Financing the Future: Firm Selection and Productivity Dynamics Under Financial Frictions

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Abstract

We build a growth model with heterogeneous firms that endogenizes productivity growth, firm selection and borrowing under financial frictions. In particular, facing an incomplete financial market with endogenous (firm-specific) interest rate spreads and borrowing constraints, firms choose productivity enhancing investments and debt with an option to default. We calibrate our model with micro data on US firms, finding that financial frictions can be very pervasive. Frictions strongly select against high-growth firms. A key trade-off emerges as relaxing frictions promotes innovation by financially healthy firms while also potentially producing financially distressed firms. We conduct experiments by analyzing several degrees of financial frictions (e.g., no debt market, no unsecured debt, no tax shield), finding a welfare swing of 5.13% between the least to the most frictional economy. Strikingly, without firm heterogeneity, this welfare gap would be more than halved to 2.45%. That is, firm heterogeneity and selection strongly increase the economy's sensitivity to frictions. We also study optimal subsidy policies across the counterfactual economies, finding that frictions increase the effectiveness of subsidies. Under optimal subsidies, the welfare gap between counterfactuals is reduced to 2.85%.

JEL Codes: G3, O4

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1 Introduction

In market economies, in addition to using internal funds, firms grow by borrowing from others and investing in risky projects to enhance their productivity. Critically, firms are heterogeneous not only in terms of their age, size, and ability to grow, but also in terms of borrowing. These observations raise an important policy question: How do industrial policies affect aggregate productivity when firms with heterogeneous growth potential compete among themselves and invest in innovation while facing financial frictions and risk of default? Answering this question requires a general equilibrium framework with the relevant firm heterogeneity, financial frictions, and endogenous innovation decisions. This paper is a major step in this direction.

In this paper, we build a firm dynamics model characterized by endogenous growth and financial frictions. Specifically, we construct a model that captures important endogenous growth and financial mechanisms including endogenous defaults, firm-specific interest rate spreads, borrowing limits, and market competition. Using this model, we study the interaction of innovation and frictions as well as the potential role of innovation subsidies in counteracting financial distortions.

The model is based on Klette and Kortum (2004) (KK) and incorporates important financial channels. In this setup, heterogeneous firms compete by improving their production technology. The novel part of our model is that firms invest in innovation amid financial frictions. In particular, firms face an incomplete financial market and have imperfect access to a defaultable short-term debt instrument. The banking sector provides this debt instrument by setting an interest rate menu based on firms' default risk. This menu is characterized by heterogeneous interest rate spreads and debt limits.

The model allows us to study the role of financial frictions in an economy populated by firms that are heterogeneous in terms of size, innovation capacity, debt, and borrowing costs. A key model trade-off is that relaxing financial frictions promotes innovation by financially healthy firms while potentially increasing the quantity of financially distressed firms. The latter phenomenon occurs as firms that would have voluntarily (firms' optimal debt leverage choice) or involuntarily (banks' debt limits) maintained less debt will benefit from laxness of the debt market and more readily increase their debt. However, a countervailing force is that the endogenous bankruptcy of less innovative firms could help in reallocating resources to more innovative firms.

We calibrate our model to the U.S. economy by targeting important moments on financials and market dynamics including small/large firm interest rate spreads and debt leverage as well as small/large young/old incumbent growth rates which we calculate using Business Dynamics Statistics (BDS) and micro-data from Orbis and Compustat. The model does a very good job matching financial moments and a decent job at recreating market dynamics moments.

We study firm dynamics under 9 versions of the US economy that vary in *degree* of financial frictions: baseline, no debt market, no recoveries in bankruptcies, no interest rate tax shield, no unsecured debt, no banking costs, no haircut in bankruptcies, perfect access to the debt market, and a bolstered access state. We find that financial frictions depress innovation and social welfare: there is a 5.13% welfare gap between the least and most frictional economies. More strikingly, the consequences of financial frictions are accentuated by the heterogeneity of firms' innovation capacities. That is, the economy would be less sensitive to financial frictions if firms were homogeneous in innovation capacity. The homogeneous economy exhibits only a 2.45% welfare gap among different frictions. The sensitivity of the heterogeneous economy is partly based on how frictions can change the economy's composition through bankruptcies.

We see that, short of completely shutting off the debt market, the friction that hurts welfare the most is the no recovery economy: when firms are liquidated without any recoveries during bankruptcies. The resulting increase in borrowing costs reduces welfare by more than 1%. On the other extreme, short of the bolstered debt economy which weakens multiple frictions, we see that welfare improves the most when all firms immediately gain access to debt. Eliminating this extensive friction increases welfare by nearly 2%.

We also study optimal subsidy policies across our 9 economies and find that subsidies are more effective when frictions are stronger: welfare gap drops to 2.85% with optimal subsidies (compared to 5.13% without policy). Specifically, subsidies diminish the bite of frictions for more innovative firms and leading to significant endogenous defaults by less innovative firms. For example, in the baseline economy, "low type" firms' endogenous default rate increases from 2.17% to 7.17% after optimal subsidies. In this process, endogenous defaults boost policy makers' capacity to promote positive selection in the economy. For example, in the baseline economy, the share of "high type" firms increases from 25.52% to 42.2% after optimal policies.

Our article contributes to the literatures on firm dynamics, financial frictions, and endogenous growth. The effects of finances on firm dynamics and capital investments have been well studied. Jermann and Quadrini (2012) study the role of financial shocks on capital-investing firms by building a model of equity rigidity, always-binding exogenous debt constraints, and no endogenous defaults in equilibrium. Cooley and Quadrini (2001) build a structural model to replicate size and age dependence of leverage, investments, and growth observed in data. Miao (2005) studies differing debt choices of firms with differing productivities and shows the importance of defaults in creating positive selection. Li et al. (2016) construct a structural model to assess the relative importance of collateral and tax benefits on firms' debt leverage choice. We contribute to this literature by introducing endogenous productivity growth and market competition.

Studies on finances and firm dynamics of endogenous growth are relatively more nascent. Malamud and Zucchi (2019) study the role of corporate savings in a quality-ladder endogenous growth model. They find that financial frictions have important compositional effects on the contribution of incumbents and entrants to growth. However, they do not endogenize the cost of raising capital and do not model corporate debt.

Laeven et al. (2015) present an endogenous growth model in which firms innovate to compete among themselves and banks innovate to improve their credit screening technologies. In our model, we also seriously model the financial sector but mainly focus on the repercussions of financial frictions on firm dynamics.

Chatterjee and Eyigungor (2020) build an endogenous growth model with a focus on studying the effects of low interest rate regimes on firm dynamics. Instead, we focus on how financial frictions distort firm dynamics and how subsidies can counteract these distortions. On the modeling side, they model firm innovation as an exogenous probability independent of firm behavior while we endogenize this probability by allowing firms to invest in innovation.

Geelen et al. (2020) build financial frictions into the KK model and conclude that relaxing financial frictions leads to less innovation by each incumbent firm. We also build our model off KK; however, we find that weakening financial frictions does not necessarily lead to depressed innovation by incumbents. Moreover, we study how frictions interact with firm types and default. Additionally, we also study how frictions affect the optimal investment subsidy.

In the firm dynamics literature without the financial margins, Acemoglu et al. (2018) share our focus on innovation types and selection as well as optimal innovation policies. There is a rich literature that studies subsidies for productivity enhancing investments (see Bloom et al. (2019) for a survey). We add to this literature by explicitly modeling financial frictions and analyzing the effectiveness of subsidies under varying degrees of frictions.

The paper is structured as follows. In section 2, we present our model. In section 3, we discuss our calibration strategy and calibrate our model to US data. In section 4, we discuss important model dynamics. In section 5, we conduct counterfactual analyses on financial frictions and subsidies. Finally, in section 6, we conclude our article.

2 Model

2.1 Household and Final Good Technology

We consider a continuous time economy, where a representative household has CRRA preferences over consumption:

$$U_0 = \int_0^\infty e^{-\rho t} \frac{C_t^{1-\vartheta} - 1}{1-\vartheta} dt, \tag{1}$$

The household maximizes its utility subject to the flow budget constraint

$$\dot{A}(t) + C(t) \leq r_f(t) A(t) + W(t) L^S, \tag{2}$$

where $A(t) = \int_{i \in \mathcal{N}(t)} V(t; i)$, $\mathcal{N}(t)$ is the set of active firms, $V(t; i)$ is the ith firm's value, $r_f(t)$ is the risk-free interest rate, L^S is the household's measure of fixed labor, $W(t)$ is the wage rate, and the usual no-Ponzi condition $\int_0^\infty e^{-r_f(t)t} A(t) dt \geq 0$. Household's maximization implies the standard Euler equation, determining the risk-free interest rate, r_f,

$$\frac{\dot{C}}{C} = \frac{r_f - \rho}{\vartheta}. \tag{3}$$

The final good, Y, is produced by aggregating intermediate good varieties, y_i, according to the following technology:

$$\ln Y = \int_{i \in \Phi} \ln y_i di, \tag{4}$$

where Φ is the set of active intermediate good product lines, the measure of which is normalized to one. The final good has two uses: (1) it is consumed by the household and (2) expended by intermediate good producers to investor toward productivity enhancement.

