

3. Prove that the first derivative of sigmoid  $f(x) = f(x)(1-f(x))$

We know that  $f(x) = \frac{1}{1 + e^{-x}}$

$$\frac{d}{dx} f(x) = \frac{d}{dx} \left[ \frac{1}{1 + e^{-x}} \right]$$

$$= \frac{d}{dx} (1 + e^{-x})^{-1}$$

$$= -(1 + e^{-x})^{-2} (-e^{-x})$$

$$= \frac{e^{-x}}{(1 + e^{-x})^2}$$

$$= \frac{1}{1 + e^{-x}} \cdot \frac{e^{-x}}{1 + e^{-x}}$$

$$= \frac{1}{1+e^{-u}} \cdot \frac{(1+e^{-u})-1}{1+e^{-u}}$$

$$= \frac{1}{1+e^{-u}} \cdot \left( \frac{1+e^{-u}}{1+e^{-u}} - \frac{1}{1+e^{-u}} \right)$$

$$= \left( \frac{1}{1+e^{-u}} \right) \cdot \left( 1 - \frac{1}{1+e^{-u}} \right)$$

$$= f(u) (1 - f(u))$$

Hence proved.