Lack of Selection and Limits to Delegation: Firm Dynamics in Developing Countries*

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Abstract

Delegating managerial tasks is essential for firm growth. Most firms in developing countries, however, do not hire outside managers but instead rely on family members. In this paper, we ask if this lack of managerial delegation can explain why firms in poor countries are small and whether it has important aggregate consequences. We construct a model of firm growth where entrepreneurs have a fixed time endowment to run their daily operations. As firms grow large, the need to hire outside managers increases. Firms' willingness to expand therefore depends on the ease with which delegation can take place. We calibrate the model to plant-level data from the U.S. and India. We identify the key parameters of our theory by targeting the experimental evidence on the effect of managerial practices on firm performance from Bloom et al. (2013). We find that inefficiencies in the delegation environment account for 11% of the income per capita difference between the U.S. and India. They also contribute to the small size of Indian producers, but would cause substantially more harm for U.S. firms. The reason is that U.S. firms are larger on average and managerial delegation is especially valuable for large firms, thus making delegation efficiency and other factors affecting firm growth complements.

Keywords: Economic development, growth, selection, competition, firm dynamics, management, entrepreneurship, creative destruction.

JEL Classifications: O31, O38, O40

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1 Introduction

Managerial delegation is essential for firm growth. In the developed world, many family-owned industrial giants, such as Walmart, The Lego Group, or Ford Motor Co., have managed to expand to hundreds of thousands of employees by relying on professional managers to run their daily operations. In contrast, firms in developing economies often shun outside managers and recruit managers exclusively among family members. Are such cross-country differences in the ease of managerial delegation important determinants of the process of firm growth? Might such limits to delegation allow small and unproductive firms in poor countries to survive because they limit the competitive pressure from more productive producers? And do they have important macroeconomic implications by reducing aggregate productivity and income per capita? In this paper, we answer these questions both theoretically and quantitatively.

To do so, we propose a macroeconomic model of firm dynamics where the need for managerial delegation takes center stage. Firms are run by entrepreneurs, who have the opportunity to increase their productivity in order to expand. Because the entrepreneur's own managerial time is a fixed factor, production features decreasing returns, and marginal profits decrease in firm size. This reduces firms' incentives to grow large. Entrepreneurs can endogenously overcome such limits to their span of control by hiring outside managers. If delegating managerial responsibilities to outside managers is riddled with problems, entrepreneurs have no incentive to invest in productivity growth as they anticipate not being able to efficiently delegate as they grow. Increases in the efficiency of delegation, therefore, raise the returns to grow large and increase aggregate productivity.

Our theory highlights an inherent complementarity between managerial delegation and firm size. Small firms do not consider the fixed managerial human capital of their entrepreneurs a drag on profitability. Only once firms reach a certain size does the entrepreneur's span of control become binding and outside managers valuable. This non-homotheticity, whereby the demand for outside managers is increasing in firm size, implies that frictions in the process of delegation affect the equilibrium distribution of firm size and the process of reallocation in a specific way. Firms with growth potential are hurt if outside managers cannot be employed efficiently and hence reduce their expansion efforts. In contrast, stagnant firms, which never grow beyond a certain size, benefit from such imperfections: they do not hire managers themselves, and they are more likely to survive, because they are shielded from the competition from their dynamic counterparts.

To quantify the importance of this mechanism, we calibrate our model to plant-level micro data from India and the U.S. Our quantitative methodology has two main features. First, we allow the structural parameters of our model to be country-specific and calibrate them to the Indian and U.S. data independently. This approach is important to address the identification problem implied by the non-homotheticity of managerial demand: Are firms in India small and is managerial delegation rare because delegation is difficult? Or do other frictions in India keep firms small and hence reduce the demand for outside managers in equilibrium? Our calibration strategy explicitly recognizes that firms in India might face higher barriers to growth (e.g., due to capital market inefficiencies or distortionary regulation), that entry costs might be higher (e.g., due to frictions in the access to start-up capital), or that many firms in India might be "subsistence entrepreneurs," who may simply lack the ability to grow their firms beyond a certain size. By allowing these features of the environment to be arbitrarily correlated with the efficiency of delegation, we refrain from attributing all differences between the U.S. and India to our mechanism of interest.

Second, we use well-identified micro-estimates as "identified moments" to calibrate our structural model (Nakamura and Steinsson, 2018). Specifically, we exploit variation in managerial practices based on the randomized experiment by Bloom et al. (2013) to estimate the production function for managerial inputs via indirect inference.[1] Bloom et al. (2013) provided a randomly selected group of Indian textile companies with management consulting to introduce American-style frontier management practices. They show this intervention increased the profitability among treatment firms: after two years, the firms that benefited from the intervention produced 9% more than firms in the control group. By explicitly using this estimated treatment effect as a moment for our structural model, we ensure our model generates the right microeconomic response to the experimental "management" intervention.

Our estimated model reveals stark differences between the U.S. and India. First, we estimate that the efficiency of delegation is indeed substantially smaller in India: a given manager is only half as efficient in an Indian firm, relative to a firm in the U.S. Second, we find the share of subsistence firms with little growth potential to be substantially higher in India. Finally, the few Indian firms with the potential to expand are substantially less efficient in doing so relative to the U.S. Such differences could, for example, reflect credit market imperfections or distortions to market entry, which prevent firms from expanding or keep innovative firms out of the market entirely.

Taken together, our estimated model implies that the Indian economy suffers from a significant lack of selection, where subsistence producers survive because firms with growth potential have low incentives to expand. Hence, the glut of small firms in India is not merely a reflection of frictions that those small firms face, but rather an indication of a lack of competition stemming from larger firms. Policies aimed at supporting small firms, for example, micro-finance programs, although potentially desirable for their redistributive properties, could be harmful by reducing the reallocation of resources from small stagnant firms to firms with growth potential.

We then use our calibrated model to quantify the importance of frictions in the delegation process to explain such differences in the process of firm dynamics between the U.S. and India. This analysis yields two main conclusions. First, we show that frictions to delegating managerial tasks in India are partly responsible for this lack of selection. If Indian firms could use outside managers as efficiently as firms in the U.S., their incentives to expand would be higher, and as a consequence aggregate productivity and income per capita would rise. Our estimates imply that such frictions can explain 11% of the income per capita difference between the U.S. and India.

Second, the complementarity of firm size and delegation implies an important interaction between the ease of delegation and other differences between India and the U.S. Although the process of firm dynamics in India does depend on the delegation environment, the implications are modest. We find that an increase in the efficiency of delegation to U.S. standards would increase average firm size by around 4% and reduce the employment share of small firms by a similar amount. If, in contrast, U.S. firms could use outside managers only as inefficiently as firms in India, the consequences would be much more severe: average firm size would decline by around 14%, and the

[1] We are very grateful to Nick Bloom and his coauthors for their willingness to share their data with us.

employment share of small firms would increase by 19%. The reason is that managerial delegation and other non-managerial factors that determine firm expansion naturally interact.

Related Literature That managerial delegation might be a key determinant of firm dynamics and macroeconomic performance goes back to the early work of Alfred Chandler (Chandler, 1990) and Edith Penrose, who argue that managerial resources are essential for firms to expand and that a scarcity of managerial inputs prevents the weeding out of small firms, because "bigger firms have not got around to mopping them up" (Penrose, 1959, p. 221). Recently, more systematic evidence for the importance of managerial inputs has accumulated. In particular, managerial practices differ systematically across countries, and firms in developed economies are larger and delegate more managerial tasks to outside managers (Bloom and Van Reenen, 2007, 2010; McKenzie and Woodruff, 2017).

We formalize and quantify the macroeconomic importance of such managerial considerations by providing a new theory of firm dynamics and the resulting firm size distribution. Our theory incorporates limits to firms' span of control, as in Lucas (1978), into a micro-founded model of Schumpeterian growth following Klette and Kortum (2004), which has been shown to provide a tractable and empirically successful theory of firm dynamics (see, e.g., (Acemoglu et al., 2018; Akcigit and Kerr, 2018; Garcia-Macia et al., 2019; Lentz and Mortensen, 2008)).[2] By explicitly allowing firms to hire outside managers, our model makes firms' span of control an endogenous variable that is jointly determined with the process of firm dynamics and the equilibrium distribution of firm size.[3]

Frictions in the market for managerial inputs are also highlighted in Caselli and Gennaioli (2013), Powell (2019), Grobovsek (2015), and Bloom et al. (2016). In contrast to our theory, all of these papers assume firm productivity is exogenous, so no interaction exists between the delegation environment and firm growth. Guner et al. (2018), Roys and Seshadri (2014) and Xi (2016) present dynamic models of (managerial) human capital accumulation but do not focus on the implications for firm dynamics. Finally, a large literature studies the internal organization of the firm; see, for example, Garicano and Rossi-Hansberg (2015) for a survey. This literature has a much richer micro structure of firms' delegation environment and the substitutability of managerial skills, but does not focus on the resulting properties of firm dynamics.[4]

Our model explicitly allows for heterogeneity in firms' innate growth potential. This heterogeneity is important to formalize the idea that limits to delegation affect the extent to which firms with

[2] As in Aghion and Howitt (1992), firm dynamics are determined through creative destruction, whereby successful firms expand through replacing other producers. See Aghion et al. (2014) for a survey of the Schumpeterian growth literature and Akcigit (2017) for the importance of firm dynamics for the process of economic growth.

[3] An overview of some regularities of the firm size distributions in India, Indonesia, and Mexico is contained in Hsieh and Olken (2014). A large literature explains cross-country differences in allocative efficiency across firms as diagnosed in Restuccia and Rogerson (2008) and Hsieh and Klenow (2009). This literature highlights credit market frictions (Buera et al. (2011); Moll (2014); Midrigan and Xu (2014)), size-dependent policies (Guner et al. (2008); Garicano et al. (2016); Gourio and Roys (2014)), monopolistic market power (Peters (2018)), and adjustment costs (Asker et al. (2014)). See Hopenhayn (2014) for a synthesis of this literature.

[4] A large empirical literature also studies family firms; see, for example, Bertrand and Schoar (2006). La Porta et al. (1999) document that family members are regularly controlling shareholders in most countries. Bennedsen et al. (2007) use variation in the gender of the CEO's firstborn child to present causal evidence that family successions hurt performance. In contrast, Mueller and Philippon (2011) argue family ownership has distinct benefits in environments of hostile labor relations.

growth potential replace stagnant, subsistence producers. Ample empirical evidence suggests such heterogeneity to be important. Schoar (2010) and Decker et al. (2014) argue some entrepreneurs are "transformative" and have the necessary skills to expand, whereas "subsistence entrepreneurs" may simply never grow independently of the environment they operate in. Hurst and Pugsley (2012) provide evidence that many firms in the U.S. intentionally choose to remain small. In the context of developing countries, Banerjee et al. (2015) and De Mel et al. (2008) stress the importance of persistent differences in growth potential. On the theoretical side, Luttmer (2011) and Lentz and Mortensen (2016) argue models without heterogeneity in growth potential are unable to explain the very rapid growth of a subset of U.S. firms.

Finally, on the methodological front, our paper adds to the recent literature in macroeconomics that uses well-identified microeconomic estimates to identify structural models (Nakamura and Steinsson, 2018). Recent examples in the literature on growth and development are Lagakos et al. (2018), Kaboski and Townsend (2011), and Brooks and Donovan (2017). To the best of our knowledge, our paper is the first to use this methodology to estimate a model of firm dynamics.

The remainder of the paper is organized as follows. In Section 2, we describe the theoretical model. Section 3 summarizes the data that we use in our quantitative analysis and discusses the identification of the model. Section 4 contains the calibration results and discusses a variety of non-targeted moments. In Section 5, we provide our main analysis to quantify the role of the delegation environment for firm dynamics and the aggregate economy. Section 6 provides various robustness checks of the main quantitative results. Section 7 concludes. All proofs and additional details are contained in the Appendix. An Online Appendix contains further results.

2 Theory

2.1 Technology, Preferences, and Static Allocations

We consider a continuous-time economy, where a representative household values the consumption of a unique final good, maximizes the stream of per-period utilities $U(C_t) = \ln(C_t)$, and discounts the future at rate ρ. Labor is supplied inelastically, and the members of the household can work as either managers or production workers. The final good Y is a Cobb-Douglas composite of a unit continuum of varieties,

$$\ln Y_t = \int_0^1 \ln y_{jt} dj, \tag{1}$$

and is used for consumption (C_t) and for productivity-enhancing investments by incumbents (R_t) and entrants ($R_{E,t}$). The aggregate resource constraint is therefore given by

$$Y_t = C_t + R_t + R_{E,t}. \tag{2}$$

To save on notation, we drop the time subscript t whenever doing so does not cause any confusion.

Producing the variety y_j requires both production workers and managerial inputs. In particular, we assume managers increase the efficiency of production workers so that firm f can produce good

j according to

$$y_{jf} = q_{jf}\phi\left(e_{jf}\right) l_{jf}, \tag{3}$$

where q_{jf} is the firm-product specific efficiency, l_{jf} is the number of production workers employed in producing intermediate good j, e_{jf} denotes the amount of managerial services that firm f allocates toward the production of good j, and $\phi\left(e_{jf}\right) \geq 1$ is an increasing function translating managerial services into physical productivity units. Letting w_P denote the equilibrium wage for production workers, the production labor cost of producing one unit of y is therefore given by $MC = \frac{w_P}{q\phi(e)}$.

Firms can produce multiple products $j \in [0, 1]$. In equilibrium, product j will be produced by the firm with the highest productivity q_{jf}. Firm f will therefore produce n_f products if it has the highest productivity in n_f product markets. We denote the producer's (i.e., the highest) productivity of variety j by q_j.

To focus on the interaction between managerial delegation and the resulting equilibrium process of firm dynamics, we keep the static market structure as tractable as possible. To do so, we assume that in each market j, the producing firm competes against a competitive fringe of potential producers that can produce variety j at marginal costs w_P/q_j.[5] Because the demand function stemming from (1) has a unitary elasticity, the producing firm engages in limit pricing and sets its price equal to the marginal costs of the competitive fringe. The gross profits after paying for production workers l_j (but before paying any managers the firm might decide to hire) are therefore given by[6]

$$\pi_j(e_j) = p_j y_j - w_P l_j = \left(\frac{\phi(e_j) - 1}{\phi(e_j)}\right) Y. \tag{4}$$

Hence, profits from variety j are increasing in the amount of managerial services e_j because managerial inputs increase physical productivity and hence profitability. For analytical convenience, we assume $\phi(e) = \frac{1}{1-e^{\sigma}}$, where $e \in [0, 1)$ and $\sigma < 1$, which implies

$$\pi(e_j) = e_j^{\sigma} Y, \tag{5}$$

that is, profits are a simple power function of managerial effort parameterized by the elasticity σ.

Managerial resources not only affect firm profitability but also the aggregate allocations. In particular (see Section OA-1.1 in the Online Appendix), aggregate output Y is given by

$$Y = Q\mathcal{M}L^P, \tag{6}$$

where $L^P = \int_0^1 l_j dj$ denotes the mass of production workers, $\ln Q = \int_0^1 \ln q_j dj$ is an index of aggregate physical productivity, and $\mathcal{M} = \left(1 - \int_0^1 e_j^{\sigma} dj\right)^{-1}$ summarizes the static effect of managerial services on aggregate productivity. Note $\mathcal{M}$ is increasing in e_j, reflecting the positive effect of managerial inputs on labor productivity at the firm level.

[5] This assumption allows us to abstract from strategic pricing decisions of firms that compete with firms of different productivity. Peters (2018) analyzes a model with strategic pricing. In terms of primitives, the fringe firms have access to the same technology as the leading firm and to a level of managerial services ϕ^{fringe}, which we normalize to unity.

[6] Note that $p_j y_j - w_P l_j = \left(1 - \frac{w_P l_j}{p_j y_j}\right) p_j y_j = \left(1 - \frac{1}{\phi(e_j)}\right) Y$ as $p_j = w_P/q_j$ and $p_j y_j = Y$.

2.2 Delegation, Span of Control, and Firms' Incentives to Grow Large

At the heart of our theory is the link between managerial delegation and firms' incentives to grow large. As in Klette and Kortum (2004), firms produce multiple products and grow by expanding into new product markets. In particular, by replacing the current producer of variety j, the firm adds new products to its portfolio and grows in sales, employment, and profits.

Because profits of each product depend directly on the amount of managerial services e, their availability is a key determinant of firms' incentives to expand. We assume firms are run by entrepreneurs, who have a fixed endowment $T < 1$ of managerial efficiency units they provide inelastically to their firms.[7] If an entrepreneur is the current producer in n markets, she will have $e_j = T/n$ units of managerial services per product. That she will want to spread her managerial time equally across all product lines follows directly from the concavity of π. The total profits of a firm of size n are hence,

$$\Pi(n) = \sum_{j=1}^{n} \pi(e_j) = n \times \pi\left(\frac{T}{n}\right) = T^{\sigma} n^{1-\sigma} Y.$$

This expression has a simple but important implication: although profits are increasing in the number of products n, they do so at a decreasing rate, because the owner's fixed endowment T limits her span of control, as in Lucas (1978). Firm size n and the entrepreneur's managerial endowment T are therefore complements and the marginal return to a unit of additional managerial resources is increasing in firm size:

$$\frac{\partial^2 \Pi(n)}{\partial n \partial T} > 0.$$

Hence, entrepreneurs with larger firms consider their fixed time endowment more of a bottleneck.

Delegation To counteract these decreasing returns, the entrepreneur can hire *outside* managers to augment her own endowment of managerial resources. This distinction between entrepreneurs and outside managers is what makes firms' span of control endogenous in our theory: while entrepreneurial human capital T is in fixed supply at the firm level, outside managers can be hired on the market. We assume the entrepreneur's and the managers' human capital are perfect substitutes and that the relative efficiency of outside managers within the firm is given by α. More specifically, if an owner of a firm of size n hires m units of managerial human capital for the production of product j, the total amount of managerial services e is given by

$$e(m) = T/n + \alpha \times m. \tag{7}$$

The parameter α is the key parameter for our analysis. It governs the efficiency of delegating tasks to outside managers, and we therefore refer to it as the *delegation efficiency*. The higher α, the more managerial services a given outside manager generates within the firm.

[7] Recall that $e < 1$ for $\phi(e) = (1 - e^{\sigma})^{-1}$ to be well-defined. It can be shown that $T < 1$ is sufficient to ensure this condition is satisfied.

We want to highlight that α is a parameter of the firm's production structure. Consider, for example, an entrepreneur in India looking to expand. One reason why the entrepreneur might decide to stay small is that the supply of sufficiently talented managers might be low. Another reason might be that the pool of managers may be fine, but he could not prevent them from shirking on the job. The former is about managerial human capital embedded in m. The latter is summarized in the delegation efficiency α.

One can think of many reasons why delegation might be less efficient in a developing economy such as India. First, a large empirical literature argues that the prevalence of efficient management practices, such as quality standards, monitoring, or meritocratic promotions, varies systematically with the level of development (see, e.g., Bloom et al. (2012) or Bloom and Van Reenen (2010)). Second, the efficiency of delegation could depend on the level of technology. For example, if delegation is complementary to IT equipment, technological differences across countries will be a source of variation in α (see, e.g., Bloom et al. (2009)). Finally, α can be interpreted as a reduced-form specification of the prevailing institutional or cultural environment. If, for example, contractual imperfections are severe or the level of trust is low, entrepreneurs might need to spend substantial amounts of their own time monitoring their managerial personnel.[8]

We assume outside managers are hired on a spot market at a given wage w_M. This assumption implies that the firm's delegation decision is static. Using (5) and (7), total profits net of managerial payments of a firm of size n are given by:

$$\Pi(n) \equiv \sum_{j=1}^{n} \max_{m_j \geq 0} \left\{ \left(\frac{T}{n} + \alpha m_j \right)^{\sigma} Y - w_M m_j \right\}. \tag{8}$$

The maximization problem in (8) defines both firms' demand for managerial inputs and their final profit function. Two properties are noteworthy. First, the entrepreneur's own managerial input T generates a well-defined extensive margin for managerial hiring. In particular, the firm only hires outside managers if the size of the firm exceeds the endogenous delegation cutoff $n^*(\alpha)$, which is given by

$$n^*(\alpha) = T \times \left(\frac{\omega_M}{\sigma \alpha} \right)^{\frac{1}{1-\sigma}}, \tag{9}$$

where $\omega_M = w_M/Y$. Hence, small firms rely purely on the time of the owner and only start delegating once they reach a size $n > n^*(\alpha)$. The cutoff $n^*(\alpha)$ is decreasing in α, as even small firms utilize outside managers if delegating is easy. Second, it is easy to verify that the optimal managerial demand per product $m(n)$, conditional on hiring, that is, if $n > n^*(\alpha)$, is given by

$$m(n) = \left(\frac{\sigma}{\omega_M} \right)^{\frac{1}{1-\sigma}} \alpha^{\frac{\sigma}{1-\sigma}} - \frac{1}{\alpha} \frac{T}{n}. \tag{10}$$

Note first that $m(n)$ is increasing in n; that is, larger firms hire more managers per product to make up for the fact that their own managerial resources are spread thinner and thinner as the firm gets larger. Hence, the demand for outside managerial resource is non-homothetic as larger

[8] In Section (OA-1.2) in the Online Appendix, we provide a simple micro-founded example, where a contractual game between the owner and outside managers leads to equation (7) and α is a combination of explicit structural parameters.

firms hire managers more intensely. Moreover, the demand for outside managers is increasing in the delegation efficiency α, holding ω_M constant.

Substituting firms' optimal delegation policies into (8) implies firm profits are given by

$$\Pi(n;\alpha) = \tilde{\pi}(n;\alpha) \times Y \quad \text{where} \quad \tilde{\pi}(n;\alpha) = \begin{cases} T^{\sigma} n^{1-\sigma} & \text{if} \quad n < n^*(\alpha) \\ T\frac{\omega_M}{\alpha} + (1-\sigma)\left(\frac{\sigma\alpha}{\omega_M}\right)^{\frac{\sigma}{1-\sigma}} n & \text{if} \quad n \geq n^*(\alpha) \end{cases}. \tag{11}$$

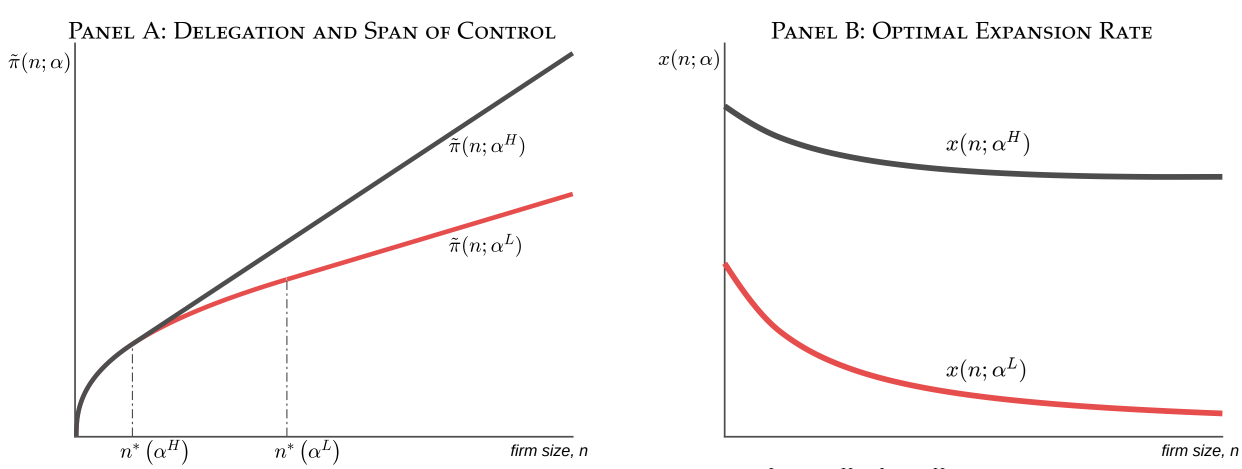
This profit function is a crucial object in our analysis, because it summarizes the firm's span of control, that is, the return to expanding into a new product market. Importantly, the possibility of delegation endogenizes the firm's span of control and makes it directly dependent on α.

In the left panel in Figure 1 we depict the profit function $\tilde{\pi}(n;\alpha)$ for two different levels of $\alpha^L < \alpha^H$. Small firms are run only by their owner and are subject to diminishing returns: as long as they do not delegate, the marginal profit from producing an additional product is declining; that is, $\tilde{\pi}(n;\alpha)$ is concave in n. Once firms reach the delegation cutoff n^* and start hiring outside managers, however, the profit function becomes linear in n. Hence, entrepreneurs overcome their limited span of control by delegating managerial tasks to outside managers.

Now consider an increase in the efficiency of delegation. This increase reduces the delegation cutoff, and smaller firms start to rely on outside managers. Importantly, an increase in α also increases the *slope* of the profit function. This links the delegation environment and the process of firm dynamics: a higher α increases firms' span of control and raises the returns to grow large.

Our model nests two workhorse models in the literature as special cases. When $\alpha = 0$, no scope exists for outside delegation. In that case, $n^* = \infty$, and all firms are subject to diminishing returns, as in Lucas (1978). In contrast, when α is sufficiently large so that $n^* < 1$, every firm delegates, the limited span of control of the owner's own time T is not a bottleneck, and firms' profit functions are linear as in the baseline version of Klette and Kortum (2004). Hence, our model offers a simple framework where the firm's span of control is endogenous and determined in equilibrium.

Figure 1: Delegation, Span of Control, and Expansion Incentives



Notes: In Panel A, we depict the profit function $\tilde{\pi}(n;\alpha)$ characterized in (11) for α^L and α^H, $\alpha^L < \alpha^H$. In Panel B, we depict the optimal expansion schedule $x(n;\alpha)$ in (14).

Firm Expansion The efficiency of delegation is a crucial determinant of firms' incentives to expand. For now, we consider the behavior of an individual firm. In Section 2.3, we embed this structure into a general equilibrium model.