The final good sectors's demand schedule for each intermediate good variety is

$$\underset{\{y_i\}_{i \in \Phi}}{\arg\max}\, P \exp\left(\int_{i \in \Phi} \ln y_i di \right) - \int_{i \in \Phi} p_i y_i, \tag{5}$$

where P is the price of the final good and p_i is the price of variety i. The above optimization results in unit-elastic demand for each variety:

$$y_i = \frac{Y}{p_i}, \tag{6}$$

where we normalize the price of the final good to one.

2.2 Intermediate Good Production

Intermediate good (product) i is produced by the monopolist who has the best (leading-edge) technology in that product line, though a single monopolist can own multiple product lines and can produce multiple intermediate goods simultaneously. Each intermediate good is produced with a linear technology based on the intermediate good producer's *variety-specific* productivity, $q_i > 0$, and labor, $l_i \geq 0$,

$$y_i = q_i l_i. \tag{7}$$

For each variety, firms compete à la Bertrand to undercut their competitors' prices. By limit pricing, only the most productive producer gets to produce the variety. The most productive firm is $\alpha > 1$ times as productive as its competitors, where α is the innovation step size. Variety i's equilibrium price is

$$p_i = \alpha \frac{W}{q_i}, \tag{8}$$

where W is the wage rate. The equilibrium production is

$$y_i = \frac{q_i Y}{\alpha W}, \tag{9}$$

and the intermediate good producer's profit from producing variety i is

$$\begin{aligned} \pi &= \frac{q_i Y}{\alpha W} \alpha \frac{W}{q_{ij}} - \frac{q_i Y}{\alpha W} \frac{1}{q_i} W, \\ &= \frac{\alpha - 1}{\alpha} Y. \end{aligned} \tag{10}$$

Lastly, note that given a fixed labor supply, L^S, the labor market clears with the wage rate

$$W = \frac{||\Phi||}{L^S \alpha} Y. \tag{11}$$

2.3 Firm Heterogeneity and Dynamics

Incumbent intermediate good producers are heterogeneous in terms of the number of intermediate goods they produce ($n \in \mathbb{N}$), access to the debt market ($a \in \{A, N\}$), debt stock ($D \in \mathbb{R}$), and innovativeness type ($k \in \{H, L\}$). A firm's type represents its innovative productivity, and high-type firms ($k = H$) are more productive at innovating than low-type firms ($k = L$). The low-type state is absorbing, and high-types exogenously transition into it with some arrival rate, $\phi_{H \to L} > 0$.

Innovation is modeled as an improvement over a product line's pre-existing productivity by the innovation step size $\alpha > 1$. Innovation is undirected meaning that the firm does not target specific intermediate varieties but instead achieves an improvement over a random variety each time.

When a firm achieves a successful innovation at a variety, it captures that product line by undercutting the prices of the previous producer. This innovation process can be described as "creative destruction" ($\tau \geq 0$) as the turnover of producers is the driving force of endogenous growth.

An incumbent firm achieves a successful innovation at the Poisson rate, $X \geq 0$, which is a function of type-specific innovation productivity $\theta_k > 0$, size n, and investment $c \geq 0$,

$$X = \left(\theta_k \frac{c}{Y}\right)^{\frac{1}{\eta}} n^{1-\frac{1}{\eta}}, \tag{12}$$

where $\eta > 1$ is the inverse innovation elasticity, $k \in \{H, L\}$, and investments are made by expending final goods. High-type firms face an innovation productivity scale $\theta_H > 0$ while low types face the scale $\theta_L > 0$, where $\theta_H > \theta_L$. Rewriting the innovation flow expression, we get that the firm's investment cost function to achieve an innovation *intensity* of $x \geq 0$ (where $x = \frac{X}{n}$) is

$$c(x) = \frac{1}{\theta_k} x^\eta n Y. \tag{13}$$

In addition to expending funds to innovate, incumbents also choose payouts. The present value of all future payouts determines the firm's value to its shareholder, the representative household which holds an aggregate basket of all firms' shares. Firms are subject to equity rigidities, modeled as quadratic equity adjustment costs similar to Jermann and Quadrini (2012). This cost structure creates a wedge between the value of the payout to the investor and the cost to the firm.

Let $p \in \mathbb{R}$ represent a firm's payout intensity (i.e., total payout over firm size). To issue pnY final goods in payouts, the firm has to expend

$$\varphi(p) = pnY + n\kappa p^2 Y, \tag{14}$$

where $\kappa > 0$ is the penalty scale. Note that the firm can choose a negative payout (i.e., an equity injection). However, the firm will have to tolerate potentially significant costs to raise the funds through negative payouts. We do not treat these costs as resource drains on the economy, and we return the expended equity adjustments costs to the household as a lump-sum payment.

Lastly, firms face a size invariant fixed cost, $\chi Y \geq 0$. Then, the firm's cash flows from production, fixed cost, innovation investments, and payout is

$$\pi n - \chi Y - c(x) - \varphi(p). \tag{15}$$

2.4 Firm's Debt Management

Firms are born without access to the debt market ($a = N$); however, they gain access at the size-specific rate $\lambda(n) \geq 0$ where $\lambda(n) = n\Lambda$ with $\Lambda > 0$. The access state ($a = A$) is absorbing, meaning once a firm gains access, it does not lose it (lest it exits the intermediate good market altogether).

In the access state, the firm has access to a defaultable short-term debt instrument, structured by the banking sector around each firm's heterogeneity and demand for debt. Namely, the firm is charged a firm-specific endogenous interest rate, $r(D, n, k) > 0$, and faces a firm-specific endogenous debt limit, $\bar{D}(n, k) \geq 0$, above which the firm cannot receive a loan.

When the firm is a going concern, its debt evolves according to the following law of motion:

$$\dot{D}(x, p; D, n, k) = -\underbrace{[\pi n - \chi Y - c(x) - r(D, n, k)D]}_{\text{Before-Tax Net Income}}(1 - \tau_{tax}) + \varphi(p). \tag{16}$$

where $r(D, n, k) = r_f + 1\{D > 0\}\delta(D, n, k)$ and $\delta(D, n, k) \geq 0$ is the interest rate spread coming from the banking problem. Note that corporate tax confers tax shield status on interest expenses. By choosing its payout and investment, the firm also chooses its debt drift above. This drift is unconstrained when the firm is not at its debt limit.

At any moment, the firm can exercise its outside option by exiting the economy. Hence, the firm's value cannot drop below zero:

$$V(D, n, k) = \max\{0, V_{\text{cont.}}(D, n, k)\}, \tag{17}$$

where $V_{\text{cont.}}(D, n, k) \in \mathbb{R}$ is the continuation value. When the firm is encumbered with debt, it can exit through a bankruptcy. By limited liability, seeking bankruptcy protection allows the firm to avoid paying its financial obligations. Given firm characteristics (D, n, k), solving the exit decision, $B(D, n, k) \in \{0, 1\}$, boils down to solving for debt levels beyond which the firm's continuation value falls below 0.

A firm could exit the economy because of shocks in four ways. First, the firm that experiences a death shock (which occurs at the arrival rate $\varepsilon \geq 0$) is forced to shut down. Second, the firm that is a going concern as a high-type can choose to default when it experiences a type transition shock (which occurs at the arrival rate $\phi_{H \to L} > 0$) that morphs the firm into a low-type firm. That is to say, there can exist firm characteristics (D, n, H) such that $B(D, n, H) = 0$ though $B(D, n, L) = 1$. Third, the firm exits when it is completely creatively destructed from the market (i.e., loses all of its product lines, where each product line is lost at the arrival rate $\tau \geq 0$). Lastly, the firm that loses a product line can declare bankruptcy despite operating other lines if it prefers defaulting to trying to repay its obligations. Specifically, there can exist firm characteristics (D, n, k) such that $B(D, n, k) = 0$ while $B(D, n - 1, k) = 1$.

2.5 Banking

The banking sector is comprised of a representative bank that borrows funds from the household and extends loans to individual debtor firms. In this process, the bank observes debtor firms' heterogeneities and offers them a loan menu comprised of firm-specific interest rates, $r(D, n, k) \geq 0$ and debt limits, $\bar{D}(n, k) \geq 0$.[1]

In the credit market, the bank's primary instrument is setting interest rate spreads (i.e., the difference between the risky borrowing rate, $r(D, n, k)$, and the risk-free rate, r_f). The bank offers its loans at a premium to compensate itself for bankruptcy risks and miscellanies banking costs.

Note that, at certain debt levels, firms lack any and all commitment to repay their loans. For example, if a very small firm receives a very large loan, the firm could optimally choose to immediately default on its obligations. Therefore, the bank would lose funds by extending a loan amount that large to a firm that small at any interest rate. In such circumstances, the bank utilizes debt limits to deny loans. Hence, in the economy,

[1]The underlying modeling choice is that the bank sets interest rates for each debt level. At debt levels for which the firm cannot credibly repay its obligations, the bank charges infinite interest rate, thus avoiding extending the loan. That is, the bank effectively chooses debt limits above which it is unwilling to extend credit. This treatment is similar to Bornstein (2020).

interest rate spreads and borrowing constraints are endogenously instituted by the bank.

