BOY surface

1 Abstract

Boy's surface is an immersion of the real projective plane in 3-dimensional space found by Werner Boy in 1901. He discovered it on assignment from David Hilbert to prove that the projective plane could not be immersed in 3-space.(from Wikipedia)

The Boy surface is a nonorientable surface that is one possible parametrization of the surface obtained by sewing a Moebius strip to the edge of a disk. Two other topologically equivalent parametrizations are the cross-cap and Roman surface. The Boy surface is a model of the projective plane without singularities and is a sextic surface. (from MathWorld)

2 Definition

It can be represented parametrically as

$$x = \frac{\sqrt{2}\cos^2 v\cos(2u) + \cos u\sin(2v)}{2 - \sqrt{2}\sin(3u)\sin(2v)}$$
$$y = \frac{sqrt2cos^2 v\sin(2u) - \sin u\sin(2v)}{2 - \sqrt{2}\sin(3u)\sin(2v)}$$
$$z = \frac{3\cos^2 v}{2 - \sqrt{2}\sin(3u)\sin(2v)}$$

for u in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ and v in $[0, \pi]$.

There exists a homotopy (smooth deformation) between the Roman surface and Boy surface as a parameter α varies from 0 to 1, where $\alpha = 0$ corresponds to the Roman surface and $\alpha = 1$ to the Boy surface.

References

[1] MathWorld bt Wolfram, `http://mathworld.wolfram.com/BoySurface.html`