We model firm growth as a stochastic process whereby the firm can choose the rate at which it improves the productivity q of a randomly selected product by $\gamma_t > 1$ and thereby replaces the existing firm. In particular, if a firm with n varieties invests R units of the final good, it expands into a new product line at rate

$$X(R;\theta,n) = \theta[R/Q]^{\zeta} n^{1-\zeta}, \tag{12}$$

where θ, which we refer to as firms' growth potential, determines the efficiency of innovation, $\zeta < 1$ parametrizes the convexity of the expansion cost function, and Q_t is the productivity index defined in (6).[9] At the same time, each product the firm currently produces is improved upon by other firms at rate τ_t. This rate of creative destruction is, of course, endogenous and determined in equilibrium, but firms take it as given.

To characterize the firm's optimal expansion policy, we need to solve for its value function. The value of a firm with n products, $V_t(n)$, solves the Hamilton-Jacobi-Bellman equation

$$r_t V_t(n) - \dot{V}_t(n) = \Pi_t(n;\alpha) - n\tau_t\left[V_t(n) - V_t(n-1)\right] + \max_X \left\{ X\left[V_t(n+1) - V_t(n)\right] - Q_t n^{\frac{\zeta-1}{\zeta}} \left[\frac{X}{\theta}\right]^{\frac{1}{\zeta}} \right\}, \tag{13}$$

where $\dot{V}_t \equiv \partial V_t/\partial t$. The right-hand side of (13) consists of three parts. First, the firm earns the flow profits $\Pi_t(n;\alpha)$ given in (11). Second, the firm might lose one of its products to other firms, which occurs at the endogenous rate of creative destruction $n\tau_t$ (because each product gets replaced at rate τ_t). Finally, the value function incorporates the option value of expansion: with flow rate X, the firm expands into a new market and experiences a capital gain of $V_t(n+1) - V_t(n)$. The associated costs of expanding into a new market stem from (12). Note the function V_t directly depends on the delegation efficiency α via the profit function.

This value function implicitly defines firms' optimal rate of expansion and productivity growth. Letting $x \equiv X/n$ denote the expansion intensity, optimality requires that

$$x_t(n;\alpha) = \theta^{\frac{1}{1-\zeta}} \zeta^{\frac{\zeta}{1-\zeta}} \times \left(\frac{V_t(n+1) - V_t(n)}{Q_t}\right)^{\frac{\zeta}{1-\zeta}}. \tag{14}$$

Naturally, the incentives to expand depend on the *marginal* return to doing so, $V_t(n+1) - V_t(n)$. This marginal return is what links firms' innovation incentives to the ease of delegation. In equation (11) and the left panel of Figure 1, we show that α determines the concavity of the profit function and hence the marginal flow profit of expansion. Because the value function inherits the properties of the profit function, α also determines the slope of the value function and hence the optimal innovation rate for firms of different sizes.

In the right panel of Figure 1, we depict the optimal innovation rate $x(n,\alpha)$. The concavity of

[9] Because we denote innovation costs in terms of the final good, the scaling variable Q is required to keep the model stationary. We also assume firms' innovation costs depend on the number of varieties n to generate deviations from Gibrat's law solely through incomplete delegation. In particular, if the profit function in (11) were linear, the specification in (12) would imply that firm growth is independent of size.

the profit and value function implies firms' expansion incentives are declining in size. An increase in α affects this schedule in two ways. First, an increase in delegation efficiency shifts the whole expansion schedule upwards. Intuitively, if firms anticipate being able to hire outside managers more efficiently once they reach the delegation cutoff n^*, their incentives to expand will already be higher today. Similarly, firms that are already delegating also increase their expansion efforts as their profitability increases. Secondly, innovation incentives increase more for larger firms, so that the schedule $x(n;\alpha)$ becomes flatter. Hence, improvements in the delegation environment are particularly important for large firms, which rely heavily on outside managers.

2.3 Firm Dynamics and Delegation in General Equilibrium

To determine the aggregate effects of the delegation environment, we now embed this model of firm growth into a general equilibrium model of firm-dynamics. At each point in time there is a set of existing firms whose innovation rates are given by (14), and a set of potential entrants that enter the economy by improving upon existing producers.

Firm Heterogeneity We explicitly allow firms to be heterogeneous in their growth potential. Formally, we assume firms differ in their innovation efficiency θ and can be either *transformative* (high, θ_H) or *subsistence* (low, θ_L) types. A firm's type is persistent and determined upon entry. Each new entrant draws a firm type $\theta \in \{\theta_H, \theta_L\}$ from a Bernoulli distribution, where

$$\theta = \begin{cases} \theta_H & \text{with probability } \delta \\ \theta_L & \text{with probability } 1-\delta \end{cases}. \tag{15}$$

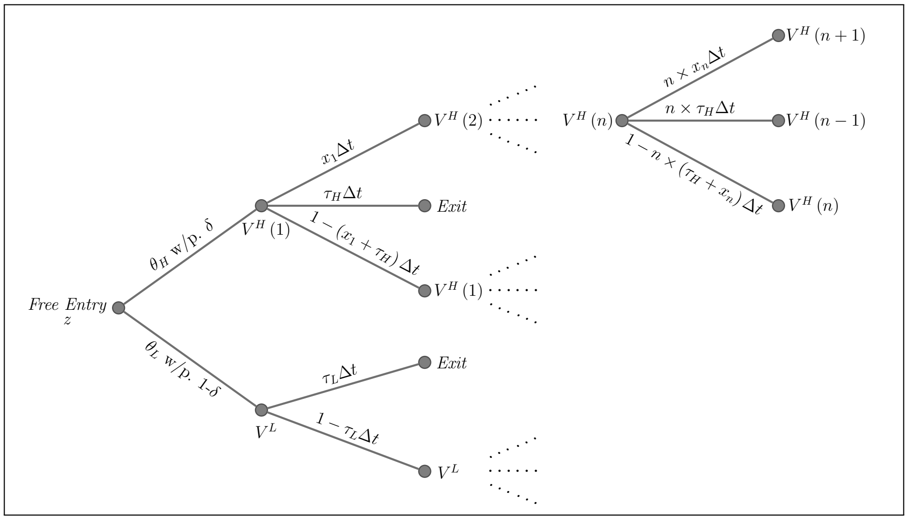
To capture the existence of subsistence entrepreneurs, we assume $\theta_L = 0$; that is, low-type firms are entirely stagnant. This polar case is conceptually useful because the sole difference in firm dynamics across countries then stems from the innovation incentives for high types - the high types' appetite for expansion is what determines the degree of selection, that is, how long it takes for low-type firms to be replaced.

In addition, we also allow firms to potentially differ in the rate at which they *lose* products due to differences in their reputation, customer loyalty, or organizational capital. Letting τ_H and τ_L be the rates at which high- and low-type firms lose a given product to other firms (both of which will be determined in equilibrium), we assume $\tau_L = \beta\tau_H$. If $\beta > 1$, low-type firms are easier to replace (or are targeted by expanding firms more intensely); if $\beta < 1$, the opposite is the case. The parameter β is one of our structural parameters that we calibrate from the data. Allowing for $\beta \neq 1$ is not conceptually important; we introduce it mostly for quantitative reasons.

To summarize, the behavior of high types is described by the optimal expansion rate in (14) and the value function in (13) (hereafter denoted by $V_t^H(n)$). Subsistence entrepreneurs, in contrast, never innovate and hence never grow beyond a single product; they exit at rate $\tau_{L,t}$. Their value function is therefore simply given by

$$r_t V_t^L - \dot{V}_t^L = \Pi_t\left(1;\alpha\right) - \tau_{L,t} V_t^L. \tag{16}$$

Figure 2: Life-Cycle Dynamics



Entry A unit mass of potential entrants attempts to enter the economy at any point in time. They use a similar innovation technology as incumbent firms, where the flow rate of entry z is related to the spending on entry efforts R_E according to $z = \theta_E [R_E/Q]^{\zeta}$. Entrants enter the economy with a single, randomly selected product. Given that an entrant becomes a high-type with probability δ, the equilibrium entry flow is given by

$$z_t(\alpha) = \underset{z}{\operatorname{argmax}} \left\{ z \left[\delta V_t^H(1) + (1-\delta) V_t^L \right] - Q_t \theta_E^{-\frac{1}{\zeta}} z^{\frac{1}{\zeta}} \right\} = \theta_E^{\frac{1}{1-\zeta}} \zeta^{\frac{\zeta}{1-\zeta}} \left[\frac{\delta V_t^H(1) + (1-\delta) V_t^L}{Q_t} \right]^{\frac{\zeta}{1-\zeta}}. \quad (17)$$

Note the equilibrium entry flow depends on the delegation environment α through firms' value function.

Figure 2 provides an overview of the life cycle dynamics in our model. Firms enter the economy with a single product and are either transformative, high-type entrepreneurs (with probability δ) or subsistence, low-type entrepreneurs (with probability $1-\delta$). The corresponding value functions are $V^H(1)$ and V^L. Within the next time interval Δt, high-type firms either expand (with probability $x_1\Delta t$), lose their only product and exit (with probability $\tau_H \Delta t$), or remain a one-product firm. In contrast, low-type firms never expand, but instead either exit (with probability $\tau_L \Delta t$) or remain in the economy by serving their initial market.

Delegation Efficiency and the Firm Size Distribution The equilibrium firm size distribution is endogenously determined from firms' expansion and entry incentives and hence depends on the delegation environment α. Let F_{nt}^H be the mass of high-type producers with n products, and let F_t^L be the mass of low-type producers (all of which have a single product). In a stationary equilibrium, these objects are constant and have simple expressions. In particular, as we show in Section A.1 in the Appendix, they are given by

$$F_n^H(\alpha) = \frac{\delta z(\alpha)}{n x(n;\alpha)} \prod_{j=1}^{n} \left(\frac{x(j;\alpha)}{\tau_H(\alpha)} \right) \text{ and } F^L(\alpha) = \frac{(1-\delta) z(\alpha)}{\tau_L(\alpha)}. \quad (18)$$

These expressions follow directly from the flow equations of the firm size distribution. Consider, for example, the case of F^L. Because subsistence firms exit the economy at rate τ_L and $z(1-\delta)$ subsistence entrepreneurs enter each instant, the equilibrium mass of low type firms is given by $(1-\delta)z/\tau_L$ as in (18). Furthermore, the aggregate rate of creative destruction is given by

$$\tau(\alpha) = \sum_{n=1}^{\infty} n x(n;\alpha) F_n^H(\alpha) + z(\alpha), \tag{19}$$

because existing producers get replaced both by other incumbent firms and new entrants. Equations (18) and (19) fully determine the equilibrium firm size distribution as a function of $x_t(n;\alpha)$ and $z_t(\alpha)$ because $\tau_L = \beta\tau_H$ and consistency requires that $\tau = \tau_H(1-F^L) + \tau_L F^L$.[10]

The expressions in (18) are useful to build intuition for how managerial delegation shapes the distribution of firm size. Recall that firm sales are proportional to the number of products n. The aggregate share of sales of firms with $n+1$ products relative to firms with n products is given by

$$\frac{(n+1)F_{n+1}^H}{nF_n^H} = \frac{x(n;\alpha)}{\tau_H(\alpha)}. \tag{20}$$

Hence, the relative importance of large producers is directly determined by the size-dependent innovation schedule $x(n;\alpha)$: the faster $x(n;\alpha)$ is declining in n, the smaller the aggregate importance of large firms. The right panel of Figure 1 therefore already suggests the link between delegation and the endogenous firm size distribution. If α is low, firms' span of control is a bottleneck for large firms, and the optimal innovation rate $x(n;\alpha)$ declines steeply in size n, as does the aggregate importance of large firms. Improvements in the efficiency of delegation therefore induce reallocation towards large producers. Similarly, the expression for the equilibrium mass of subsistence firms F^L shows why inefficiencies in the process of delegation reduce selection and keep low-type firms alive: by harming large firms more than small firms, they reduce creative destruction more than the entry rate. Environments where delegation is difficult therefore enable low-type firms to survive. In our quantitative analysis, we show these intuitions carry through once we take general equilibrium effects into account.

Creative Destruction and Aggregate Growth The rate of creative destruction is also the driver of aggregate growth in our economy. Recall that each successful innovation increases productivity by the step size γ_t. Because the rate of creative destruction is the rate at which such innovations take place, the aggregate growth rate of the productivity index Q_t is given by (see Appendix A.2)

$$g_t(\alpha) \equiv \frac{\dot{Q}_t}{Q_t} = \ln(\gamma_t) \times \tau_t(\alpha). \tag{21}$$

This expression highlights the relationship between delegation and aggregate growth. In our model, more efficient delegation increases aggregate growth through its effect on expansion and entry and hence creative destruction. Whether such increases in the rate of growth are persistent, depends on the behavior of the step size γ_t. As far as the process of firm dynamics is concerned, we do not

[10]Note F_t^L is the share of products that are produced by subsistence entrepreneurs as they produce one product each.

have to take a stand on γ_t, because our model permits a stationary firm-size distribution even if the step size γ_t varies over time; see Section A.3 of the Appendix, where we prove this property formally. However, to quantify the effect of delegation on long-run productivity differences, we consider a model where γ_t is endogenous and the long-run distribution of income across countries is stationary. Hence, differences in α between the U.S. and India will result in level differences, not growth differences (see Section 5 below).

2.4 The Labor Market Equilibrium for Outside Managers

To complete the characterization of the equilibrium, we need to specify the supply and demand of managerial inputs. The demand for outside managers results from firms' optimal hiring decisions. Because of the non-homotheticity of managerial demand, larger firms delegate more intensely, and the aggregate demand for managerial inputs depends on the endogenous firm size distribution. Using the optimal hiring rule in (10), a firm with $n \geq n^*$ products hires a total of $nm(n)$ managerial efficiency units. The demand for outside managers, H^{OM}, is therefore given by

$$H^{OM} = \sum_{n=1}^{\infty} \mathbb{1}(n \geq n^*) m(n) n F_n(\alpha) = \sum_{n=1}^{\infty} \underbrace{\mathbb{1}(n \geq n^*) \left(\left(\frac{\sigma}{w_M / Y} \right)^{\frac{1}{1-\sigma}} \alpha^{\frac{\sigma}{1-\sigma}} n - \frac{T}{\alpha} \right)}_{\text{Managerial demand given firm size}} \underbrace{F_n(\alpha)}_{\text{Effect of FSD}}, \tag{22}$$

where $F_n = F_n^H + \mathbb{1}(n = 1) F^L$. This expression highlights two important determinants of managerial demand. Holding the firm size distribution constant, aggregate demand is increasing in α. In addition, because managerial demand is non-homothetic, the firm size distribution $F_n(\alpha)$ itself also affects managerial demand directly: if firms are small, outside managers are in low demand because small firms can be run by their owners. This dependence on $F_n(\alpha)$ highlights an important identification challenge that our empirical strategy has to address: Do we see few outside managers in India because delegation is difficult? Or do other frictions keep Indian firms small, and hence no managers are required?

To model the supply of managerial workers, we assume each individual is endowed with a single efficiency unit of production labor and h units of managerial human capital, distributed according to $G(h)$. Individuals make their occupational choice to maximize total earnings; that is, individual i works as an outside manager if $h_i w_M > w_P$. Labor market clearing therefore requires that

$$H^{OM} = \int_{h \geq \frac{w_P}{w_M}} h g(h) dh, \tag{23}$$

where $g(h)$ is the density associated with $G(h)$.

In our application, we assume h is drawn from a Pareto distribution, namely $G(h) = 1 - \left(\frac{\vartheta-1}{\vartheta}\mu_M\right)^{\vartheta} \times h^{-\vartheta}$. Here, μ_M parametrizes the average level of managerial skills, and $\vartheta > 1$ governs the heterogeneity in managerial talent. Using this functional form, the labor market clearing condition in (23) is given by

$$H^{OM} = \left(\frac{\vartheta - 1}{\vartheta} \mu_M \right)^{\vartheta} \left(\frac{w_M}{w_P} \right)^{\vartheta - 1} \frac{\vartheta}{\vartheta - 1}. \tag{24}$$

Note the supply of outside managers is increasing in the relative wage with an elasticity of $\vartheta - 1$. Moreover, holding relative wages fixed, the supply of managerial skills is increasing in the average level of managerial human capital μ_M.

An equilibrium in our economy is then defined in the following way:

Definition 1 *Consider the environment described above. A dynamic equilibrium path is characterized by a time path of*

$$\left[p_{jt}, y_{jt}, \{V_t^H(n)\}_{n=1}^{\infty}, V_t^L, \{x_t(n)\}_{n=1}^{\infty}, z_t, w_{t,M}, w_{t,P}, \{F_{nt}^H\}_{n=1}^{\infty}, F_t^L, r_t, g_t\right]_{t=0}^{\infty},$$

such that (i) p_{jt} and y_{jt} maximize monopoly profits in (4)*, (ii) the value functions $V_t^H(n)$ and V_t^L are given by* (13) *and* (16)*, (iii) the innovation rates $x_t(n)$ are optimal and given in* (14)*, (iv) the entry rate z_t satisfies* (17)*, (v) $w_{t,P}$ and $w_{t,M}$ clear the labor market for production and managerial labor, (vi) the numbers of firms of each size $[F_{nt}^H, F_t^L]$ are consistent with the flow equations in Section A.1 in the Appendix, (vii) the interest rate r_t satisfies the household's Euler equation, and (viii) the aggregate productivity growth rate is consistent with* (21)*.*

2.5 Taking Stock

We have developed a theory to link the efficiency of delegation to firms' growth incentives and hence the process of firm dynamics and the equilibrium firm size distribution. At the heart of our model is the insight that a higher efficiency of delegation endogenously increases firms' span of control and hence their incentives to grow large.

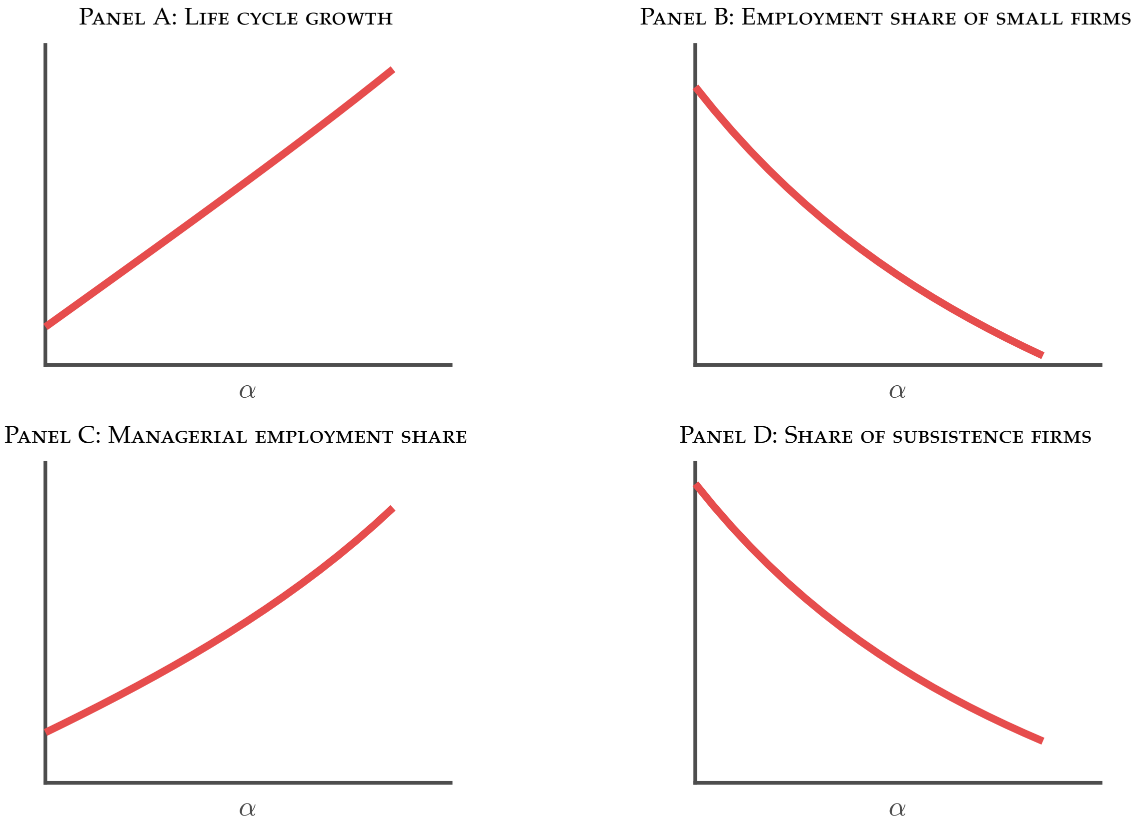
To summarize the effects of an increase in delegation efficiency α, consider Figure 3, where we depict the qualitative relationships between α and various equilibrium outcomes.[11] Panel A shows a positive relationship between delegation efficiency and firms' life-cycle growth. This follows directly from the resulting increase in firms' expansion incentives, in particular for large firms. This faster growth at the firm level shifts the firm-size distribution to the right so that the employment share of small firms declines (Panel B). These changes at the firm level are accompanied by changes in the labor market. In particular, the employment share of outside managers is increasing in α both because firms' demand for managers increases and because the firm size distribution shifts to the right, which further increases managerial demand, because large firms are manager intensive (Panel C). Finally, because firms are heterogeneous in their growth potential, an increase in α will also be accompanied by selection. Because subsistence entrepreneurs are small in equilibrium, they do not benefit from the opportunity to hire managers. In contrast, they *lose* from improvements in delegation efficiency because they are less likely to survive (Panel D).

These patterns are qualitatively consistent with stylized facts on firm dynamics in poor countries where firms are small and do not grow, subsistence producers are abundant, and outside managers are rare. Importantly, the glut of small, stagnant firms in poor countries might not solely reflect frictions these firms face, but may also result from more productive firms not being able to overcome

[11] Although these relationships stem from our quantitative model and we currently do not have an analytical proof, we have yet to find a counterexample. Hence, we suspect these comparative static results hold true regardless of the particular parametrization of the model.

limits to their span of control. Improvements in the efficiency of delegation enable firms with growth potential to overcome these decreasing returns and speed up the aggregate selection process. In the remainder of this paper, we analyze whether this mechanism can quantitatively account for the observed differences in the firm size distribution between the U.S. and India and whether it has important implications for differences in income per capita.

FIGURE 3: TAKING STOCK: DELEGATION, SELECTION, AND FIRM DYNAMICS



Notes: The figure summarizes the qualitative implications of changes in the delegation efficiency α for firms' life cycle growth (Panel A), the employment share of small firms (Panel B), the managerial employment share (Panel C), and the equilibrium share of low-type firms (Panel D).

3 Data and Calibration Strategy

3.1 Data

In this section we briefly describe the main data sources. A detailed description is contained in Section B.1 of the Appendix.

Establishment level data for the U.S. and India: We calibrate our model to data for the manufacturing sector of the U.S. and India. For the U.S. we rely on publicly available data for the population of manufacturing plants from the Business Dynamics Statistics (BDS). The BDS is provided by the U.S. Census Bureau and compiled from the Longitudinal Business Database (LBD), which provides data on employment and age for each establishment with paid employees. We focus on the data from 2012.

Analyzing data for the manufacturing sector in India is less straightforward, because no single database provides this information. To capture the entirety of the manufacturing sector, we follow Hsieh and Klenow (2014) and Hsieh and Olken (2014) and combine the Annual Survey of Industries (ASI) and the survey of the unorganized manufacturing sector from the National Sample Survey (NSS). The ASI focuses on the formal sector and covers all establishments employing 10 or more workers using electric power or employing 20 or more workers without electric power. The NSS, every five years, surveys a random sample of the population of manufacturing establishments outside the sampling frame of the ASI. Hence, the firms in the NSS are decidedly smaller and mostly informal - more than 80% of plants have at most two employees and less than 1% have more than 15 employees (see Table 10 in the Appendix, where we report the firm size distribution in the NSS). We merge these two datasets using the sampling weights provided in the data and focus on the year 2010, which is the latest year for which both datasets are available.

For our analysis, we treat this union of the ASI and NSS data as representing the population of manufacturing firms in India. To provide direct evidence for the representativeness of these data, we compared them with the Indian Economic Census, which is a complete count of all economic units in India. As we show in Section B.1 in the Appendix, the cross-sectional firm size distributions of the ASI/NSS sample and the Economic Census are very similar. We cannot rely on the Economic Census for our main analysis, because it does not contain information on firm age and hence cannot be used to estimate the employment life cycle or to measure firm entry.

Table 1 contains some basic descriptive statistics about the distribution of establishment size in the U.S. and India.[12] Expectedly, the importance of large firms differs enormously. In the U.S., two-thirds of manufacturing employment is concentrated in establishments with at least 100 employees, and only one-third of the establishments have fewer than four employees. In India, more than 9 out of 10 establishments have fewer than four employees, and they account for more than half of aggregate employment. Because the Indian data are collected at the level of the establishment, our benchmark analysis focuses on individual establishments. We conduct robustness checks using firm-level data for the U.S. in Section 6.

Data on Managerial Employment: To measure managerial employment, we rely on national census data provided by the IPUMS project. We focus on male workers in the manufacturing industry working in private-sector jobs. We always use the most recent data available, which is 2004 in the case of India and 2010 in the case of the U.S. Our theory stresses the importance of *outside* managers. We, therefore, classify employees as managers if they are assigned the occupational code "Legislator, Senior official, and manager" *and* they are hired as wage workers instead of being, for example, unpaid family members or the owner themselves. As shown in the last column of Table 1, in the U.S. roughly 12.5% of employees satisfy this criterion. In India, less than 2% are employed as

[12] Recently, Rotemberg and White (2017) argued the data in the U.S. and India differ in terms of data cleaning strategies. These concerns are less relevant for our study because we only rely on sample averages of the reported employment data and do not utilize information on any higher moments, which are important for the measurement of misallocation. We did recalculate all estimation moments after dropping firms in the top and bottom 2% of the employment distribution (both in the population of firms and conditional on age) and found this decision had little effect on our analysis.