Consider the case of a k-type firm with size n applying for loan amount $D > 0$. If the firm does not suffer from a commitment problem, the bank extends the loan at a firm-specific interest rate spread, $\delta(D, n, k) \geq 0$.

The bank funds this loan by borrowing from the household at the risk-free rate, r_f. The bank is able to borrow at the risk-free rate because it extends loans to a continuum of firms, perfectly diversifying its credit risks. Hence, the bank's total gains and losses are known with certainty, meaning that it can structure its loans such that it always honors its obligations to the household.

The debtor firm defaults with some flow rate. When a bankruptcy occurs, the firm has some size-dependent liquidation value, potentially allowing the bank to recover some funds.

Specifically, the bank sells loans in default to the representative asset management company (AMC) at a discount. The AMC uses bankrupted firms' leading technologies to produce and collect profits. However, the AMC does not use its technology portfolio to invest in innovation, implying that each product line's value for the AMC is

$$\int_t^{\infty} \pi_h (1 - \tau_{tax}) e^{-h(r_f + \tau)} dh = \frac{\pi_t (1 - \tau_{tax})}{r_f - g + \tau}, \tag{18}$$

where the closed-form solution on the right holds at the balanced growth path where the economy grows at the rate, $g \geq 0$. The sale of firm technology between the bank and AMC is negotiated with a take-it-or-leave-it offer where the bank gets to make the offer with probability $1 - \xi \in [0, 1]$. The Nash Bargaining price for each transacted product line is

$$\mu_t = (1 - \xi) \frac{\pi_t (1 - \tau_{tax})}{r_f - g + \tau}. \tag{19}$$

Importantly, this market structure ensures that endogenous defaults do not produce inactive product lines as production still continues for bankrupted varieties. Alternatively, the bank can liquidate the firm exogenously and recover a size-invariant value $\mu_0 Y \geq 0$.

Let $E[\Phi_D(D, n, k)] \geq 0$ represent the expected ratio of unrecovered funds over the loan's principal. Capturing other sources of banking costs, some amount, ν, is expended by the bank for every unit of loan it extends. Then, when granting a loan, D, to the the firm characterized by (n, k), the bank's profit function is

$$\pi_B(D,n,k) = \underbrace{D(1+r_f+\delta(D,n,k))}_{\text{Debtor Firm's Obligations}} - \underbrace{D\mathbb{E}[\Phi_D(D,n,k)]}_{\substack{\text{Expected Losses}\\ \text{from Bankruptcies}}} - \underbrace{D(1+r_f)}_{\substack{\text{Bank's}\\ \text{Obligations}}} - \underbrace{D\nu}_{\substack{\text{Banking}\\ \text{Costs}}} . \tag{20}$$

The bank is risk-neutral and prices its loans competitively. Hence, the equilibrium spread is determined as the rate that sets the bank's profits to zero:

$$\delta^*(D,n,k) = \underbrace{\nu}_{\substack{\text{Banking}\\ \text{Costs}}} + \underbrace{\mathbb{E}\left[\Phi_D(D,n,k)\right]}_{\text{Expected Loss Ratio}}, \tag{21}$$

Observe that the equilibrium credit spread function, when charged over a continuum of firms, ensures that the bank's income from firms less its other costs exactly equals its obligations to the household.

The bank prices its loan conscious of the different liquidation values under different bankruptcy scenarios. Let $1_\phi = 1_{k=H}B(D,n,L)$ be the indicator variable representing whether firm characterized by (D,n,k) would declare bankruptcy when transitioned to a low-type. Let $1_\tau = B(D,n-1,k)$ be the indicator variable representing whether the firm would default if it shrinks by a size. Then, the bank faces an expected loss ratio equaling

$$\mathbb{E}[\Phi_D(D,n,k)] = \underbrace{(\varepsilon+\phi_{H\to L}1_\phi)}_{\substack{\text{Flow Rate}\\ \text{of Size } n\\ \text{Default}}} \underbrace{\left[\frac{D-\max\{\mu_0,\mu n\}}{D}\right]^+}_{\substack{\text{Ratio of}\\ \text{Unrecovered}\\ \text{Debt at Size } n}} + \underbrace{\tau n 1_\tau}_{\substack{\text{Flow Rate}\\ \text{of Size } n-1\\ \text{Default}}} \underbrace{\left[\frac{D-\max\{\mu_0,\mu(n-1)\}}{D}\right]^+}_{\substack{\text{Ratio of}\\ \text{Unrecovered}\\ \text{Debt at Size } n-1}}, \tag{22}$$

where $[]^+$ stands for the positive part of the expression in the brackets. That is, the bank does not recover more funds in the bankruptcy than it lended

In order to deny loans that would lead to the firm's defaulting with certainty, the bank sets debt limits as follows:

$$\bar{D}(n,k) = \max\{D : V_{Firm}(D,n,k) \geq 0\} \tag{23}$$

Importantly, default rates and recoveries are not functions of the firm's *current* decisions. Instead, they are a function of the culmination of *past* actions and shocks (i.e, the firm's current state). Therefore, the interest rate schedule is characterized completely by setting interest rates for possible firm heterogeneities (D,n,k) without needing to price the firm's current choices of x and p.

2.6 Entry and Exit

As we focus on incumbent financial frictions, we model the entrant's problem more parsimoniously. Potential entrants are homogeneous until entry and boast a mass of one. They invest in innovation to capture a product line and enter the market, at which time they enter without access to debt, without debt/savings (i.e., $D = 0$), hold one product line, and draw an innovativeness type. Each entrant has probability $m_{E,H} \in [0, 1]$ of becoming a high type and probability $1 - m_{E,H}$ of becoming a low type.

The entrant's innovation dynamics is similar to the incumbent's. The following is the entrant innovation cost function:

$$c_E(z) = \frac{1}{\theta_E} z^{\eta} Y, \tag{24}$$

where z is the arrival rate of successful innovations and $\theta_E > 0$ is the inverse innovation productivity scale.

By choosing only their innovation intensity as follows, entrants maximize their expected value:

$$z \left[m_{E,H} V(0, 1, H) + (1 - m_{E,H}) V(0, 1, L) \right] - c_E(z). \tag{25}$$

Potential entrant innovation presents the last source of incumbent exit in our model.

2.7 Firm Value

Recursively, firm value is

$$\begin{aligned} r_f V(D,n,k,a) - \partial_t[V(D,n,k,a)] \\ = \max\Bigg\{0, \max_{x,p}\Bigg\{ & pnY + \frac{\partial V}{\partial D}(D,n,k,a)\dot{D}(x,p;D,n,k,a) - \varepsilon V(D,n,k,a) \\ & + xn\left[V(D,n+1,k,a) - V(D,n,k,a)\right] + \tau n\left[V(D,n-1,k,a) - V(D,n,k,a)\right] \\ & + 1_{k=H}\phi_{H\to L}\left[V(D,n,L,a) - V(D,n,H,a)\right] + 1_{a=N}\lambda(n)\left[V(D,n,k,A) - V(D,n,k,N)\right]\Bigg\}\Bigg\} \end{aligned} \tag{26}$$

s.t.

$$0 \geq D \tag{27}$$

for $a = N$, and

$$\bar{D}(n,k) \geq D \tag{28}$$

for $a = A$, and $\dot{D}$ as in (16).

The outer maximization is the firm's exit decision. The inner maximization is the firm's innovation and payout decisions if it continues to operate. The first term accounts for the contribution of payouts to firm value. The second term accounts for how firm value changes as a result the change in debt. The third term accounts for the exogenous death shock's effect on firm value. The fourth term accounts for the value of innovation, and the fifth term captures the value lost due to creative destruction. The sixth line accounts for value destruction when high types transition into low types. The last term accounts for the value of gaining access to the debt market.

2.8 Aggregate Growth and Welfare

Proposition 1. The growth rate of the economy is equal to

$$g = \ln(\alpha)\tau \tag{29}$$

Definition 1. A stationary equilibrium of this economy is a tuple

$$\{y_i, p_i, W, V, B, x, p, z, \delta, \bar{D}, \mu, g, r_f\}$$

such that y_i and p_i maximize the ith product line's profits as in (9) and (8), W clears the labor market as in (11), V is the incumbent firm value function as in (26) subject to (28) and (27), respectively, as well as the debt drift as in (43), $B(D, n, k)$ underpinning voluntary market exits in accordance with (17), $x(D, n, k, a)$ and $p(D, n, k, a)$ optimizing incumbent firm value $V(D, n, k, a)$, z maximizing entrant value as in (25), $\delta(D, n, k)$ maximizing the bank's expected profits in (20) as in (21), $\bar{D}$ ensuring that a loan is extended only when $\delta(D, n, k)$ has a finite solution as in (23), μ captures the collateral value as in (19), g is given by (29), and r_f is determined in accordance with the Euler in (3).

3 Calibration

We calibrate our model to the United States by targeting important moments of the economy calculated conditional on size and age.

3.1 Data

We use the United States Census Bureau's Business Dynamics Statistics to compute moments on growth and exit (2009-2018). Moreover, we employ Bureau van Dijk's Orbis dataset to calculate moments on debt (2003-2019). Lastly, we use the Compustat dataset to calculate moments on shareholder payouts (2009-2018).

