Graph $z = \dfrac{xy}{x^2 + y^2}$

1 Abstract

The function $z = f(x, y) = \dfrac{xy}{x^2 + y^2}$ is a famous example such that it is not continuous at the origin. If we consider a line $x = 0$ then the limit is

$$\lim_{x \to 0} f(x, 0) = \lim_{x \to 0} \frac{x \cdot 0}{x^2 + 0^2} = 0,$$

but if we consider the limit along a line $y = x$ then

$$\lim_{x \to 0} f(x, x) = \lim_{x \to 0} \frac{x^2}{x^2 + x^2} = \frac{1}{2}.$$

At $(0, 0, \pm\frac{1}{2})$, there occur Whitney's umbrellas.