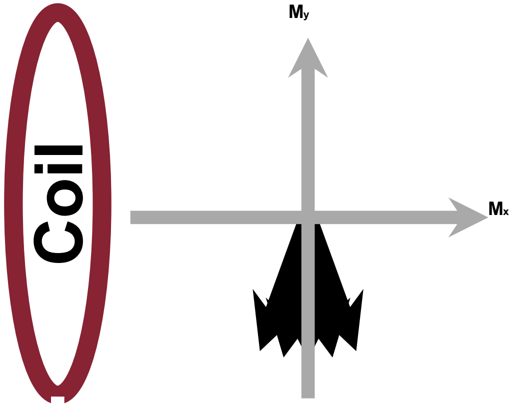


Rotating Frame

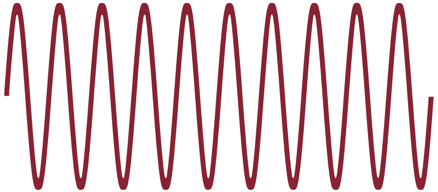


Intravoxel Dephasing

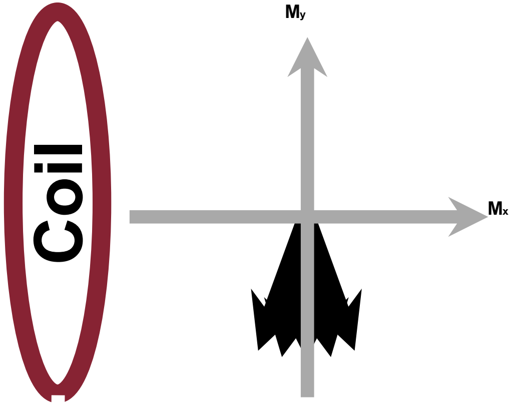
Homogenous
Intravoxel Field



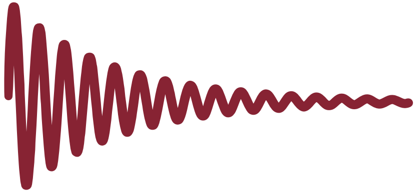
Spin Coherence



Inhomogenous
Intravoxel Field

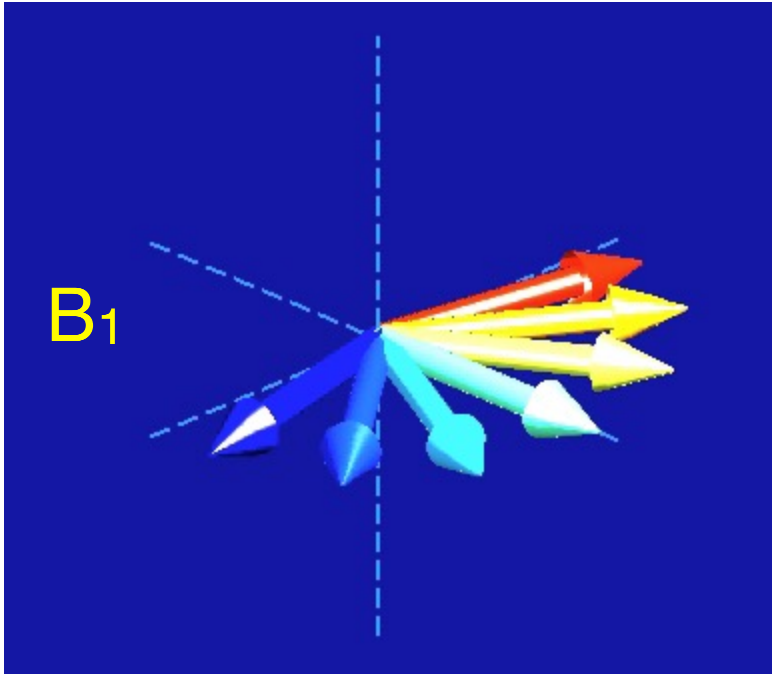


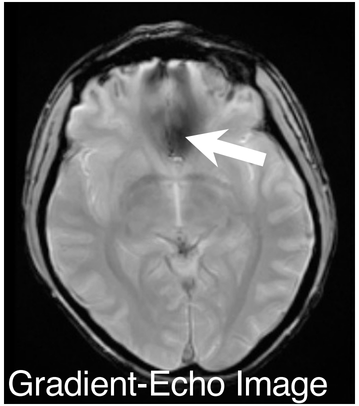
Spin Dephasing



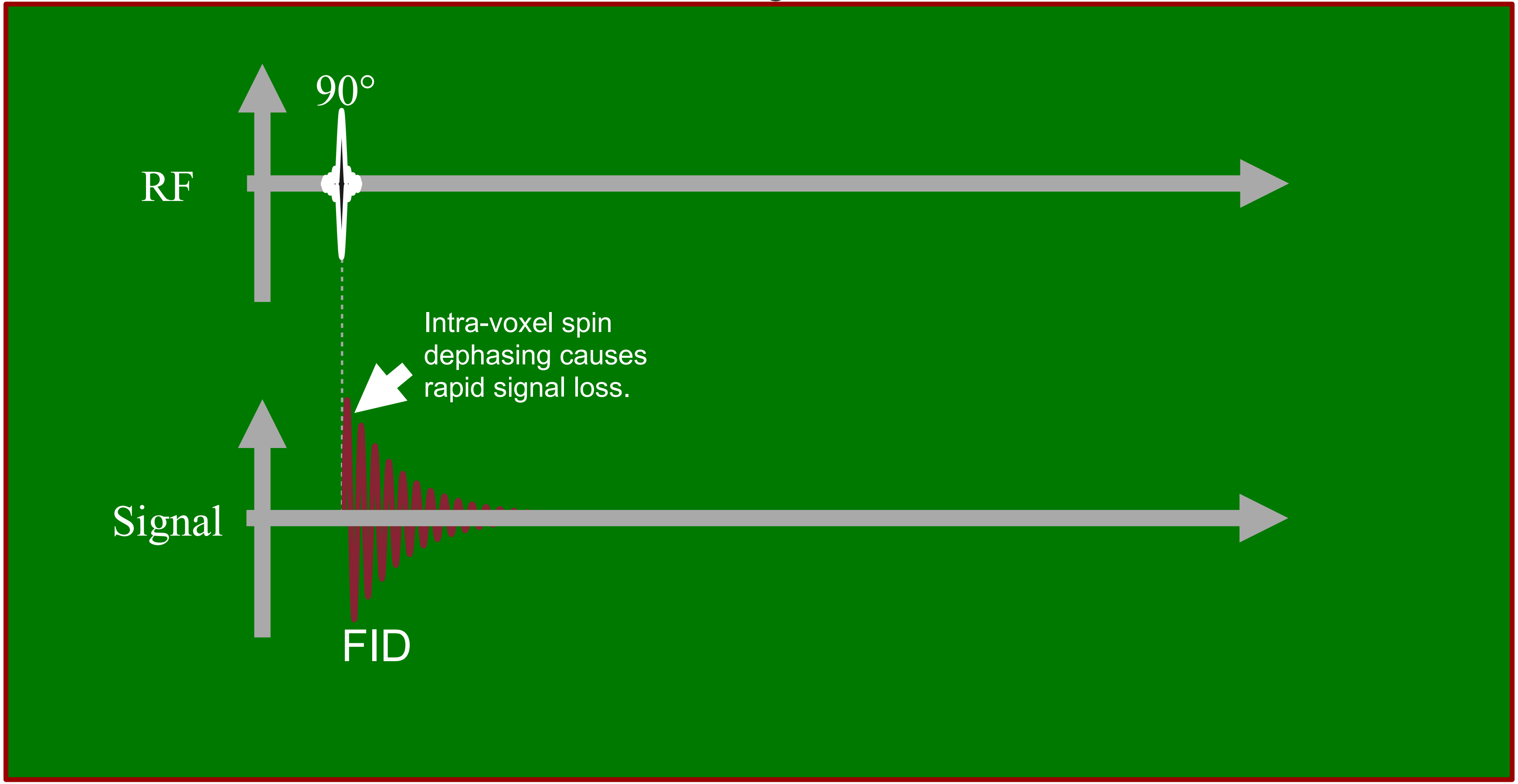
T2 Relaxation

- Frequency Variations cause dephasing

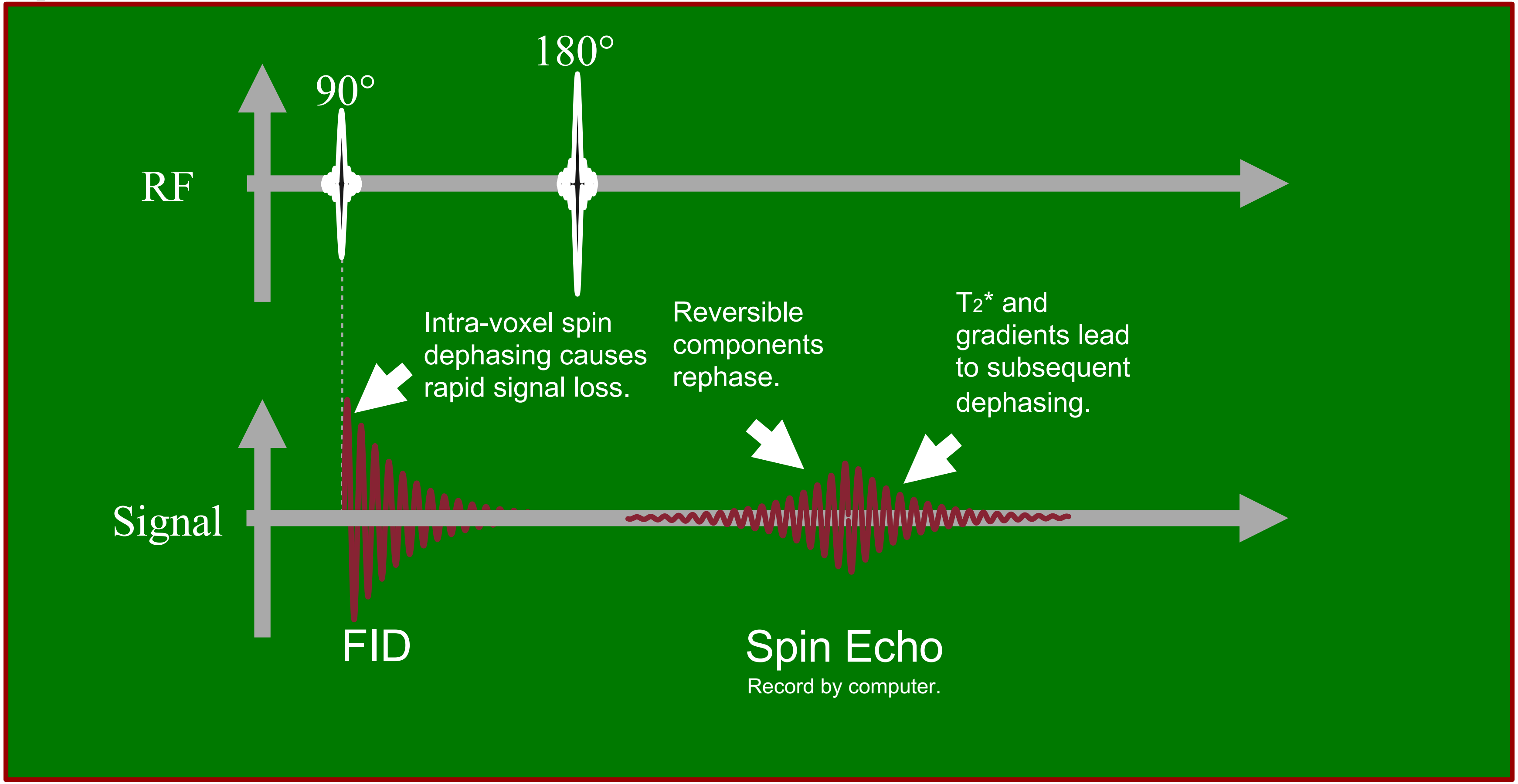




Free Induction Decay

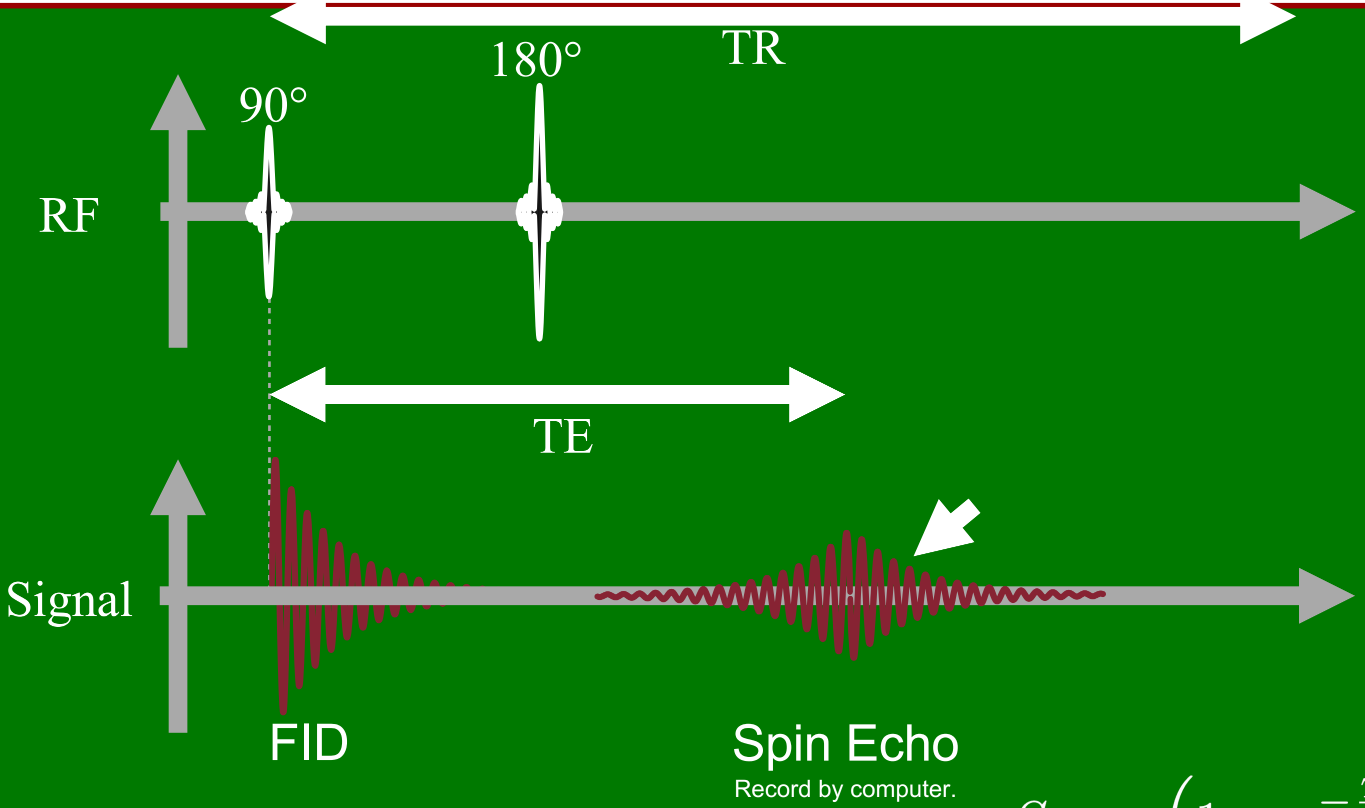


Spin Echo



180°
90°
RF