Table 1: Establishment Size and Managerial Employment in the U.S. and India

	Establishment Size					Empl. share of outside managers
		1 - 4 employees		$\geq$ 100 employees		
	Average empl.	Share	Empl. share	Share	Empl. share	
U.S.	42.7	32.8%	1.8%	8.8%	65.5%	12.5%
India	2.7	93.0%	54.8%	0.1%	18.6%	1.65%

Notes: The table contains summary statistics from the firm size distribution in the U.S. and India. The U.S. data come from the BDS in 2012, and the data for India comes from the NSS and ASI in 2010. In the last column, we report the share of outside managers, that is, all workers who are classified as managers according to the occupation classification ISCO and who are hired as wage workers. These data stems from IPUMS.

outside managers.

Insisting on outside managers is important. In the U.S., roughly 14% of the labor force is classified as managers according to their occupational code. The majority, namely 91%, are wage workers and hence outside managers in the sense of our theory. By contrast, in India only 14% of individuals working in a managerial occupation are wage workers. The remainder are either entrepreneurs themselves or unpaid family members. Hence, Indian firms acquire managerial services mostly from their owners or close family members. This pattern is very much the exception in the U.S.

An important implication of our model is that firms' demand for outside managers is non-homothetic: larger firms have higher managerial employment shares. In Table 2, we show such non-homotheticities to be norm in the Indian firm-level data.[13] Whereas firms with one to four employees have essentially no managerial personnel, firms with more than 100 employees have managerial employment shares exceeding 10%. The aggregate managerial share as measured from the firm-level data is 2.9%, which is reasonably close to the 1.65% reported in IPUMS. Below, we show the predictions of our model are also quantitatively in line with Table 2.

Table 2: Non-homothetic managerial demand in India

	Number of employees							Full Sample
	1-4	5-9	10-19	20-49	50-99	100-999	+1000	
Share of managers	0.002	0.017	0.043	0.077	0.079	0.101	0.147	0.029

Notes: The table reports the share of managerial employment among firms of a given size (columns 1 - 7) and for the aggregate economy (last column). The data combines the NSS data from 1995 and the ASI data from 1999. 1995 is the only year where we observe managerial hiring in the NSS data, and 1999 is the closest year for which we have access to the ASI data.

Although measuring such non-homotheticities from the firm-level data is natural, doing so has the disadvantage that we cannot report Table 2 for the U.S. (because the BDS data do not have

[13]The definition of outside managers is similar between the firm-level data and the data on IPUMS. The firm-level data have an employment category "supervisory and managerial staff." This category contains everyone who holds positions of supervision and management and who are working proprietors and managers when paid a regular salary. This category is distinct from the category "working proprietors," that comprises all owners who are actively engaged in the work of the enterprise and all unpaid working proprietors. We use the managerial employment share from IPUMS as our main calibration target to ensure the classification is consistent between the U.S. and India.

information on managerial employment). In Section B.1 in the Appendix (see in particular Figure 10), we use data from the Current Population Survey (CPS), which shows managerial hiring is also non-homothetic in the U.S. In addition, because the IPUMS data for India (but not for the U.S.) contain information on the size of the establishment individuals work in, we also corroborate the results reported in Table 2 using the data from IPUMS.

3.2 Identification and Calibration

Our model has 12 structural parameters:

$$\Omega \equiv \{\underbrace{\alpha, \sigma, T, \mu_M, \vartheta}_{\text{Management}}, \underbrace{\theta, \theta_E, \zeta, \delta, \beta}_{\text{Firm dynamics}}, \underbrace{\gamma_t, \rho}_{\text{Macro}}\}.$$

Five parameters are directly related to the demand for and supply of managerial services: the delegation efficiency (α), the managerial output elasticity (σ), the owners' own human capital (T), and the distribution of managerial skills (μ_M and ϑ). The process of firm dynamics is captured by the expansion and entry efficiencies (θ and θ_E), the convexity of the cost function ζ, the share of high-type entrants (δ), and the difference in type-specific creative destruction rates (β). Finally, the remaining "macro" parameters include the innovation step size (γ_t) and the discount rate (ρ).

As highlighted above, we estimate most of these parameters separately for the U.S. and India. We restrict three parameters to be the same across countries: ρ, ζ, and ϑ. We fix ρ and ζ exogenously and calibrate the remaining parameters by minimizing the distance between several empirical moments and their model counterparts.[14] In particular, let M^E denote the vector of S empirical moments and let $M(\Omega)$ denote the vector of model-simulated moments. We then chose Ω to minimize the absolute relative deviation between the model and data; that is, we solve

$$\min_{\Omega} \sum_{m=1}^{S} \frac{|M_m^E - M_m(\Omega)|}{|M_m^E|}.$$

Even though our parameters are calibrated jointly, below we provide a heuristic description of the relationship between the parameters and specific moments. In Appendix B.2, we give a more formal identification discussion and verify these relationships numerically using a sensitivity matrix, where we report the elasticity of each moment used in the internal calibration with respect to the parameters of the model (see Table 14 in Section B.5 in the Appendix).

Note we allow the innovation step size γ_t to be country-specific and time-varying. In particular, we allow for the Indian economy to be along a transition path; that is, catching-up with the U.S. Concerning the firm size distributions, we estimate the parameters under the assumption that the distributions are stationary. As we show formally in Section A.3 of the Appendix, our model implies the firm-size distribution will remain stationary during the transition, that is, despite the fact

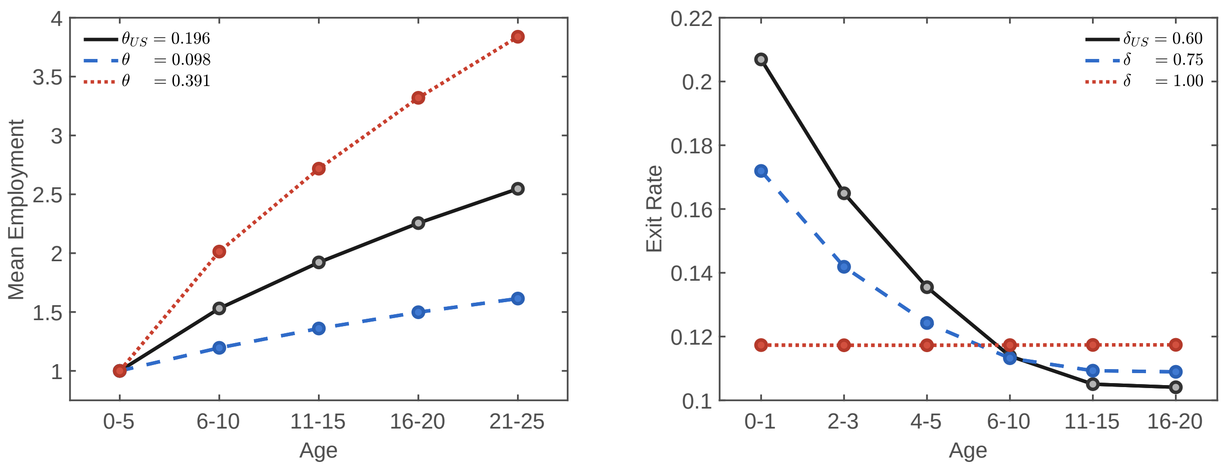
[14] Because we do not have data on spending on innovation, we do not attempt to estimate the curvature of the expansion cost function, ζ. Instead, we follow the microeconomic literature, whose estimates imply a quadratic cost function, namely, $\zeta = 0.5$. See Akcigit and Kerr (2018) and Acemoglu et al. (2018), who discuss this evidence in more detail. In Section 6, we provide a battery of robustness checks. We set the discount rate ρ equal to 5%.

that the aggregate economy has not yet reached a BGP.[15] We can therefore calibrate all parameters independently of γ_t. In Section 5.2, we describe in detail how we discipline the evolution of γ_t.

Firm Dynamics: Identifying θ, δ, β, and θ_E. The expansion efficiency θ is mostly identified from the profile of firms' life-cycle growth. This is seen in Panel A of Figure 4, where we depict average employment by age for different values of θ, holding all other parameters fixed. The higher θ, the faster firms grow conditional on survival. To identify the share of high-type producers δ, we focus on the age profile of exit rates *conditional* on firm size. Without type heterogeneity, the likelihood of exit would be independent of age conditional on size. In the data, however, such conditional exit rates are strongly decreasing in firm age (see, e.g., Haltiwanger et al. (2013)). Through the lens of our model, this pattern is rationalized through endogenous selection, whereby the share of low-type firms within a given cohort declines as the cohort ages. This is shown in Panel B of Figure 4, where we display the exit rate of small firms by age for different values of δ. Without any heterogeneity, that is, $\delta = 1$, the conditional exit hazard is flat. The parameter β, which determines how quickly low-type firms lose market share, is identified from the aggregate employment share of old firms. Intuitively, because high-type firms are older on average, the aggregate size of old cohorts is informative about this parameter. Finally, the entry efficiency θ_E is identified from the aggregate entry rate.

FIGURE 4: IDENTIFICATION OF δ AND θ

PANEL A: THE LIFE-CYCLE — PANEL B: EXIT RATE OF SMALL FIRMS BY AGE



Notes: The left panel shows the employment life-cycle, that is, average employment by age, for different values of θ. The right panel shows the exit rate of one-product firms by age for different values of δ. The black line depicts the U.S. calibration (i.e., $\theta_{US} = 0.196$ in the left panel and $\delta_{US} = 0.60$ in the right panel). The other lines are obtained by varying θ (left panel) or δ (right panel) while keeping the rest of the parameters constant.

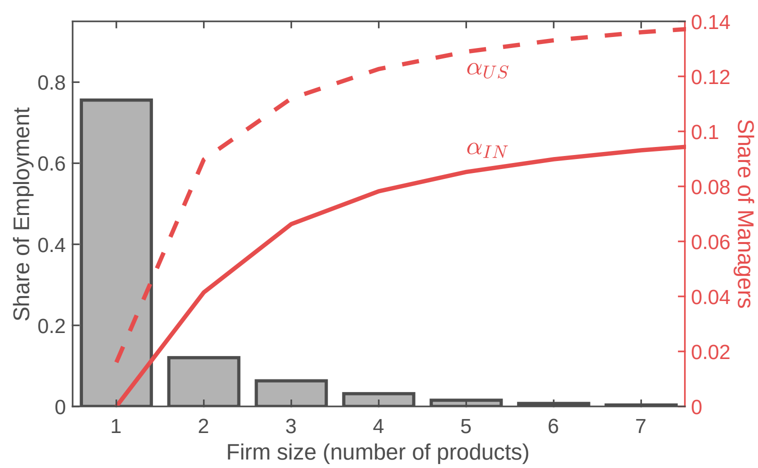
Identifying the delegation efficiency α. The delegation efficiency α is a crucial parameter of our analysis. Because α directly affects firms' managerial demand, we aim to identify it from the aggregate employment share of outside managers. Doing so, however, requires us to address an important

[15]Empirically, the firm size distribution in India is relatively stable over time, despite the fast convergence in income per capita (see Section OA-2.2 in the Online-Appendix).

identification problem. Because the share of managers is increasing in firm size, the firm size distribution directly affects the aggregate managerial employment share. For example, consider Figure 5, where we plot the managerial employment share by firm size and the employment distribution in India from our calibrated model. Holding α constant, the managerial share is higher for larger firms. More importantly, holding firm size fixed, the equilibrium managerial share is increasing in α. Because the aggregate managerial share is the integral of the firm level managerial shares with respect to the employment distribution, we have to distinguish whether managerial delegation in India is rare because delegating is difficult or whether other frictions keep Indian firms small and hence reduce the share of outside managers in the aggregate.

To credibly identify the efficiency of delegation α, we therefore need to simultaneously match the aggregate managerial employment share and the firm size distribution. Our model and calibration strategy allows us to do so. In particular, recall that the equilibrium firm size distribution is determined from firms' expansion schedules x_n and the entry rate z (see (18) and (19)). And by allowing the fundamental determinants of x_n and z, namely, the firm-dynamics parameters θ, δ, β, and θ_E, to vary between the U.S. and India in an unrestricted way, our calibration can match the firm size distribution using these parameters and identify α from the residual variation in managerial employment shares between the U.S. and India.[16]

FIGURE 5: IDENTIFICATION OF α



Notes: This figure shows the share of managers by firm size for two values of α and the calibrated Indian firm size distribution.

Identifying the "management elasticity" σ. We identify the parameter σ from the relationship between firm profits and managerial efficiency e. Using the profit function in (11) and the optimal

[16]Differences in high types' growth potential θ could - in a reduced-form way - capture differences in capital market efficiency that prevent Indian firms from investing (see, e.g., Cole et al., 2016) or size-dependent policies, whereby Indian firms might be subject to steeper (implicit) tax rates (see, e.g., Hsieh and Klenow (2014), Guner et al. (2008), Ulyssea (2018), or Bento and Restuccia (2017)). Similarly, inefficiencies in the allocation of start-up capital, bureaucratic red tape, or frictions in the labor market might induce more subsistence firms to enter in India ($\delta_{IND} < \delta_{US}$) or entry costs to be higher ($\theta^E_{IND} < \theta^E_{US}$).

amount of managerial efficiency $e = (\alpha\sigma/\omega_M)^{\frac{1}{1-\sigma}}$, profits can be written as

$$\tilde{\pi}(n) = (1-\sigma)\, e^{\sigma} n + \sigma T e^{-(1-\sigma)}. \tag{25}$$

Equation (25) highlights that σ governs the relationship between managerial services e and firm profits. In fact, if firms' managerial demand were homothetic, that is, if T were equal to zero, σ would exactly be the elasticity of profits with respect to e holding firm size n constant.

An ideal way to estimate σ is to exploit exogenous variation in managerial inputs at the firm-level and subsequent changes in firm profitability. We, therefore, estimate σ via indirect inference and target the experimental evidence on the relationship between management practices and firm performance from Bloom et al. (2013).[17] The authors provided free consulting on the efficacy of 38 management practices to a set of randomly chosen textile establishments in India. These practices, which are standard in U.S. firms, centered on factory operations, formalized quality control and inventory practices, and changes in human resource management, such as performance-based incentive pay. Using the random assignment of this managerial intervention, Bloom et al. (2013) estimate the treatment effect of managerial practices on subsequent output growth using the specification

$$\ln Output_{i,t} = \beta \times TREAT_{i,t} + f_i + \epsilon_{i,t}, \tag{26}$$

where $TREAT_{i,t}$ takes the value of 1 for the treatment plants starting one month after the end of the intervention period, and f_i are a full set of plant fixed effects. They estimate (26) at the weekly level and find a treatment effect of 9% for a horizon of 100 weeks.

We use this treatment effect as an "identified moment" to identify σ (Nakamura and Steinsson, 2018). To implement this experiment in our model, we need to take a stand on what the treatment means in our theory, that is, how we translate the ordinal nature of the treatment into a cardinal increase in managerial services e among treated firms. Our strategy is as follows. In our model, firms' managerial environment is fully summarized by their managerial services e. We, therefore, relate firm f's optimally chosen managerial services e_f to the share of practices that firm f chooses to adopt, which we denote by MP_f. Note that like e in our theory, the adoption decision of the managerial practices in the experiment was also endogenous. In particular, the experimental intervention provided management consulting but left the eventual choice of which practices to adopt to the firms. Bloom et al. (2013, p. 22) explicitly report that the adoption decision "was endogenous and it presumably varied with the cost-benefit calculation for each practice."

To link the unobservable e_f to the observable MP_f, we consider the measurement equation $e_f = \upsilon MP_f^{\varrho}$, where υ and ϱ are positive parameters. Letting MP_{IND}^{Treat} be the share of managerial practices adopted by Indian firms after the treatment and MP_{IND} be the share among control plants implies

$$\frac{e_{IND}^{Treat}}{e_{IND}} = \left(\frac{MP_{IND}^{Treat}}{MP_{IND}}\right)^{\varrho}.$$

For a given parameter ϱ, we can therefore infer the change in managerial service e due to the

[17] See also Bruhn et al. (2018) for a related management intervention for small and medium enterprises in Mexico.

treatment from the change in managerial practices MP. To determine ϱ, we use data on differences in managerial practices between the U.S. and India and the model-implied differences in managerial services, e_{IND} and e_{US}. In particular, letting MP_{US} denote the share of practices adopted by U.S. firms, our measurement equation implies $\frac{e_{IND}}{e_{US}} = \left(\frac{MP_{IND}}{MP_{US}}\right)^{\varrho}$. Hence, we can map the observed change in managerial practices among treatment firms to the change in e as

$$\ln\left(\frac{e_{IND}^{Treat}}{e_{IND}}\right) = \varrho \times \ln\left(\frac{MP_{IND}^{Treat}}{MP_{IND}}\right) = \frac{\ln\left(e_{IND}/e_{US}\right)}{\ln\left(MP_{IND}/MP_{US}\right)} \times \ln\left(\frac{MP_{IND}^{Treat}}{MP_{IND}}\right). \tag{27}$$

In the microdata of the experiment, we find $MP_{IND} = 0.25$; that is, prior to the treatment, Indian firms adopt roughly one fourth of the managerial practices. The treatment increases the adoption rate to $MP_{IND}^{Treat} = 0.63$. Given that all of these practices "have been standard for decades in the developed world" (Bloom et al., 2013, p. 43), we assume firms in the U.S. adopt all these practices; that is, $MP_{US} = 1$.[18] Furthermore, for a given calibration of our model, we can calculate e_{IND} and e_{US}. We can then use (27) to calculate e_{IND}^{Treat}.[19]

As we describe in detail in Section B.3 in the Appendix, our implementation takes the endogeneity of e_{IND}^{Treat} explicitly into account. In particular, we have to take a stand on *how* the experiment induced treatment firms to increase their e. Because the intervention provided information on how to use such managerial practices optimally, we model the treatment as a proportional increase in the productivity of treated firms' endogenous managerial services. Specifically, we assume treated firms' total managerial resources are given by ξe, and we choose ξ such that ξe^{Treat} coincides with the value implied by (27), where e^{Treat} denotes the optimal choice of e given ξ. In Section B.3 in the Appendix, we show ξ is given by $\xi = \left(e_{IND}^{Treat}/e_{IND}\right)^{1-\sigma}$. Importantly, we keep all general equilibrium variables constant in order to implement a partial equilibrium analysis consistent with the experiment.

We then relate this increase in managerial services to the resulting profits to estimate σ. Specifically, we take 50 firms from the very top of the firm size distribution of our calibrated Indian economy (consistent with the sample selection in Bloom et al. (2013)), treat them with the management intervention as described above, simulate their evolution for 100 weeks, and then estimate the treatment effect according to (26) in the model-generated data. Whereas Bloom et al. (2013) estimate (26) using physical output as a measure of firm performance, we focus on total profits as the dependent variable in our model counterpart. We do so because profits are at the heart of our theory linking managerial services to firm performance.

Because the experiment was only conducted for firms in India, this strategy forces us to assume σ is common across countries.[20] Because of the importance of this parameter, we also implement a complementary identification strategy that does not rely on the experimental evidence, but only

[18] In Section B.3 in the Appendix, we use the reported management scores from Bloom and Van Reenen (2007) (which are available both for firms in the U.S. and for firms in India pre-treatment) to provide additional corroborating evidence for our assumption that $MP_{US} = 1$.

[19] To give a concrete example, our baseline calibration implies Indian firms utilize only 71% as many managerial services as firms in the U.S.; that is, $e_{IND}/e_{US} = 0.71$. Together with $MP_{US} = 1$, $MP_{IND} = 0.25$, and $MP_{IND}^{Treat} = 0.63$, (27) implies $e_{IND}^{Treat}/e_{IND} = 1.26$; that is, we infer the endogenous adoption of managerial practices from 0.25 to 0.63 corresponds to a 26% increase in managerial efficiency in treatment firms.

[20] Bloom et al. (2016) use managerial scores to estimate production functions for managerial inputs across countries. They find the coefficients on the managerial scores to be very similar across countries.

uses standard accounting data. The standard intuition from a constant elasticity production function suggests the output elasticity should be related to relative cost shares. The same intuition is true in our model: the higher σ, the larger the share of managerial compensation relative to profits. More specifically, our model implies

$$\frac{w_M nm\left(n\right)}{\Pi\left(n\right)} = \frac{\sigma}{1-\sigma}\left(1-\frac{Tw_M}{\sigma\alpha\Pi\left(n\right)}\right), \tag{28}$$

where $w_M nm$ and $\Pi\left(n\right)$ denote total managerial payments and profits, respectively. Note that if firms had to rely only on outside managers, that is, if $T=0$, the demand for outside managers would be homothetic and σ would reflect the relative compensation share. In our model, this mapping is slightly more complicated, but (28) shows the managerial compensation share is directly affected by σ. Because we can measure this moment both for the U.S. and India, this approach allows us to estimate σ separately for both countries. As we discuss in Section 6, these approaches lead to similar results. In particular, the estimates for σ are almost identical between the U.S. and India and only slightly higher than the estimates implied by our indirect inference strategy.

Identifying the remaining management parameters μ_M, ϑ and T. As we discuss in Section B.2 in the Appendix, all allocations in the model depend only on $\mu_M \times \alpha$. To separately identify the efficiency of managers within firms α from the supply of managerial skills μ_M, we require variation in the demand for managerial skills, holding managerial human capital fixed. Intuitively, we would want to observe the same manager working with both the U.S. and the Indian α. We mimic this experiment by using data from the New Immigrant Survey (NIS), which contains information about the pre- and post-migration occupations of recent immigrants to the U.S and has recently been used by Hendricks and Schoellman (2017). In Section B.4 in the Appendix, we show in detail how we can use the managerial employment share of Indian migrants in India relative to their managerial employment share in the U.S. to identify μ_M and α separately. Intuitively, Indian immigrants to the U.S. are almost as likely to work in managerial occupations as U.S. residents. However, they are much more likely to have worked in managerial jobs *prior* to emigrating. This finding implies that the average managerial human capital of the non-selected, non-migrant Indian population is lower than in the U.S. These two moments separately identify α and μ_M and allow us to perform our counterfactual, where we change the delegation efficiency α while holding the supply of managerial skills μ_M constant.

To identify the dispersion of the managerial skill distribution, ϑ, we note that it can be directly calibrated to match the dispersion in managerial earnings. In particular, the model implies the variance of log managerial earnings to be given by ϑ^{-2}. Finally, the owner's time endowment T is a fixed factor, and firm profits are a renumeration for the provision of these services. We therefore calibrate T by targeting the entrepreneurial profit share, which is given by

$$\frac{\text{Aggregate Profits}}{\text{Total Sales}} = \frac{\sum_n \Pi(n) F_n}{Y} = \sum_n \tilde{\pi}(n) F_n, \tag{29}$$

where $F_n = F_n^H + \mathbb{1}(n=1)F^L$ is the number of firms with n products and $\tilde{\pi}(n)$ is increasing in T, holding aggregate prices fixed (see (11)).

4 Estimation Results

In this section, we discuss our estimation results. Section 4.1 contains the structural parameters and targeted moments. In Section 4.2, we show our model is also consistent with a variety of non-targeted moments. Finally, in Section 4.3, we use our estimated model to assess why firms in India are small.

4.1 Calibrated Parameters and Targeted Moments

Tables 3 and 4 contain the calibrated parameters and the targeted moments. For convenience, Table 3 also reports the main target for the respective parameter even though the parameters are calibrated jointly. For the U.S., we estimate seven parameters and for India, we estimate eight parameters.

TABLE 3: ESTIMATED PARAMETERS

Parameter	Interpretation	Target	U.S.	India
		Panel A. Internal Calibration		
		Firm Dynamics		
θ	Expansion efficiency	Employment life-cycle	0.196	0.058
δ	Share of high types	Exit profile by age (cond. on size)	0.602	0.111
β	Relative creative destruction	Empl. share of old firms	4.659	2.852
θ_E	Entry efficiency	Entry rate	0.100	0.099
		Managerial Environment		
α	Delegation efficiency	Managerial employment share	0.433	0.206
μ_M	Average managerial human capital	Occupational sorting by immigrants	$1.000^{\dagger}$	0.405
ϑ	Dispersion of managerial human capital	Var of ln managerial earnings	1.429	1.429^{*}
σ	Managerial output elasticity	Treatment effect of Bloom et al. (2013)	0.468^{*}	0.468
T	Entrepreneurial time endowment	Average entrepreneurial profit share	0.159	0.267
		Panel B. External Calibration		
ζ	Convexity of expansion costs		0.50	0.50
ρ	Discount rate		0.05	0.05

Notes: The table reports the parameter values that yield the model moments reported in Table 4. We denote normalized parameters by "$\dagger$" and parameters that we do not estimate by "$*$".

Consider first Table 3. The top panel shows that 90% of entering firms in India are subsistence entrepreneurs. In contrast, entrants in the U.S. are about six times as likely to be high types ($\delta_{US} \approx 6 \times \delta_{IND}$) and such firms are around 3.5 times as efficient in expanding into new markets as their Indian counterparts ($\theta_{US} \approx 3.5 \times \theta_{IND}$). At the same time, the costs of creating such superior firms are almost the same between the U.S. and India ($\theta_{E,US} \approx \theta_{E,IND}$). Economically, we find these estimates plausible in that they capture the myriad reasons why firms in India might not expand (e.g., due to the presence of credit constraints or size-dependent policies) or why unproductive firms are abundant upon entry (e.g., because of low opportunity costs of entrepreneurship in India).

The next panel contains our estimates of the delegation environment. Our estimation implies delegation in the U.S. to be twice as efficient as in India ($\alpha_{US} \approx 2 \times \alpha_{IND}$). As highlighted above, this low estimate of α_{IND} is conditional on the other determinants of the firm size distribution, namely, θ, δ, and θ_E. In fact, if we only calibrated our model to the Indian firm-dynamic moments in Panel A, but kept the delegation efficiency at the U.S. level, the managerial employment share would be around 5%, that is, exceeding the level observed in India. Hence, although the fact that firms in India are small accounts for a sizable part of the lower share of managerial inputs, a less efficient delegation environment α is also required to explain the data.

