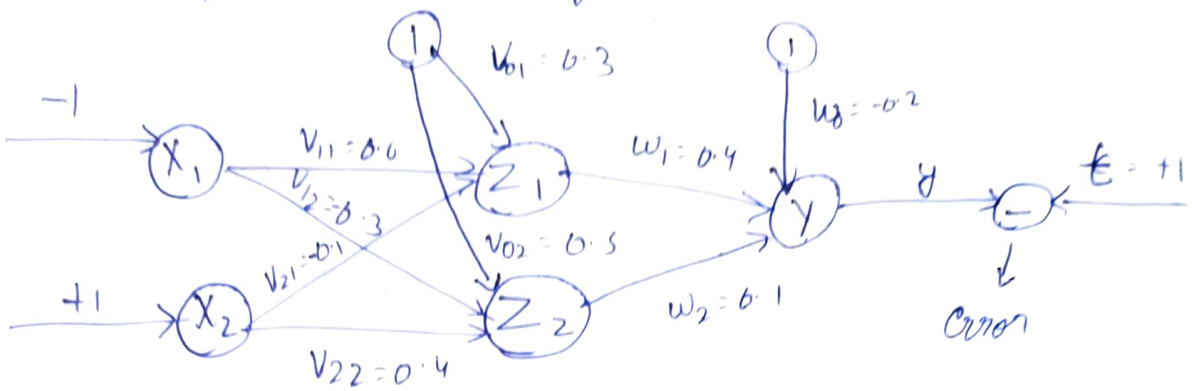
1. Solve a numerical problem in back propagation.



~~Initial weights~~ :

target output $t = +1$

learning rate $\alpha = 0.25$

Initial weights :-

$$\begin{cases} \{V_{01}, V_{11}, V_{21}\} = [0.3, 0.6, -0.1] \\ \{V_{02}, V_{12}, V_{22}\} = [0.5, -0.3, 0.4] \\ \{W_0, W_1, W_2\} = [-0.2, 0.4, 0.1] \end{cases}$$

$$[X_1, X_2] = [-1, +1]$$

sigmoid function $f(x) = \frac{1}{1+e^{-x}}$

$$f'(x) = f(x)(1 - f(x))$$

Step 1 calculation at the hidden layer (Z_1, Z_2)

$$Z_{in1} = 0.3 \times 1 + (-1 \times 0.6) + (+1 \times -0.1)$$
$$= 0.3 - 0.6 - 0.1$$
$$= -0.4$$

$$Z_{in2} = 0.5 \times 1 + (-1 \times 0.3) + (1 \times 0.4)$$
$$= 0.5 + 0.3 + 0.4 = 1.2$$

$$z_1 = f(z_{in1}) = \frac{1}{1+e^{-z_{in1}}} = \frac{1}{1+e^{0.4}}$$

$$z_1 = 0.4013$$

$$z_2 = f(z_{in2}) = \frac{1}{1+e^{-z_{in2}}} = \frac{1}{1+e^{-1.2}}$$

$$z_2 = 0.7685$$

Step 2 calculation at the output layer.

$$y_{in} = z_1 w_1 + z_2 w_2 + 1 \times w_0$$

$$= 0.4013 \times 0.4 + 0.7685 \times 0.1 + 1 \times -0.2$$

$$y_{in} = 0.03737$$

$$y = f(y_{in})$$

$$= \frac{1}{1+e^{-0.03737}}$$

$$y = 0.5093$$

Step 3 compute the error between output layer and hidden layer.

$$\text{error } (\delta_{on}) = (t-y) f'(y)$$

$$= (1-0.5093)((0.5093)(1-0.5093))$$

$$\delta_{on} = 0.1226$$

Step 4 change in weight between output and hidden layer.

$$\Delta w_1 = \alpha \delta_{on} z_1 \qquad \Delta w_0 = \alpha \delta_{on} 1.$$

$$\Delta w_2 = \alpha \delta_{on} z_2$$

$\Delta w_1 = 0.25 \times 0.1226 \times 0.4013$

$= 0.01229$

$\Delta w_2 = 0.25 \times 0.1226 \times 0.7685$

$= 0.02355$

$\Delta w_0 = 0.25 \times 0.1226$

$= 0.03065$

Step 5 : Calculate the error between hidden layer and the input layer.

error $(\delta_i h_{z_1}) = \delta_{oh} \times w_1 \times f'(z_1)$

$\delta_i h_{z_2} = \delta_{oh} \times w_2 \times f'(z_2)$

$\delta_i h_{z_1} = 0.1226 \times 0.4 \times ((0.4013)(1-0.4013))$

$= 0.01178$

$\delta_i h_{z_2} = 0.1226 \times 0.1 \times ((0.7685)(1-0.7685))$

$= 0.00218$

Step 6 : Change in weight between hidden layer and input layer

$\Delta V_{01} = \alpha \delta_i h_{z_1} x_1 = 0.25 \times 0.01178 \times 1 = 0.00294$

$\Delta V_{11} = \alpha \delta_i h_{z_1} x x_1 = 0.25 \times 0.01178 \times -1 = -0.00294$

$\Delta V_{21} = \alpha \delta_i h_{z_1} x x_2 = 0.25 \times 0.01178 \times 1 = 0.00294$

$\Delta V_{02} = \alpha \delta_i h_{z_2} x 1 = 0.25 \times 0.00218 \times 1 = 0.00054$

$\Delta V_{12} = \alpha \delta_i h_{z_2} x x_1 = 0.25 \times 0.00218 \times -1 = -0.00054$

$\Delta_{22} = \alpha \delta_i h_{z_2} x x_2 = 0.25 \times 0.00218 \times 1 = 0.00054$

Step 7 : Calculation of the final weights.

$V_{01}(new) = V_{01} + \Delta V_{01} = 0.3 + 0.002945 = 0.30294$

$V_{11}(new) = V_{11} + \Delta V_{11} = 0.6 + (-0.000294\,5) = 0.59705$

$V_{12}(new) = V_{12} + \Delta V_{12} = -0.3 + (-0.000054\,5) = -0.300054$

$V_{02}(new) = V_{02} + \Delta V_{02} = 0.5 + 0.000054\,5 = 0.500054$

$V_{21}(new) = V_{21} + \Delta V_{21} = -0.1 + 0.002945 = -0.097055$

$V_{22}(new) = V_{22} + \Delta V_{22} = 0.4 + 0.000545 = 0.400545$

$w_0(new) = w_0 + \Delta w_0 = -0.2 + 0.03065 = 0.1065$

$w_1(new) = w_1 + \Delta w_1 = 0.4 + 0.01229 = 0.41229$

$w_2(new) = w_2 + \Delta w_2 = 0.1 + 0.02355 = 0.12355$