Intra-voxel spin dephasing causes rapid signal loss.
Reversible components rephase.
T_2* and gradients lead to subsequent dephasing.
Signal
FID
Spin Echo
Record by computer.

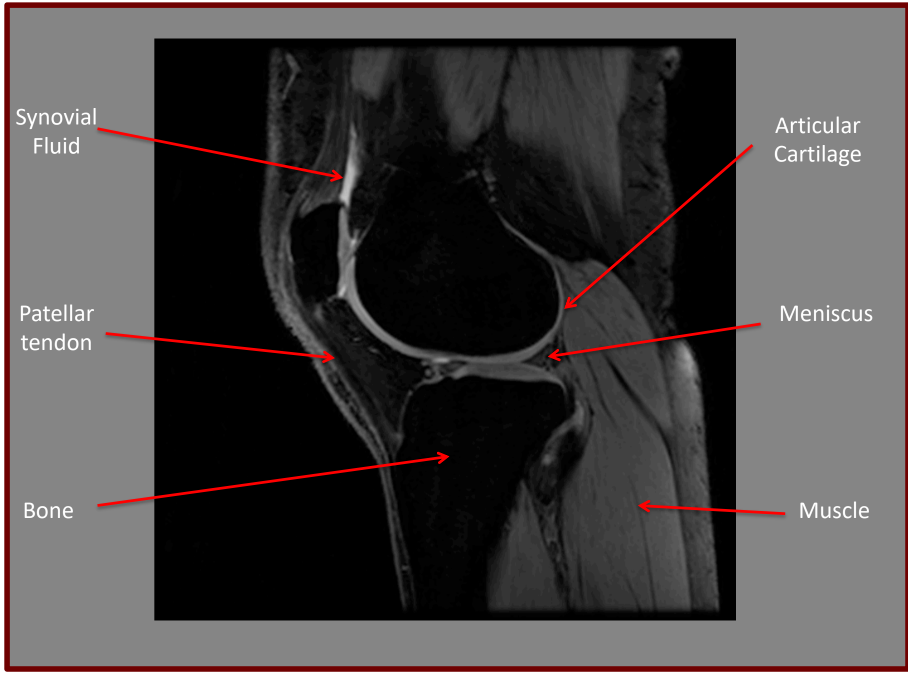
Spin Echo



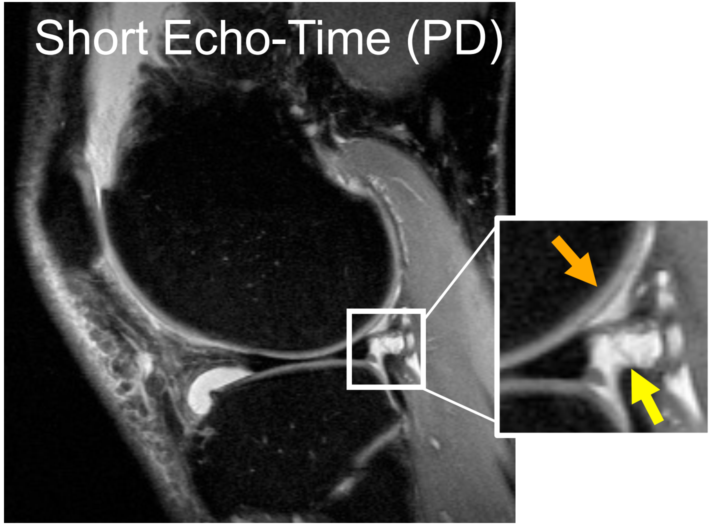
$$S = \rho\left(1 - e^{-\frac{TR}{T_1}}\right) e^{-\frac{TE}{T_2}}$$

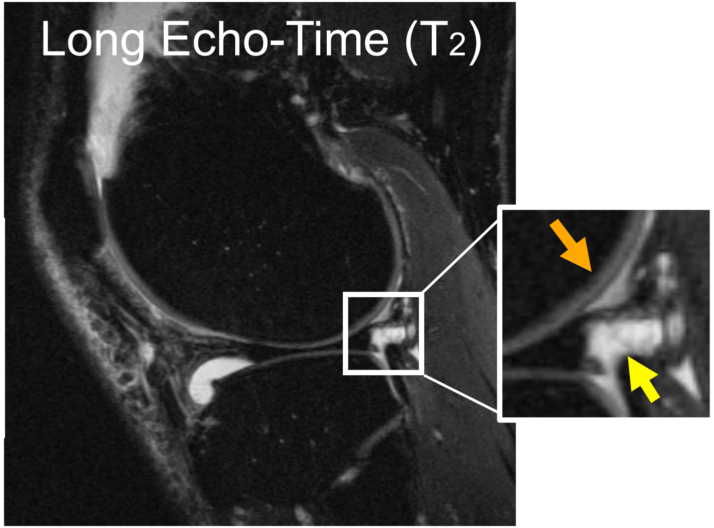
MSK Tissue Relaxation Properties

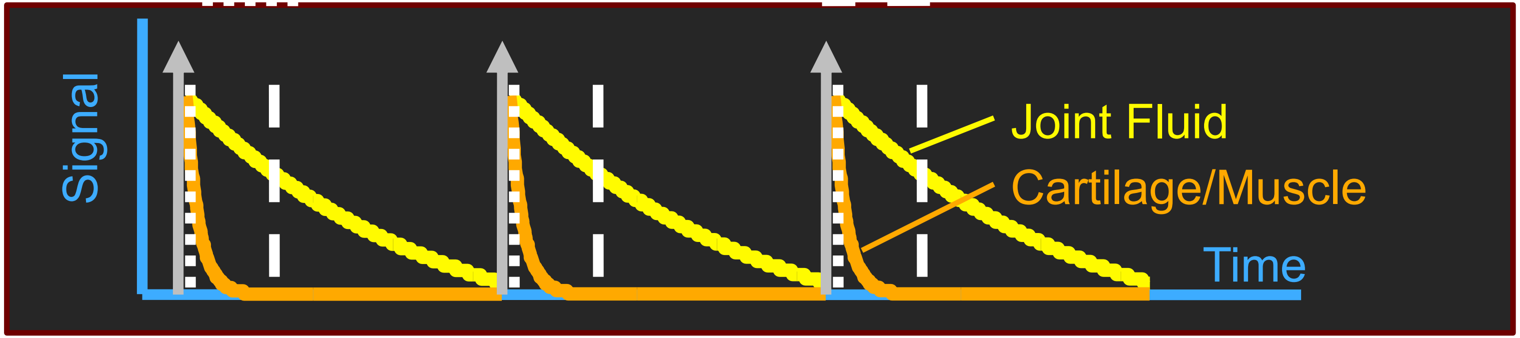
Tissue	PD (Relative to 110M water)	T1 (ms)	T2 (ms)
Bone	0.2	150	< 0.05
Tendon/ Ligament	0.25	500	2-6
Meniscus	0.35	750	8-12
Cartilage	0.75	1000	35-45
Muscle	0.7	1200	35-45
Fat	0.9	300-400	50
Fluid	1.0	>2000	>700



