

ROMAN surface

1 Abstract

The Roman surface, is a quartic nonorientable surface The Roman surface is one of the three possible surfaces obtained by sewing a Möbius strip to the edge of a disk. The other two are the Boy surface and cross-cap, all of which are homeomorphic to the real projective plane (Pinkall 1986). (fromMathWorld)

The center point of the Roman surface is an ordinary triple point with $(\pm 1, 0, 0) = (0, \pm 1, 0) = (0, 0, \pm 1)$, and the six endpoints of the three lines of self-intersection are singular pinch points, also known as pinch points. The Roman surface is essentially six cross-caps stuck together and contains a double infinity of conics. (fromMathWorld)

The Roman surface is the quintic surface of revolution given by the equation

$$(x^2 + y^2 + z^2 - k^2)^2 = \{(z - k)^2 - 2x^2\}\{(z + k)^2 - 2y^2\}.$$

that is closely related to the ding-dong surface. It is so named because the shape of the lower portion resembles that of a Hershey's Chocolate Kiss. (MathWorld)

2 Definition

It can be represented parametrically as

$$\begin{aligned}x(u, v) &= a \sin(2u) \sin^2 v \\y(u, v) &= a \sin u \sin(2v) \\z(u, v) &= a \cos u \sin(2v),\end{aligned}$$

for u in $[0, 2\pi)$ and v in $[-\pi/2, \pi/2]$, where a is a constant.

References

- [1] MathWorld by Wolfram, <http://mathworld.wolfram.com/RomanSurface.html>

$$z = \frac{xy}{x^2 + y^2}$$

$$z = \frac{x^2y}{x^4 + y^2}$$