We also estimate that managers in the U.S. have more human capital ($\mu_{M,US} > \mu_{M,IND}$). We infer this result from the fact that the share of managers among Indian immigrants in the U.S. is 12.7% (hence very similar to the overall manager share in the U.S.), but they are much *more* likely than the Indian population to work as managers prior to migrating. Therefore, the unselected population in India has a comparative disadvantage in managerial occupations.

TABLE 4: MOMENTS: MODEL VS. DATA

	U.S.		*India*	
	Data	Model	Data	Model
Firm Dynamics				
Entry rate (%)	7.35	7.35	5.60	5.60
Exit profile by age (cond. on size)	1.59	1.59	1.11	1.09
Employment life-cycle	2.55	2.55	1.11	1.12
Employment share of old firms (%)	9.70	6.94	7.75	6.42
Managerial Environment				
Managerial employment share (%)	12.5	12.5	1.65	1.65
Treatment effect from Bloom et al. (2013) (%)	n/a	n/a	9.00	9.00
Relative managerial share of Indian migrants	n/a	n/a	2.08	2.08
Average entrepreneurial profit share (%)	21.0	21.0	48.3	46.2
Variance of ln manager earnings	0.49	0.49	0.45*	0.49

Notes: The table reports both the data moments and the corresponding moments in the model for the U.S. and India. We define "old" and "young" firms as firms of age 21 - 25 years and 1-5 years, respectively. We define small firms as firms with 1-4 employees in the data and with a single product in the model. The employment life cycle is the the relative size of old firms relative to young firms. The conditional exit profile is the exit rate of young, small firms relative to old, small firms. See Section B.1 in the Appendix for details. "*" denotes that the moment is not targeted in the calibration.

In Table 4, we report the targeted moments. The first two columns contain the U.S. calibration. Our model is able to rationalize most moments well. In particular, it matches the observed employment life cycle (whereby firms of age 21-25 years are about 2.5 times as large as firms younger than 5 years), the aggregate entry rate, and the differences in exit rates (whereby small young firms, which exit at a rate of 22% per year, are around 1.6 times as likely to exit as small old firms, which have an exit rate of 14%). The model slightly underestimates the aggregate employment share of old firms.[21]

[21] One reason is that in our model growth is only driven by the extensive margin of adding products. Hence, the process of growth and the resulting exit hazard are tightly linked. If we allowed for growth on the intensive margin (e.g., through quality innovations within existing product lines as in Akcigit and Kerr, 2018, or Garcia-Macia et al., 2019), we could

The model also matches the aggregate share of managerial workers of 12.5% reported in Table 1, an entrepreneurial profit share of about 20%, and the dispersion of log managerial earnings.[22] Although we assume ϑ is identical across countries for simplicity, the dispersion of log managerial earnings in India is essentially the same as in the U.S.[23]

The model is similarly successful in matching the moments of the Indian economy reported in columns 3 and 4. In particular, it replicates the essentially flat life-cycle of Indian establishments, the low share of aggregate managerial employment, and that young establishments exit almost at the same rate as old establishments. As is the case for the U.S. calibration, the model slightly underestimates the share of old firms in the economy.[24] Also note firms in India have a much higher share of entrepreneurial profits than firms in the U.S., because most firms in India are small, and the majority of their sales are attributed to entrepreneurial compensation for the provision of the fixed factor T.

Finally, our model is able to replicate the treatment effect of Bloom et al. (2013). This property is important, because to credibly quantify the aggregate effects of changes in the efficiency of delegation, it is reassuring that our model is quantitatively consistent with well-identified microeconomic evidence on the dynamic effects of changes in managerial efficiency at the firm-level. Matching the estimated treatment effect requires an estimate of σ around 0.47. As discussed in detail above, for our baseline analysis, we restrict σ to be the same across countries. In Section 6, we discuss an alternative strategy where we estimate σ from accounting data and allow it to be country-specific.

4.2 Non-targeted Moments

Our model also performs well in matching a variety of non-targeted moments. In particular, we focus on the non-homotheticity of managerial demand, firms' survival hazards, and the number of products firms sell. Additionally, we also discuss some qualitative patterns in the delegation decisions of Indian firms based on a regression analysis and compare them with the predictions of our theory.

Non-homothetic Managerial Demand A key mechanism of our model is that large firms endogenously increase their span of control by hiring outside managers. In particular, larger firms are more likely to hire any outside managers, and they hire more per product, conditional on hiring. Because the Indian data report managerial hiring at the firm-level, we can look for these implications in the data.

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break this link.

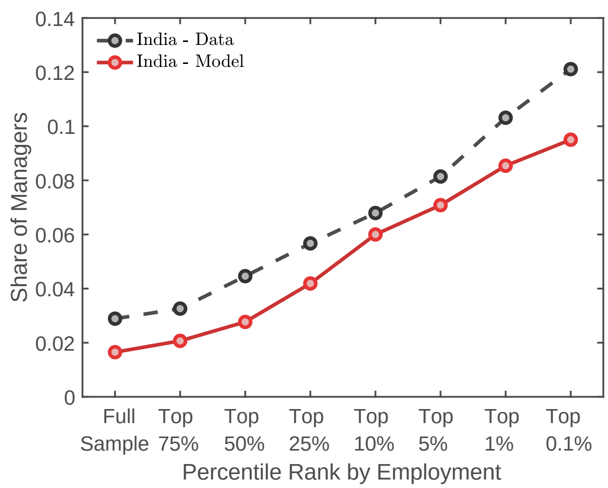
[22] Empirically, we target the dispersion of residual log managerial earnings after controlling for industry fixed effects. To be able to compare India and the U.S. we use data for 2005 for the U.S.

[23] Our distributional assumption of managerial human capital implies the average wage of managers relative to production workers within a country is given by $\vartheta/(\vartheta-1)$. When we look at this implication in the micro-data, we find that managers in the U.S. (India) earn a premium of 0.59 log points (0.67 log points). Both of these are lower than the model-implied premium given the estimate of ϑ, which is 1.19 log points. Because ϑ plays the role of a labor supply elasticity, we prefer to target the dispersion in wages, which is more directly related to the scope of selection. In Section 6, we discuss how different assumptions about this supply elasticity affect our results.

[24] At first glance, the fact that old firms have roughly the same aggregate employment share in the U.S. and India might be surprising. The reason is that the aggregate employment share of *very* old firms is much higher in the U.S. In the U.S. (India), the share of firms older than 25 years is 55% (15%). See Sections OA-2.1 and OA-2.2 in the Online Appendix for details.

Our model predicts both the extensive and intensive margin of managerial hiring well. Regarding the extensive margin, our model implies that firms that run their operations without outside managers account for 72% of aggregate employment in India. Empirically, we find this moment to be 77.5% in the Indian micro data. In Figure 6, we show that our model is also quantitatively consistent with the relationship between managerial employment shares and firm size conditional on hiring. To compare the model and the data (which we reported below in Table 2), we focus on the quantiles of the firm size distribution. In particular, going from right to left, we plot the share of managerial employment among the largest 0.1%, the largest 1%, the largest 5% of firms, and so on. Hence, by going from right to left, we trace out the average managerial share as a function of the firm size distribution. At the far left, we report the share among the 100% largest firms, which is simply the entire sample of firms. Hence, in the data, the managerial share is the sample average of 2.9% (see Table 2), and in the model, it is 1.65%, our calibration target from the IPUMS data. Figure 6 shows that our model replicates the "delegation-firm size" relationship observed in India very well even though we do not target it explicitly.

Figure 6: Managerial Demand by Firm Size



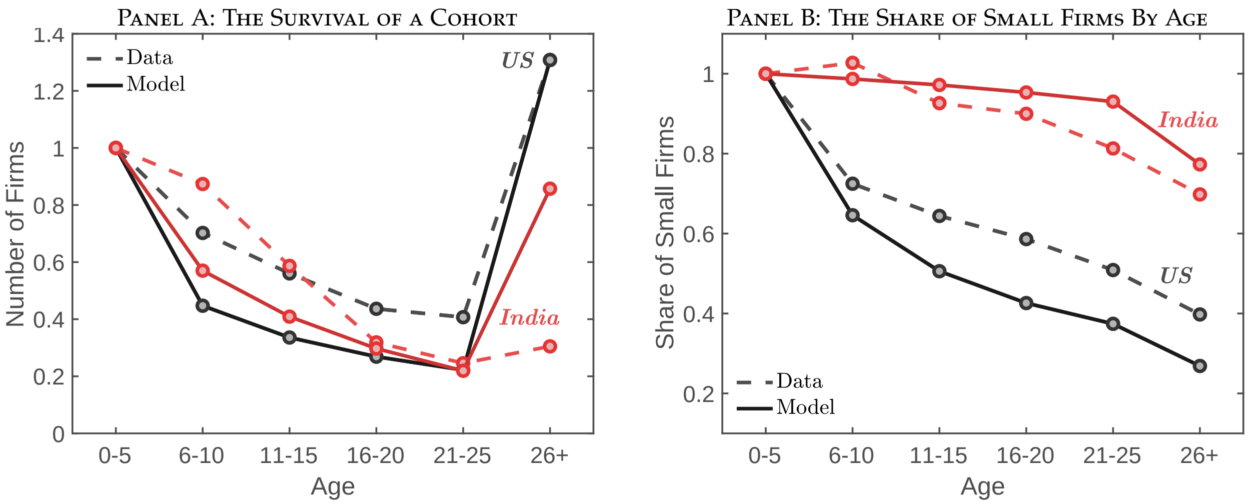
Notes: This figures shows the employment share of managers among firms in the top x% of the firm-size distribution for $x = 0.1\%, 1\%, 5\%, ...$ We report the data using a black dashed line and the model using a red solid line. See also Table 2 for a summary of the data.

Survival Hazards In Figure 7, we compare our model with two measures of the degree of selection. In Panel A, we depict the survival rate, that is, the size of a given age cohort relative to the entering cohort. The rate of firm survival is reasonably similar in the U.S. and India – both in the data and in the model.[25] In Panel B, we show the share of small firms by age (relative to their share among young firms). While the share of small firms in the U.S. declines to 40% by the age of 25, the vast

[25] As for the category of 26+ firms: Note that the survival rate is the *accumulated* stock of surviving firms, that are older than 26 years. Hence, even though the U.S. exit rates are only slightly lower than those in India, the small differences in the flow of exit add up to a sizable difference in the stock of old firms. See also Figures 2 and 3 in Hsieh and Klenow (2014), who show exit rates are only slightly lower in the U.S. but that the aggregate employment share of old firms is vastly larger in the U.S.

majority of old firms in India are still small. Our model again replicates these patterns reasonably well.

Figure 7: Firm Selection



Notes: Panel A depicts the share of firms by age relative to the share of firms in the youngest age category. Panel B shows the share of small firms by age. We show the data using solid lines and the model using dashed lines. In the U.S., small firms are firms with one to four employees. In India, small firms are firms with one employee.

The Distribution of Products In our model, a firm is a collection of product lines. Our calibration focuses only on employment data to measure firm size and does not use data at the product level. Both the U.S. and the Indian data, however, contain information on the number of five-digit product codes in which individual firms are operating.[26] In Figure 8, we plot the distribution of firm-level product counts in the data and the model. Our model matches this aspect of the data remarkably well, despite the fact that this moment is not targeted. In particular, the vast number of Indian firms indeed produce only a single product.

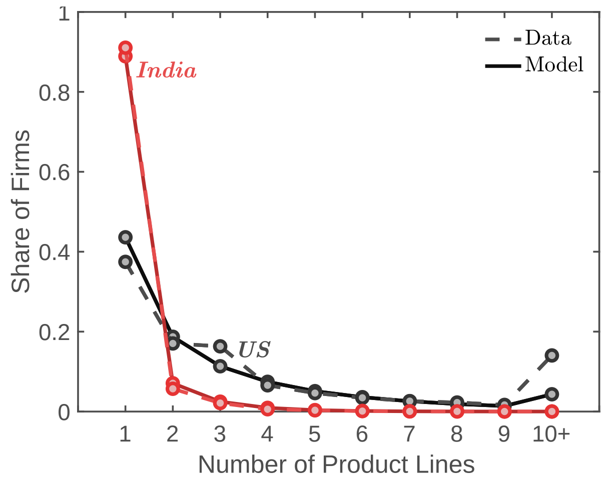
Qualitative Predictions on Delegation in the Indian Micro Data Finally, we can look at some qualitative predictions of our theory.[27] Our theory implies firms do not hire outside managers if their size falls short of the delegation cutoff, that is, if $n < n^* = T\left(\frac{\omega_M}{\sigma\alpha}\right)^{\frac{1}{1-\sigma}}$. Hence, firms are more likely to delegate if (i) firm size n increases, (ii) the delegation efficiency α is larger, and (iii) the owner's inelastically provided managerial human capital T is smaller.

To take these predictions to the data, we follow Bloom et al. (2013, p. 4), who argue that for Indian textile firms, "managerial time was constrained by the number of male family members. Non-family members were not trusted by firm owners with any decision-making power, and as a result, firms did not expand beyond the size that could be managed by close (almost always male) family members." Hence, we take the size of the entrepreneur's family as a proxy for T. Moreover, we use regional variation in trust within India to proxy for variation in α. The latter is calculated from the World Values Survey as the share of people providing the answer "Most people can be

[26] The data for the U.S. firms come from Acemoglu et al. (2018)

[27] See Section B.6 for the details of the empirical analysis. There we also provide an explicit derivation of the regression equations based on our theory.

FIGURE 8: THE DISTRIBUTION OF PRODUCTS



Notes: The figure shows the distribution of the number of products by firm in the data (dashed line) and the model (solid line).

trusted" within the Indian state where the firm is located. This measure of trust is the one most commonly used in the literature (see, e.g., La Porta et al. (1997)).

We then regress firms' managerial hiring decisions on firm size, household size, and regional trust in 22 Indian states. We always control for the market of a firm, that is, whether the firm is urban or rural, firm age, state-level GDP per capita, and two-digit sector fixed effects. Due to space constraints, we only report the estimated equation; the full analysis can be found in Appendix B.6. We find that:

$$\mathbb{1}(\boldsymbol{Firm\ hires\ managers}) = \underset{(0.003)^{***}}{0.039} \times \boldsymbol{Firm\ Size} \underset{(0.001)^{**}}{-0.003} \times \boldsymbol{Family\ Size} \underset{(0.006)^{***}}{+0.013} \times \boldsymbol{Trust},$$

where *"Firm Size"* and *"Family Size"* are the logarithms of the number of employees and household members, respectively. Hence, as predicted by our theory, firm size and regional trust correlate positively, whereas family size correlates negatively, with the probability of hiring an outside manager. These results are consistent with Bloom et al. (2012) who, using data from a survey on managerial practices, show firms in high-trust areas delegate more decision power to managers.

Our model also has implications for the relationship between family size and firm size. In our model, managerial resources within the family, T, are the constraining factor for firm size. This constraint, however, is less important the higher the delegation efficiency α becomes. Hence, although family size should be a predictor of firm size, the effect should be particularly strong in regions where trust, and hence the possibility of delegation, is less developed. We can test this prediction from the interaction between trust and family size. This approach allows us to include a full set of state-fixed effects in the regression to control for all characteristics (including the level of trust) that are constant within Indian states. As before, we also control for the location of the firm (rural vs. urban), firm age, and two-digit sector fixed effects. We find that

$$\boldsymbol{Firm\ Size} = 0.812 \times \boldsymbol{Family\ Size} - 1.329 \times \boldsymbol{Family\ Size} \times \boldsymbol{Trust}.$$

$(0.278)^{***}$ $(0.758)^{*}$

Hence, the correlation between family size and firm size is positive and particularly strong in low-trust regions. Through the lens of our model, this pattern is due to the imperfections in delegation in those regions.

4.3 Why Are Indian Firms Small? The Role of Selection and Creative Destruction

The estimated model allows us to give a structural interpretation of the observed differences in firm dynamics between the U.S. and India. Our theory stresses that two key determinants are the extent of selection and the rate of creative destruction. Although neither of these mechanisms is directly observable, we can measure them through the lens of the model.

In Table 5, we report a set of statistics from the stationary distribution. First, note our calibration implies that creative destruction in the U.S. is twice as large as in India. At first glance, it seems surprising that we infer large differences in creative destruction despite the fact that both aggregate entry and exit rates and firms' survival probabilities by age are quite similar (see Figure 7). The key to reconciling these facts is to realize that the underlying distributions of firm size are vastly different between the U.S. and India. Recall that the number of exiting firms is the *product* of the mass of firms operating in a single market and the rate of creative destruction. The fact that exit rates are quite similar despite the fact that many firms in India are small and hence close to the exit threshold implies creative destruction in India has to be substantially smaller. Conversely, most creative destruction in the U.S. takes place in infra-marginal markets where firms lose market share without exiting.

Table 5: Creative Destruction and Selection

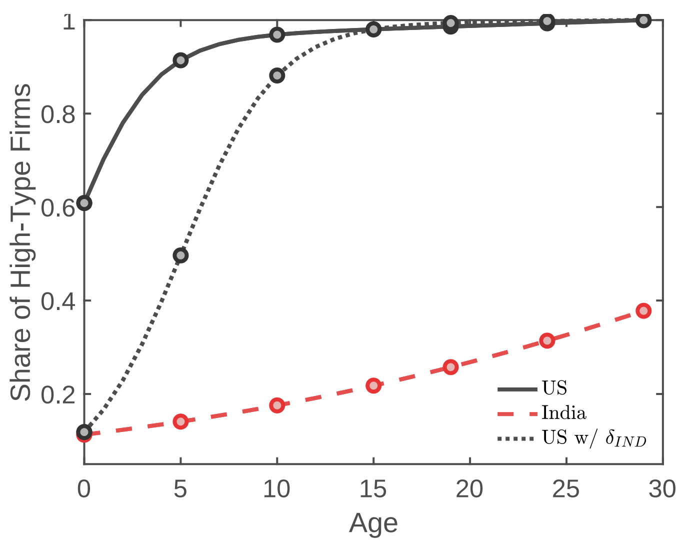
	India	U.S.
Rate of creative destruction, τ	0.054	0.124
Share of high-type firms upon entry (δ)	0.111	0.602
Long-run share of high-type firms	0.344	0.946
Long-run employment share of high-type firms	0.470	0.983
Long-run share of high-type firms among firms of age 21-25	0.301	1.000

Notes: The table contains various equilibrium objects from the stationary distribution of the calibrated models. The models are parametrized according to Table 3.

In the remaining rows of Table 5, we report different aspects of the degree of selection. In the stationary distribution of the U.S., around 95% of firms are high-type firms (compared to 60% at the time of entry), and they have a combined employment share of 98%, because they are bigger on average. In India, even in the long-run, high-type firms account for only 34% of firms and 47% of aggregate employment. This slower weeding-out process of low-type firms in India is also highlighted by the fact that even among old firms, more than two-thirds of them are subsistence entrepreneurs. This finding is in stark contrast to the U.S., where the population of old firms consists only of high types.

In Figure 9, we display the dynamics of this "shake-out" process by tracing out the share of high-type firms within a cohort at different ages. Not only is the share of high-type firms in the U.S. significantly greater among the entering cohort, they also grow much faster, creating a much stronger selection force. This selection process is dampened in India: even among 30-year-old plants, more than half are low-type firms. Importantly, this lack of selection in India is not only due to fact that few high-type firms exist to begin with. To illustrate this distinction, we simulate a counterfactual cohort of U.S. firms that starts with the initial type distribution of India, that is, where the initial share of high-type firms is δ_{IND}. Figure 9 shows that differences in growth incentives of high-type firms in the U.S. and India are a key aspect of the selection dynamics: by the age of 15, this counterfactual cohort in the U.S. would again be populated by mostly high-type firms.

Figure 9: Endogenous Selection



Notes: The figure shows the share of high-type firms by age both for the India calibration (red line) and for the U.S. calibration (black line). It also shows the counterfactual share of high-type firms by age if the initial share of high-type firms in a cohort in the U.S. is given by its Indian counterpart δ_{IND}. All calibrated parameters are taken from Table 3.

5 The Aggregate Importance of Delegation Efficiency

To what extent are differences in the efficiency of delegation responsible for the observed differences in firm dynamics and aggregate economic performance between the U.S. and India? To answer these questions, we study a counterfactual Indian economy where we increase α from α_{IND} to α_{US} while keeping the rest of the parameters at their calibrated levels. We first quantify the effects on firm-level outcomes. We then turn to the aggregate effects and study the link between α and aggregate income differences.

Table 6: Increasing the Delegation Efficiency in India: Firm-level implications

	Panel A: Equilibrium outcomes					
	Average	$n = 1$	$n = 2$	$n = 3$	$n = 4$	$n = 5$
Expansion rate $x(n;\alpha)$	+23.69%	+14.36%	+18.97%	+20.73%	+21.49%	+21.87%
Entry intensity $z(\alpha)$	+1.44%					
Creative destruction τ	+4.11%					
Share of outside managers	+138%					
	Panel B: Implications for firm dynamics					
Average firm size	+3.59%					
Share of high-type firms	+3.21%					
Empl. share of small firms	−3.00%					
	Effects by age					
	<=5	6-10	11-15	16-20	21-25	+26
Average firm size	2.17%	2.20%	2.31%	2.54%	2.89%	4.98%
Share of small firms	-0.08%	-0.29%	-0.58%	-0.97%	-1.51%	-5.00%

Notes: The table reports the changes in various equilibrium outcomes after increasing the delegation efficiency in India from α_{IND} to α_{US}. "Small firms" are those with a single product. All changes refer to changes in the stationary distribution.

5.1 Delegation Efficiency and Firm Dynamics

The firm-level implications are summarized in Table 6. In Panel A, we focus on the changes in firm expansion, entry, and creative destruction. Incumbents' expansion incentives are much more responsive than the entry margin. While firms' expansion rates increase by 24%, on average, the entry intensity increases only by 1.4%. These differences arise because outside managers are complementary to firm size and therefore are not very important for subsistence firms, which never grow. This complementarity also implies the expansion rate of large firms is particularly responsive.

At the aggregate level, however, the increase in creative destruction is much closer to the change in the entry intensity. The reason is that the market share of high-type firms in India is relatively small, so the majority of creative destruction is accounted for by new entrants. Finally, the equilibrium employment share of outside managers would more than double to 3.9%. Note this is still well below the level in the U.S, because Indian firms are still substantially smaller than their U.S. counterparts.

In Panel B, we report the implications for the resulting process of firm-dynamics. If Indian firms could employ outside managers as efficiently as firms in the U.S., average firm size would increase by 3.6%, the share of high-type firms would increase by 3.2%, and the importance of small producers would decline by 3.0%.[28] The last two rows of Panel B show these changes stem mostly from older firms, which are, on average, larger and hence more likely to rely on outside managers. Quantitatively, firms between 21 and 25 years old experience an employment increase by 2.9% and their share of single-product firms decline by 1.5%. The reason why these effects are small compared

[28]Our calibrated model predicts that firms in the U.S. are, on average, roughly 2.5 times as large as firms in India. This number is not comparable to the empirical size difference of 15.8 as reported in Table 1. The reason is that in our model, entrants in the U.S. start at the same size as entrants in India. Empirically, entrants in the U.S. have, on average, 13.7 employees, whereas entrants in India have 2.5. Entrants in the U.S. are therefore 5.5 times as large as entrants in India. Hence, relative to the initial size difference, U.S. firms are 15.8/5.5 = 2.8 times as large as firms in India.

to the increase in high types' expansion rate, $x(n;\alpha)$, is again due to the lack of selection because even among old firms, the majority of firms in India are subsistence producers. The effect of α on the process of firm dynamics in India is therefore modest.

The Importance of Complementarities The results in Table 6 highlight the interaction between the ease of delegation and other aspects of the economy. In particular, improvements in the efficiency of delegation are more potent if high-type firms are plentiful and those firms can expand easily. To see that this intuition is indeed correct, Table 7 presents the U.S. analogue of Table 6.[29] Compared to the results for the Indian economy, we find that a decrease in the efficiency of delegation in the U.S. to the Indian level would affect firm growth substantially. In particular, the rate of creative destruction decreases by 25%, average firm size declines by 14%, and the employment share of small firms increases by 19%. Similarly, the effects on managerial hiring are also larger in the U.S. If outside managers were as inefficient as their Indian counterparts, the equilibrium managerial share would decline from 12.5% to 5.4%. The reason for such stark differences is that high-type firms are abundant in the U.S. and their expansion costs are low. Preventing these dynamic entrepreneurs from growing affects the process of firm dynamics substantially.

Table 7: Decreasing Delegation Efficiency in the U.S.

Panel A: Equilibrium outcomes			*Panel B: Implications for firm dynamics*			
Average Expansion rate	Entry intensity	Creative Destruction	Average Firm Size	Share of high type firms	Empl. Share small firms	Share of managers
-28.7%	-10.0%	-24.9%	-13.6%	-0.3%	+19.1%	-57.0%

Notes: The table reports the changes in various equilibrium outcomes after decreasing the efficiency of delegation in the U.S. from α_{US} to α_{IND}. "Small firms" are those with a single product. All changes refer to changes in the stationary distribution.

5.2 Delegation Efficiency and Aggregate Income Differences

How important are frictions to delegating decision power in Indian firms for the gap in income per capita gap between India and the U.S.? To answer this question, we need to specify the evolution of the step size γ_t. Because we can estimate all other parameters of the model independently, our earlier results do not depend on these assumptions in any way.

We consider a parametrization of our model where the distribution of income between the U.S. and India is stationary in the long-run. More specifically, we assume the Indian economy (by being technologically backward relative to the U.S.) benefits from "catch-up" growth and a higher step-size γ. To capture this intuition in a parsimonious way, we assume the Indian step-size $\gamma_{IND,t}$ is related to the technological gap $Q_{US,t}/Q_{IND,t}$ and given by

$$\gamma_{IND,t} = \gamma_{US} \times \left(\frac{Q_{US,t}}{Q_{IND,t}}\right)^{\lambda}, \tag{30}$$

[29] For brevity, we only report the aggregate outcomes. The results by firm size and firm age are available upon request.

where $\lambda \geq 0$ and γ_{US} is the step size for the U.S., which we assume is constant.[30] Equation (30) captures – in a reduced-form way – the presence of knowledge spillovers. If $\lambda > 0$, the lower the relative technology in India, the higher the innovation step size. If $\lambda = 0$, no advantages from backwardness exist.