T2 Contrast


Short Echo-Time (PD)

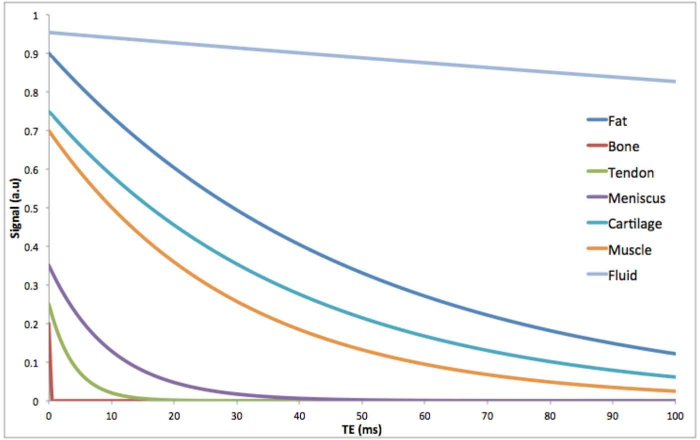

Long Echo-Time (T2)

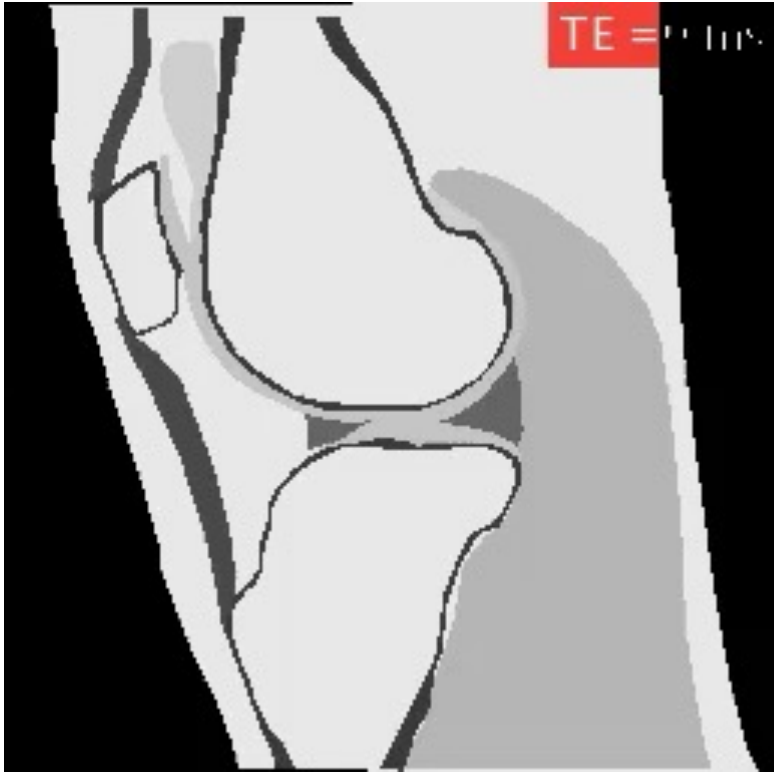

Signal
Joint Fluid
Cartilage/Muscle
Time

Signal Contrast – T2 Weighting

$$S(TR,TE) = M_0\left(1 - e^{-TR/T_1}\right)e^{-TE/T_2}$$

	Short TE	Long TE
Short TR	T1 Weighting	
Long TR	PD Weighting	T2 Weighting





Signal Contrast – T2 Weighting

$$S(TR,TE) = M_0\left(1 - e^{-TR/T_1}\right)e^{-TE/T_2}$$

	Short TE	Long TE
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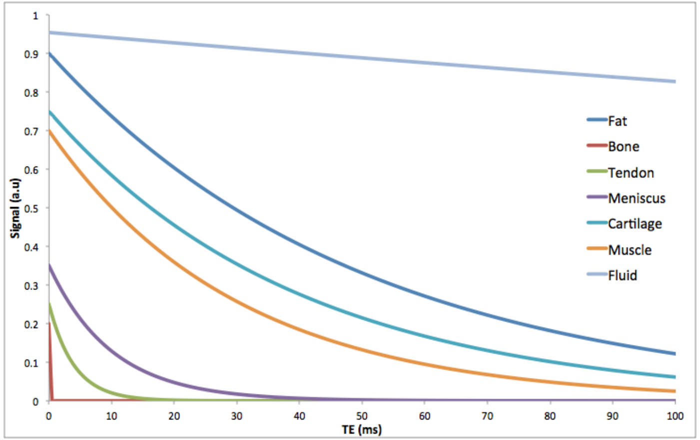
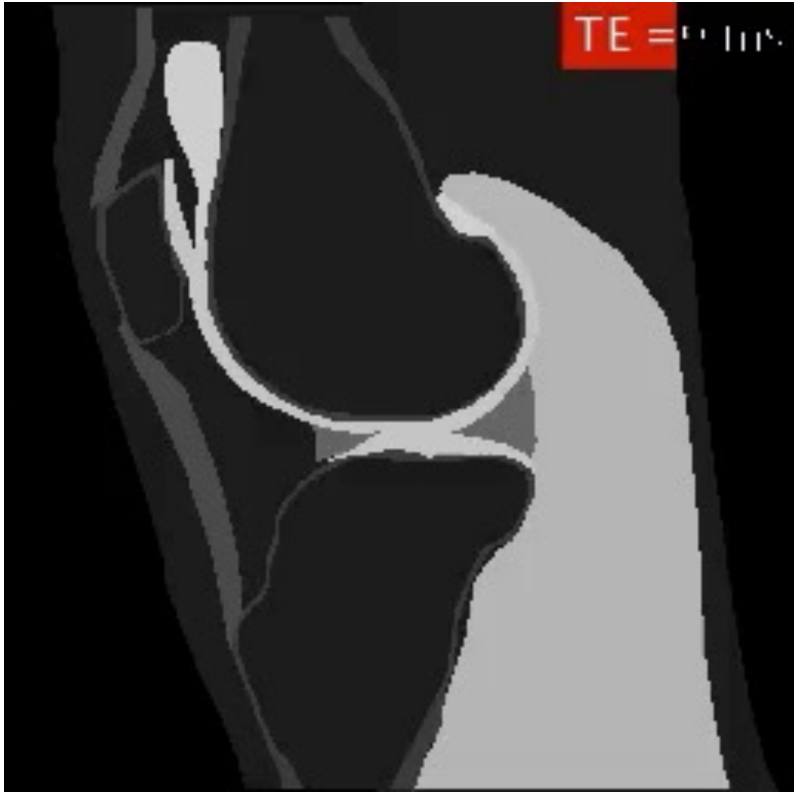
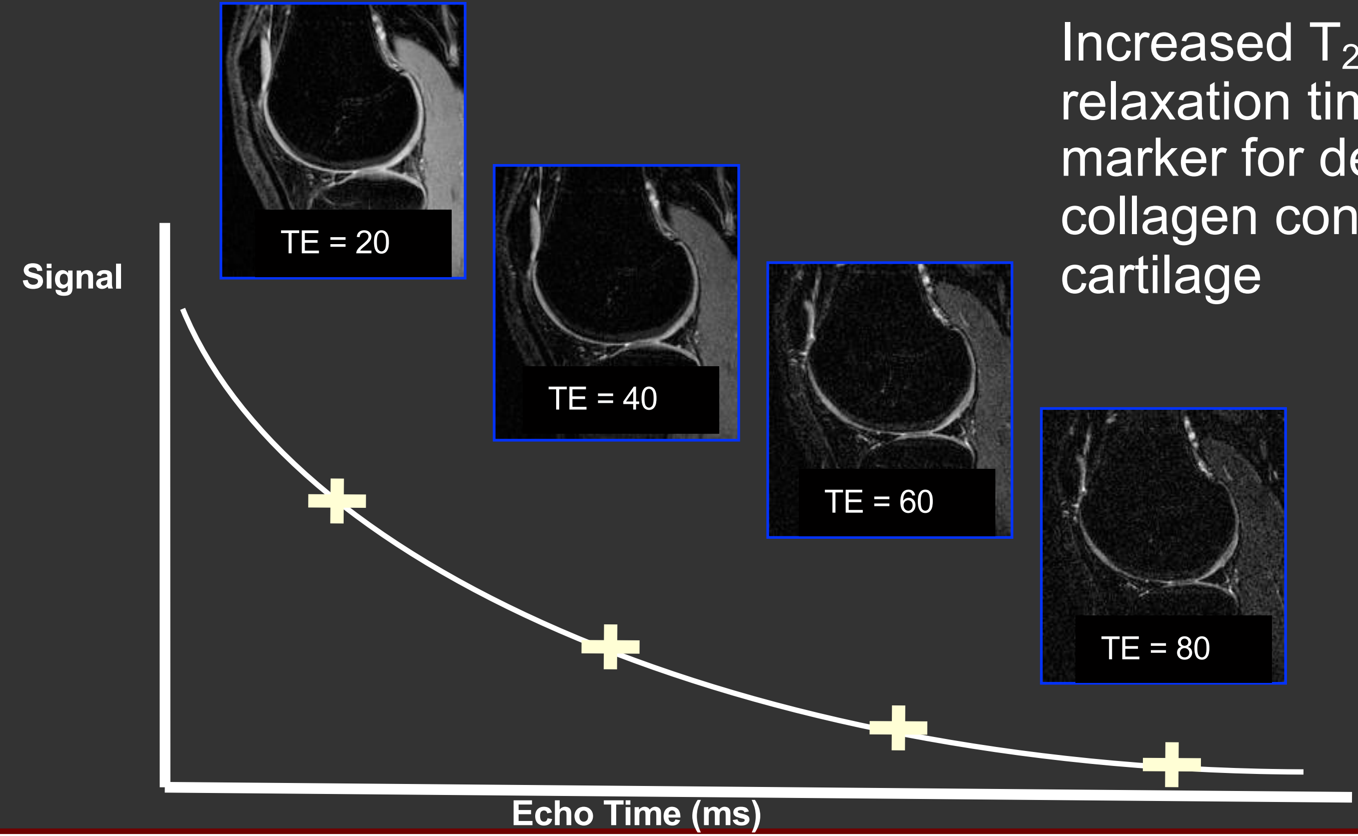




