

ECON 413 — Cheat Sheet:

Bridging RBC & NK Models to VARs

Purpose & Big Picture

Structural models (RBC/NK) give mechanism-first predictions for how variables move after a one-time “nudge.” VARs are data-first systems that summarize how variables co-move over time. The common language is the **impulse response function (IRF)**—the path of a variable after a shock. This sheet maps *model ideas* \rightarrow *VAR choices* \rightarrow *IRF patterns* \rightarrow *interpretation*.

From Model Logic to a VAR (no calculus)

- **Rule-of-thumb solution:** near a steady state, think $today \approx numbers \times yesterday + shock$. This encodes *persistence* and *comovement*.
- **Map model objects to observables:**
 - Output Y : real GDP (use log growth). Inflation π : CPI/PCE (log difference, annualized if quarterly).
 - Hours N : total hours or employment. Real wage w/P : compensation deflated by CPI/PCE.
 - Policy rate i : federal funds rate (or a shadow rate at the ZLB). Productivity A : output per hour or a TFP proxy.
- **VAR approximation:** with a few lags, these observables satisfy a finite-lag VAR. Estimate it, then *identify* shocks so IRFs can be read as technology, monetary, demand, or cost-push disturbances.

Identification Without Fear

A. Cholesky (Timing) Identification — “Who Moves First?”

Goal: Convert *correlated* same-period forecast errors into *uncorrelated, named shocks* by imposing a within-quarter timing order that matches your economic story.

Plain-language algorithm (same date t):

1. Pick an **order** of variables. The first in line keeps its whole surprise; call that a structural shock.
2. For the second, *subtract* the piece explained by the first’s surprise; the leftover is its structural shock.
3. For the third, subtract what the first *and* second explain; continue down the list.

Interpretation: first-mover can adjust contemporaneously; later movers react within the quarter.

Default classroom stories:

- *RBC technology block:* $A \rightarrow Y \rightarrow N \rightarrow w$. Tech news hits measured productivity first; output, hours, and wages respond within the quarter.
- *NK monetary block:* $Y \rightarrow \pi \rightarrow i$. The central bank observes output/inflation surprises and adjusts the policy rate quickly; prices/quantities do not fully reset within the quarter.

Two-variable cartoon: With order $[\pi, i]$, the monetary shock is the *leftover* in i after removing the part that merely reacts to this quarter’s inflation surprise. Swap the order and you swap the story. **Strength:** transparent. **Cost:** the *ordering is the assumption*.

B. Other identification options (short list)

- **Long-run restrictions:** technology shocks have *permanent* effects on productivity (or Y per worker); monetary/demand shocks do not.
- **Sign restrictions:** constrain IRFs over selected horizons (e.g., a monetary tightening: $i \uparrow$ on impact; $Y, \pi \downarrow$ with delay).
- **External instruments (proxy-SVAR):** use outside surprise series (e.g., high-frequency policy shocks) correlated with the target shock only.

Fast Recipes: Variables, Orders, and Expected Fingerprints

Goal	Variables (examples)	Cholesky order	IRF “fingerprint”
Technology (RBC supply)	A, Y, N, w (or A, Y, w)	$A \rightarrow Y \rightarrow N \rightarrow w$	$Y, C, I \uparrow$; real wage \uparrow ; π flat/ \downarrow ; <i>permanent</i> level effect in A . Hours N often \uparrow but model-dependent. Responses tend to be swift, often monotone.
Monetary policy (NK)	Y, π, i (add a commodity price index if needed)	$Y \rightarrow \pi \rightarrow i$	Tightening: $i \uparrow$ on impact; $Y, \pi \downarrow$ with delay; IRFs often <i>hump-shaped</i> ; <i>no permanent</i> real level effect.
Demand/IS (NK)	Y, π, i (or add fiscal/credit proxy X)	$Y \rightarrow \pi \rightarrow i$	Positive demand: $Y \uparrow, \pi \uparrow$; Taylor rule $\Rightarrow i \uparrow$; real effects fade in the long run.
Cost-push (NK)	π, Y, i, mc (marginal cost / wages)	$mc \rightarrow \pi \rightarrow Y \rightarrow i$	$\pi \uparrow, Y \downarrow$; policy usually tightens; short-run output-inflation trade-off appears.

VAR Hygiene Before Interpreting

- **Stationarity:** use log growth for real activity; inflation as log differences; rates often in levels (demean if needed).
 - **Deterministics:** include a constant; add a trend only if levels clearly trend.
 - **Lag length:** choose via AIC/BIC; quarterly work commonly uses 2–4 lags. Check residual autocorrelation.
 - **Sample windows:** regimes matter (Volcker, ZLB, COVID). Consider subsamples or a shadow rate at the ZLB.
 - **Scaling:** report IRFs to a 1-s.d. shock or normalize monetary shocks to a 1 pp policy-rate move.
 - **Stability & errors:** roots inside the unit circle; inspect residuals for autocorrelation/heteroskedasticity.
- Classic pitfalls (with fixes):**
- *Price puzzle:* inflation rises after a tightening. Add a commodity price index; use sign restrictions or an external instrument.
 - *Over-differencing:* differencing already stationary variables erases long-run information. Plot first.
 - *Too many lags in small samples:* wide bands and wobbly IRFs. Prefer parsimony and robustness checks.

Reading IRFs Like an Economist

- **Sign & timing:** who moves first and how quickly do others follow?
- **Shape:** monotone (RBC tech often) vs. hump-shaped (NK monetary often); any overshoot?

- **Persistence:** does it die out or shift the long-run level (permanence)?
- **Comovement:** do Y, C, I, N, π, i align with the model's mechanism?
- **Uncertainty:** 68% bands are fine for classroom work; 90% for stricter evidence.
- **FEVD:** which shock explains most of Y 's variance at 1, 4, 12 quarters?

One-Minute Talk Track for Presentations

1. **Mechanism:** “Our model says shock X should create these co-movements over this horizon.”
2. **Design:** “We estimated a VAR on [variables], identified X using [Cholesky order / long-run / signs / instrument].”
3. **Evidence:** “IRFs show sign/shape/persistence that *match* / *conflict* with the model's fingerprint.”
4. **Next step:** “We would tweak [variables, lags, sample, identification] or adjust the model mechanism.”

Bottom line: The VAR is a mirror for your RBC/NK story. If the reflection looks off, change the lighting (identification), the lens (variables/lags), or the face paint (model features), then compare IRFs again.