Importantly, the formulation in (30) implies income differences between the U.S. and India will be constant in the long-run. Along a BGP where $g = \ln(\gamma_{US})\tau_{US} = \ln(\gamma_{IND})\tau_{IND}$, equation (30) yields

$$\ln\left(\frac{Q_{IND,t}}{Q_{US,t}}\right) = \frac{\ln \gamma_{US} - \ln \gamma_{IND}}{\lambda} = \frac{\ln \gamma^{US}}{\lambda} \times \left(\frac{\tau_{IND} - \tau_{US}}{\tau_{IND}}\right). \tag{31}$$

This expression highlights that the long-run distribution of technology Q across countries is stationary and determined by differences in creative destruction. Differences in delegation efficiency α, by affecting the rate of creative destruction, therefore manifest themselves in level differences, not in growth differences in the long run. During the transition, an increase in α increases the growth rate of $Q_{IND,t}$. In addition, a change in α has static consequences because it increases the amount of managerial efficiency units, $\mathcal{M}_t$, and hence increases income per capita, holding the level of Q_t fixed (see (6)).

To quantify the strength of these forces, we consider an experiment where in 2010 the delegation efficiency in India increases unexpectedly and permanently from α_{IND} to α_{US}. We then trace out the dynamic evolution of the Indian economy. To do so, we need to calibrate γ_{US}, λ, and the initial productivity differences between the U.S. and India. We assume the U.S. economy is on a BGP, and choose γ^{US} to match a growth rate of 2%, given the rate of creative destruction reported in Table 5. India, in contrast, is still catching up to the U.S. economy. Empirically, relative productivity in the U.S., vis-à-vis India, decreased substantially from about 4 in 1985 to 3.2 in 2005 (see Section B.2 in the Appendix, in particular Figure 11). We therefore calibrate λ and the relative productivity between the U.S. and India in 1985, $Q_{IND,1985}/Q_{US,1985}$, to match these time-series dynamics. This exercise implies $\lambda = 0.296$.[31]

In Table 8, we summarize the aggregate implications of this experiment. In Panel A, we report the implications for the growth rate of the technology index Q_t. On impact, the growth rate increases by about 0.16 percentage points in 2010. Over time, this growth rate differential between the baseline and the counterfactual Indian economy declines, and in the long run, both countries grow at the same rate. In Panel B, we calculate the cumulative effect of this higher growth rate on the (relative) level of Q_t. In 2000, the technology in India is about 26.6% of the U.S. level. Our baseline estimates imply that long-run technological differences between the U.S. and India would be 49.5%. If delegation in India were as seamless as in the U.S., relative technology in India would be equal to 52%. Hence, limits to delegation can account for $\frac{51.9-49.3}{100-49.3} \approx 5.0\%$ of the long-run technological gap between the U.S. and India.

[30] Taking the U.S. as the frontier economy is purely for simplicity. Suppose there is an exogenous technological frontier $Q_{F,t}$, which grows at rate g. Suppose the step size in country c is given by (30) *relative* to this frontier, that is, $\gamma_{c,t} = \gamma \times (Q_{F,t}/Q_{c,t})^{\lambda}$. If the U.S. economy has already reached its BGP, (30) holds with $\gamma_{US} = g/\tau_{US}$.

[31] Whereas we use plant-level data from the manufacturing sector for the firm-related moments, here we rely on data about aggregate TFP. As long as relative TFP in the manufacturing sector, $TFP^{Manu}_{IND}/TFP^{Manu}_{US}$, shows the same rate of catch-up, our analysis will be valid. If aggregate TFP in India were to show faster catch-up (e.g., due to the reallocation of workers out of agriculture), our estimate of λ would be upward biased and we would underestimate the aggregate consequences of changes in α - see equation (31).

The effects on income per capita, shown in Panel C, are larger. In the long run, an increase in the efficiency of delegating managerial tasks would increase relative income per capita in India from 51.7% to around 57.0%. This increase accounts for $\frac{57.0-51.7}{100-51.7} \approx 11\%$ of the aggregate gap in income per capita. The effects are larger because of the static effects captured by $\mathcal{M}$. In particular, the magnitudes of the static effects of better delegation and the dynamic effects operating through higher creative destruction are roughly equal.[32] For completeness, we also report the long-run change in consumption per capita in Panel D, which - in contrast to the comparison of income per capita - also takes the resources spent on entry and expansion efforts into account.

Table 8: Increasing Delegation Efficiency in India: Macroeconomic Implications

Year:	2000	2010	2020	2030		∞
		Panel A: Productivity growth g_Q				
Baseline	3.00%	2.85%	2.72%	2.62%	...	2.00%
$\alpha = \alpha_{US}$	3.00%	3.01%	2.84%	2.71%	...	2.00%
		Panel B: Relative productivity Q_{IND}/Q_{US}				
Baseline	26.6%	29.2%	31.6%	33.8%	...	49.5%
$\alpha = \alpha_{US}$	26.6%	29.2%	32.0%	34.6%	...	52.0%
		Panel C: Relative income pc y_{IND}/y_{US}				
Baseline	27.8%	30.5%	33.0%	35.3%	...	51.7%
$\alpha = \alpha_{US}$	27.8%	32.6%	35.5%	38.3%	...	57.0%
		Panel D: Relative consumption c_{IND}/c_{US}				
Baseline	29.1%	31.9%	34.6%	36.9%	...	54.1%
$\alpha = \alpha_{US}$	29.1%	33.9%	37.0%	39.9%	...	59.5%

Notes: The table reports the aggregate implications of an increase in the efficiency of delegation in India from α_{IND} to α_{US} in the year 2010. We report the rate of growth of the productivity index Q_t (Panel A), the differences in Q_t between the U.S. and India (Panel B), the differences in income per capita (Panel C), and the differences in consumption per capita (Panel D). These results are based on an estimate for λ of 0.296 (see Section B.2 in the Appendix).

6 Robustness

In this section, we discuss the robustness of our results. For each specification, we recalibrate both the U.S. and the Indian economy and redo our analysis. Overall, we find our main conclusions are fairly robust. All results are reported in Table 9. We report the implied levels of creative destruction in both countries (columns 1 and 2) as a summary statistic of the respective calibrations and the changes in creative destruction, relative technology and income, average firm size, and the share of small firms among 21- to 25 year-old firms in India due to an increase in α to the U.S. level. In Panel A of Table 9, we report our baseline results for comparison.

[32] Additionally, the increase in α also reduces the number of production workers as individuals sort into managerial occupations. Quantitatively, the number of production workers declines by about 2.3% along the BGP.

To summarize: our baseline calibration is qualitatively robust across the different alternatives we consider. The most important parameters are the "management elasticity" σ, the elasticity of labor supply, and the dispersion of managerial human capital ϑ.

TABLE 9: ROBUSTNESS

	τ_{IND}	τ_{US}	Change in ... due to the increase from α_{IND} to α_{US}				
			τ_{IND}	Q_{IND}/Q_{US}	y_{IND}/y_{US}	Avg. firm size	Share of small firms
Panel A. Baseline Calibration							
	0.054	0.124	4.11%	5.05%	10.25%	3.59%	-1.51%
Panel B. Estimating country-specific σ from accounting information							
	0.057	0.129	4.83%	6.88%	13.94%	0.63%	-1.00%
Panel C. Estimating country-specific σ from Bloom et al. (2013) and accounting information							
	0.056	0.111	5.31%	8.11%	15.25%	2.30%	-1.45%
Panel D. Entry elasticity ζ_e							
$\zeta_e^L = 0.4$	0.054	0.124	3.77%	4.63%	9.76%	3.70%	-1.49%
$\zeta_e^H = 0.6$	0.054	0.124	4.55%	5.58%	10.88%	3.44%	-1.53%
Panel E. Convexity of expansion technology ζ							
$\zeta^L = 0.4$	0.054	0.122	4.19%	5.19%	10.94%	2.15%	-1.08%
$\zeta^H = 0.6$	0.054	0.127	3.90%	4.70%	9.25%	5.04%	-2.05%
Panel F. Estimation with firm level data							
	0.054	0.116	4.12%	5.12%	10.47%	2.90%	-1.43%
Panel G. Strength of knowledge diffusion λ							
$\lambda^L = 0.217$	0.054	0.124	4.11%	6.95%	12.24%	3.59%	-1.51%
$\lambda^H = 0.423$	0.054	0.124	4.11%	3.51%	8.63%	3.59%	-1.51%
Panel H. Elastic labor supply in the manufacturing sector							
$\Delta L/L = 2\%$	0.054	0.124	5.62%	6.87%	14.62%	3.78%	-1.79%
$\Delta L/L = 5\%$	0.054	0.124	7.88%	9.54%	21.29%	4.08%	-2.22%
Panel I. Dispersion in managerial human capital ϑ							
	0.052	0.120	1.56%	2.13%	3.32%	6.45%	-0.70%

Notes: Panel A contains our baseline results based on the parameters reported in Table 3. In Panels B and C, we estimate σ based on accounting information and allow it to differ across countries. In Panel D, we consider two different values for the elasticity of the entry technology, $\zeta_e^L = 0.4$ and $\zeta_e^H = 0.6$. In Panel E, we consider two different values for the convexity of the innovation function, $\zeta^L = 0.4$ and $\zeta^H = 0.6$. In Panel F, we report the results when we calibrate the model for the U.S. economy to firm-level moments. In Panel G, we consider two values for λ, which controls the strength of the knowledge diffusion in step size for India, $\lambda^L = 0.217$ and $\lambda^H = 0.423$. These values are chosen such that the speed of convergence (in terms of half-life) is 25% longer (λ^L) and 25% shorter (λ^H) compared to the baseline Indian economy. In Panel H, we allow the total workforce to increase by 2% or 5% in response to the change in α. In Panel I, we consider a value for ϑ of 2.24.

Alternative estimates of σ: Our baseline estimates of σ are identified from the estimated treatment effect of the managerial intervention of Bloom et al. (2013). A concern with this strategy is that we had to restrict σ to be constant across countries. In Panels B and C, we report the results from an alternative strategy that addresses these limitations. In Panel B, we consider a calibration, which does not rely on the experimental results of Bloom et al. (2013), but instead uses the share of managerial compensation in total profits to identify σ (see (28)).[33] Because we observe this moment in both countries, this strategy allows us to let σ vary across countries. Our calibrated model is able to match this moment in both countries. We estimate that $\sigma_{IND} = 0.51$ and $\sigma_{US} = 0.67$, which are higher than our baseline estimate of $\sigma = 0.47$. In Panel C, we use both the estimated treatment effect and the managerial compensation shares as moments, and we find $\sigma_{IND} = 0.46$ and $\sigma_{US} = 0.67$. These estimates for σ amplify the aggregate consequences of an increase in α.

Entry: In our benchmark specification, we assume entrants have access to the same innovation technology as incumbent firms; that is, the cost function has an elasticity governed by $\zeta_e = \zeta = 0.5$. To assess the importance of this parameter, we recalibrate our model, both for the U.S. and India, while setting ζ_e to alternative values. The higher the value of ζ_e, the more responsive are entrants to changes in the value of entry. As shown in Panel D, if we set ζ_e to 0.4 (0.6), the effects of improving the efficiency of outside managers are smaller (larger). In terms of income per capita, our baseline results decrease (increase) by 0.5 percentage points. As expected, a higher entry elasticity reduces the effect on average firm size.

Convexity of incumbents' expansion technology: Similarly, we studied how the convexity of the expansion cost function for incumbent firms changes our results. Interestingly, the results are the opposite of the ones found in Panel D: the higher (lower) the elasticity of incumbent innovation, the weaker (stronger) the response of aggregate income and creative destruction to changes in α. The reason is that, in India, entrants account for most creative destruction. The higher the incumbent expansion elasticity, the more entrants are crowded out. Although such a higher elasticity increases average firm size, it actually reduces the aggregate impact of changes in α.

Firm-Level Analysis: For our baseline analysis, we have focused solely on establishment-level data. We did so to ensure comparability between the U.S. and India, because we cannot link individual establishments to specific firms in the Indian data. Panel F shows this choice has no substantial implications for our conclusions - the counterfactual implications of an increase in α are quantitatively similar when we calibrate the U.S. parameters to firm-level moments.[34]

Strength of Knowledge Diffusion: Our benchmark analysis estimates the diffusion parameter λ from the time series of TFP differences between India and the U.S.. Our estimate implies a half-life of around 50 years. We considered two alternative values for λ that increase (reduce) the speed of convergence by 25%. This parameter only affects aggregate income differences and not the firm size distribution. Panel G of Table 9 shows that a faster transition speed (i.e., a high level of λ) decreases the impact of α on productivity and income differences. This follows directly from (31),

[33] In Section B.1 in the Appendix, we discuss in detail how we measure this moment.

[34] The model is able to match the firm-level moments quite well. The main difference between establishments and firms at the horizon of age 21-25 is the life-cycle, the aggregate employment share, and the relative exit rate. The life-cycle is slightly steeper, the employment share is lower (because very old firms are much bigger than very old establishments), and the relative exit rate of young firms is higher than that of older establishments, because old firms exit less frequently than older establishments. Moreover, the aggregate entry rate is slightly lower at the firm level. In Section OA-2.1 in the Online Appendix, we provide more details on the establishment-firm comparison for the U.S.

which shows that Q_{IND}/Q_{US} is less sensitive to changes in τ if λ is large. The quantitative results are, however, in the ballpark of our baseline estimates.

Elastic Labor Supply: In our main analysis, we treated aggregate labor supply as exogenous and hence non-responsive to an increase in α. If an increase in delegation efficiency in the manufacturing sector raises productivity, we might expect the manufacturing sector to draw in workers from the rest of the economy. In Panel H, we report the results when we assume the total workforce in the manufacturing sector increases by 2% or 5% when α is increased to the U.S. level. Allowing for elastic labor supply amplifies our results because an increase in the workforce increases creative destruction and hence reduces income differences.

Dispersion in Managerial Human Capital ϑ: For our baseline estimates, we use the dispersion in log managerial earnings to calibrate the dispersion in managerial human capital ϑ. Our assumption regarding the managerial skill distribution implies that average managerial earnings relative to those of production workers are given by $\vartheta/(\vartheta-1)$ (see also footnote 23). The managerial earnings premium of 0.59 log points in the U.S. implies a higher ϑ value of 2.24. Panel I shows the results based on this higher value. This parameter is quite important in that the change in relative income per capita due to the increase in α declines from 10% to 3.3%. The main reason is that a higher ϑ makes the labor supply of managers more elastic. A given change in α therefore induces a sharper decline in the number of workers, which in turn tends to lower profits and hence weakens the effect on expansion, entry, and creative destruction.

7 Conclusion

Are inefficiencies in delegating managerial tasks to outside managers an important determinant of the process of firm dynamics and aggregate income in poor countries? To answer this question, we proposed a novel model of firm growth that highlights the interaction between managerial delegation, firms' incentives to expand, and aggregate productivity. Our theory predicts an inherent complementarity between the efficiency of delegation and firm size, because delegation only becomes necessary once firms reach a certain scale. If firms anticipate they will not be able to delegate efficiently once they grow large, their incentives to expand are throttled. At the micro-level, this implies most firms stay small. At the macro-level, it reduces the extent of reallocation, allows stagnant, subsistence producers to survive, and lowers aggregate productivity.

To quantify the strength of this mechanism, we calibrate our model to plant-level data from India and the U.S. To credibly identify the link between managerial inputs and firms' incentives to expand, we estimate our structural model to the experimental evidence on the relationship between management practices and firm performance reported in Bloom et al. (2013).

We draw three lessons from our quantitative analysis. First, we find that the Indian economy suffers from a lack of selection, which allows subsistence firms to survive. The glut of small firms in poor countries may therefore not result from frictions these firms face, but rather may be a sign that other, more dynamic firms do not grow sufficiently. Policies targeted at small firms could therefore end up supporting stagnant producers and have unintended consequences.

Second, we find that inefficiencies in delegating managerial tasks have non-trivial macroeconomic implications. Our estimates imply that a given manager is only half as efficient in an Indian firm, relative to a firm in the U.S. If Indian firms could use managers as efficiently as U.S. firms, income per capita difference between these two countries would be 11% lower. This increase is due to both static and dynamic effects that are of roughly equal size.

Finally, we find a strong complementarity between delegation efficiency and other factors affecting firm growth. Whereas an increase to U.S. standards would increase average firm size in India only modestly, firms in the U.S. would shrink substantially if they had to operate with the delegation environment common in India. Hence, for improvements in the efficiency of delegation to have sizable effects in India, other determinants of firm growth also need to be addressed: even if one of its wheels is fixed, a car cannot run when the rest of its wheels remain broken.

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Appendices

A Theoretical Appendix

A.1 Firm Size Distribution

Let $\nu^H_{n,t}$ denote the share of high-type firms with n products, and F^j_t be the number of firms of type j. Then, firm size distribution of the economy can be represented by the following differential equations:

$$\frac{\partial F^H_t \nu^H_{1,t}}{\partial t} = z_t \times \delta - F^{H,t}\nu^H_{1,t}\tau_{H,t} \tag{32}$$

$$\frac{\partial F^H_t \nu^H_{n,t}}{\partial t} = \left[\nu^H_{n-1,t}\left(n-1\right)x_{n-1,t} + \nu^H_{n+1,t}\tau_{H,t}\left(n+1\right) - \nu^H_{n,t}n\left(\tau_{H,t}+x_{n,t}\right)\right] \times F^H_t. \tag{33}$$

$$\frac{\partial F^L_t}{\partial t} = z_t \times (1-\delta) - F^{L,t}\tau_{L,t}. \tag{34}$$

and the requirement that $\nu^H_{n,t}$ be a proper distribution, $\sum_{n=1}^{\infty}\nu^H_{n,t} = 1$.

Equation (32) states that the number of one-product high type firms is given by the difference between entering high-type firms and exiting high-type firms. Recall that $\tau_{j,t}$ denotes the rate at which a firm of type j loses a given product at each point in time. Similarly, equation (33) is an accounting equation for the net-change in the number of high type firms with n products. Finally, (34) is the analogue of (32) for low-type firms, which always have a single product.

Proposition 1 *Consider a stationary equilibrium and let the flow of entry z and high-type firms' expansion rates $\{x_n\}_{n=1}^{\infty}$ at stationary equilibrium be given. The distribution of high-type firms is*

$$\nu^H_n = \frac{n^{-1}\frac{\tau_H}{x_n}\prod_{j=1}^{n}\left(\frac{x_j}{\tau_H}\right)}{\sum_{s=1}^{\infty}s^{-1}\frac{\tau_H}{x_s}\prod_{j=1}^{s}\left(\frac{x_j}{\tau_H}\right)}, \tag{35}$$

the measure of high- and low-type firms is

$$F^H = \frac{\delta z}{\tau_H} \times \left[\sum_{n=1}^{\infty}\frac{\tau_H}{nx_n}\prod_{j=1}^{n}\left(\frac{x_j}{\tau_H}\right)\right] \quad and \quad F^L = \frac{(1-\delta)z}{\tau_L}, \tag{36}$$

the aggregate rate of creative destruction is

$$\tau = z \times \left[\delta\sum_{s=1}^{\infty}\prod_{j=1}^{s}\left(\frac{x_j}{\tau_H}\right) + 1\right], \tag{37}$$

and the type-specific creative destruction rates are

$$\tau_H = \tau - z\left(1-\delta\right)\left(\frac{\beta-1}{\beta}\right) \quad and \quad \tau_L = \beta\tau - z\left(1-\delta\right)\left(\beta-1\right). \tag{38}$$

Proof. By setting the time derivatives to zero in (32), (33) and (34), stationary firm size distribution is described by the following equations

$$\begin{aligned} F^H \nu_1^H \tau_H &= z \times \delta & (39)\\ \nu_n^H n\left(\tau_H + x_n\right) &= \nu_{n-1}^H \left(n-1\right) x_{n-1} + \nu_{n+1}^H \tau_H \left(n+1\right) & (40)\\ F^L \tau_L &= z \times \left(1-\delta\right) & (41)\end{aligned}$$

Let ν_1^H and τ be given. First note that consistency requires that the total amount of innovation has to be equal to the total rate of creative destruction:

$$\tau = \tau_H(1-F^L) + \tau_L F^L \tag{42}$$

Then, by using (41), (42) and $\tau_L = \beta\tau_H$, we get

$$\tau_H = \tau - z\left(1-\delta\right)\left(\frac{\beta-1}{\beta}\right) \quad \text{and} \quad \tau_L = \beta\tau - z\left(1-\delta\right)\left(\beta-1\right). \tag{43}$$

Next, by using (39) - (41), we calculate F^L, F^H, and $\{\nu_n\}_{n=2}^{\infty}$.

Lemma 1 *The distribution of high types takes the following form*

$$\nu_n^H n = \frac{\prod_{j=1}^n x_j}{\tau_H^n} \frac{\tau_H}{x_n} \nu_1^H. \tag{44}$$

Proof. Substituting (44) in (39) - (41) shows that if ν_n^H satisfies (44), it satisfies all the flow equations in (39) - (41). ■

This implies that $1 = \sum_{n=1}^{\infty} \nu_n^H = \nu_1^H \sum_{n=1}^{\infty} \frac{1}{n} \frac{\tau_H}{x_n} \prod_{j=1}^n \left(\frac{x_j}{\tau_H}\right)$, so that (44) reads

$$\nu_n^H = \frac{1}{n} \frac{\prod_{j=1}^n x_j}{\tau_H^n} \frac{\tau_H}{x_n} \frac{1}{\sum_{s=1}^{\infty} \frac{1}{s} \frac{\tau_H}{x_s} \prod_{j=1}^s \left(\frac{x_j}{\tau_H}\right)}. \tag{45}$$

Then, from (39) and (41), we have

$$F^H = \frac{\delta z}{\tau_H} \times \left[\sum_{n=1}^{\infty} \frac{1}{n} \frac{\tau_H}{x_n} \prod_{j=1}^n \left(\frac{x_j}{\tau_H}\right)\right] \quad \text{and} \quad F^L = \frac{\left(1-\delta\right) z}{\tau_L}.$$

Hence, we only need to determine τ, which we get from (19) as

$$\tau = \sum_{n=1}^{\infty} n x_n \nu_n^H F^H + z = \left[\sum_{n=1}^{\infty} \delta \left(\prod_{j=1}^n \left(\frac{x_j}{\tau_H}\right)\right) + 1\right] z. \tag{46}$$

Together with (43), one can show that (46) has a unique solution for τ. ■

A.2 Derivation of Equation (21)

We can express $\ln Q_t$ after an instant Δt as

$$\begin{aligned} \ln Q_{t+\Delta t} &= \int_0^1 \left[\tau_t \Delta t \ln\left(\gamma_t q_{jt}\right) + \left(1-\tau_t\Delta t\right) \ln q_{jt}\right] dj \\ &= \tau_t \Delta t \ln\left(\gamma_t\right) + \ln Q_t \end{aligned}$$

where second and higher order terms in Δt are omitted. By subtracting $\ln Q_t$ from both sides, dividing by Δt, and taking the limit as $\Delta t \to 0$, we get

$$g_t = \frac{\dot{Q}_t}{Q_t} = \lim_{\Delta t \to 0} \frac{\ln Q_{t+\Delta t} - \ln Q_t}{\Delta t} = \ln(\gamma_t)\,\tau_t.$$

A.3 Transitional Dynamics with Stationary Firm Size Distribution

Proposition 2 *Suppose that the firm-size distribution at time t coincides with the stationary distribution characterized in Proposition 1. Then, for any path of the step size γ_t, there is an equilibrium path, where (i) the firm size distribution remains stationary, (ii) all aggregate variables grow at the same rate $\ln(\gamma_t)\tau_{BGP}$, where τ_{BGP} is the constant rate of creative destruction rate at the stationary equilibrium.*

Proof. Note that in the stationary equilibrium of the model described in Online Appendix OA-1.3, the step size γ_t does not affect any expressions. Hence, we need to show that there exists an interest rate path r_t such that C_t, Q_t and Y_t grow at the same rate during the transition. If this was the case, firms' innovation and entry choices would not change and the distribution would remain stationary. It is easy to see that interest rate path

$$r_t = \ln(\gamma_t)\tau_{BGP} + \rho$$

serves the purpose. Recall that consumption decisions of the household yield the usual Euler equation which implies that

$$r_t = g_{C,t} + \rho$$

so that under the proposed interest rate path, $g_{C,t} = \ln(\gamma_t)\tau_{BGP}$. Moreover $g_{Q,t} = \ln(\gamma_t)\tau_{BGP}$ as shown in Appendix A.2. Lastly we have $Y_t = Q_t \mathcal{M}_t L_{P,t}$. Since $\mathcal{M}_t$ and $L_{P,t}$ are constant at the proposed equilibrium, this implies that $g_{Y,t} = g_{Q,t}$. Therefore all growing variables grows at the same rate. ■

B Empirical Appendix

B.1 Data

In this section we provide more information about our data sources.

Establishment- and Firm-level Information for the U.S. We use data from the Business Dynamics Statistics (BDS). BDS is a product of the U.S. Census Bureau. The BDS data are compiled from the Longitudinal Business Database (LBD). The LBD is a longitudinal database of business establishments and firms covering the years between 1976 and 2012. We focus on the manufacturing sector in 2012. The data are publicly available at http://www.census.gov/ces/dataproducts/bds/.