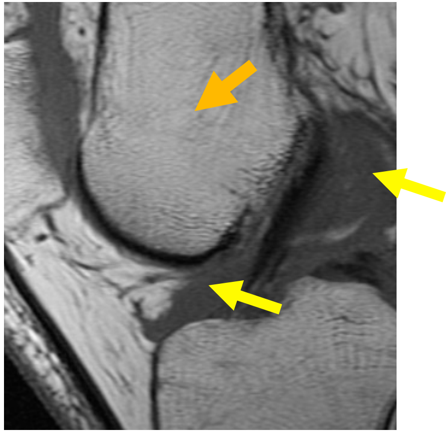
T2 Mapping

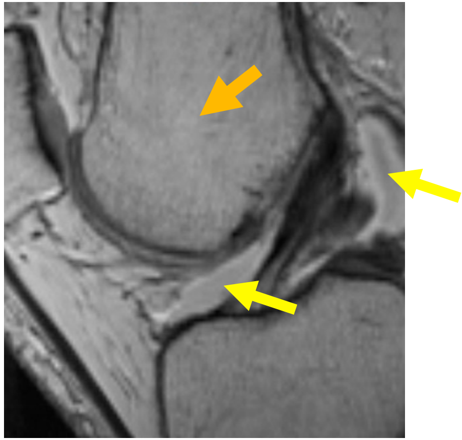


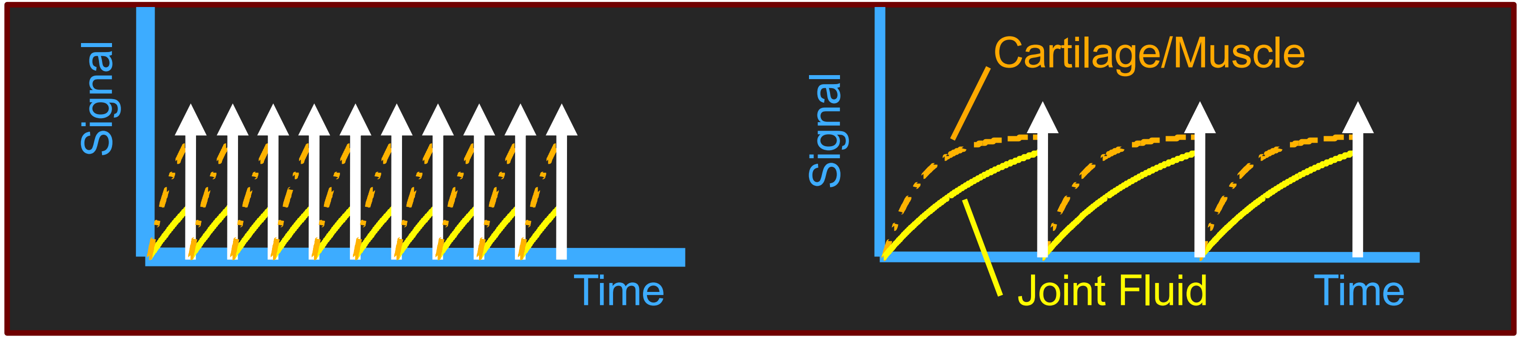
Increased T_2 relaxation time is a marker for decreased collagen content in cartilage

Dardzinski BJ, et al. Radiology, 205: 546-550, 1997.

T1 Contrast



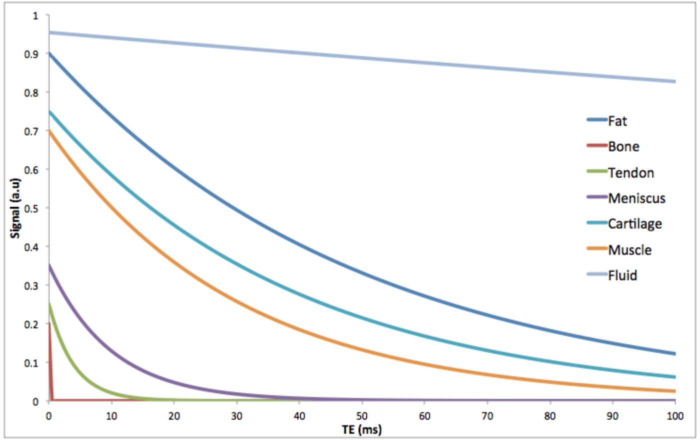


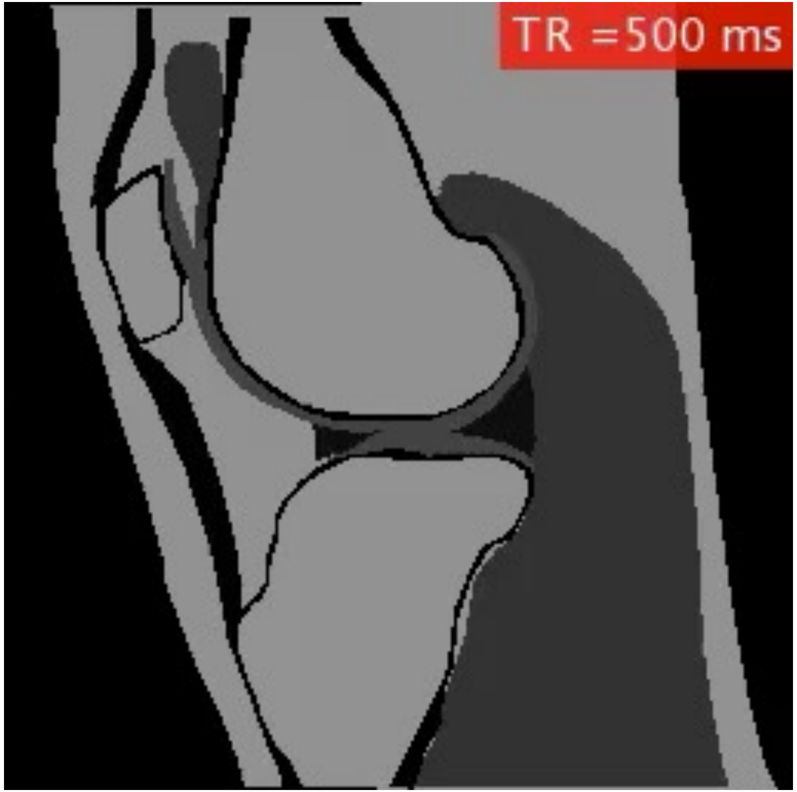


Signal Contrast – T1 Weighting

$$S(TR,TE) = M_0\left(1 - e^{-TR/T_1}\right)e^{-TE/T_2}$$

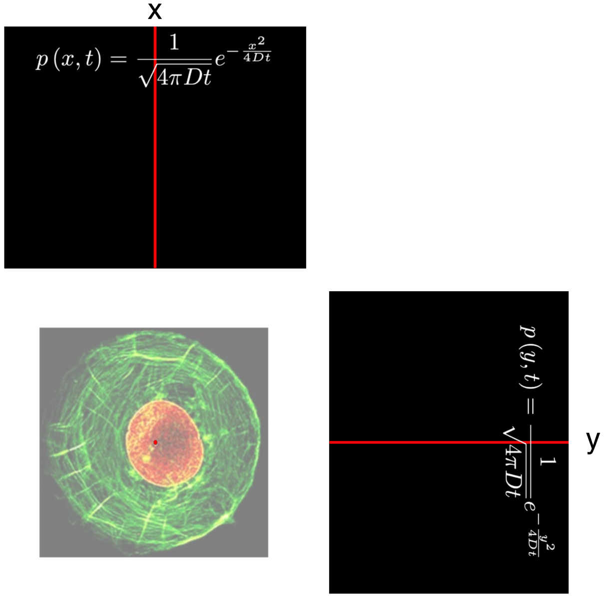
	Short TE	Long TE
Short TR	T1 Weighting	
Long TR	PD Weighting	T2 Weighting





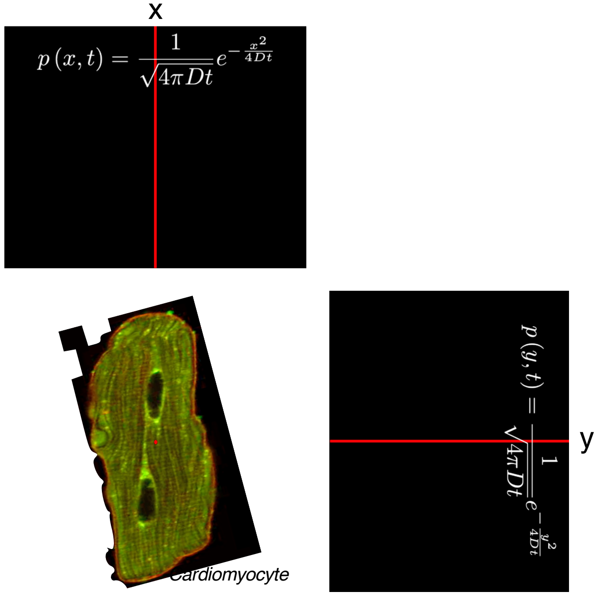
Relaxation - Diffusion

Free Diffusion



$p(x,t) = p(y,t)$

Restricted Diffusion

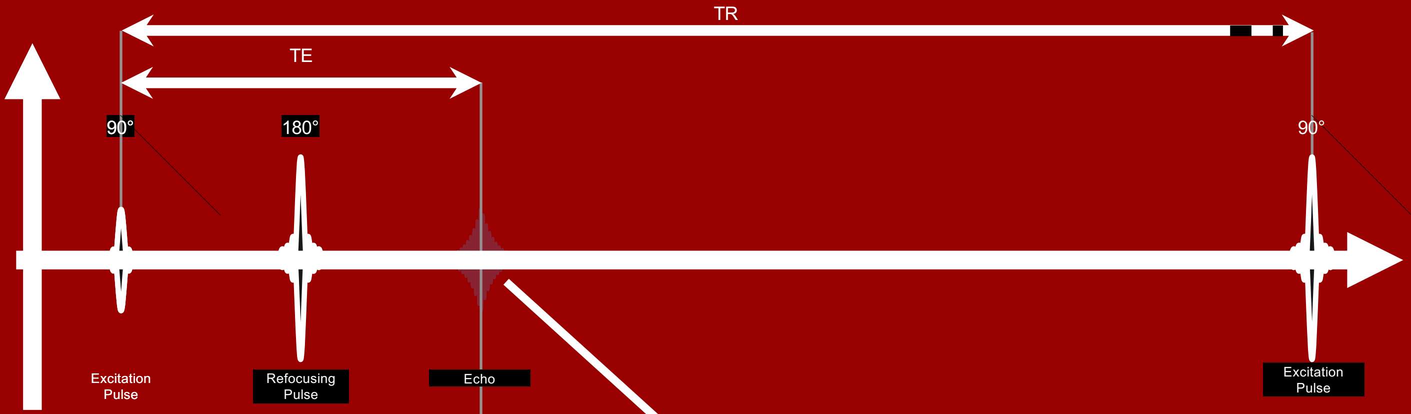


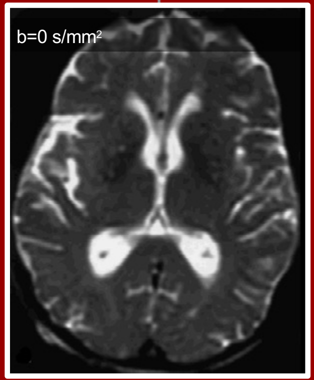
$p(x,t) \neq p(y,t)$

Diffusion in biological tissues is typically anisotropic.