3.2 Identification

Financial Dynamics: $\boldsymbol{\kappa}$, $\boldsymbol{\nu}$, $\boldsymbol{\xi}$, $\boldsymbol{\mu_0}$, $\boldsymbol{\chi}$, and $\boldsymbol{\Lambda}$. The equity adjustment penalty, κ, determines firm policies on payouts. Hence, we identify the parameter by targeting the mean payout (conditional on posting a positive payout). Because we cannot observe the payout behavior of private firms, we calculate this moments with Compustat firms. However, as public firms are considerably larger than most firms in the economy, we calculate our model's corresponding moment by using only the largest firms.

Fixed cost, χ, diminishes firms' ability to service their debt and thus help determine their debt limits. This effect is especially strong for small firms. Therefore, we identify the parameter by targeting the average leverage of small firms as calculated from Orbis.

Liquidation haircut, ξ, determines bank's recoveries from each product line that the bankrupt debtor owns and thus influences debt limits. Thus, we identify the parameter by targeting large firms' average leverage.

Fixed recovery value, μ_0, strongly influences the interest rates and debt limits of the smallest firms. Therefore, we target small firms' average interest rate spread (in conjunction with their leverage).

Banking costs, ν, affects interest rate spreads irrespective of size. Hence, we identify the parameter by targeting large firms' average interest rate spread (in conjunction with small firms' spreads).

Lastly, debt access delay, Λ, is important in determining what portion of firms are debtors. We identify the parameter by targeting debtor fractions across sizes.

Market Dynamics: $\boldsymbol{\theta_H}$, $\boldsymbol{\theta_L}$, $\boldsymbol{\theta_E}$, $\boldsymbol{m_{E,H}}$, $\boldsymbol{\phi_{H\to L}}$, $\boldsymbol{\alpha}$, and $\boldsymbol{\varepsilon}$. We identify market dynamics parameters with moments on firm growth and turnover. Entrant innovation productivity, θ_E, helps determine entry. Thus, we identify it by targeting the 5-year (employment) share of entrants as calculated from BDS statistics.

The disparity between high, θ_H, and low, θ_L, type productivities lead to different growth rates for types. Additionally, entrant share of high types $m_{E,H}$ and the rate of their transition into low types, $\phi_{H\to L}$, lead to different type concentrations in different ages and sizes. Hence, we target growth rates of small-young, small-old, and large-old firms as calculated from BDS statistics. The aggregate implication of these four parameters is underpinning the creative destruction, which especially affects small firms' exit rate. Thus, we also target small firms' exit rate.

Death shock, ε, affects the exit rate of incumbents, uniformly. Hence, by targeting large firms' exit rate (in conjunction with small firms' exit), we identify the parameter.

Finally, step size, α, determines aggregate growth given some aggregate creative destruction. Having already underpinned creative destruction, we target aggregate growth to identify α.

Discount, Elasticities, and Tax: $\boldsymbol{\rho}$, $\boldsymbol{\vartheta}$, $\boldsymbol{\eta}$, and $\boldsymbol{\tau_{tax}}$. We calibrate $\rho = 0.02$, $\vartheta = 2$, and $\eta = 2$ as in Acemoglu et al. (2018). Moreover, we calibrate $\tau_{tax} = 0.15$ as in Geelen et al. (2020).

Table 1: External Calibration

#	Parameter	Description	Value
1	ρ	Discount Rate	0.02
2	ϑ	Inverse Inter. Elasticity of Subs.	2.00
3	η	Inverse Innovation Elasticity	2.00
4	τ_{tax}	Tax Rate	0.15

3.3 Method

We externally and internally calibrate the following parameters:

$$\{\underbrace{\rho, \vartheta, \eta, \tau_{tax}}_{\text{External}}, \underbrace{\kappa, \nu, \xi, \mu_0, \chi, \Lambda, \theta_H, \theta_L, \theta_E, \alpha, m_{E,H}, \phi_{H\to L}, \varepsilon}_{\text{Internal}}\} \tag{30}$$

We iterate over the 13 internally calibrated parameters to minimize the distance between the 14 model and data generated moments:

$$\sum_{i=1}^{14} \frac{w(i)|\,\text{model}(i) - \text{data}(i)|}{\frac{1}{2}|\,\text{model}(i)| + \frac{1}{2}|\,\text{data}(i)|}, \tag{31}$$

where model(i) is the ith moment as calculated from our model, data(i) is the ith moment as calculated from data, and $w(i)$ is the ith moment's weight.

The general strategy of selecting the characterization of internal parameters is (1) to solve hundreds of thousands of equilibria with combinations generated from Sobol sequences, (2) to perform annealing algorithms on the most promising combinations from Sobol, and (3) to perform local optimization on the most promising calibrations from annealing.

3.4 Computational Algorithm

Given any parametrization, we can solve for the steady state equilibrium with three nested loops. In the outer-most loop, we solve a fixed point algorithm on creative destruction, τ. In the middle loop, we solve for default decisions, $B(D, n, k)$, debt limits, $\bar{d}(n, k)$, and interest rate spreads, $\delta(D, n, k)$, that satisfy the firm's exit and the bank's problems. In the inner-most loop, we solve for firm value and optimal innovation and payout.

When the two inner-most loops converge, we get the solution to the firm value's Hamilton-Jacobi-Bellman equation conditional on our creative destruction guess in the outer-most loop. Then, deriving the Kolmogorov forward equation, we solve for the steady state firm distribution. Having solved optimal innovation policies and distributions, we calculate the implied creative destruction and update the outer-most loop as appropriate.

4 Results

4.1 Parameters

Table 2 lists our calibration results. We can summarize financial variables as follows. The payout penalty scale (κ) implies that, at any instant, if a firm allocated all of its profit from sales to payouts, 5.2% of its payout budget will have been lost to adjustment costs. The fixed cost (χ) equals 80% of a size one firm's profits from sales. Liquidation discount (ξ) implies that each product line's liquidation value is equal to 1.4 times the annual profit from sales from that line. Fixed recovery (μ_0) is significantly larger, comprising 4.4 times the annual profit from sales from holding one product line. The banking costs (ν) parameter implies that one third ($\frac{1}{3}$) of the average interest rate spread is attributable to the bank's miscellanies costs of originating and monitoring its loans. Debt access speed (Λ) implies that if a firm were to not grow after its entry, on average, it would take 20 years to gain access to the debt market.

Market parameters can be summarized as follows. Hight-type incumbents are 3.7 times ($\frac{\theta_H}{\theta_L} \approx 3.7$) as productive as low type incumbents who are themselves slightly more productive ($\frac{\theta_L}{\theta_E} \approx 1.09$) than potential entrants. Note that firms' optimal innovation budgets are non-linear functions of their productivities as well as other important factors like their financial health. Therefore, the average high-type incumbent innovates with an intensity 4.8 as much as the average low-type incumbent's intensity though it is 3.7 times as productive. Innovation step size (α) implies that profit from sales equal 18.5% sales. High type share of entrants ($m_{E,H}$) and transition rate ($\phi_{H \to L}$) imply that a vast majority of firms enter the market as high types and, on average, each high type firm becomes a low-type in 5.7 years. Finally, death shock (ε) leads to 1% of firms getting shutdown for exogenous reasons, which means that endogenous exits explain 90.3% of small firms' exits.

Table 2: Internal Calibration

#	Parameter	Description	Value
1	κ	Payout Penalty Scale	0.369
2	χ	Fixed Cost	0.149
3	μ_0	Fixed Recovery	0.700
4	ξ	Liquidation Discount	0.791
5	ν	Banking Costs	0.010
6	Λ	Debt Access Rate	0.050
7	θ_H	H-Type Innovation Productivity	0.268
8	θ_L	L-Type Innovation Productivity	0.073
9	θ_E	Entrant Innovation Productivity	0.067
10	α	Innovation Step Size	1.227
11	$m_{E,H}$	Share of H-Types in Entry	0.811
12	$\phi_{H \to L}$	Transition Rate into L-Type	0.174
13	ε	Death Shock	0.010

4.2 Fit

Table 3 shows how closely our model can hit targeted moments. The model does a very good job at hitting financial moments and a decent job at capturing growth/turnover moments. Data suggest that a bigger fraction of large firms have debt compared to small firms while small firms (conditional on being debtors) are, in fact, more highly leveraged than large firms. Our model captures this dynamic exactly, hitting the intensive margin (debtor leverage) and the extensive margin (debtor fraction) very closely. Moreover, the model closely captures the fact that small firms' interest rates are higher than large firms' rates.

Although the model does not hit the dramatic growth of small firms, it replicates the striking growth disparity between young and old firms. Ultimately, the model hits the aggregate growth almost exactly, exit rates very closely, and the entry moment partially.