For our analysis, we utilize the following four moments from the U.S. data: (i) the cross-sectional relationship between age and size, which we refer to as the life-cycle, (ii) the aggregate employment share by age, (iii) the exit rate as a function of age *conditional on size*, and (iv) the rate of entry. For our main analysis we focus on establishments. The BDS reports both aggregate employment and the number of establishments by age. This allows us to calculate the first two moments. The BDS also directly reports both entry and exit rates for each size-age bin. The entry rate at the establishment level is calculated as the number of new establishments at time t relative to the average number of establishments in t and $t-1$. Similarly, the exit rate at the establishment level is calculated as the number of exiting establishments in t relative to the average number of establishments in t and $t-1$. The corresponding information is also reported at the firm level. In particular, the BDS reports the number of exiting firms for different size-age bin. Note that all establishments owned by the firm must exit for the firm to be considered an exiting firm. As for firm entry, we treat firms of age 0 as an entering firm. Because a firm's age is derived from the age of its establishments, this implies that we treat firms as entering firms only if all their establishments are new. In Section OA-2.1 in the Online Appendix we provide detailed descriptive statistics about the dynamic process at both the firm- and establishment-level.

Establishment-Level Information for India As explained in the main body of the text, we construct a representative sample of the Indian manufacturing sector by combining data from the Annual Survey of Industries (ASI) and the National Sample Survey (NSS), which - every five years - has a special module to measure unorganized manufacturing establishments. We use cross-sectional data from 2010. In contrast to the U.S., both the ASI and NSS are based on establishments and we cannot link establishments to firms. With the majority of employment being accounted for by very small producers, multi-establishment firms are unlikely to be important for the aggregate in India. Firms in the NSS account for 99.2% of all establishments and for 76% of manufacturing employment. In Table 10 we report the size distribution of establishments in the NSS. More than 80% of plants have at most 2 employees and only 5% have more than 5 employees. Note that the NSS data contains some large firms: 1.5% of plants have more than 10 employees and roughly 0.25% have more than 20 employees. These plants are sampled in what is called "Segment 9" of the data, which is reserved for such large firms.

Table 10: The employment distribution in the NSS

Number of employees							
1-2	3-5	6-9	10-14	15-19	20-24	25-49	>50
82.10%	13.49%	2.90%	0.87%	0.36%	0.11%	0.10%	0.05%

Notes: The table reports the share of firms in the respective size category in the NSS data in 2010. We use the sampling weights provided in the data to aggregate the number of firms.

Comparison of the NSS/ASI Data with the Economic Census In our analysis we follow the literature to treat the combination of the NSS and ASI data as measuring the population of firms (see for example Hsieh and Olken (2014) or Hsieh and Klenow (2014)). To provide further evidence for the validity of this choice, we now compare this data to the Indian Economic Census (EC). The EC is a complete count of all economic units in the country. While the ASI/NSS is collected in the year 2010, no EC was conducted in 2010. We therefore report a comparison with the EC in 2005 and 2013. Given that the ASI/NSS focuses on manufacturing plants, we also select manufacturing firms from the EC.

In Table 11 we compare the firm size distribution as measured by these three datasets. We report the share of plants, the share of employment and the average plant size for different size categories. The main take-away from Table 11 is that the distributions from the EC and our ASI/NSS are very similar. There are slightly more firms with 1-4 employees in our ASI/NSS sample and hence their aggregate employment share is consequentially also larger. Note however, that the ASI/NSS sample contain less firms and therefore less employment in the 5-9 category. The share of firms and employment in firms with less than 10 employees is almost identical between the EC and the NSS/ASI data. Also note that the distribution of average firm size within size classes is very similar.

Non-homothetic Demand for Outside Managers In Figure 10 we provide additional evidence for the non-homothetic pattern of managerial demand reported in Table 2. While Table 2 is based on

Table 11: Comparison of NSS/ASI and Economic Census

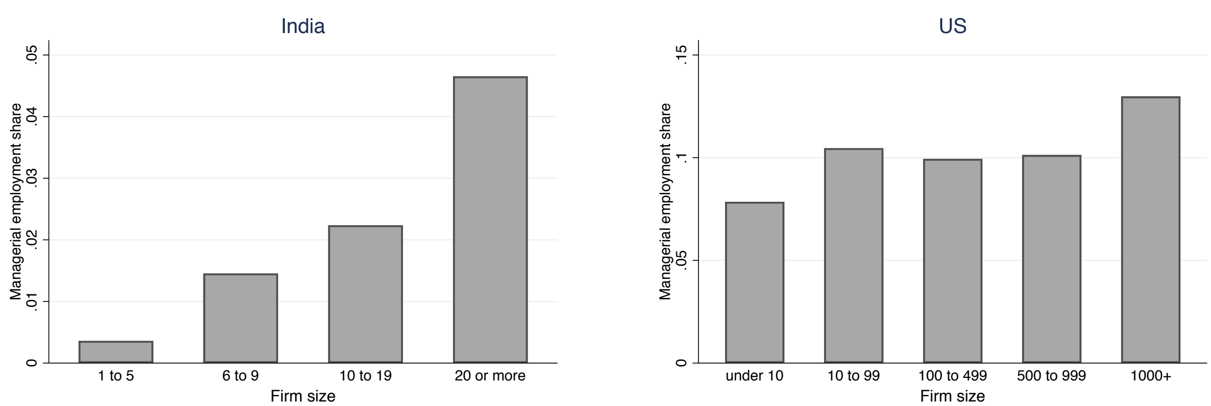
Size	Share of firms			Share of employment			Average firm size		
	EC '05	EC '13	ASI/NSS	EC '05	EC '13	ASI/NSS	EC '05	EC '13	ASI/NSS
1-4	89.82%	90.08%	92.97%	49.01%	49.26%	54.76%	1.7	1.6	1.6
5-9	8.24%	7.88%	4.91%	17.24%	16.12%	11.61%	6.5	6.0	6.3
10-19	0.93%	1.05%	1.43%	3.92%	4.54%	7.04%	13.1	12.8	13.0
20-49	0.55%	0.60%	0.42%	5.19%	5.95%	4.58%	29.3	29.5	29.1
50-99	0.24%	0.22%	0.13%	5.19%	4.91%	3.56%	67.6	67.0	69.9
100-249	0.16%	0.11%	0.09%	7.45%	5.66%	4.86%	142.0	146.5	149.3
250-499	0.04%	0.03%	0.03%	4.70%	3.83%	3.56%	329.8	336.1	346.7
500-999	0.01%	0.01%	0.01%	2.87%	3.35%	3.41%	664.3	678.7	683.9
1000+	0.01%	0.01%	0.01%	4.43%	6.38%	6.61%	2208.1	2256.3	2452.7

Notes: This table contains summary statistics of the firm size distribution as measured by the NSS/ASI in 2010, the Economic Census in 2005 and the Economic Census in 2013. For the NSS/ASI sample we use the sampling weights provided in the data.

firm-level data, Figure 10 uses individual data from IPUMS (for India) and the Current Population Survey (the U.S). In both datasets we observe individuals occupation, whether they work as a wage worker and the size of the firm in which they work. Hence, we can compute the share of people who are classified as outside managers conditional on working in firms in a particular size bin.

The left panel of Figure 10 shows that the non-homotheticity of managerial demand is not only present in the firm-level data but also pervasive in the data from IPUMS. The results are also (roughly) quantitatively in line with our measurement from the firm-level data reported in Table 2. The right panel documents the same relationship for the U.S. Again, we find robust evidence for managerial demand to be non-homothetic. Note that the share of outside managers in the CPS data is quantitatively similar to what we measure IPUMS. There we found a managerial share of 12.5%. Note also that our model predicts that the non-homotheticity should be less pronounced in the U.S., where the delegation efficiency α is high relative to the owners' managerial supply T - see e.g., equation (10).

Figure 10: Non-homothetic Demand for Outside Managers



Notes: The left (right) panel shows the share of workers working as outside managers for different firm size bin in India (the U.S.). The India data stems from IPUMS in 2004. The U.S. data stems from the CPS and we averaged the annual data for 2005-2016.

Data on Managerial Compensation and Profits for the U.S. We identify σ from the share of managerial compensation in aggregate profits *before* managerial payments [see equation (47)]. To estimate this moment, we use two data sources. From NIPA we can retrieve a measure of aggregate profits in the manufacturing industry. Specifically, we start with aggregate corporate profits, which are directly measured in NIPA. The BEA's featured measure of corporate profits -profits from current production - provides a comprehensive and consistent economic measure of the income earned by all U.S. corporations. As such, it is unaffected by changes in tax laws, and it is adjusted for non- and misreported income. We then add to this measure non-farm proprietors' income in the manufacturing sector, which provides a comprehensive and consistent economic measure of the income earned by all U.S. unincorporated non-farm businesses.

To measure managerial wages, we augment the information in NIPA from information in the census. While NIPA reports compensation for workers, managerial payments are not directly recorded in NIPA. To calculate the managerial wage bill, we therefore use the U.S. census data. In the census we have micro data on labor compensation and occupations at the micro level. Hence, we calculate the *share* of managerial payments in the total wage bill and apply that share to the aggregate compensation data in NIPA. According to the census, managerial compensation amounts to roughly 22% of total wages. Recall that the managerial employment share in the U.S. is about 12.5% so that managerial wages are relatively high. We then calculate the share of managerial compensation (CSM) in aggregate profits net of managerial wages as

$$CSM = \frac{\text{Managerial Compensation}}{\text{Corporate Profits + Nonfarm Proprietor's Income + Managerial Compensation}},$$

where "Managerial Compensation" is simply 22% of the total labor compensation in NIPA. We also calculate a second measure of CSM, where we do not include "Nonfarm Proprietor's Income." We calculate CSM before the Great Recession, because we were concerned about corporate profits being very low during the financial crisis. CSM is quite volatile. It ranges from 65% in 2001 to 33% in 2006. For our calibration we focus on the average across the years 2000 - 2007, which is 50%. If we do not include "Nonfarm Proprietor's Income", the numbers are very similar and only slightly larger, ranging from 69% in 2001 to 35% in 2006. The average is 53%. Hence, it is not essential for us to take "Nonfarm Proprietor's Income" into account.

Data on Managerial Employment and Earning: To measure managerial employment and earnings in the U.S. and India, we employ national Census data from the IPUMS project. We focus on the most recent year, which is 2010 for the U.S. and 2004 for India. For each country we get a sample from the census, which has detailed information about personnel characteristics. In particular we observe each respondent's education, occupation, employment status, sex, and industry of employment. We focus on male workers in the manufacturing industry working in private-sector jobs.

The list of occupations according to ISCO is contained in Table 12. To qualify as a manager in the sense of our theory, two characteristics have to be satisfied. First, the respective individual has to work as a "Legislator, senior official, and manager." In order to focus on managers, which are agents of a firm owner, i.e., outside managers, we *also* require workers to be wage workers and not working on their own account or to be unpaid family members. This information is also contained in the IPUMS census data in the variable "worker type." As we showed in Table 1 above, it is important to take these differences into account as poor countries have a higher share of people working on their own account (or as a family member) *conditional* on being classified as a manager according to ISCO.

Table 12: List of Occupations according to ISCO

Legislators, senior officials, and managers	Plant and machine operators and assemblers
Professionals	Elementary occupations
Technicians and associate professionals	Armed forces
Clerks	Other occupations, unspecified or n.e.c.
Service workers and shop and market sales	Response suppressed
Skilled agricultural and fishery workers	Unknown
Crafts and related trades workers	NIU (not in universe)

Notes: Table 12 contains the occupational categories available in the IPUMS data. A necessary condition for someone to be classified as an outside manager is to be assigned the occupational title "Legislators, senior officials, and managers." See the main body of the text for the additional requirements.

B.2 Identification of the Model

We will now discuss the identification of our model in more detail. In total, there are 11 parameters to identify[35]:

$$\Omega \equiv \{\alpha, \sigma, T, \mu_M, \vartheta, \theta, \theta_E, \delta, \beta, \gamma^{US}, \lambda\}.$$

In Section A.1, we discussed how the distribution of firm size is determined given the optimal innovation and entry rates $\{x_n\}_{n=1}^{\infty}$ and z. More specifically, $\{x_n\}_{n=1}^{\infty}$ and z determine the aggregate innovation rate τ and these three objects together uniquely pin down the joint distribution of age and size, i.e., the entire process of firm-dynamics. The four parameters that affect this process directly are $(\theta, \theta_E, \beta, \delta)$. We therefore use the following four firm-level moments to calibrate these parameters: (i) the life cycle, i.e., the relative size of firms of age 21-25 to firms of age 1-5, (ii) the share of aggregate employment accounted for by firms of age 21-25, (iii) the relative exit rate of 1-5 year old firms relative firms of age 21-25 conditional on size, and (iv) the entry rate. Intuitively, the slope of the life-cycle is informative about θ, which determines the level of incumbent's innovation effort. As β effectively controls the size of old cohorts (by determining the speed with which high-type firms exit), it is related to the aggregate importance of old cohorts in the economy, i.e., the relative employment share of old firms. The exit hazard conditional on size is informative about the degree of selection. If there was no type heterogeneity, the exit rate would only be a function of size. To the extent that older firms are positively selected, they are less likely to exit conditional on size. The ex-ante heterogeneity δ determines how strong this effect can be. Finally, the entry rate is informative about θ_E.

We then use several moments related to managerial employment patterns - namely the compensation of managers relative to corporate profits, the entrepreneurial share in total compensation, the dispersion of managerial wages, and managerial employment shares - to identify σ, T, ϑ, α and μ_M. Consider first σ, the elasticity of profits with respect to managerial services.[36] In the model, the total compensation for managerial personnel relative to aggregate profits (before managerial payments) is given by

$$\frac{w_M H^M}{\Pi + w_M H^M} = \frac{\sum_{n=1}^{\infty} w_M \times n \times m(n) \times \varphi_n}{\sum_{n=1}^{\infty} e(n)^{\sigma} Y \times n \times \varphi_n},$$

where $\varphi_n = F^H \nu_n^H$ and $\varphi_1 = F^H \nu_1^H + F^L$ is the endogenous firm size distribution. By using $m(n) = T\alpha^{-1} \times \max\{0, (n^*)^{-1} - (n)^{-1}\}$, $\omega_M \equiv \frac{w_M}{Y} = \sigma\alpha\left(\frac{n^*}{T}\right)^{1-\sigma}$ and $e(n) = T\max\{n^{-1}, (n^*)^{-1}\}$, we get

[35] Recall that we calibrate ζ and ρ outside of the model.

[36] Although the specific ordering of parameters in the identification discussion is not essential, it facilitates the argument.

that

$$\frac{w_M H^M}{\Pi + w_M H^M} = \sigma \frac{\sum_{n=1}^{\infty} (n^*)^{1-\sigma} \left(\max\left\{0, \frac{1}{n^*} - \frac{1}{n}\right\}\right) \times n \times \varphi_n}{\sum_{n=1}^{\infty} \left(\max\left\{\frac{1}{n}, \frac{1}{n^*}\right\}\right)^{\sigma} \times n \times \varphi_n}. \tag{47}$$

Hence, conditional on n^* and the firm size distribution, (47) only depends on σ.

To determine T, we target the share of income accruing to entrepreneurs after paying for their factors of production. As entrepreneurs are the residual claimants on firm profits, this moment is simply given by

$$\begin{aligned}\frac{\Pi}{Y} &= \sum_{n=1}^{\infty} \left[e(n)^{\sigma} - \omega_M m(n)\right] \times n \times \varphi_n \\ &= T^{\sigma} \sum_{n=1}^{\infty} \left[\left(\max\left\{n^{-1}, (n^*)^{-1}\right\}\right)^{\sigma} - \sigma n^* \max\left\{0, \frac{1}{n^*} - \frac{1}{n}\right\}\right] \times n \times \varphi_n,\end{aligned}$$

which is directly informative about T for given n^*, φ_n, and σ.

The shape parameter of skill distribution ϑ can be identified directly from the dispersion of managerial earnings. To see this, note that the earnings of a manager with relative skill h is $w_M h$. The distribution of managerial earning is therefore given by

$$P\left[w_M h > x | h \geq \frac{w_P}{w_M}\right] = \left(\frac{w_P / w_M}{x / w_M}\right)^{\vartheta} = \left(\frac{w_P}{x}\right)^{\vartheta},$$

which is pareto with shape ϑ and location w_P. Defining the relative managerial earnings $y \equiv \ln\left(\frac{w_M h}{w_P}\right)$, we get $P(y \leq y_0) = 1 - e^{-\vartheta y_0}$, so that

$$var(y) = var\left(ln\left(\frac{w_M h}{w_P}\right)\right) = var\left(ln\left(w_M h\right)\right) = \vartheta^{-2}.$$

Hence, we can calibrate ϑ directly to the variance of log managerial earnings.

Finally, we identify α and μ_M by using the share of managers in the whole economy *and* among Indian immigrants to the U.S. economy. Let χ denote the equilibrium managerial employment share which is given by

$$\chi = P\left[h_M w_M \geq w_P\right] = \left(\frac{\frac{\vartheta-1}{\vartheta}\mu_M}{w_P / w_M}\right)^{\vartheta} = \left(\frac{\vartheta - 1}{\vartheta}\mu_M \frac{\sigma\alpha}{\omega_P}\left(\frac{n^*}{T}\right)^{1-\sigma}\right)^{\vartheta}.$$

Using the expression for total managerial demand, the equilibrium condition for the managerial labor market can be written as

$$\mu_M \alpha = (\chi)^{-\frac{\vartheta-1}{\vartheta}} \times \sum_{n \geq n^*}^{\infty} T\left(\frac{1}{n^*} - \frac{1}{n}\right) \times n \times \varphi_n. \tag{48}$$

Hence, given n^*, T, ϑ, and φ_n, we can directly determine $\mu_M \times \alpha$ from the data on the share of managers in the whole population (i.e., χ). To separate the effect of managerial human capital (μ_M) from delegation efficiency (α), we use data on managerial employment pattern of Indian immigrants. Because our approach uses additional data and because all allocations in the model only depend on $\mu_M \times \alpha$, we discuss the details of our strategy in Section B.4. Once we identify μ_M, we get α from (48).

Lastly we use moments regarding aggregate dynamics of the economies to pin down γ and λ. In particular, we calibrate the step-size for U.S., γ^{US}, to fit the aggregate growth rate as $g = \ln\left(\gamma^{US}\right)\tau$

and U.S. is assumed to be on the balanced growth path. In the case of India, step size is partly determined by the productivity gap between U.S. and India and λ parametrizes the importance of this channel on step size [see (30)]. By using (21) and (30), we can write the *change* of relative productivity differences $Z_t \equiv \frac{Q_{US,t}}{Q_{IND,t}}$ as

$$g_{Z,t} = \frac{\dot{Z}_t}{Z_t} = \left\{ \ln(\gamma^{US}\tau_{US,t} - \tau_{IND,t}\left[\ln(\gamma^{US}) + \lambda \ln(Z_t)\right] \right\} \tag{49}$$

Therefore, given γ^{US} and the aggregate rates creative destruction for U.S. and India, we can infer λ from the dynamics of relative productivity differences between the U.S. and India.

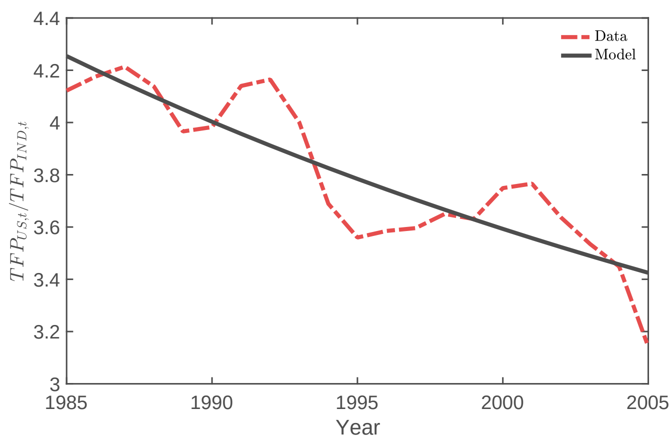
To relate Z_t to the data, note that empirically we observe total factor productivity as implied by the Penn World Tables. Given that total population size is normalized to unity, our model implies that TPF is given by $TFP = Y = Q\mathcal{M}L^P$ (see (6)). Hence, relative TFP is given by

$$\frac{TFP_{t,US}}{TFP_{t,IND}} = Z_t \times \frac{\mathcal{M}_{t,US}L^P_{t,US}}{\mathcal{M}_{t,IND}L^P_{t,IND}}.$$

Note that if the firm-size distribution is stationary, both $\mathcal{M}_{t,c}$ and the sectoral allocation of labor $L^P_{t,US}$ are constant. Hence, the change in measured relative TFP, $TFP_{t,US}/TFP_{t,IND}$, is exactly $g_{Z,t}$ given in (49) and hence can be used to calibrate λ.

In Figure 11 we depict the evolution of relative TFP levels between the U.S. and India between 1985 and 2005. It is clearly seen that India is catching up as relative TFP differences decline from 4 in 1985 to roughly 3.2 in 2005. We therefore calibrate λ and level of relative productivity in 1985, Z_{1985}, to minimize the distance (as measured by the sum of squared residuals) between the model and the data. The resulting fit is also displayed in Figure 11.

Figure 11: Identification of λ: TFP Differences between the U.S. and India



Notes: The figure shows the observed relative TFP between the U.S. and India (dashed) and the one implied by the model (solid).

B.3 Identifying the managerial output elasticity σ

In this section we describe in detail how we estimate the managerial output elasticity σ using indirect inference. As explained in Section 3.2, our measure of firms' managerial environment is their total managerial services $e = T/n + \alpha \times m$ (see (7)). This object is endogenous through firms' choice of outside managers m. While e is not directly observable, we assume that it is related to the observable share of managerial practices firms adopt. We refer to the share of practices firm f adopts as MP_f. In particular, we assume that e and MP_f are related via the measurement equation $e_f = vMP_f^{\varrho}$. As explained in Section 3.2, we can use the pre-treatment information on the share of adopted practices in the U.S. and India and the model-implied differences in e in our U.S. and India calibration to identify ϱ. Given ϱ, we can then express the model-implied change in total managerial services e due to the treatment, e_{IND}^{Treat}, as a function of observables ($MP_{IND}, MP_{US}, MP_{IND}^{Treat}$) and the equilibrium objects in our calibration (e_{IND}, e_{US}) - see equation (27). In our baseline calibration, we infer that the treatment increased total managerial efficiency among treatment plants by 26% (see footnote 19).

Because e is endogenous, we have to take a stand *how* the experiment induced firms to increase e by 26%, i.e., which structural parameter changed. We assume that the experiment increases the total efficiency of managerial services e by a multiple $\xi > 1$. Hence, if a treatment firm hires m units of managerial human capital on the market, it generates $\xi e = \xi(T/n + \alpha \times m)$ units of managerial services in the firm. This formalization captures the main spirit of the experiment in that the intervention provided information about how to make management more efficient via the provision of consulting services, but left the actual adoption of such managerial practices up to the treatment firms.

In practice we implement this procedure in the following way. Given the partial equilibrium nature of the experiment, treatment firms chose their optimal quantity of efficiency units of outside managers according to (8) taking the higher return to managerial services ξ as given. Formally, the optimal number of outside managers treatment firms hire, $m(\xi)$, is implicitly defined by

$$\left[m_j(\xi)\right]_{j=1}^{n} = \underset{m_j \geq 0}{\operatorname{argmax}} \sum_{j=1}^{n} \left\{ \left(\xi \left(\frac{T}{n} + \alpha m_j \right) \right)^{\sigma} Y - w_M m_j \right\}. \tag{50}$$

The solution to this problem is given by (see (10))

$$m(n;\xi) = \left(\frac{\sigma}{\omega_M} \right)^{\frac{1}{1-\sigma}} (\xi\alpha)^{\frac{\sigma}{1-\sigma}} - \frac{1}{\alpha}\frac{T}{n} \tag{51}$$

and the associated number of managerial services, $e(n;\xi)$ is given by

$$e(n;\xi) = \xi(T/n + \alpha m(n;\xi)) = \left(\frac{\xi\alpha\sigma}{\omega_M} \right)^{\frac{1}{1-\sigma}}. \tag{52}$$

This implies that

$$\frac{e_{IND}^{Treat}}{e_{IND}} = \frac{e(n;\xi)}{e(n)} = \frac{(\xi\alpha\sigma/\omega_M)^{\frac{1}{1-\sigma}}}{(\alpha\sigma/\omega_M)^{\frac{1}{1-\sigma}}} = \xi^{1/(1-\sigma)}, \tag{53}$$

so that the required productivity increase ξ for treatment firms to increase their level of managerial efficiency from e_{IND} to e_{IND}^{Treat} is given by $\xi = \left(\frac{e_{IND}^{Treat}}{e_{IND}}\right)^{1-\sigma}$.

To understand our strategy to estimate the σ, suppose that all other structural parameters were given. In this hypothetical case, where we would only estimate σ, our algorithm would be the following:

1. Guess a value of σ and solve the equilibrium of the model.
2. The model then implies equilibrium values for e_{IND} and e_{US}.
3. Given (e_{IND}, e_{US}), an assumption on the adoption of such managerial practices in the U.S., MP_{US} and the estimated increase in managerial practices for treatment firms, $\frac{MP_{IND}^{Treat}}{MP_{IND}}$, we can use (27) to calculate e_{IND}^{Treat}
4. Given e_{IND}^{Treat}, we can calculate ξ according to $\xi = \left(\frac{e_{IND}^{Treat}}{e_{IND}}\right)^{1-\sigma}$

5. Given ξ we then perform the management experience in our model.

   (a) We select 100 firms (50 for the treatment and 50 for the control group) from the top 0.01% of the size distribution from our India calibration. This selection procedure based on size mimics the selection procedure in Bloom et al. (2013), who note that the experimental firms had "about 270 employees, assets of 13 million, and sales of 7.5 million a year. Compared to U.S. manufacturing firms, these firms would be in the top 2% by employment and the top 4% by sales, and compared to India manufacturing they are in the top 1% by both employment and sales (Hsieh and Klenow 2010)" (Bloom et al., 2013, p. 9). Because we calibrate our model to the population of Indian firms (i.e. including firms in the NSS), firms with 270+ employees correspond to the top 0.01% of the firm size distribution. Our calibrated model implies that this set of firms coincides with firms of $n = 7$ products.

