Möbius band

1 Abstract

A Möbius band (also spelled Mobius or Moebius), is a surface with only one side (when embedded in three-dimensional Euclidean space) and only one boundary. The Möbius band has the mathematical property of being unorientable. It can be realized as a ruled surface. Its discovery is attributed to the German mathematicians Johann Benedict Listing and then independently August Ferdinand Möbius in 1858, though a structure similar to the Möbius band can be seen in Roman mosaics dated circa 200–250 AD.

An example of a Möbius strip can be created by taking a paper strip and giving it a half-twist, and then joining the ends of the strip to form a loop.

(Wikipedia)

2 Definition

One way to represent the Möbius strip as a subset of three-dimensional Euclidean space is using the parametrization:

$$\begin{aligned} x(u,v) &= \left(1 + \frac{v}{2}\cos\frac{u}{2}\right)\cos u \\ y(u,v) &= \left(1 + \frac{v}{2}\cos\frac{u}{2}\right)\sin u \\ z(u,v) &= \frac{v}{2}\sin\frac{u}{2} \end{aligned}$$

where $0 \leq u < 2\pi$ and $-1 \leq v \leq 1$.

References

[1] `https://en.wikipedia.org/wiki/Mobius_strip`