MRI permits probing the diffusion coefficient along any vector direction.

Diffusion





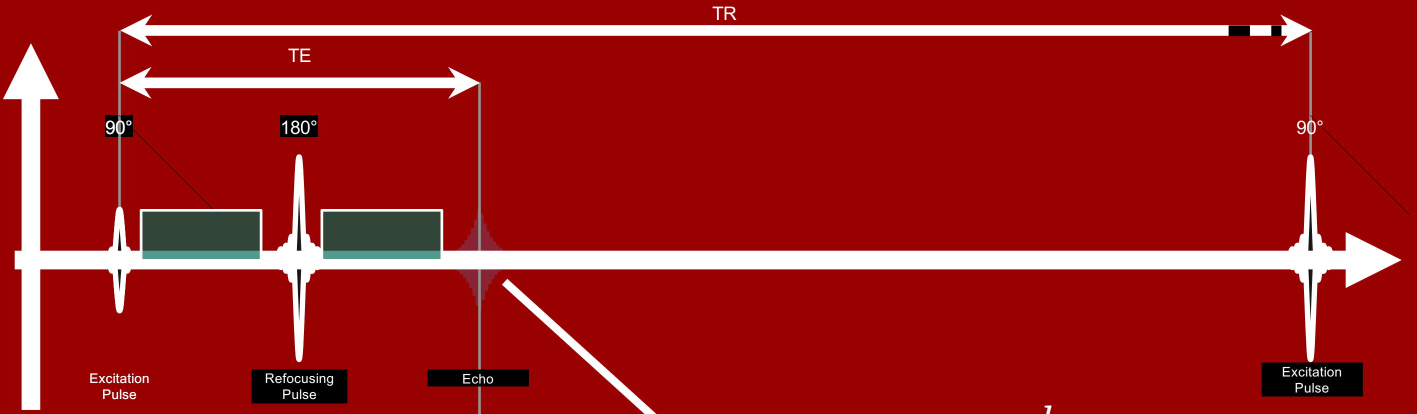
$$S = S_0$$

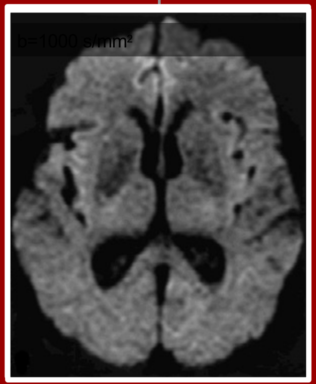
(Non-Diffusion Weighted Imaging)

$$S_0 = \rho\left(1 - e^{-\frac{TR}{T1}}\right) e^{-\frac{TE}{T2}}$$

http://mriquestions.com/what-is-the-b-value.html

Diffusion





$$S = S_0 e^{-\frac{b}{D}}$$

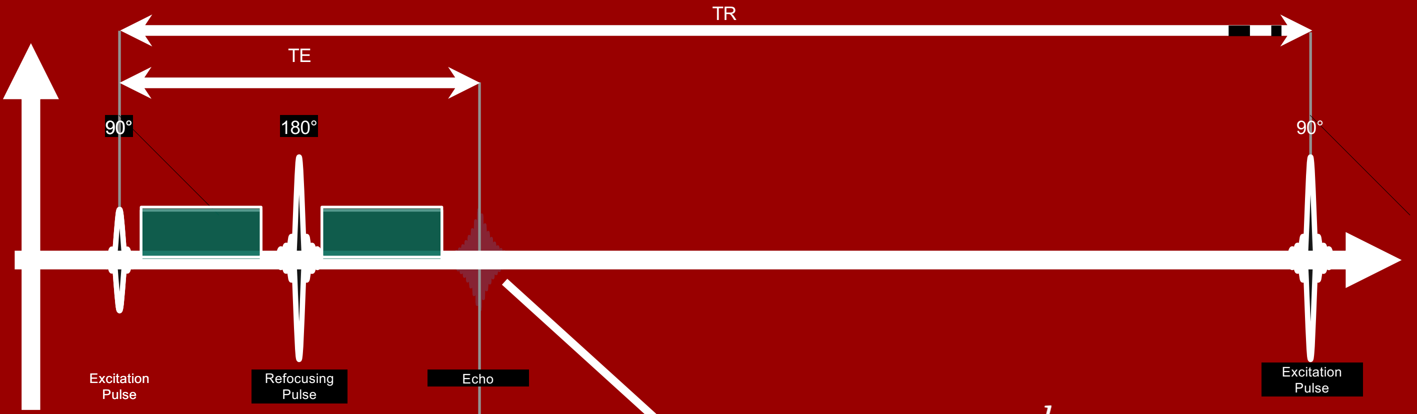
(Diffusion Weighted Imaging)

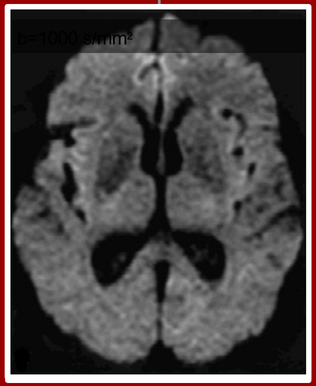
$$b = \gamma^2 G^2 \delta^2 \left(\Delta - \frac{\delta}{3}\right)$$

(Controls Diffusion Weighting)

This is what we control on the scanner.

Diffusion



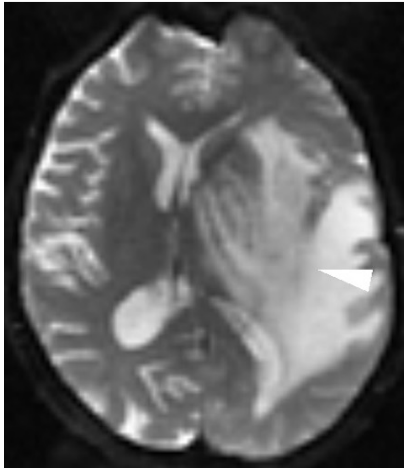


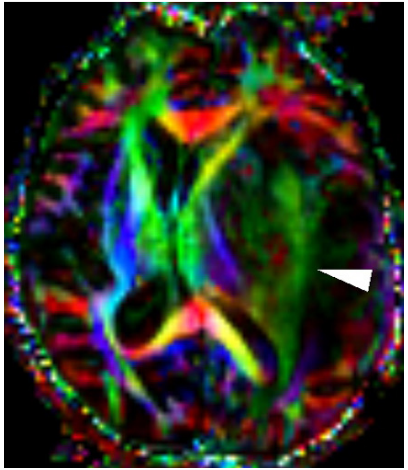
$$S = S_0 e^{-\frac{b}{D}}$$ (Diffusion Weighted Imaging)

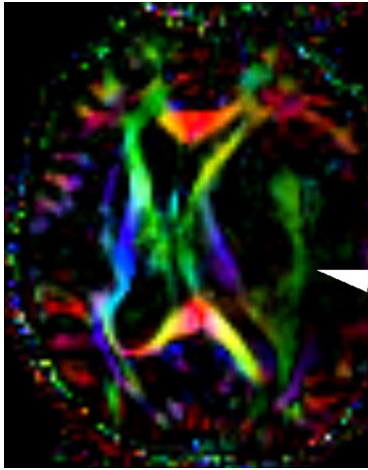
$$b = \gamma^2 G^2 \delta^2 \left(\Delta - \frac{\delta}{3} \right)$$ (Controls Diffusion Weighting)

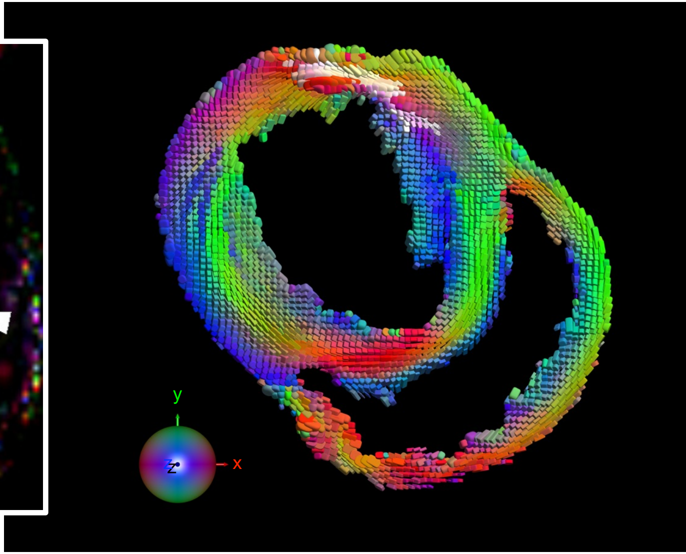
This is what we control on the scanner.

Diffusion Tensor Imaging









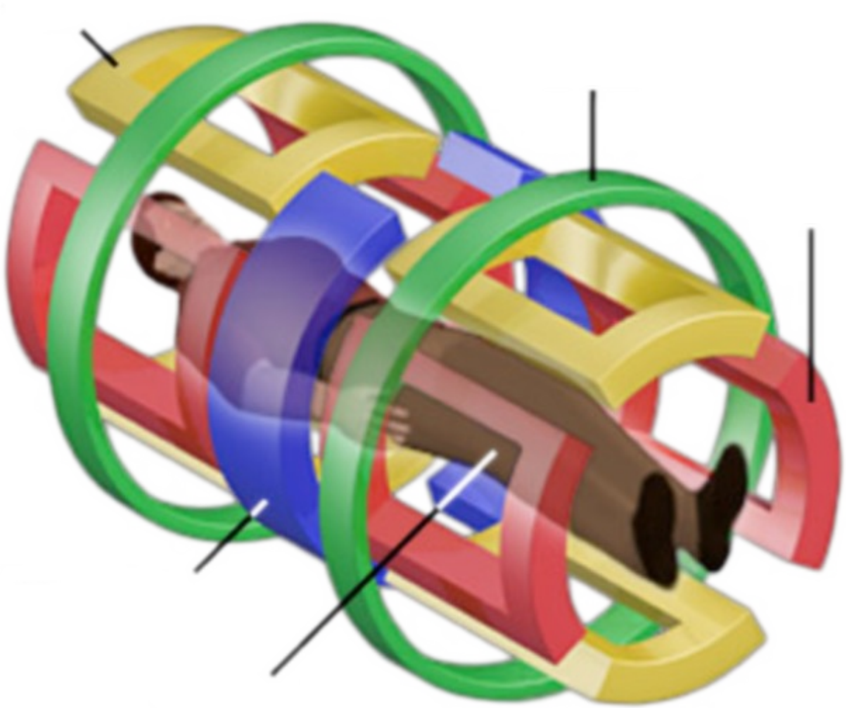
Other Relaxation Mechanisms

- Magnetization Transfer
  - Chemical Exchange
- Magnetic Susceptibility
  - Iron Imaging
- Flow and Perfusion
  - Angiography
  - Perfusion Imaging
- Contrast Agents
  - Gd/Iron-oxide
  - Hyperpolarized