   (b) We then scale the total managerial efficiency of treatment firms by ξ to induce the required increase in managerial efficiency e and simulate their life-cycle for 100 weeks. Note that treatment firms are free to change their number of outside managers at any point at the equilibrium wage rate w_M of the baseline economy to mimic the partial equilibrium nature of the experiment. For the entire 100 weeks, managerial services in treatment firms have a productivity advantage of ξ.

   (c) We then measure profits for all 100 weeks according to (8) for both treatment and control firms. For control firms, profits gross of innovation spending are given by (25). For treatment firms, profits are given by

   $$\tilde{\pi}^{Treat}(n) = (1-\sigma)\, e\,(n;\xi)^{\sigma}\, n + e\,(n;\xi)^{-(1-\sigma)}\, \sigma \xi T.$$

   Hence, treatment firms have higher profits for three reasons: (1) they hire more managerial service given their size ($m\,(n;\xi) > m\,(n)$), (2) they receive a direct benefit of being able to use e more efficiently ($\xi > 1$) and (3) they will on average be larger as their innovation incentives increase. While (1) and (2) are static effects, (3) is a dynamic effect

6. Given the model-generated data on $\tilde{\pi}^{Treat}(n)$ and $\tilde{\pi}(n)$ we then run the regression in (26), i.e. we estimate the specification

   $$\ln \tilde{\pi}_{i,t} = \beta_0 + \beta_1 \times TREAT_{i,t} + \epsilon_{i,t} \tag{54}$$

   and recover the treatment effect $\hat{\beta}_1$. Note that in our regression there is no need to use firm-fixed effects as all firms with $n > 1$ are high-type firms and all firms have the same size n. As explained in Section 3.2 we choose profits as our measure of firm-performance, while Bloom et al. (2013) focus on physical output. Bloom et al. (2013) do not estimate a treatment effect based on profits.

7. To average out the sampling variation in our estimate, we replicate this procedure 250 times and calculate the model-implied treatment effect

   $$\hat{\beta}^{Treat} = \frac{1}{250}\sum_{i=1}^{250} \hat{\beta}_1^{(i)}. \tag{55}$$

8. If $\hat{\beta}^{Treat}$ is equal to the empirically observed value of 9%, we stop. Otherwise we go back to step 1 with a different guess for σ.

Recall that in order to infer e_{IND}^{Treat}, we had to assume a particular value for the share of practices adopted by firms in the U.S., MP_{US} (see (27)). For our baseline calibration, we assumed that firms in the U.S. adopt all such practices as these practices "have been standard for decades in the developed world" (Bloom et al., 2013, p. 43). From the experimental micro-data, we can provide some additional evidence for this assumption. In the experimental data for Indian firms, we observe two objects related to the firms' managerial environment: the share of particular practices the firm implements and the management score from Bloom and Van Reenen (2007). The management score is only measured pre-treatment but the practices are observed pre- and post-treatment. Using the pre-treatment variation of managerial practices and managerial scores across the Indian firms and the estimated changes in managerial practices due to the treatment, we can predict the average change in the firms' managerial score induced by the intervention. More specifically, we first run the cross-sectional regression

$$BVR_f = \beta + \gamma \times MP_f + \epsilon_f, \tag{56}$$

where BVR_f is the management score from Bloom and Van Reenen (2007) and MP_f is the share of adopted managerial practices. We then predict the change in the BVR score due to the treatment according to

$$E[BVR_f|Treatment] = E[BVR_f] + \hat{\gamma} \times \left(E[MP_f|Treatment] - E[MP_f]\right), \tag{57}$$

where $\hat{\gamma}$ is estimated coefficient from (57). The average BVR score among Indian firms before the treatment is 2.6. Using the estimated coefficient $\hat{\gamma}$ and the change in managerial practices due to the treatment $E[MP_f|Treatment] - E[MP_f]$, we find that the treatment increases the BVR score among treatment firms, $E[BVR_f|Treatment]$, depending on how we treat outliers in the regression, to 2.84 on the low end and 3.12 on the high end. The average BVR score among U.S. firms is equal to 3.28. Hence, this exercise suggests that the treatment closes the "management gap" as measured by BVR scores by $\frac{2.84-2.6}{3.28-2.6} = 35\%$ on the low end and $\frac{3.12-2.6}{3.28-2.6} = 76\%$ on the high end.

We can compare this number to the implications of our model. Our baseline calibration implies that the treatment increases e from $e_{IND} = 0.203$ by 26% to $e_{IND}^{Treat} = 0.255$. Our calibration also implies that $e_{US} = 0.286$. Hence, Indian firms use 71% the amount of managerial services as firms in the U.S. and the treatment increases managerial services to 89% of the U.S. level. Hence, the treatment reduces the "management gap" by $\frac{0.252-0.203}{0.286-0.203} \approx 59\%$.

B.4 Identifying Managerial Skill Supplies μ_M

To decompose differences in the managerial environment in India and the U.S. into supply and demand factors, we start out with 4 parameters: $(\mu_{M,US}, \alpha_{US}, \mu_{M,IND}, \alpha_{IND})$. Without loss of generality we can normalize $\mu_{M,US} = 1$. Since $\mu_{M,c} \times \alpha_c$ is identified from the equilibrium managerial employment shares [see (48)], we require one additional equation to determine the relative managerial human capital in India, $\mu_{M,IND}$. To do so, we use data on employment patterns of immigrants from India to the U.S.

Let χ_c be the managerial share of the native population in country c. Let χ_{IND}^M be the managerial employment share in the population of Indian migrants in India (i.e., pre-migration). Let χ_{US}^M be the managerial employment share in the population of Indian migrants in the U.S. (i.e., post-migration). Suppose that the distribution of managerial ability of Indians who migrate to the U.S. is distributed Pareto with shape ϑ and mean $\hat{\mu}_{M,IND}$. If $\hat{\mu}_{M,IND} = \mu_{M,IND}$, migration is orthogonal to managerial skills. If $\hat{\mu}_{M,IND} > \mu_{M,IND}$, migrants have, on average, a comparative advantage in managerial work. Given these assumptions it follows that

$$\chi_c = \tilde{\vartheta} \left(\omega_M^c\right)^{\vartheta} \left(\mu_{M,c}\right)^{\vartheta} \quad \text{and} \quad \chi_c^M = \tilde{\vartheta} \left(\omega_M^c\right)^{\vartheta} \left(\hat{\mu}_{M,c}\right)^{\vartheta}$$

where $\tilde{\vartheta} = \left(\frac{\vartheta-1}{\vartheta}\right)^{\vartheta}$ and ω_M^c is the relative managerial wage $\frac{w_M}{w_P}$ in country c. Hence,

$$\frac{\mu_{M,IND}}{\mu_{M,US}} = \underbrace{\left(\frac{\chi_{US}^M}{\chi_{US}}\right)^{1/\vartheta}}_{\text{uncorrected ratio}} \times \underbrace{\left(\frac{\chi_{IND}}{\chi_{IND}^M}\right)^{1/\vartheta}}_{\text{selection correction term}} . \tag{58}$$

The first term in (58) compares migrants and U.S. natives in the U.S. economy, i.e., holding α constant. Differences in managerial employment are therefore interpreted as differences in human capital. The second term accounts for selection into migration: if immigrants are positively selected on their managerial skills, i.e., $\chi_{IND}^M > \chi_{IND}$, the observed differences in outcomes in the U.S. *under*estimate the differences in skills in the population. The last term in equation (58) corrects for that potential selection.

We want to note that this identification strategy relies on occupational sorting being based on skills - both before and after migrating. If for example Indian migrants face excessive frictions to enter managerial positions (relative to other jobs), their observed managerial employment share is lower than their skills warrant. In that case we would conclude that they have relatively little human capital. See for example Hsieh et al. (2019) for an elaboration of this point. Alternatively, migrants could have been *more* likely to work as managers prior to migrating relative to their innate skills.[37] If, for example, migrants stem from families, which are richer and more likely to own a business, migrants might have worked as managers before simply because of their family connection. In that case migrants might not be selected on their managerial skill but rather representative of the population at large. If that was the case, we would erroneously conclude that the U.S. population had a comparative advantage in managerial occupations. Again we want to stress that our identification strategy will correctly recover $\alpha \times \mu$. The information in (58) is only used to separately identify α and μ.

Given that we already calibrated ϑ and we already used χ_{IND} and χ_{US} in our calibration. χ_{US}^M is directly observable in the U.S. Census, because we see the employment structure among recent Indian immigrants. Finally, χ_{IND}^M can be estimated from the New Immigration Study, which explicitly asks immigrants about the occupations *prior* to migration [see Hendricks and Schoellman (2017)].

The data to quantify (58) is contained in Table 13. Column 1 and 3 report the managerial share in the U.S. and India, respectively. In column 2 we report the managerial share among Indian immigrants in the U.S. To ensure that this population is informative about the human capital of recent Indian migrants, we restrict the sample to migrants that arrived in the U.S. within the last 5 years. The managerial share in this population is given by 12.7%. In the last column we exploit information from the New Immigration Study to measure the share of migrants that used to work as managers in India. We find that roughly 6.1% of them worked as outside manager.

The sample size for estimating the managerial share of migrants in India, χ_{IND}^M, is only 403, i.e., quite small. To judge the robustness of our results, we report the implied differences in delegation quality $\frac{\alpha_{US}}{\alpha_{IND}}$ as a function of the point estimate of χ_{IND}^M. We treat the other empirical objects in (58), as fixed as these are precisely estimated. We construct the confidence intervals for $\frac{\alpha_{US}}{\alpha_{IND}}$ using a Bootstrap procedure, where we repeatedly draw samples with replacement from the New Immigration Study data and calculate χ_{IND}^M. The results of this exercise are contained in Figure 12. We find that the confidence interval [1.7,2.7] contains the relative delegation efficiency of the U.S. with 90% probability. We also want to stress that this uncertainty *only* affects the decomposition of the implied counterfactual into the human capital and the delegation efficiency component, as all allocation only depend on $\mu_{M,c}\alpha_c$.

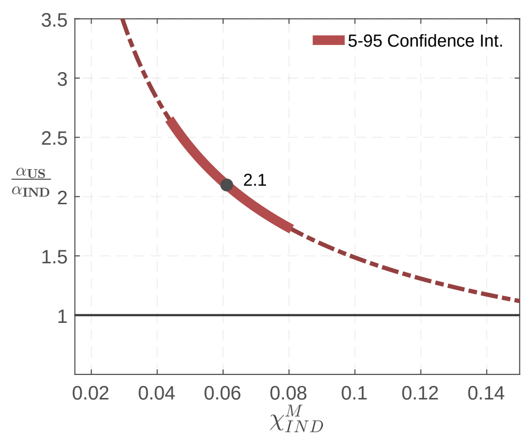
[37] We are grateful to one of our referees to suggest this possibility.

Table 13: Identification of Managerial Skills: Managerial Employment Shares

	U.S.		India	
Sample	Male, 20-60 years, employed			
Population	U.S. population χ_{US}	Indian migrants χ^M_{US}	Indian population χ_{IND}	Indian migrants χ^M_{IND}
Managerial share	12.5 %	12.7 %	1.65%	6.11%
Data source	U.S. Census	U.S. Census	Indian Census	New Immigration Study

Notes The table contains estimates for the managerial employment share in the native population of the U.S. (column 1), the population Indian immigrants in the U.S. (column 2), the native population in India (column 3), and the sample of Indian migrants to the U.S. *in* India (column 4). For the definition of outsider managers, see Table 1 and the discussion there. χ_{US} and χ^M_{US} are calculated from the U.S. census and χ_{IND} from the Indian census. χ^M_{IND} is calculated from the data of the New Immigration Study. We refer to Hendricks and Schoellman (2017) for a detailed description of the data. For the New Immigration Study we use the occupational codes "10 to 430: executive, administrative and managerial" and "500 to 950: management related" as referring to managers. We also insist on the individual having received a salary (instead of, for example, being self-employed).

Figure 12: Calibrating $\frac{\alpha_{US}}{\alpha_{IND}}$



Notes: The figure depicts the resulting $\frac{\alpha_{US}}{\alpha_{IND}}$ as a function of χ^M_{IND}. Our point estimate for the immigrants' managerial share in India (6.1%) yields a relative delegation quality of 2.11. The 5-to-95 confidence interval around that value ranges from about 1.7 to 2.7.

B.5 Moment Sensitivity

In Table 14 we report a sensitivity matrix, which contains the elasticity of each moment used in the internal calibration (rows) with respect to the parameters of the model (columns). Specifically, we report percentage change in the moment for a 1% change in the parameter from its benchmark calibrated value, while keeping the rest of the parameters at their benchmark values. We report the average elasticities based on +1% and -1% changes. This provides useful information about how the parameters influence the model counterpart of targeted moments. For brevity, we report the matrix for our India calibration. The sensitivity matrix for the U.S. calibration is available upon request.

B.6 Reduced-Form Evidence based on Variation across Indian Establishments

In Section 4.2, we reported some basic patterns on managerial hiring and firm size from the Indian micro data and discussed how they relate to our theory. This section describes this analysis in more

Table 14: Moment Sensitivity

		δ	β	T	$\alpha \times \mu$	θ	θ_E	σ
M1.	Entry rate	-0.02	-0.04	0.60	0.04	0.05	1.25	-0.90
M2.	Mean empl. of 21-25-year-old firms	0.09	0.12	0.18	0.002	0.27	0.04	-0.31
M3.	Empl. share of 21-25-year-old firms	-0.05	-0.29	-0.25	-0.04	-0.23	-0.33	0.46
M4.	Rel. exit rate of small 21-25-year-old firms	0.11	0.18	0.09	0.01	0.01	0.18	-0.13
M5.	Share of managers	0.02	0.32	0.25	1.08	0.54	-0.42	-0.89
M6.	Share of entrepreneurial profit	-0.002	-0.13	0.42	-0.03	-0.32	0.17	-0.43
M7.	Treatment effect of Bloom et al. (2013)	0.09	1.88	-1.62	-1.52	3.25	-2.44	-0.96

Notes: The table presents the elasticity for each moment used the internal calibration for India with respect to the parameters of the model. In particular, we report percentage change in the moment for a 1% change in the parameter from its benchmark value in the Indian calibration, while keeping the rest of the parameters at their benchmark values. We report the average elasticities based on +1% and -1% changes. To identify α and μ separately, we use the manager share among Indian migrants before and after emigrating to the U.S. See Section B.4 for more information.

detail.

Our empirical investigation mainly focuses on the implications of the two parameters of our model: (i) entrepreneur's time endowment T and (ii) delegation efficiency α. In the theory, time endowment of entrepreneurs T has the interpretation that it can neither be sold on the market, nor is there any need to monitor. The NSS data for 1995 contain information on the size of the family of the establishment's owner. As long as family members require less monitoring time than outside managers, we can think of family size as inducing variation in the time endowment T. As for the delegation efficiency α, we will rely on the variation in trust across 22 Indian states. The Indian micro data contain information about the state in which the respective establishment is located. Additionally, we extract information on the general level of trust between people at the state level from the World Value Surveys. The World Values Survey is a collection of surveys based on representative samples of individuals and provides an index of trust in different regions of India. The primary index we use is derived from the answers to the question "Generally speaking, would you say that most people can be trusted, or that you can not be too careful in dealing with people?". Following Bloom et al. (2012) and La Porta et al. (1997), the regional trust index is constructed as the percentage of people providing the answer "Most people can be trusted" within the state where the firm is located. This is the most common measure of trust used in the literature. While this variable is not directly aimed at eliciting the (perceived) quality of the prevailing legal environment, it fits well into our theoretical framework as long as trust reduces the required time the owner needs to spend to incentivize outside managers. See also Bloom et al. (2012), who also use this variable to proxy the efficiency with which decisions can be delegated.

In Table 15, we look at some of the implications of our theory based on the above-mentioned proxies. We first focus on the extensive margin of managerial hiring. In the model, a firm hires an outside manager only when its size n is above a certain (endogenous) threshold which we denote as n^*

$$n^* \equiv T \times \left(\frac{\omega_M}{\sigma\alpha}\right)^{\frac{1}{1-\sigma}}.$$

For the purpose of the empirical analysis, in addition to firm size n, suppose that firms also differ in (i) owner's time endowment T and (ii) delegation efficiency α. Then, the extensive margin of managerial hiring decision for firm f can be summarized as

$$\begin{aligned}\mathbb{1}\left[Manager_f > 0\right] &= \mathbb{1}\left[n_f \geq n_f^*\right] \\ &= \mathbb{1}\left[n_f \geq T_f \times \left(\frac{\omega_M}{\sigma\alpha_f}\right)^{\frac{1}{1-\sigma}}\right] \\ &= \mathbb{1}\left[\log n_f - \log T_f + \frac{1}{1-\sigma} \times \log \alpha_f + const. \geq 0\right],\end{aligned}$$

where subscript f indicates firm specific values and *const*. includes all terms that are not firm specific. This relation can be converted to an *estimable* one by introducing some stochasticity. In particular, by introducing a uniformly distributed random variable, which can be considered as measurement error, to the RHS of the above equation and taking the expectation of both sides, we get

$$\mathbb{P}\left(Manager_f > 0\right) = \beta_0 + \beta_1 \log n_f - \beta_2 \log T_f + \beta_3 \log \alpha_f. \tag{59}$$

This equation implies that the likelihood of hiring a manager should be increasing in firm size and delegation efficiency and declining in the owner's time endowment. To test these predictions empirically, we estimate the coefficients of (59) by using the proxy variables mentioned above.[38] Column 1 of Table 15 summarizes the results. It suggests that the predictions of the model regarding extensive margin of managerial hiring are in line with the data: empirically large firms and firms in states with favorable trust measures are more likely to hire outside managers, while firms with larger families abstain from hiring outside managerial personnel holding firm size constant.

Table 15: Managerial Hiring, Firms Size and Growth in India

	Dependent Variable				
	Manager > 0	Log empl (Manager > 0)		Log empl	
Log Empl	0.039***				
	(0.003)				
Log HH Size	-0.003**	0.927***	0.812***	0.224***	0.235***
	(0.001)	(0.306)	(0.278)	(0.033)	(0.032)
Trust	0.013**	3.264**		0.094	
	(0.006)	(1.628)		(0.174)	
Log HH Size* Trust		-1.694**	-1.329*	0.036	0.028
		(0.818)	(0.758)	(0.093)	(0.090)
State FE	N	N	Y	N	Y
N	178,999	2,350	2,350	178,999	178,999
R^2	0.04	0.42	0.50	0.18	0.20

Notes: Robust standard errors in parentheses. *** p<0.01, ** p<0.05, * p<0.10. All regressions include 2-digit fixed effects, the age of the establishment, year dummies, and a dummy variable for the establishment to be in a rural area as control variables. For the regressions that do not include state-level fixed effects, log GDP per capita at the state level is included as a control variable. "Log Empl" denotes the (log of) total employment at the establishment. "Log HH size" denotes the (log of) the size of the household of the establishment's owner. This variable is only available for the NSS data. "Trust" is the measure of trust at the state level, which we calculate from the World Value Surveys. The dependent variables are: an indicator of managerial hiring (column 1), log employment conditional on managerial hiring (columns 2 - 3), log employment (columns 4-5).

These static determinants of managerial hiring have dynamic implications relating to firms' expansion incentives and hence firm size. In particular, conditional on hiring managers, growth

[38]Note that (59) implies a linear probability model and its parameters can be estimated using OLS. We also include additional control variables in the regression. Details are given in the notes under Table 15.

incentives and hence firm size are increasing in delegation efficiency. Our theory implies that delegation efficiency α and the owner's time endowment T are substitutes, i.e., we should expect a tighter link between family size and firm size in low-trust regions. Columns 2 and 3 show that this is the case. First, similar to Bloom et al. (2013), we also find a tight relationship between firm size and family size. We interpret this correlation as family members substituting for the scarcity of available outside managers. Furthermore, the coefficient on the interaction term is negative, which means that the positive relationship between firm size and family size is weaker in regions where trust is higher and hence delegation is more efficient.[39] In column 3, we replicate these results with state-fixed effects to control for all time-invariant regional characteristics.

In columns 4 and 5, we redo the analysis of columns 2 and 3 for the whole sample of firms, i.e., we do not condition on delegation. Again we find a positive correlation between the size of the family and firm size. Note that the effect of trust for the entire sample of firms is much weaker. This is consistent with our theory, which implies that delegation efficiency only matters for the firms that actually delegate. For firms without outside managers (i.e., firms with $n < n^*$), growth incentives are only determined by the owner's time endowment T.

Finally, we replicated the entire analysis of Table 15, which controlled for 2-digit sector fixed effects, with 3-sector fixed effects. The results are contained in Table 16. It is seen that results are similar. The only exception are the results in columns 2 and 3, which are conditioned on managerial hiring and hence have a small sample size[40]. While all point estimates are of the same sign, they are not significantly different from zero.

Table 16: Managerial Hiring, Firms Size and Growth in India: Robustness

	Dependent Variable				
	Manager $>$ 0	Log empl (Manager $>$ 0)		Log empl	
Log Empl	0.040***				
	(0.003)				
Log HH Size	-0.004***	0.389	0.394*	0.207***	0.220***
	(0.001)	(0.248)	(0.231)	(0.030)	(0.030)
Trust	0.012*	0.570		-0.008	
	(0.006)	(1.300)		(0.160)	
Log HH Size* Trust		-0.443	-0.359	0.062	0.040
		(0.658)	(0.614)	(0.086)	(0.084)
State FE	N	N	Y	N	Y
N	178,999	2,350	2,350	178,999	178,999
R^2	0.05	0.58	0.63	0.28	0.30

Notes: Robust standard errors in parentheses. *** p<0.01, ** p<0.05, * p<0.10. All regressions include 3-digit fixed effects, the age of the establishment, and a dummy variable for the establishment to be in a rural area as control variables. For the regressions that do not include state level fixed effects, log GDP per capita at the state-level is included as a control variable. "Log Empl" denotes the (log of) total employment at the establishment. "Log HH size" denotes the (log of) the size of the household of the establishment's owner. This variable is only available for the NSS data. "Trust" is the measure of trust at the state level, which we calculate from the World Value Surveys. The dependent variables are: an indicator of managerial hiring (column 1), log employment conditional on managerial hiring (columns 2 - 3), log employment (columns 4-5).

[39] In a separate regression, not shown here, we also control for the assets of the firm as both family size and the level of regional trust could be correlated with the supply of capital to the firm. The results are very similar.

[40] Given the small sample size, finer controls for sector fixed effect leave less variation in the data for the relations we are interested in.

Online Appendix for "Lack of Selection and Limits to Delegation: Firm Dynamics in Developing Countries"

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OA-1 Online Appendix - Theory

OA-1.1 Static Equilibrium

Consider the equilibrium in the product market. At each point in time, each product line j is produced by a single firm with productivity q_{jt}. We normalize the price of aggregate output Y to one. As firms set a price equal to $p_{jf} = q_{jf}^{-1} w_t$ we get that

$$\ln(Y) = \int_0^1 \ln(y_j)dj = \int_0^1 \ln(p_j y_j)dj - \int_0^1 \ln(p_j)dj = \ln(Y) - \ln(w_P) + \int_0^1 \ln(q_j)dj$$

which implies $w_P = Q \equiv \exp\left[\int_0^1 \ln q_j dj\right]$. The production function [see equation (3)] also implies that

$$L^P = \int_0^1 l_j dj = \int_0^1 \frac{y_j p_j}{q_j p_j} dj = \frac{Y}{w} \int_0^1 \mu_j^{-1} dj, \quad \text{(OA-1)}$$

where L^P is the aggregate demand for production labor. Using that $\mu_j = \frac{1}{1-e(n_j)^{\sigma}}$, where n_j is the number of products the producer of product j has in its portfolio, (OA-1) implies that $L^P = \frac{1}{\mathcal{M}} \frac{1}{\omega_P}$, where $\omega_P = \frac{w_P}{Y}$ and $\mathcal{M}$ is given by

$$\mathcal{M} = \left[1 - \sum_{n=1}^{\infty} (e(n))^{\sigma} \times n \times \left(\nu_n^H F^H + \nu_n^L F^L\right)\right]^{-1},$$

where function $e(.)$ is defined in (7), ν_n^i and F^i are the size distribution and the measure of i-type firms, $i \in \{H, L\}$, respectively (see Proposition 1).

OA-1.2 A Simple Microfoundation for α

In this section, we provide a simple example of how α could depend on various institutional parameters in an economy. Please note that none of the analysis in the main text depends on this particular example. This example is provided to fix ideas.

Suppose that both managers and entrepreneurs each have one unit of time at their disposal. While the latter can provide T units of effort during that time interval, managers can provide 1 unit of effort. Suppose that the provision of managerial effort is subject to contractual frictions. For simplicity, assume that the manager can decide to either provide effort or shirk, in which case he adds no usable services to the firm. The firms can translate each unit of managerial effort into η units of managerial services.

While the manager's effort choice is not contractible, the entrepreneur can monitor the manager to prevent him from shirking. If the entrepreneur spends s units of her time monitoring the manager, she will catch a shirking manager with probability s. Whenever the manager shirks and gets caught, the entrepreneur can go to court and sue the manager for the managerial wage w. In particular, the court (rightly) decides in the entrepreneur's favor with probability κ. Hence one can think of κ as parameterizing the efficiency of the legal system. Finally, the demand for shirking arises because shirking carries a private benefit bw, where $b < 1$.[41]

It is straightforward to characterize the equilibrium of this simple game. If the entrepreneur spends s units of her time monitoring the manager, the manager does not shirk if and only if

$$w \geq bw + w(1 - \kappa s),$$

where $(1 - \kappa s)$ is the probability that the manager gets paid despite having shirked. Clearly the owner will never employ a manager without inducing effort. Hence, the owner will spend $s = b/\kappa$ units of time monitoring the manager. The overall amount of managerial services in product line j is therefore given by[42]

$$e_j = \frac{T}{n} - m_j s + \eta m_j \quad = \quad \frac{T}{n} + \left(\eta - \frac{b}{\kappa}\right) \times m_j = \frac{T}{n} + \alpha(\kappa, \eta, b) \times m_j. \tag{OA-2}$$

Hence, α measures precisely the net increase in managerial services through delegation. In particular, the delegation efficiency is increasing in the firm's efficiency to employ managers (η) and in the state of the contractual environment (κ), because monitoring and the strength of the legal system are substitutes. Note also that the whole purpose of delegation is to increase a firm's managerial resources, so that firms will never hire a manager if $\alpha(\kappa, \eta) \leq 0$. Hence, whenever managers are sufficiently unproductive or the quality of legal systems is sufficiently low, firms will never want to hire outside managers because owners need to spend more of their own time to prevent the opportunistic behavior of managers than they gain in return.