Table 3: Moments

#	Moment	Data	Model
1	Payout Mean (+, Very Large)	0.081	0.075
2	Credit Spread (Small)	0.030	0.029
3	Credit Spread (Large)	0.022	0.023
4	Debtor Leverage (Small)	0.696	0.691
5	Debtor Leverage (Large)	0.400	0.351
6	Debtor Fraction (Small)	0.368	0.358
7	Debtor Fraction (Large)	0.969	0.866
8	5-Year Entrant Share	0.107	0.144
9	Exit (Small)	0.092	0.103
10	Exit (Large)	0.018	0.019
11	Growth (Small-Young)	0.231	0.122
12	Growth (Small-Old)	0.104	0.040
13	Growth (Large-Old)	0.005	0.005
14	Aggregate Growth	0.022	0.022

*Large/Small firms have 1-4 and 5+ employees, respectively.
**Young/Old firms are of ages 1-5 and 6-25, respectively.
***Very Large firms are the top 10% by size.
****The payout moment is conditioned on positive payout.

We do not target firm age and size-age distributions directly. However, our model's distribution closely parallels the distribution in data as can be seen in Table 4.

Table 4: Firm Distribution

Moment	Data	Model
Small-Young Portion	0.169	0.254
Small-Old Portion	0.257	0.276
Large-Old Portion	0.213	0.252
Young Portion	0.252	0.320
Old Portion	0.469	0.528

4.3 Equilibrium

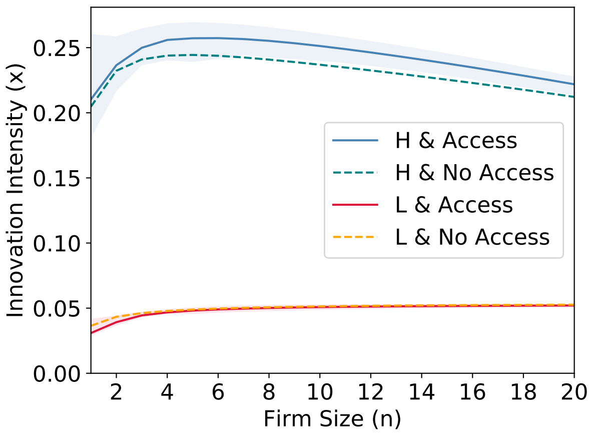
Table 5 shows important equilibrium objects. Low types hold 2.5 times as many product lines as high types. Moreover, only 25.6% of incumbents are high types. Considering that 81.1% of entrants are high types, the economy exhibits a strong negative selection. Additionally, high types hold a small fraction of the total debt. This result is not surprising

Table 5: Equilibrium Objects

#	Statistics	Value
1	H-Type Share of Product Lines	25.2%
2	L-Type Share of Product Lines	63.72%
3	AMC Share of Product Lines	11.06%
4	Default Rate (%, H, Cond. on Debt, Endog.)	0.14%
5	Default Rate (%, L, Cond. on Debt, Endog.)	2.17%
6	H-Type Share of Debt	22.1%
7	H-Type Share of Debt Increase	63.0%
8	H-Type Share of Incumbents	25.55%
9	Agg. Growth	2.18%
10	Agg. Creative Destruction	0.1069
11	Incumbent Creative Destruction	0.0833
12	Entrant Creative Destruction	0.0235
13	Avg. (Incumbent) Innov. Intensity	0.0936
14	Avg. Innov. Intensity (H)	0.2157
15	Avg. Innov. Intensity (L)	0.0453

#	Statistics	Value
16	Avg. Innov. Intensity (H & Access)	0.2108
17	Avg. Innov. Intensity (H & No Access)	0.2242
18	Avg. Innov. Intensity (L & Access)	0.0465
19	Avg. Innov. Intensity (L & No Access)	0.0403
20	Debtor Share of Incumbents	52.38%
21	Debtor Share of Incumbents (H)	27.8%
22	Debtor Share of Incumbents (L)	60.81%
23	Avg. Debtor Leverage	38.34%
24	Avg. Debtor Leverage (H)	35.87%
25	Avg. Debtor Leverage (L)	39.1%
26	Avg. Spread	2.44%
27	Avg. Spread (H)	1.47%
28	Avg. Spread (L)	2.71%

Figure 1: Innovation in Access v. No Access

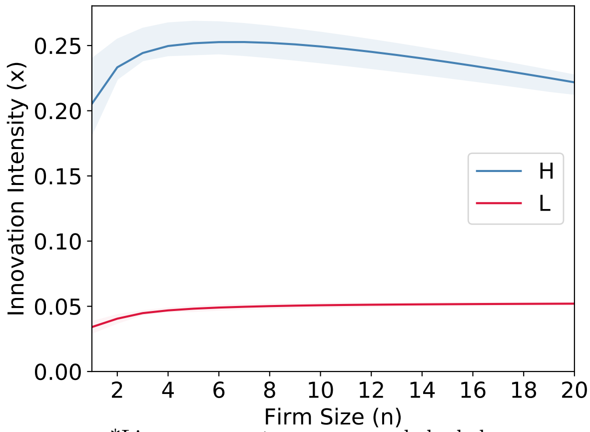


*Lines represent averages, and shaded regions represent ranges.

as high types are relatively rare. However, this disparity is magnified by the fact that a vast majority of high types are without access to debt while a majority of low types have access. In an economy with negative selection and time delay in gaining access to debt, the aforementioned debt distribution across types is expected.

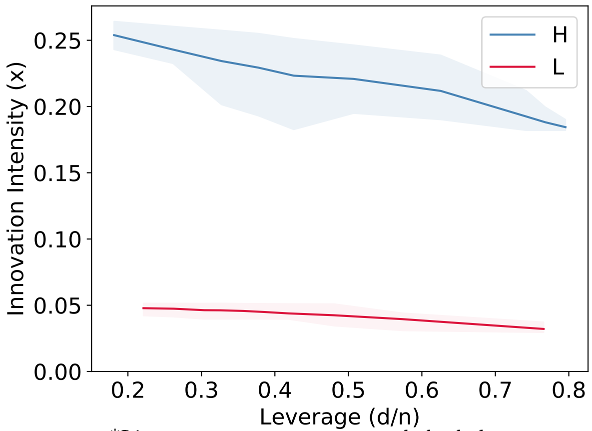
Average innovation intensity statistics seemingly suggest that gaining access to debt lowers high-types' innovation. However, those statistics are confounded by differences in firm sizes. Figure 1 shows that, conditional on size, the average high type with access to debt innovates more than the average high type without access. But, especially in smaller sizes, the left tail of high types with access actually innovates less than high types without access, capturing the “debt overhang” problem: increased levels of debt diminishing firms’ financial health and thus stymying investments. As seen in the same figure, debt overhang is a bigger problem for low types as their average access innovation is below their no access innovation. The presence of the debt overhang is empirically and structurally borne out in papers like Lamont (1995) and Hennessy (2004),

Figure 2: Innovation Across Size



*Lines represent averages, and shaded regions represent ranges.

Figure 3: Innovation Across Leverage

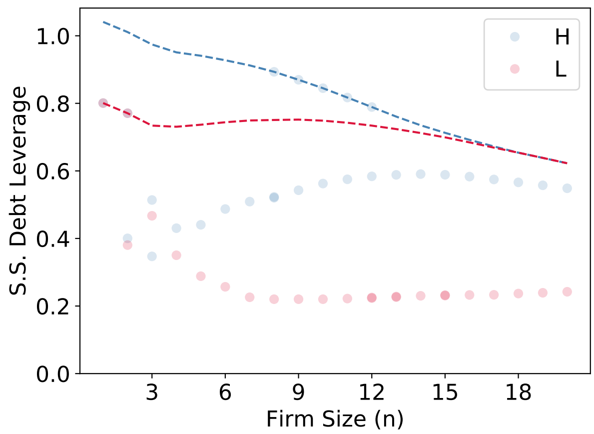


*Lines represent averages, and shaded regions represent ranges.

Figure 3 also points to the debt overhang problem as innovation intensities generally fall when debt leverage increases. Figure 2 shows that innovation intensity increases with size and that the model admits a rich firm distribution even conditional on size and type (which are two of the four state variables).