MR IMAGING

Spatial Encoding

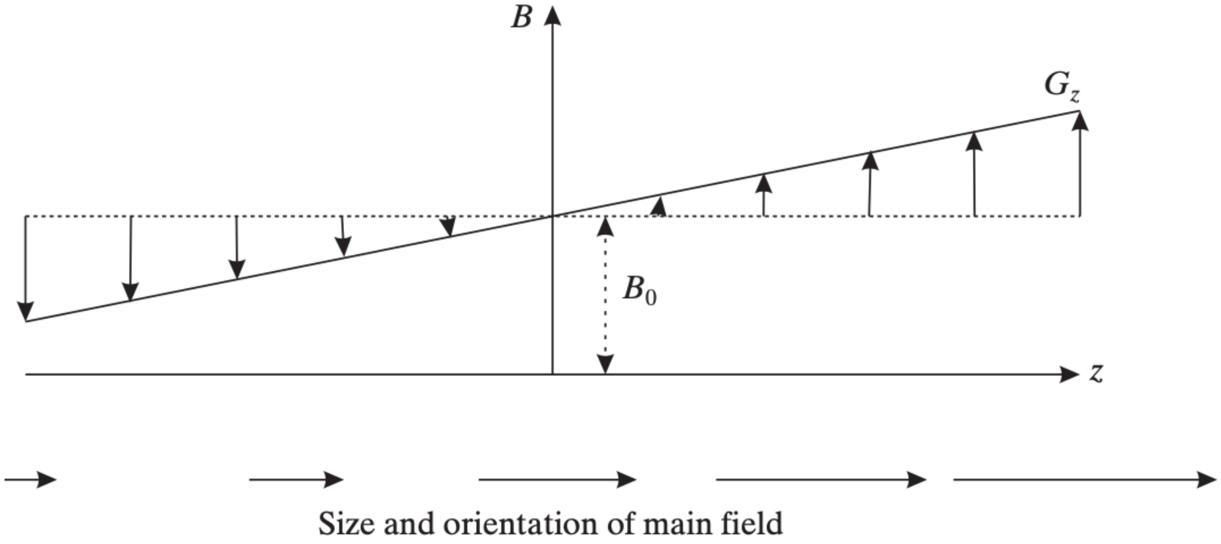
- Gradients are a:
  - Small
    - <5G/cm (±0.0075T @ edge of 30cm FOV)
  - Spatially varying
    - Linear gradients
  - Time varying
    - Slewrate Max. ~150-200mT/m/ms
  - Magnetic field
    - Adds/Subtracts to the B_0 field
  - Parallel to B_0



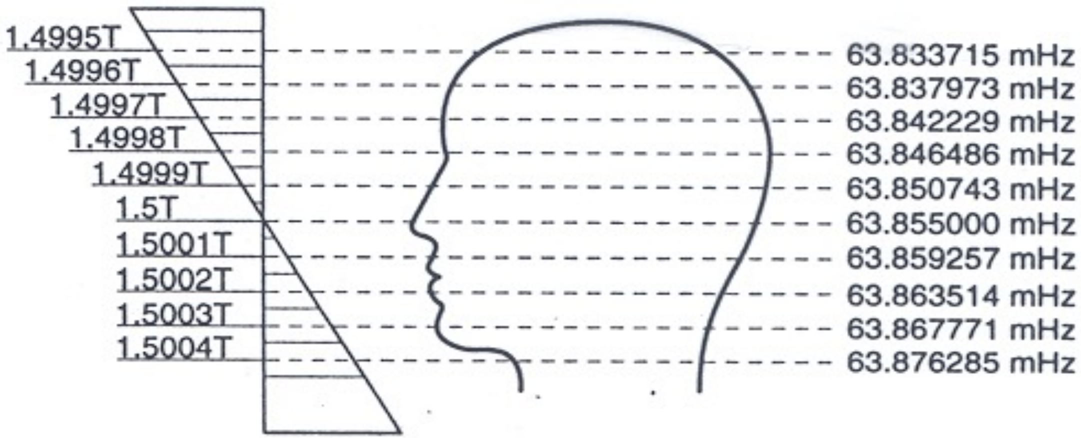
Slice Select Gradient

- Gradients Adds/Subtracts to the B_0 field

$$\nu(z) = \gamma(B_0 + G_z z)$$

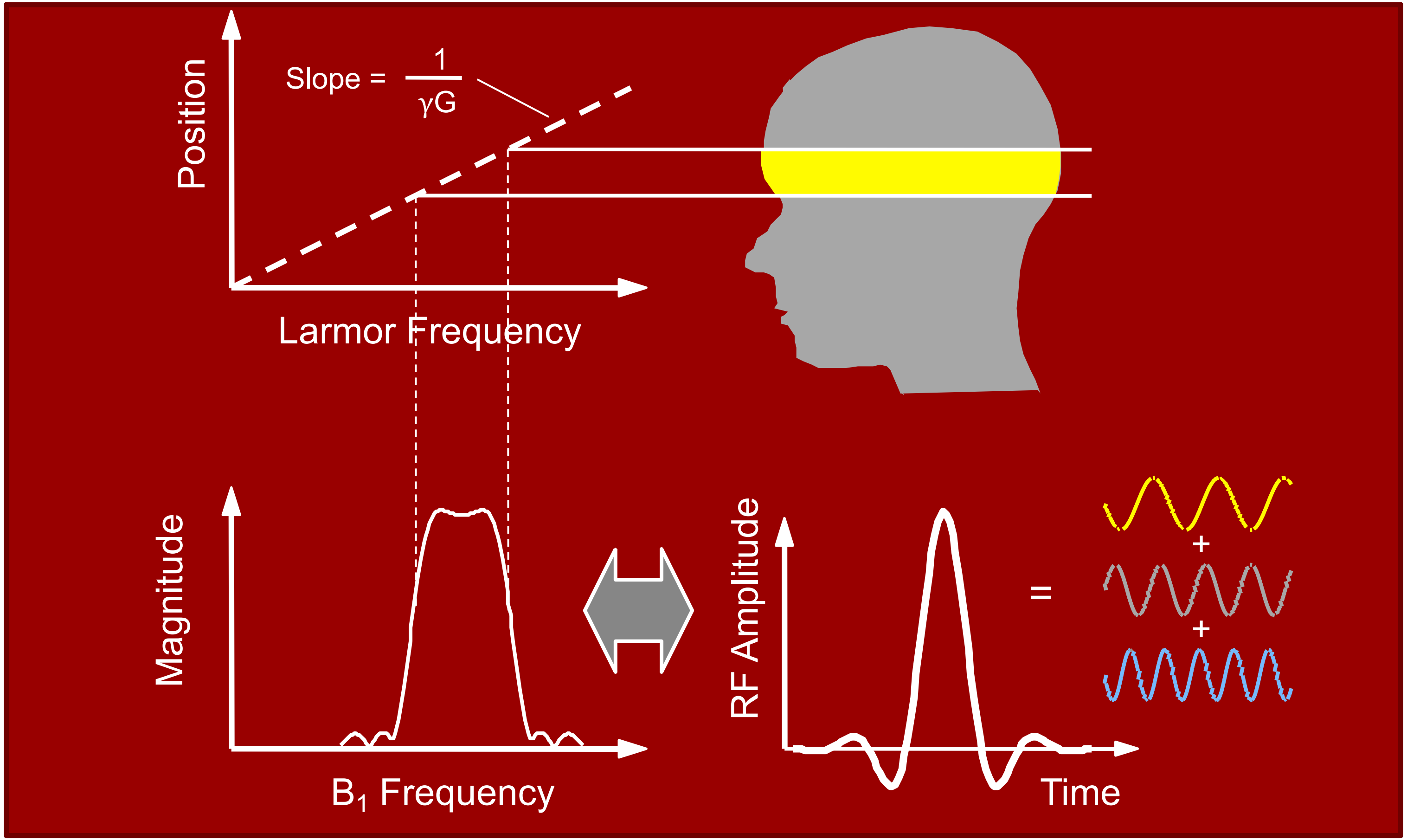


Slice Select Gradient



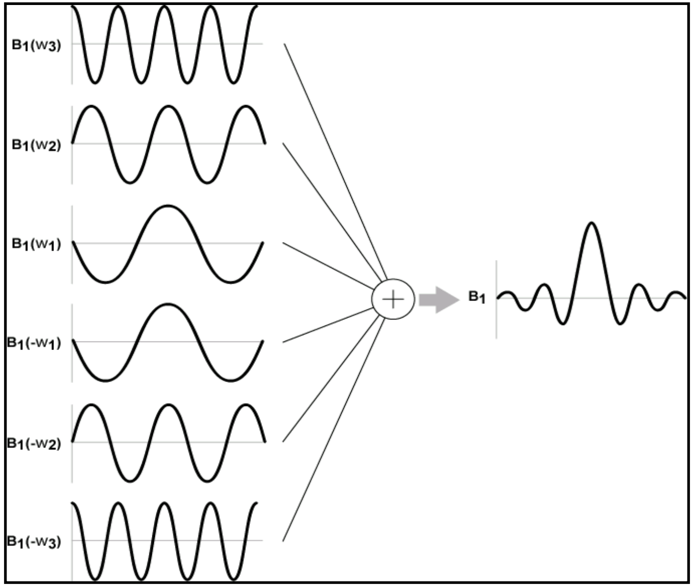
The effects of the main magnetic field and the applied slice gradient. In this example, the local magnetic field changes in one-Gauss increments accompanied by a change in the processional frequency from chin to the top of the head.

Slice Select Gradient



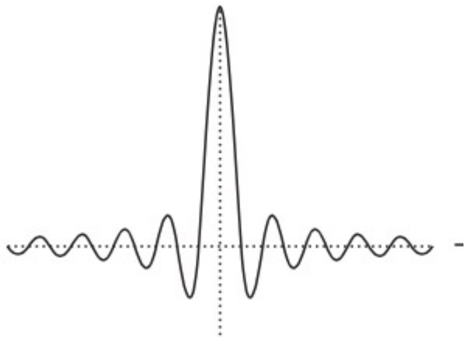
Selective RF Excitation

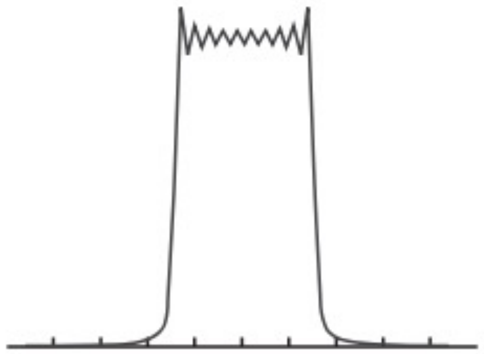
Recall frequency of RF excitation has to be equal or in resonance with spins