OA-1.3 Stationary Equilibrium of the Model

In this section, we describe the stationary equilibrium of the model in detail. To do so, we proceed in two steps.

Step 1 Fix $s \equiv (n^*, \omega_P)$ where n^* and ω_P are delegation cut-off and normalized wage rate for production workers, respectively. By using (37) and (38), we can write the rate of destruction for high types $\tau_H(s)$ as

$$\tau_H(s) = z(s) \times \left\{ \left[\delta \sum_{h=1}^{\infty} \prod_{j=1}^{h} \left(\frac{x_j(s)}{\tau_H(s)} \right) \right] + 1 - (1-\delta) \left(\frac{\beta - 1}{\beta} \right) \right\}, \tag{OA-3}$$

where $[x_j(s)]_{j=1}^{\infty}$ is the optimal innovation policy by high types implicitly defined in (13) and $z(s)$ is the optimal entry rate. We focus on a solution where $x_j < \tau_H$ for all τ_H. This is a sufficient condition for a stationary solution.[43] We will show below that such a solution exists for all s provided that θ_E is large enough.

Let $v_H(n)$ be normalized value function (normalized with Y_t) of a high-type firm depicted in

[41] The necessity for the private benefit being proportional to the wage arises in order to make the contract stationary.

[42] Note that we do not require that $s < T$, i.e., we do not require the owner to perform the monitoring himself. We rather think of managerial efficiency units to be perfect substitutes within the firm, i.e., an owner can hire a manager to monitor other managers.

[43] A necessary condition is that there exists $\hat{n}$ with $x_j < \tau_H$ for all $j > \hat{n}$.

(13).[44] At BGP where both C_t and Y_t grows at the same rate and $\dot{v}_{H,t} = 0$, it can be written as

$$\rho v_H(n) \quad = \quad \max_{x_n} \left\{ \tilde{\pi}(n; n^*) - \omega_p \theta^{-\frac{1}{\zeta}} n x_n^{\frac{1}{\zeta}} + x_n n \left[v_H(n+1) - v_H(n) \right] + \tau_H n \left[v_H(n-1) - v_H(n) \right] \right\}.$$

where we use the fact that $w_p = Q$ to substitute $\frac{Q}{Y}$ with ω_P and $r = \rho + g$ from household problem.[45] By rearranging terms and explicitly imposing the restriction $x_j < \tau_H$, we can write v_H as

$$v_H(n) \quad = \quad n \times \max_{x_n < \tau_H} \left\{ \frac{\frac{\tilde{\pi}(n;n^*)}{n} - \omega_p \theta^{-\frac{1}{\zeta}} x_n^{\frac{1}{\zeta}} + x_n v_H(n+1) + \tau_H v_H(n-1)}{\rho + (x_n + \tau_H) n} \right\}.$$

Now consider the function $b(n) \equiv \frac{v_H(n)}{n}$, which - by using the above equation - can be written as

$$b(n) = \max_{x_n < \tau_H} \left\{ h(n, x_n) + \frac{x_n(n+1)}{\rho + (x_n + \tau_H) n} b(n+1) + \frac{\tau_H(n-1)}{\rho + (x_n + \tau_H) n} b(n-1) \right\}, \tag{OA-4}$$

where $h(n, x_n) \equiv \frac{\frac{\tilde{\pi}(n;n^*)}{n} - \omega_p \theta^{-\frac{1}{\zeta}} x_n^{\frac{1}{\zeta}}}{\rho + (x_n + \tau_H) n}$.

We will show that the right-hand side of (OA-4) satisfies Blackwell's sufficient conditions for a contraction. To see this, define the operator T by

$$(Tf)(n) \equiv \max_{x_n < \tau_H} \left\{ h(n, x_n) + \frac{x_n(n+1)}{\rho + (x_n + \tau_H) n} f(n+1) + \frac{\tau_H(n-1)}{\rho + (x_n + \tau_H) n} f(n-1) \right\}. \tag{OA-5}$$

Hence, b can be defined as a fixed point of T, i.e., a function such that $(Tb)(n) = b(n)$. First, note that $h(n, x_n)$ is bounded [see (11)] so that T maps the space of continuous bounded functions into itself (Berge's Maximum Theorem). Moreover, for any continuous bounded functions f, g with $f(n) \leq g(n)$ for all $n \in Z^{++}$, we have

$$\begin{aligned} (Tf)(n) \quad &= \quad \max_{x_n < \tau_H} \left\{ h(n, x_n) + \frac{x_n(n+1)}{\rho + (x_n + \tau_H) n} f(n+1) + \frac{\tau_H(n-1)}{\rho + (x_n + \tau_H) n} f(n-1) \right\} \\ &\leq \quad \max_{x_n < \tau_H} \left\{ h(n, x_n) + \frac{x_n(n+1)}{\rho + (x_n + \tau_H) n} g(n+1) + \frac{\tau_H(n-1)}{\rho + (x_n + \tau_H) n} g(n-1) \right\} \\ &= \quad (Tg)(n), \end{aligned}$$

so that the monotonicity condition is satisfied. Lastly, for any continuous bounded function f and $a \geq 0$,

$$\begin{aligned} (T[f+a])(n) \quad &= \quad \max_{x_n < \tau_H} \left\{ h(n, x_n) + \frac{x_n(n+1)}{\rho + (x_n + \tau_H) n} \left[f(n+1) + a \right] + \frac{\tau_H(n-1)}{\rho + (x_n + \tau_H) n} \left[f(n-1) + a \right] \right\} \\ &\leq \quad \max_{x_n < \tau_H} \left\{ h(n, x_n) + \frac{x_n(n+1)}{\rho + (x_n + \tau_H) n} f(n+1) + \frac{\tau_H(n-1)}{\rho + (x_n + \tau_H) n} f(n-1) \right\} + \Omega a \\ &= \quad (TF)(n) + \Omega a \end{aligned}$$

where

$$\Omega \equiv \max_{x_n < \tau_H} \left\{ \frac{(x_n + \tau_H) n}{\rho + (x_n + \tau_H) n} + \frac{x_n - \tau_H}{\rho + (x_n + \tau_H) n} \right\} < 1.$$

[44] We drop the dependence of the value function on s for notational clarity.

[45] See Section OA-1.1 for details.

Hence, the operator T satisfies the discounting condition, so that T is a contraction mapping and therefore possesses a unique fixed point [Stokey et al. (1989)], which is continuous in s and τ_H. Moreover, the expression inside the max operator in (OA-5) is continuous in x_n and strictly concave so that Berge's Maximum Theorem implies that the set of maximizers x_n^* is a continuous function of s and τ_H. The equilibrium entry rate z is fully determined from v_H and v_L [see (17)] and hence also a continuous function of s and τ_H.[46]

Hence, equation (OA-3) is continuous in τ_H. To see that there exists a fixed point for τ_H, note that the RHS is bounded away from zero because $z(s) > 0$ and that it is bounded from above. To see that, note that $\sum_{h=1}^{\infty}\prod_{j=1}^{h}\left(\frac{x_j(s)}{\tau_H(s)}\right)$ is bounded in a stationary equilibrium and that z is bounded [see (17)]. Hence, there exits a fixed point for τ_H. Moreover, because z is increasing in θ_E for a given s and τ_H, (OA-3) implies that for each s there is θ_E large enough such that this fixed point satisfies $\tau_H > x_n$.

Step 2 We can now represent the whole model in terms of labor market clearing conditions. The Cobb-Douglas final good production function together with the market structure described in Section 2.1 implies that the total number of production workers hired for variety j by a producer, who is active in n markets, is given by[47]

$$l_j = [\omega_P \mu(e)]^{-1} = \omega_P^{-1} \times (1 - e(n)^{\sigma}).$$

Using firms' optimal delegation policy and aggregating over the firm size distribution yields the aggregate demand for production workers is given by

$$H^P = \left[1 - \sum_{n=1}^{\infty}\left(\max\left\{\frac{T}{n}, \frac{T}{n^*}\right\}\right)^{\sigma} \times n \times \varphi_n\right] \times \omega_P^{-1} \tag{OA-6}$$

Similarly, firms' managerial demand function implies that the aggregate demand for managers is given by

$$H^M = \sum_{n\geq n^*}^{\infty} n \times m(n) \times \varphi_n = \left(\frac{\sigma}{\omega_M}\right)^{\frac{1}{1-\sigma}} \alpha^{\frac{\sigma}{1-\sigma}} \sum_{n\geq n^*}^{\infty} n\varphi_n - \frac{T}{\alpha}\sum_{n\geq n^*}^{\infty}\varphi_n. \tag{OA-7}$$

Given Step 1, we can calculate the firm size distribution $\varphi_n(s) = \nu_n^H(s)F^H(s) + \nu_n^L(s)F^L(s)$ from Proposition 1. From (24), (OA-6), and (OA-7), the labor market clearing conditions for managers and production workers can then be written by

$$0 = \left(\frac{\vartheta - 1}{\vartheta}\mu_M\right)^{\vartheta}\left(\frac{(n^*)^{1-\sigma}\sigma\alpha}{T^{1-\sigma}\omega_P}\right)^{\vartheta-1}\frac{\vartheta}{\vartheta-1} - \frac{T}{\alpha}\sum_{n>n*}\left(\frac{1}{n^*} - \frac{1}{n}\right) n\varphi_n(s) \tag{OA-8}$$

$$0 = 1 - \left(\frac{\vartheta - 1}{\vartheta}\mu_M\right)^{\vartheta}\left(\frac{(n^*)^{1-\sigma}\sigma\alpha}{T^{1-\sigma}\omega_P}\right)^{\vartheta} - \frac{1}{\omega_P}\left[1 - \sum_{n=1}^{\infty}\left(\max\left\{\frac{T}{n}, \frac{T}{n^*}\right\}\right)^{\sigma} n\varphi_n(s)\right] \tag{OA-9}$$

where two equations depend only on $s \equiv (n^*, \omega_P)$. Note that $\varphi_n(s)$ is continuous in z, τ_H and x_n. Therefore, from Step 1, left-hand-side of both equations are continuous in (n^*, ω_P). Solution to the system of equation given by (OA-8) and (OA-9) constitutes an equilibrium for our economy.

[46] Recall that $v_L(1) = \frac{\pi(1)}{\rho+\tau_L}$, where $\tau_L = \beta \times \tau_H$.

[47] To see this, note that $Y = p_j y_j = \frac{w_P}{q_j} q_j \mu(e_j) l_j$ and $\omega_P = w_P/Y$.

OA-2 Online Appendix - Empirical Analysis

OA-2.1 Firms vs. Establishments in the U.S. Manufacturing Sector

In this section we compare the process of firm-dynamics across U.S. manufacturing firms and establishments. Table OA-1 provides some summary statistics about the size-distribution of firms and establishments in the U.S. The average manufacturing firm in the U.S. has 51 employees, while the average establishment only 43. It is also the case that large firms have multiple establishments (firms with more than 1000 employees have on average 13) so that large firms account for half of total employment. There is a lower concentration at the establishment level in that establishments with more than 1000 employees account for less than one-fifth of aggregate employment in manufacturing in the U.S.

Table OA-1: Descriptive Statistics: U.S. Micro Data

Size	Firms No.	Firms Avg. Employment	Firms Agg. Share	Firms No. of Establishments	Firms Exit rate	Establishments No.	Establishments Avg. Employment	Establishments Agg. Share	Establishments Exit rate
1-4	86936	2.30	1.65	1.00	13.22	93038	2.31	1.78	16.50
5-9	48178	6.68	2.66	1.00	3.46	54281	6.73	3.02	4.20
10-19	37942	13.80	4.33	1.01	2.66	45803	14.01	5.30	3.10
20-49	32555	30.92	8.31	1.05	2.27	44085	31.90	11.62	2.40
50-99	13516	67.94	7.58	1.21	2.03	21582	71.54	12.75	1.90
100-249	8914	139.90	10.30	1.61	1.59	16476	155.76	21.20	1.00
250-499	3167	280.96	7.35	2.47	0.92	5444	348.72	15.68	0.50
500-999	1720	503.49	7.15	3.94	0.29	2120	677.19	11.86	0.30
1000+	2423	2531.92	50.67	12.68	0.25	984	2068.2	16.81	0.30
Aggregate	235351	51.44	100		6.53	283813	42.66	100	7.3

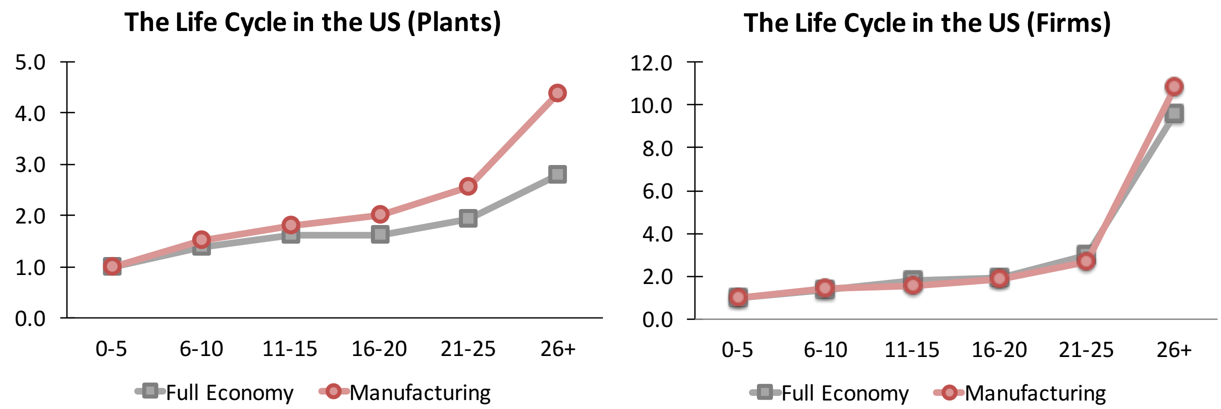
Notes: This table contains summary statistics for U.S. manufacturing firms and establishments in 2012. The data are taken from the BDS.

We now turn to the implied dynamics. Because we focus on cross-sectional data, the information on firm (establishment) age is crucial for us. For establishments, the definition of age is straightforward. Birth year is defined as the year a establishment first reports positive employment in the LBD. Establishment age is computed by taking the difference between the current year of operation and the birth year. Given that the LBD series starts in 1976, the observed age is by construction left censored at 1975. In contrast, firm age is computed from the age of the establishments belonging to that particular firm. A firm is assigned an initial age by determining the age of the oldest establishment that belongs to the firm at the time of birth. Firm age accumulates with every additional year after that. In Figure OA-1 we show the cross-sectional age-size relationship for establishments (left panel) and firms (right panel) in the U.S.

Not surprisingly, the life-cycle is much steeper for firms, especially for +26-year-old firms, as firms grow both on the intensive margin at the establishment level and the extensive margin of adding establishments to their operation.

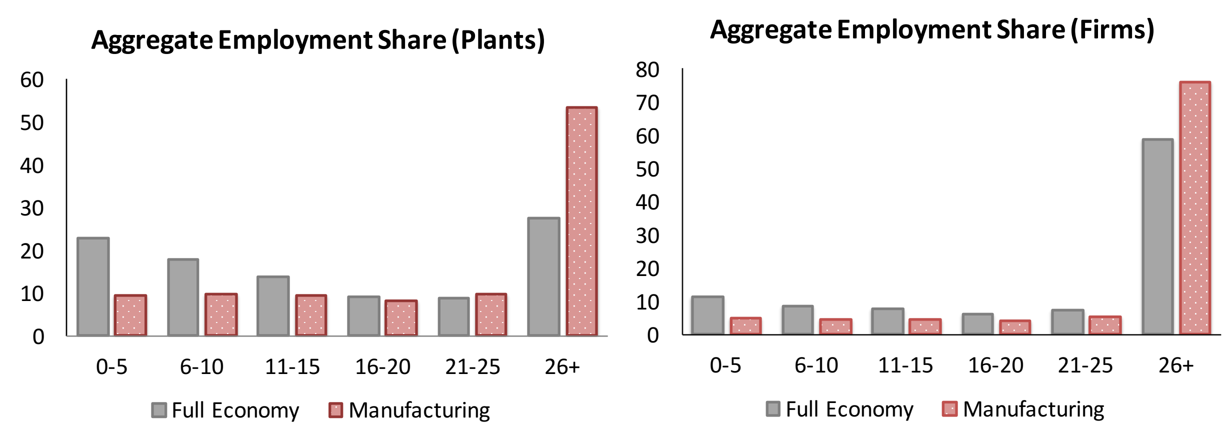
In Figure OA-2 we show the aggregate employment share of establishments and firms of different ages. As suggested by the life-cycle patterns in Figure OA-1, old firms account for the bulk of employment in the U.S. However, the relative importance of old establishments/firms is somewhat less pronounced because of exit, i.e., while the average firm/establishment grows substantially by age conditional on survival, many firms/establishments have already exited by the time they would have been 20 years old. Nevertheless, firms (establishments) older than 25 years account for 76% (53%) of employment in the manufacturing sector.

Figure OA-1: Life Cycle of Establishments and firms in the U.S.



Notes: The figure contains the cross-sectional age-size relationship for establishments (left panel) and firms (right panel) in the U.S. The data are taken from the BDS and we focus on the data for 2012. We depict the results for both the manufacturing sector and the entire economy.

Figure OA-2: The employment share by age of establishments and firms in the U.S.

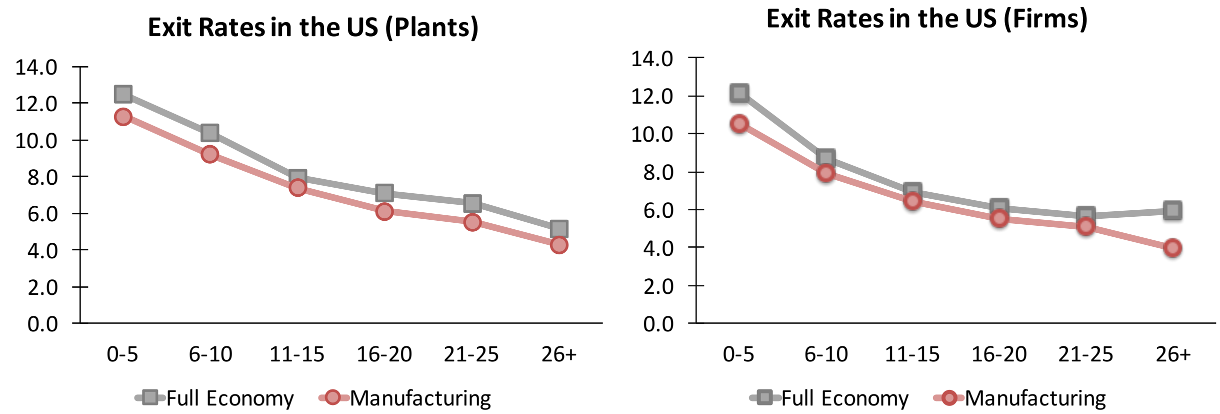


Notes: The figure contains the aggregate employment share of establishments (left panel) and firms (right panel) in the U.S. as a function of age. The data are taken from the BDS and we focus on the data for 2012. We depict the results for both the manufacturing sector and the entire economy.

This pattern of exit is depicted in Figure OA-3. There we show annual exit rates for firms and establishments as a function of age. The declining exit hazard is very much suggestive of a model of creative destruction, whereby firms and establishments grow as they age (conditional on survival) and exit rates are lower for bigger firms/establishments.

An important moment for us is the age-specific exit rate conditional on size. It is this moment that will identify the importance of selection. In a model without heterogeneity, size will be a sufficient statistic for future performance, so that age should not predict exit conditional on size. However, if the economy consists of high- and low-type entrepreneurs, old firms are more likely to be composed of high types conditional on size. Hence, the size-specific exit rate by age is monotone in the share of high types by age. In Figure OA-4 we report this schedule for both establishments

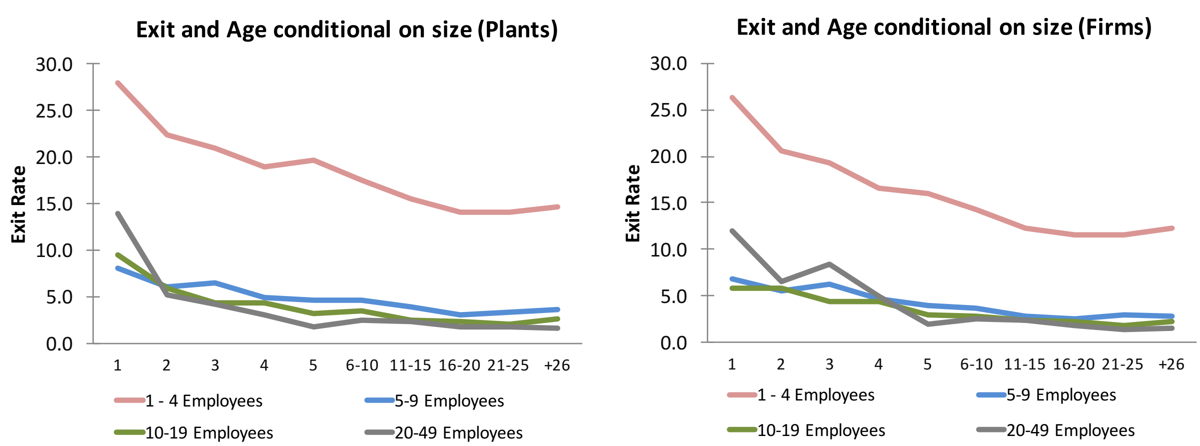
Figure OA-3: The Exit Rates of Establishments and Firms in the U.S. by Age



Notes: The figure contains the exit rates of establishments (left panel) and firms (right panel) in the U.S. as a function of age. The data are taken from the BDS and we focus on the data for 2012. We depict the results for both the manufacturing sector and the entire economy.

and firms. The data show a large degree of age-dependence (conditional on size). The schedules for small firms and establishments look almost identical. This is reassuring because small firms are almost surely single-establishment firms, so that a firm-exit will also be a establishment-exit and vice versa.

Figure OA-4: Size-dependent exit rates of establishments and firms in the U.S. by age



Notes: The figure contains the conditional exit rates by size of establishments (left panel) and firms (right panel) in the U.S. as a function of age. The data are taken from the BDS and we focus on the data for 2012. We depict the results for the manufacturing sector.

OA-2.2 Establishments in the Indian Manufacturing Sector

In this section we provide more descriptive evidence about the underlying process of firm dynamics in the manufacturing sector in India. Table OA-2 contains descriptive statistics for our sample of Indian manufacturing establishments. For comparison, we organize the data in the same way as in the left panel of Table OA-1, which contains the results for manufacturing establishments in the

U.S. It is clearly seen that the establishment-size distribution in India is concentrated on very small firms. The average establishment has fewer than 3 employees and more than 50% of aggregate employment is concentrated in establishments with at most 4 employees. Such establishments account for 93% of all establishments in the Indian manufacturing sector. A comparison of establishment size distribution for the years 1995 and 2010 in Table OA-3 suggests that these patterns are stable over time.

Table OA-2: Descriptive Statistics: Indian Micro Data

Size	No.	Avg. Employment	Aggregate Employment Share
1-4	15957296	1.56	54.76
5-9	843091	6.26	11.61
10-19	243868	12.98	6.96
20-49	70834	29.22	4.55
50-99	23242	69.89	3.57
100-249	14898	149.31	4.89
250-499	4701	346.69	3.58
500-999	2283	683.86	3.43
1000+	1232	2452.65	6.65
Aggregate	17161445	2.65	100.00

Notes: This table contains summary statistics for establishments in the Indian manufacturing sector in 2010. The data are taken from the ASI and the NSS. To calculate the number of firms, we use the sampling weights provided in the data.

Table OA-3: Establishment Size Distribution in India

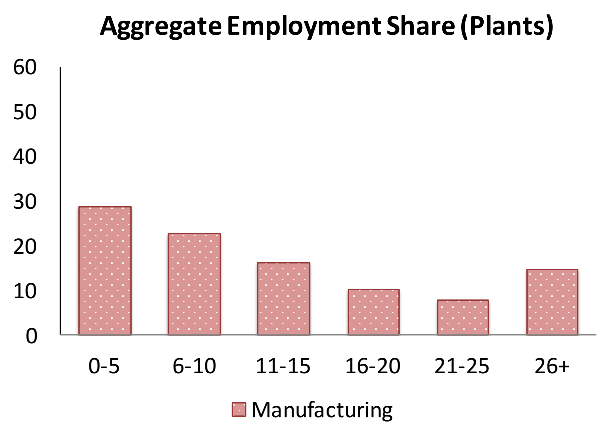
	Plant Size				
	1-4	**5-9**	**10-19**	**20-49**	**50+**
1995	0.9171	0.0631	0.0143	0.0035	0.0020
2010	0.9297	0.0491	0.0143	0.0042	0.0027

Notes: This table presents the share of establishments for different size bins in India, for the years 1995 and 2010. Size bins are constructed based on number of employees.

Figure OA-5 reports the aggregate employment share by age for Indian manufacturing establishments and is hence comparable to Figure OA-2 for the U.S.

It is clearly seen that the aggregate importance of old firms is very small in India. While firms that are older than 25 years account for 55% of employment in the U.S., the corresponding number is less than 20% in India. This is a reflection of the shallow life-cycle in India and not of there being fewer old firms in the Indian economy.

Figure OA-5: The employment share by age of establishments in India



Notes: The figure contains the aggregate employment share of manufacturing establishments in India as a function of age. The data are taken from the ASI and the NSS and we focus on the data for 2010. We combine the two data sets using the sampling weights provided in the micro data.