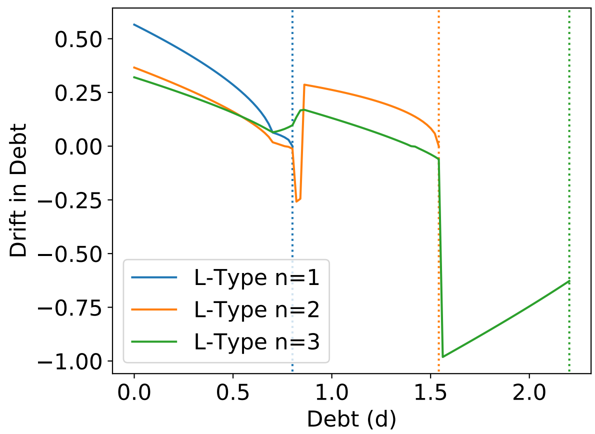
Figure 4 plots steady state leverages across size and type. We see that leverage targets (which in our model is analogous to steady states) are not unique for all firms, with each size and type having its own target(s). In fact, even conditional on size and type, steady state leverage is not always unique: with high leverage steady states forming due to high debt in the firm’s recent past. For example, consider a low type firm of size n=2 in Figure 5. This firm has two steady state debt levels, one at size n=1’s debt limit and another at its own size’s debt limit. When the firm’s size becomes n=2, if it has less debt than n=1’s debt limit, it adjusts its debt to the smaller steady state leverage. If it has barely more debt than n=1’s limit, it once again adjusts its debt to the smaller steady state leverage. However, if the firm already has debt sufficiently larger than n=1’s limit, it starts adjusting towards the larger steady state level. These dynamics replicate

Figure 4: Steady State Leverage and Size



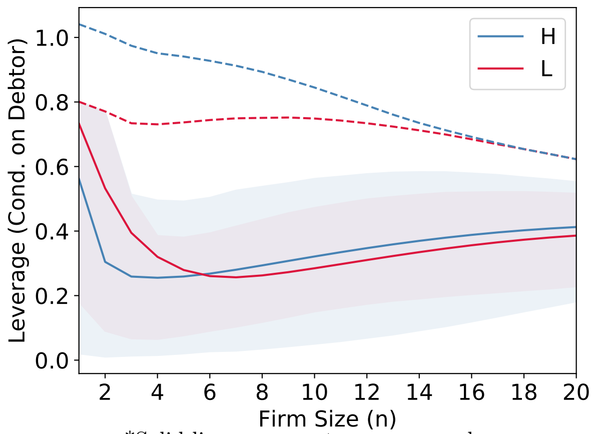
*Dashed lines represent debt limits.

Figure 5: Debt Adjustment



*Vertical dashed lines represent debt limits.

Figure 6: Leverage and Size



*Solid lines represent averages, and shaded regions represent ranges. Dashed lines represent debt limits.

an important feature described and modeled in Hennessy and Whited (2005).[2]

Another important dynamic is asymmetric adjustment speeds as noted in Faulkender et al. (2012) with over-levered firms generally adjusting more quickly than under-levered firms. Our model replicates this empirical trend. For example, consider a low type firm of size n=3 in Figure 5. This firm's steady state leverage is n=2's debt limit. If the firm is slightly to the left of the limit, the firm adjusts towards its target very slowly. However,

[2] "...leverage exhibits hysteresis, in that firms with high lagged debt use more debt than otherwise identical firms." Hennessy and Whited (2005)

slightly to the right, the firm's adjustment speed is much larger.

State-dependent leverage targets and varying adjustment speeds create a healthy debt distribution which is captured by Figure 6 which plots the distribution of leverages across size and type. The model is able to create a robust range of leverages even after conditioning on state variables due to path dependence.

Figure 6 also shows that high and low types have different endogenous debt limits.[3] Given some size and debt, a high type firm is more valuable because its innovation option value is greater. Hence, it remains solvent at larger debt amounts than the low type firm can.

5 Financial Frictions

In this section, we study the economic consequences of financial frictions by varying the degree of frictions. We consider 4 economies with stronger frictions than our baseline economy and 4 economies with weaker frictions. As the following counterfactual economies are subject to multiple channels of frictions, a conclusive pair-wise friction "scoring" is not always possible. However, generally, the following list introduces our counterfactual economies in descending order of financial frictions.

No Access to Debt: $\boldsymbol{\Lambda \to 0}$. In this economy, no firm has access to debt. Hence, they fund their expenditures with their internal funds and/or equity injections.

No Default Recoveries: $\boldsymbol{\mu_0 = 0}$ and $\boldsymbol{\xi = 1}$. In this economy, the bank does not have any leverage over the asset management company during the liquidation of bankrupt firms. Therefore, the bank cannot count on any recoveries and charges (potentially) higher risk premium.

No Interest Tax Shield. In this economy, the government continues corporate taxation but revokes the interest deduction. Thus, firms' effective debt-financing costs (potentially) increase.

No Unsecured Debt: $\boldsymbol{\bar{D}(n, k) = \max\{\mu_0, \mu(n-1)\}}$. In this economy, the bank does not lend more funds than it is assured to recover. That is, firms' liquidation values

[3] Importantly, collateral value does not drive this result as collateral value is not type dependent. However, if it were, the wedge between debt limits could be even greater.

become exogenous limits on debt, mimicking Jermann and Quadrini (2012) as well as many DSGE models with exogenous borrowing constraints.

No Liquidation Haircut: $\xi = 0$. In this economy, firms can be liquidated at their full value to the AMC. Hence, firms (potentially) face lower risk premium.

No Banking Costs: $\nu = 0$. In this economy, the bank does not incur miscellaneous costs to extend debt. Therefore, interest rates (potentially) uniformly fall.

Full Access to Debt: $\Lambda \to \infty$. In this economy, every firm has full access to the debt market. Hence, firms of all ages and sizes can debt-finance their investments.

Bolstered Access to Debt: $\Lambda \to \infty$, $\xi = 0$, and $\nu = 0$. In this economy, every firm has immediate access to debt, liquidations are not done at a discount, and the bank does not face miscellaneous costs. Hence, firms' ability to debt-finance is (potentially) enhanced.

As illustrated in Figure 7, counterfactual frictions can be placed on a scale with No Access having the strongest frictions and Bolstered Access having the weakest.

Figure 7: Friction Scale

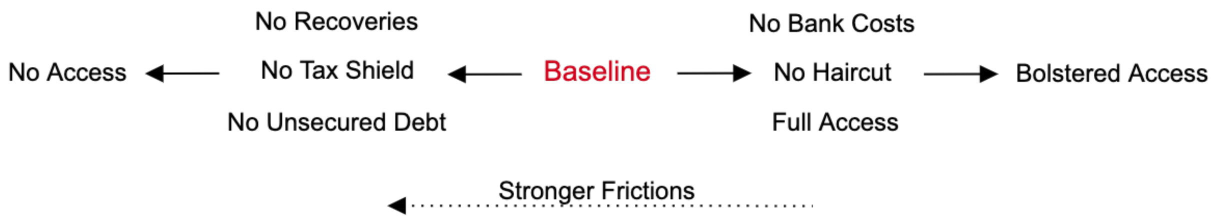


Table 6 summarizes equilibrium objects across different friction scenarios. Note that the "Baseline" column is equivalent to Table 5 in Section 4.3. Each row of Table 6 is colored to represent the relative magnitude of each statistic across frictions (i.e., colors are comparable only within each row). For example, as the high type share of product lines increases with weaker frictions, the first row of the table gets darker when reading from left to right.

Firms' leverages increase significantly when financial frictions are weakened (see rows 23 and 24 in Table 6). Therefore, it is not surprising that firms also endogenously default more frequently when frictions weaken (see rows 4 and 5). That is, frictions actually depress endogenous defaults. Intuitively, weakened financial frictions give firms enough rope to hang themselves.

Though endogenous defaults increase for both types, low-type firms endogenously default considerably more frequently than high-type firms. High types expect to grow more, leading to greater firm values than low types (all else equal). Hence, they remain solvent at larger debt amounts than low types do. Moreover, as high types successfully grow, it becomes easier to service their pre-existing debt, bolstering their financial health. However, as low types tend to shrink more than high types, debt can quickly suffocate them. Hence, the endogenous default rate ends up being significantly greater for low types.

Endogenous defaults are a channel of positive selection for the economy. Weaker frictions lead to an exodus of low types, reallocating resources to high types (see rows 1 through 3 in Table 6 or Figure 8). Improvement in firm composition leads to average innovation increasing as frictions weaken (see row 13 or Figure 9). However, incumbents' *total* contribution to creative destruction has a more secular trend (see row 11). The discrepancy between the *average* and *total* trends is caused by endogenous defaults' leading to a larger share of the economy controlled by firms that do not innovate (i.e., AMC's holdings, see row 3 or Figure 8). That is, though the average incumbent's innovation intensity increases, in total, incumbents hold fewer product lines, resulting in the aforementioned secular trend. However, weakening frictions has a much clearer effect on entrants who increase their innovative investments because becoming an incumbent becomes more valuable (see row 12).

Figure 10 decomposes the contribution of high type incumbents, low type incumbents, and entrants to GDP growth in excess of their respective contributions in the baseline economy. Consistent with the selection story, the contribution of high types significantly increases when frictions relax but the contribution of low types decreases.