Build RF pulse from sum of narrow frequency range

Slice Profile





In plane Encoding

- I'd previously shown you that

$$M_{xy}(t) = M_0 \sin\alpha\, e^{-j(2\pi \nu_0 t - \phi)}$$

- Adding in relaxation

$$M_{xy}(t) = M_{xy}(0^+) e^{-j(2\pi \nu_0 t - \phi)} e^{-t/T_2}$$

Readout Gradient

- If a gradient is applied across the x-direction

$$\nu(x) = \gamma(B_0 + G_x x)$$

- This signal is thus described by

$$s(t) = A \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} M_{xy}(x, y; 0^+) e^{-j2\pi(\nu_0 + \gamma G_x x)t} e^{-t/T_2(x,y)}\, dx\, dy\,,$$

$$= e^{-j2\pi\nu_0 t} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} A M_{xy}(x, y; 0^+) e^{-t/T_2(x,y)} e^{-j2\pi\gamma G_x x t}\, dx\, dy\,.$$

Readout Gradient

- Let us define spin density, f(x,y) as

$$f(x,y) = AM(x,y;0^+)e^{-t/T_2(x,y)}$$

- Receiver signal can thus be modeled as

$$s(t) = e^{-j2\pi\nu_0 t}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\,dx\,dy$$

- With **No** gradients, the baseband signal is

$$s_0(t) = e^{+j2\pi\nu_0 t}s(t)$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\,dx\,dy$$

Readout Gradient

- Let us define spin density, f(x,y) as

$$f(x,y) = AM(x,y;0^+)e^{-t/T_2(x,y)}$$

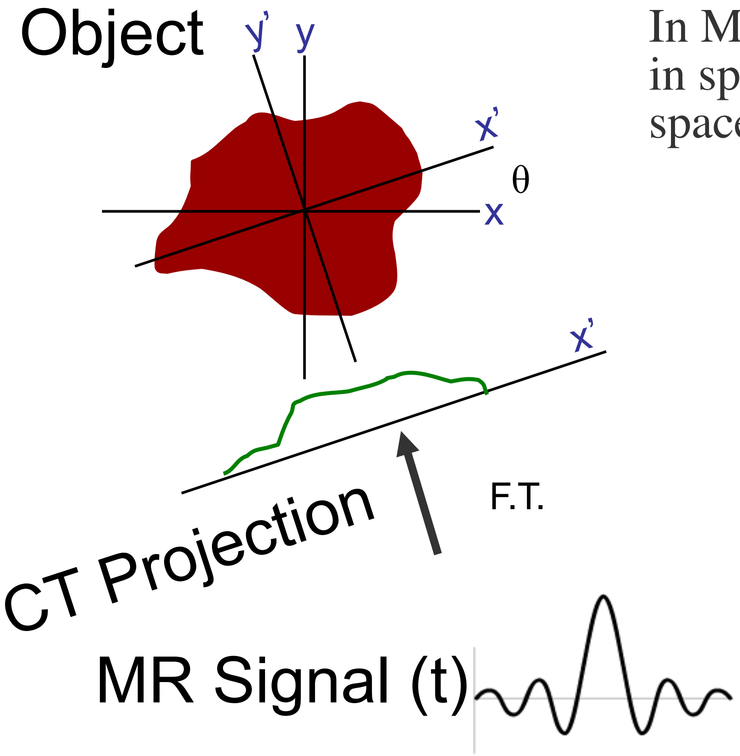
- Receiver signal can thus be modeled as

$$s(t) = e^{-j2\pi\nu_0 t}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\,dx\,dy$$

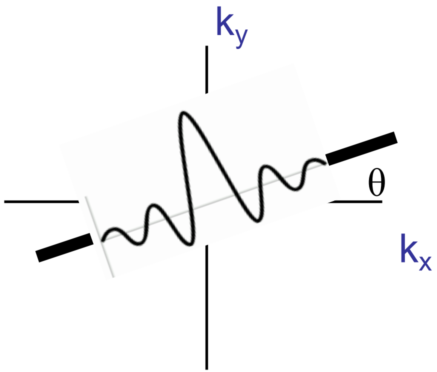
- With gradients, the demodulated signal becomes

$$s_0(t) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)e^{-j2\pi\gamma G_x x t}\,dx\,dy$$

Central Section Theorem in MRI



In MR, echo gives a radial line in spatial frequency space (k-space).



Interesting - Time signal gives spatial frequency information of $m(x,y)$

Gradient Echo

Phase Encoding

- We can apply a similar gradient along ‘y’. Although there is no readout during this pulse (because there is no ADC window at this time), we see that the phase accumulated during this pulse is given by

$$\phi_y(y) = -\gamma G_y T_p y$$

- Incorporating this into our baseband signal

$$s_0(t) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y) e^{-j2\pi \bar{\gamma} G_x x t} e^{-j2\pi \bar{\gamma} G_y T_p y} \, dx \, dy$$

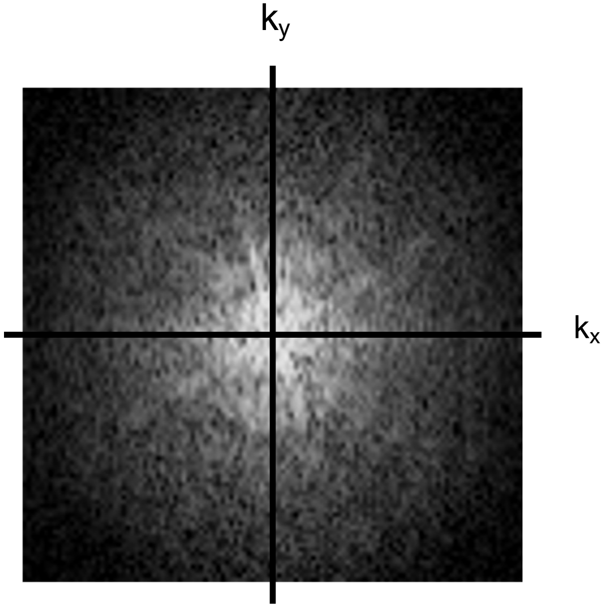
Signal Equations

- For a single spin: $$\phi = \gamma(x \int G_x dt + y \int G_y dt)$$
- Represent as exponential: $$s = e^{-i\gamma(x \int G_x dt + y \int G_y dt)}$$
- Sum over many spins: $$s = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \rho(x,y) e^{-i\gamma(x \int G_x dt + y \int G_y dt)} dx dy$$
- Signal equation:

$$s = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \rho(x,y) e^{-i(k_x x + k_y y)} dx dy \qquad k_{x,y}(t) = \frac{\gamma}{2\pi} \int_0^t G_{x,y}(\tau) d\tau$$

$$s(t) = FT[\rho(x,y)]|_{k_x(t), k_y(t)}$$

2D Image Reconstruction



Frequency-space
(k-space)

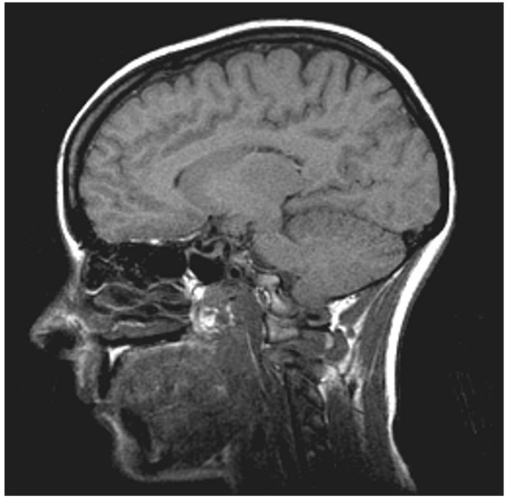
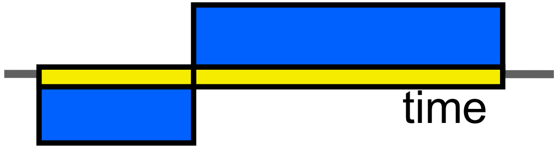