Figure 8: Selection

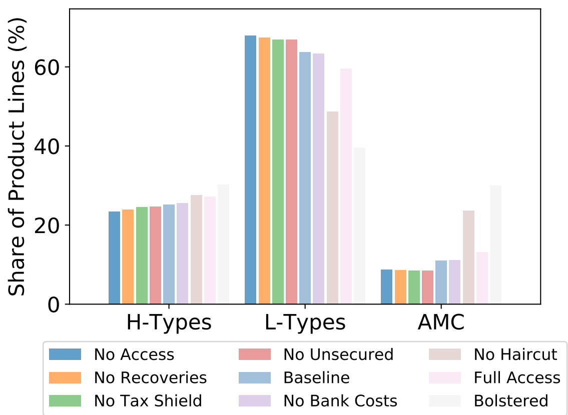


Figure 9: Incumbent Innovation

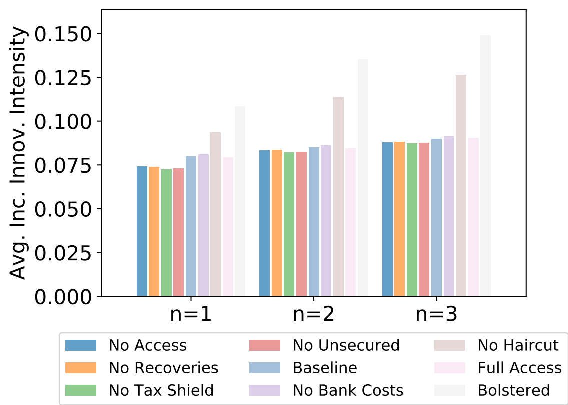


Figure 10: Growth Decomposition

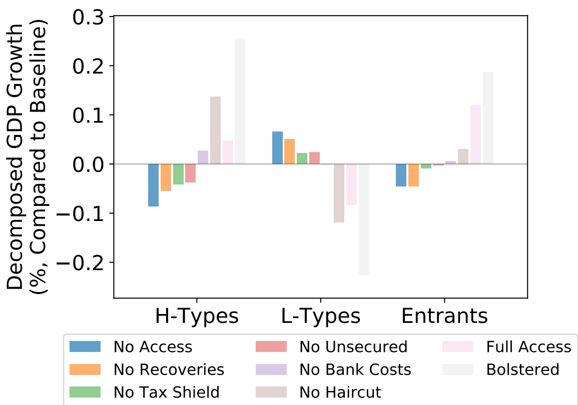


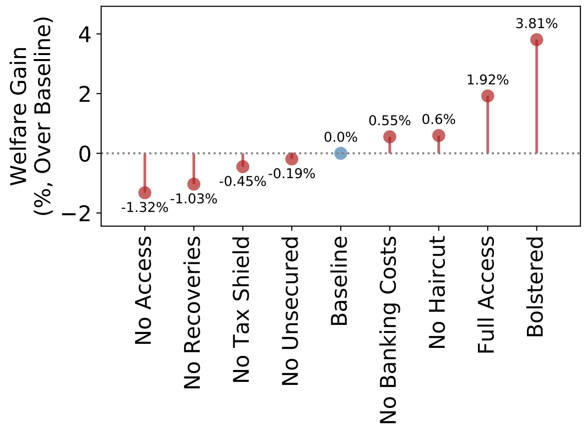
Table 6: Equilibrium Objects Across Frictions

#	Statistics	No Access	No Recoveries	No Tax Shield	No Unsecured	Baseline	No Bank Costs	No Haircut	Full Access	Bolstered
1	H-Type Share of Product Lines	23.4%	23.9%	24.5%	24.6%	25.2%	25.6%	27.6%	27.2%	30.3%
2	L-Type Share of Product Lines	67.88%	67.42%	66.9%	66.85%	63.72%	63.3%	48.71%	59.59%	39.65%
3	AMC Share of Product Lines	8.7%	8.68%	8.56%	8.51%	11.06%	11.1%	23.68%	13.16%	30.05%
4	Default Rate (%, H, Cond. on Debt, Endog.)	-	0.05%	0.02%	0.0%	0.14%	0.22%	2.58%	0.12%	4.98%
5	Default Rate (%, L, Cond. on Debt, Endog.)	-	0.02%	0.01%	0.0%	2.17%	2.34%	8.3%	2.58%	7.0%
6	H-Type Share of Debt	-	27.2%	22.3%	16.0%	22.1%	21.8%	29.9%	28.0%	39.3%
7	H-Type Share of Debt Increase	-	80.0%	58.0%	52.9%	63.0%	63.4%	59.2%	71.9%	72.2%
8	H-Type Share of Incumbents	22.29%	22.51%	23.08%	23.16%	25.55%	25.93%	33.03%	29.3%	42.72%
9	Agg. Growth	2.12%	2.13%	2.15%	2.17%	2.18%	2.21%	2.23%	2.27%	2.4%
10	Agg. Creative Destruction	0.1036	0.1045	0.1055	0.1061	0.1069	0.1084	0.1092	0.111	0.1174
11	Incumbent Creative Destruction	0.0823	0.0831	0.0824	0.0827	0.0833	0.0846	0.0842	0.0816	0.0847
12	Entrant Creative Destruction	0.0212	0.0213	0.0231	0.0234	0.0235	0.0237	0.0249	0.0293	0.0326
13	Avg. (Incumbent) Innov. Intensity	0.0901	0.091	0.0901	0.0904	0.0936	0.0952	0.1103	0.0939	0.1211
14	Avg. Innov. Intensity (H)	0.2142	0.2165	0.2135	0.2134	0.2157	0.2176	0.2212	0.2082	0.2206
15	Avg. Innov. Intensity (L)	0.0473	0.0465	0.0448	0.045	0.0453	0.0456	0.0474	0.0417	0.0451
16	Avg. Innov. Intensity (H & Access)	-	0.2101	0.2076	0.2073	0.2108	0.2133	0.2168	0.2082	0.2206
17	Avg. Innov. Intensity (H & No Access)	0.2142	0.2283	0.2235	0.2238	0.2242	0.2251	0.2292	-	-
18	Avg. Innov. Intensity (L & Access)	-	0.0479	0.0458	0.046	0.0465	0.0469	0.0492	0.0417	0.0451
19	Avg. Innov. Intensity (L & No Access)	0.0473	0.0395	0.0402	0.0405	0.0403	0.0402	0.0423	-	-
20	Debtor Share of Incumbents	-	57.39%	55.88%	56.77%	52.38%	51.84%	38.32%	99.53%	98.78%
21	Debtor Share of Incumbents (H)	-	27.74%	27.66%	27.83%	27.8%	27.75%	27.23%	98.71%	97.72%
22	Debtor Share of Incumbents (L)	-	66.01%	64.35%	65.5%	60.81%	60.27%	43.8%	99.87%	99.56%
23	Avg. Debtor Leverage	-	20.93%	22.3%	21.9%	38.34%	44.26%	72.33%	43.78%	73.35%
24	Avg. Debtor Leverage (H)	-	26.47%	21.7%	15.98%	35.87%	39.96%	65.61%	39.13%	66.7%
25	Avg. Debtor Leverage (L)	-	19.4%	22.48%	23.57%	39.1%	45.63%	75.64%	45.89%	78.4%
26	Avg. Spread	-	3.53%	1.15%	1.01%	2.44%	1.56%	2.47%	2.8%	2.2%
27	Avg. Spread (H)	-	2.26%	1.13%	1.01%	1.47%	0.55%	1.32%	1.56%	1.11%
28	Avg. Spread (L)	-	4.0%	1.16%	1.01%	2.71%	1.84%	2.97%	3.28%	2.9%

*Each row is colored to represent the relative magnitude of each statistic across frictions (i.e., colors are comparable only within each row).

5.0.1 Welfare Comparisons

Figure 11: Welfare Compared to Baseline



*Steady state distributions.

The aggregate effects of mechanisms described above are captured in the welfare comparisons in Figure 11. Welfare numbers should be interpreted as additional (percent) consumption the household would require over its baseline consumption to be indifferent between staying in the baseline economy and moving to the new economy (i.e., consumption equivalence). Both the baseline and the new economy are assumed to be in steady state in this analysis.

Unsurprisingly, stronger financial frictions lead to worse welfare results. Between the most and least frictional economies, there is a 5 percentage point welfare swing, attesting to the economy's sensitivity to financial frictions.

Having access to debt can be very important for firms as they face rigidities in adjusting their equity positions. In addition to (at least partially) offsetting equity rigidities, debt can also be relatively cheap due to its tax shield status. Hence, firms are incentivized to invest toward innovation as their borrowing costs are diminished by the tax deduction. Moreover, because firms have recovery values during bankruptcies, they can access cheap funds when their debt leverage is low, further incentivizing investments.

Our model predicts closing down the debt market would wipeout 1.32% in welfare. The model also predicts that if recovery were not possible, welfare would plummet by more than a percent as borrowing costs rise substantially. Moreover, without the interest rate tax deduction, welfare would drop by 50 basis points due to the increase in the *effective* borrowing costs.

Firms' technology is more valuable to them than to the bank. So, when the bank enforces that it must always be able to fully recover its loan under all possible shocks, endogenous defaults are precluded. Risk premium declines to zero, but welfare drops by 20 basis points as firms' ability to increase their debt leverage diminishes.

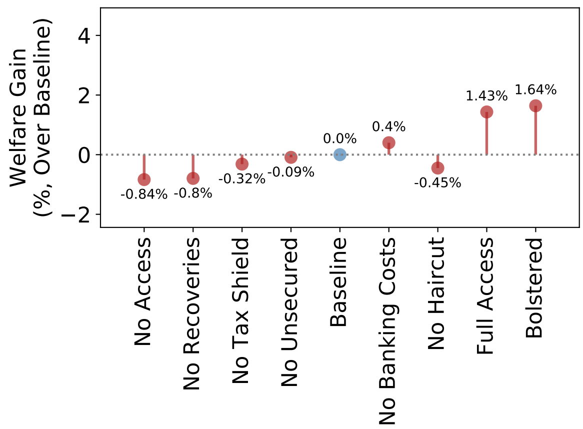
Without banking costs, despite a secular increase in endogenous defaults, average interest spreads decline significantly (though not by the amount that eliminating banking costs would have applied in partial equilibrium). This cheapening has the opposite effect of eliminating the tax shield and welfare increases by more than 50 basis points.