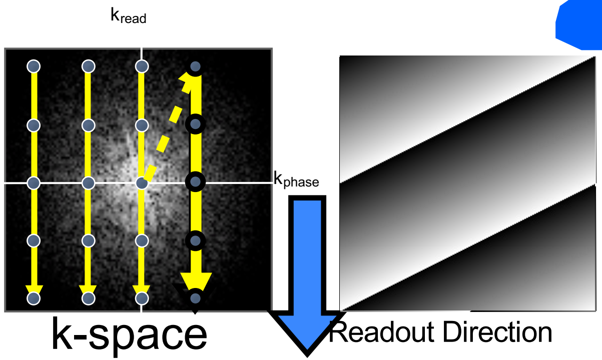
Image space

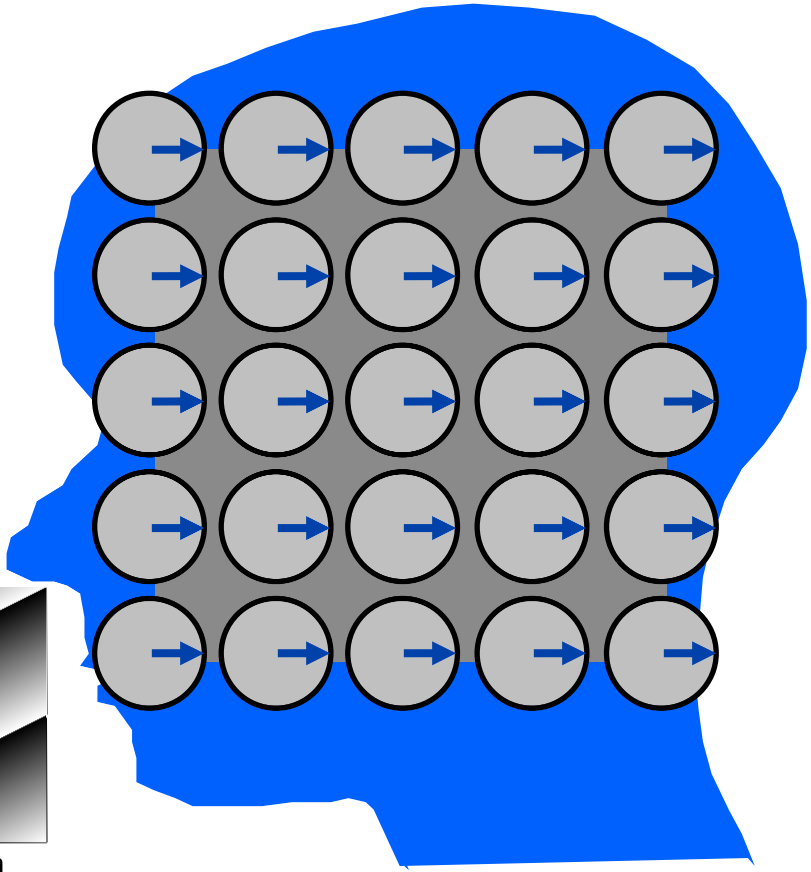
Image Formation and Sampling

Readout Gradient



Phase-Encode Gradient

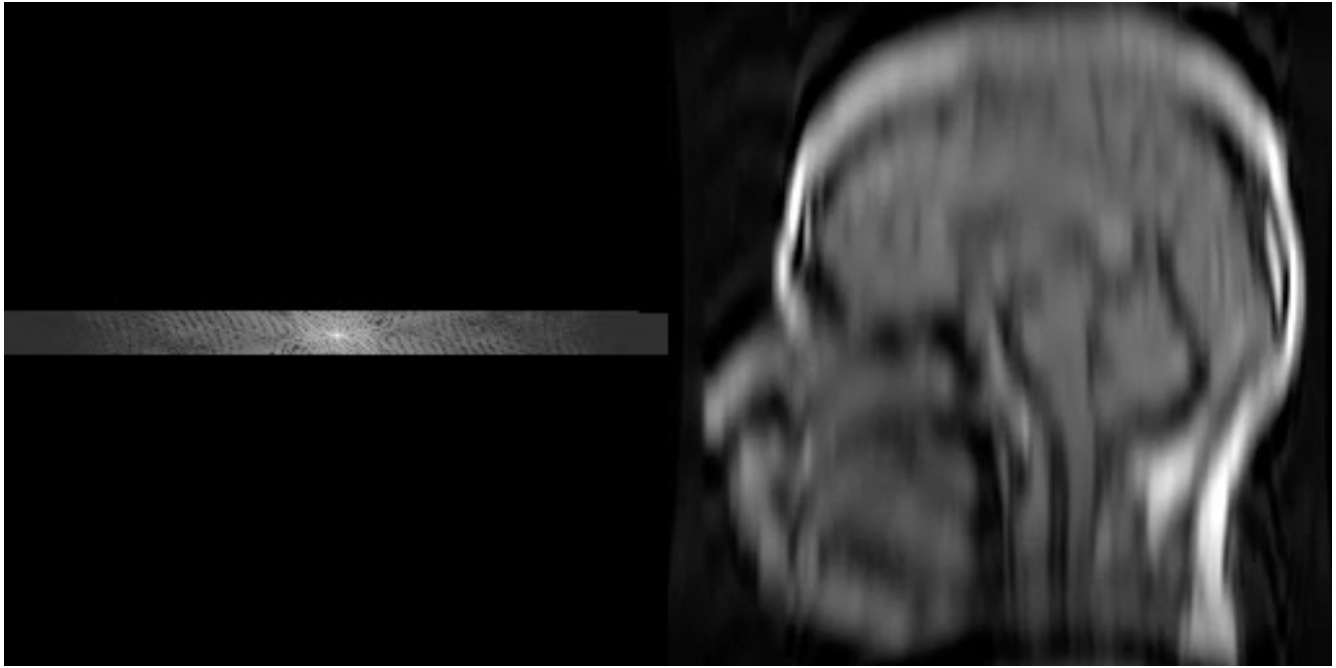




2D FT Reconstruction

Data Acquisition "k" space

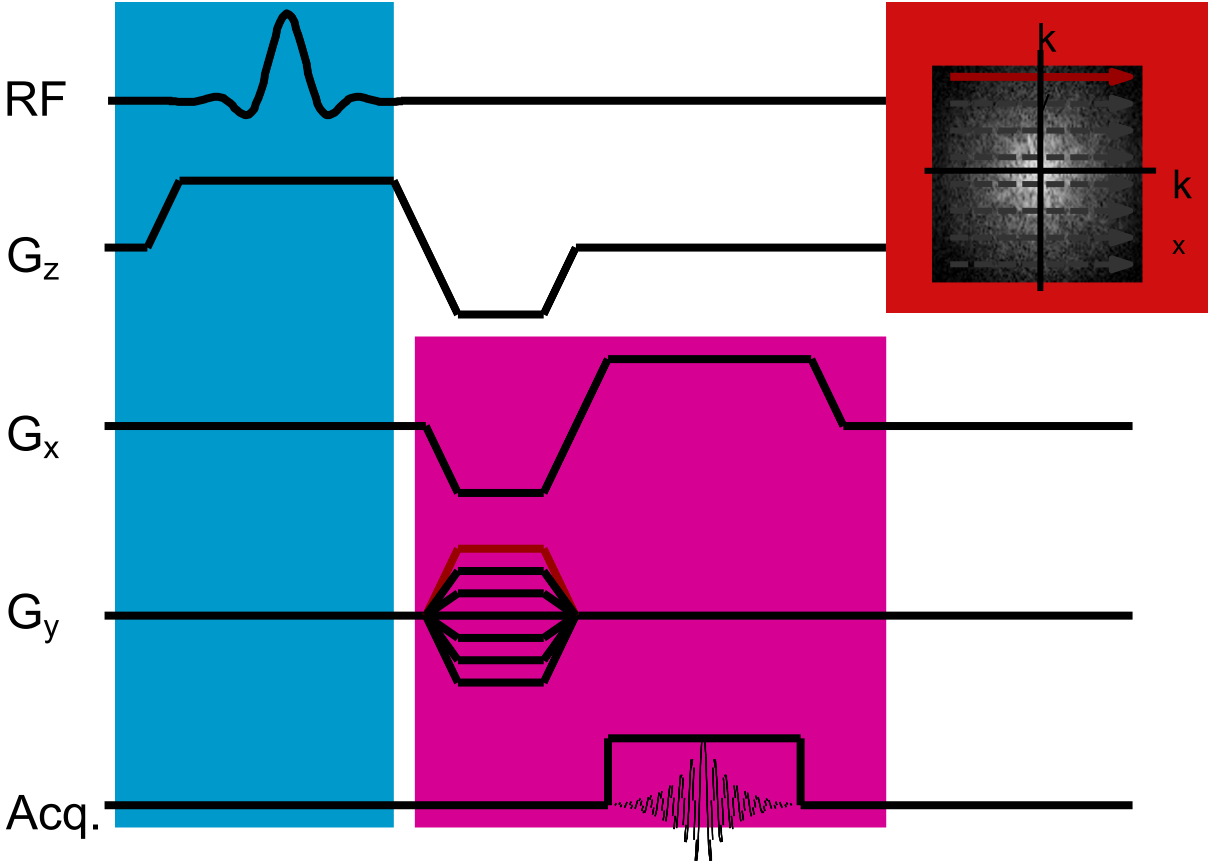
Image Space



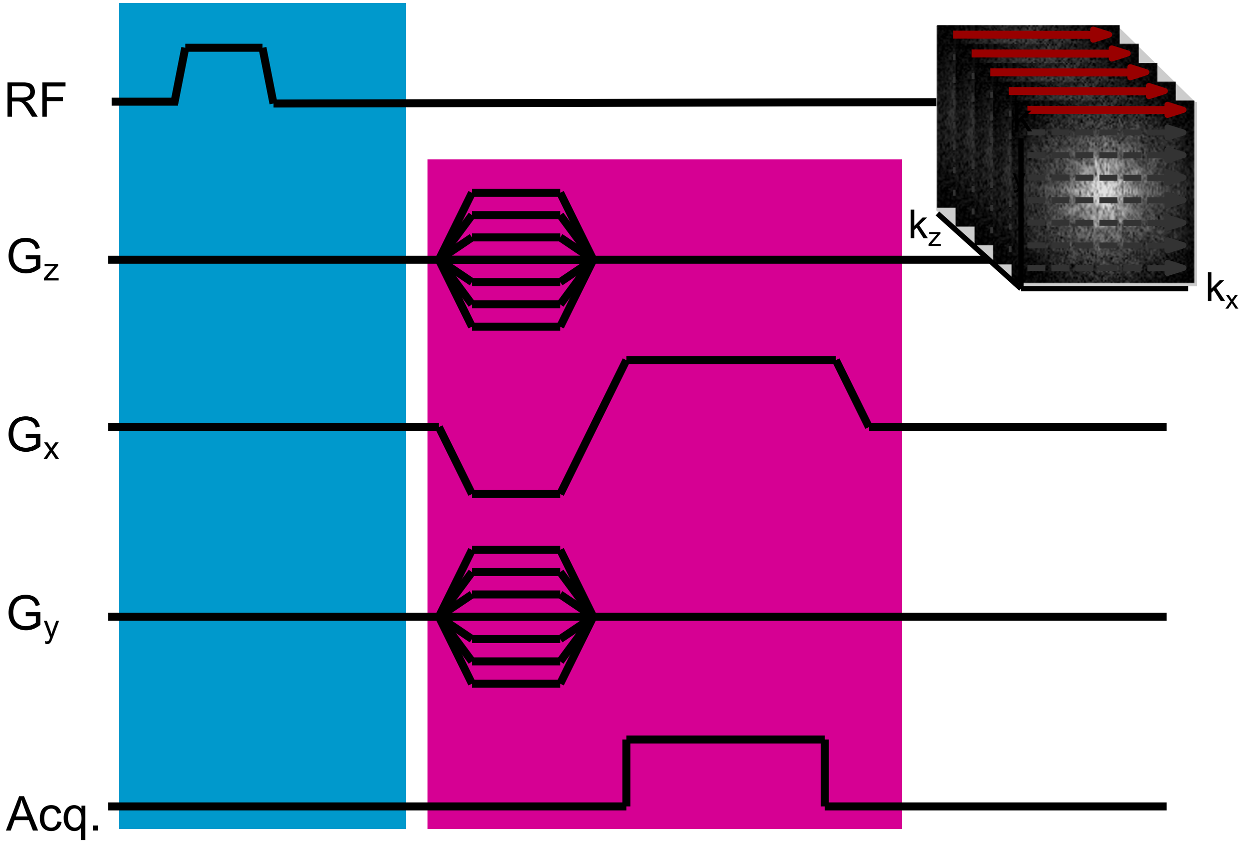
Fourier Transform

$\Delta x = 1/(2k_{max})$

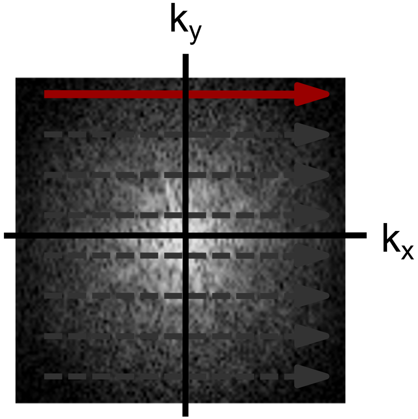
2D Imaging - Pulse Sequence



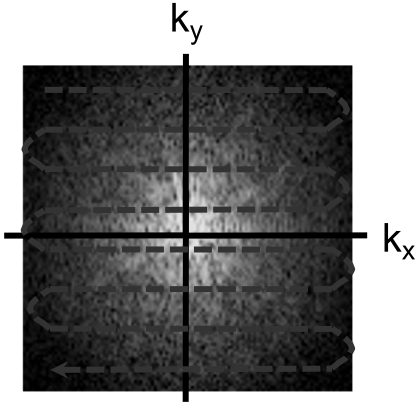
3D Pulse Sequence



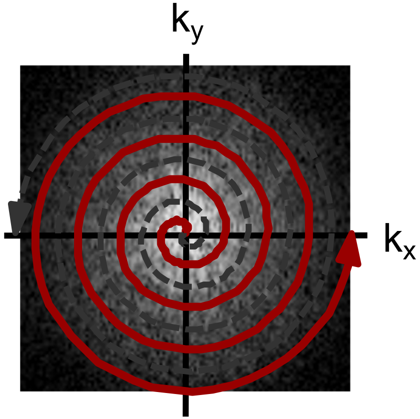
k-Space Trajectories



2D Fourier Transform



Echo-Planar



Spiral