If product lines could be recovered by the bank without a haircut, endogenous default rates would increase significantly as firms are green-lighted to take on significantly more debt. However, interest spreads increase only marginally, as the downside risk of bankruptcies are diminished by boosted recoveries. Interestingly, high types' spreads decline while low types' spreads increase. The aggregate effect end up being a 60 basis improvement in welfare.

Full access to debt diminishes incumbents' contribution to growth while increasing the entrants' (see rows 11 and 12 in Table 6). Gaining immediate debt access increases firm values, incentivizing entrants to invest more heavily. Explaining the decrease in incumbent innovation is slightly more complicated. The base of endogenous default enlarges though the rates remain very similar. The point is, with more firms in debt, even the same endogenous default rate would increase the mass of product lines that are in default. Therefore, the mass of product lines of innovating incumbents decreases, which causes a decline in incumbents' contribution to growth despite an increase in average incumbent innovation investment.

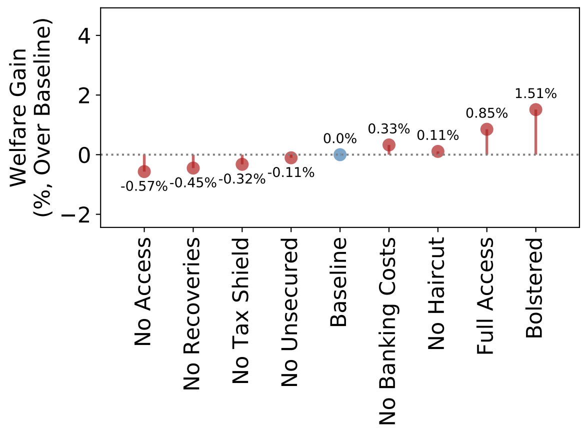
Bolstered access to debt combines no banking costs, haircut-less recoveries, and full debt access. The simultaneous realization of these three weaker frictions creates welfare of more than 3.8%, which is actually greater than the linear combinations of each individual scenario's welfare improvement. That is, the interaction of these weaker frictions further increases welfare. This force is especially strong because consumption has diminishing returns for the household and, without benefits from interactions, simultaneously alleviating the three frictions would produce less welfare than their independent linear combination.

Figure 12: L-Types Only



*Steady state welfare comparisons.

Figure 13: H-Types Only



*Steady state welfare comparisons.

5.0.2 Importance of Type Heterogeneity

Firm heterogeneity (in terms of innovation productivity) increases welfare's sensitivity to frictions. Figure 12 represents the exercise in Figure 11 with one twist: all firms are low types. Similarly, Figure 13 captures welfare comparisons when all firms are high types. Together, Figures 11, 12, and 13 have the same vertical scale to facilitate easier comparisons. A clear trend emerges: the two homogeneous economies are less sensitive to frictions than our heterogeneous economy.

Note that high and low type firms have a dramatic gap in their average innovation outputs (see rows 14 and 15 in Table 6). In fact, this gap is especially large in the heterogeneous economy (see rows 14 and 15 in Tables 6 as well as Tables 1 and 2 in Appendix B). High types in the heterogeneous economy innovate more than they would in the high-type-only economy. At the same time, low types in the heterogeneous economy innovate less than they would in the low-type-only economy. The general idea is that having more high types in the economy increases total creative destruction, depressing the innovation incentive of each firm. Also, recall that endogenous defaults are an important selection channel, strongly influencing the composition of the economy.

Figure 17 (16) compares the contribution of high (low) types in the heterogeneous and high-type-only (low-type-only) economies in each friction to their respective baselines. Similarly, Figure 15 (14) compares percentage point change in high (low) types' share of the economy in the heterogeneous and high-type-only (low-type-only) economies in each friction to their respective baselines.

Figure 14: Shift in L-Type Share

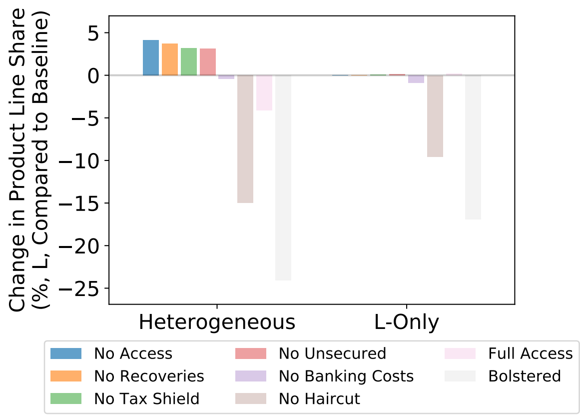


Figure 15: Shift in H-Type Share

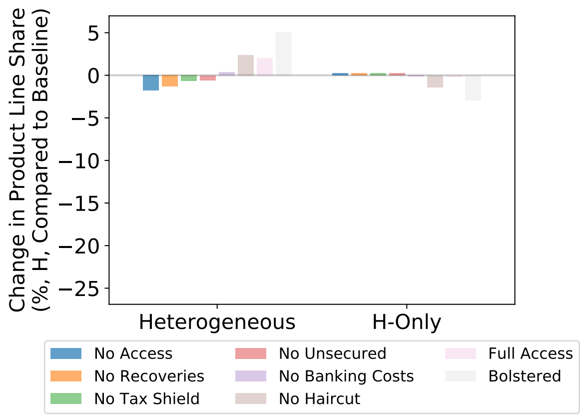


Figure 16: L-Type Fueled Growth

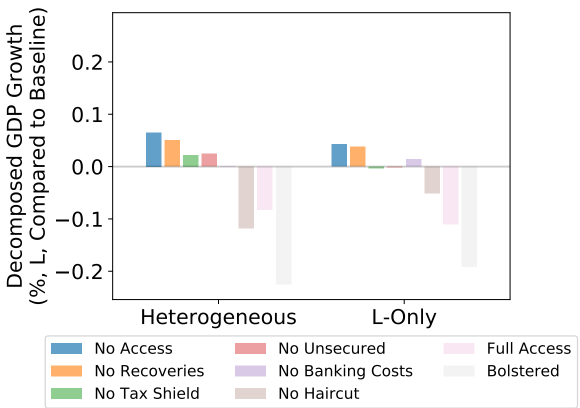
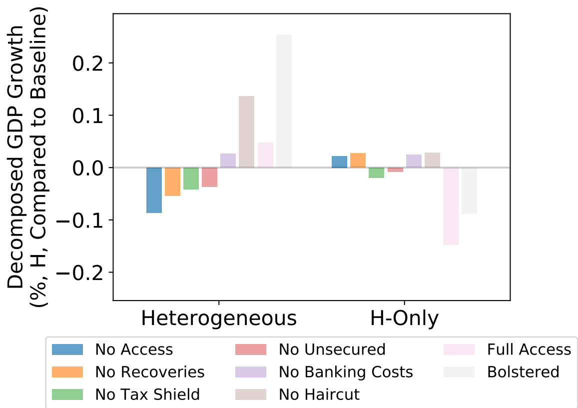


Figure 17: H-Type Fueled Growth



We see that low types' contributions are quite similar in the heterogeneous and homogeneous economies. However, high types' contributions differ substantially. While in the heterogeneous economy, weakening frictions strongly increases the contribution of high types to growth, the trend in the homogeneous economy is noisy (if not reversed). The main driver is that the high-type share of the economy actually increases in the heterogeneous economy while declining in the high-type-only economy. In the heterogeneous economy, the pitfalls of weaker frictions chiefly plague low types, increasing the role of high types in the economy.

5.1 Policy

We study the interaction of financial frictions and a straightforward, canonical innovation policy: a flat investment subsidy rate unconditional on size or type. Given each friction, we compute and implement the welfare maximizing subsidy rate. Then, we compare the optimal policy outcomes of different frictions.

Table 7 is equivalent to Table 6 after optimal policies are instituted. Optimal policies lower average debt leverages across frictions and types while mostly leaving the share of debtor firms unchanged except for high type firms (see rows 20 to 25 in Tables 6 and 7). Optimal subsidies increase growth by more than 2 percentage points across each friction.

Figure 18: Endog. Default Before Policy

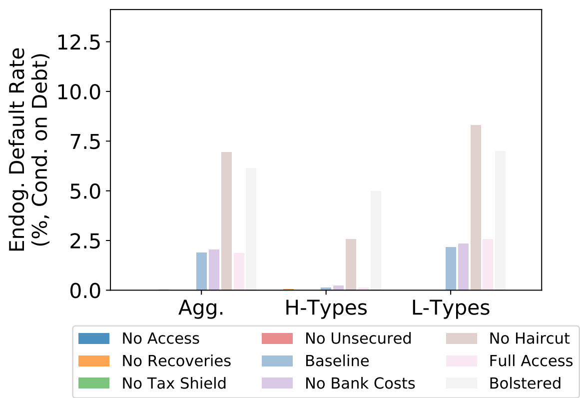


Figure 19: Endog. Default After Policy

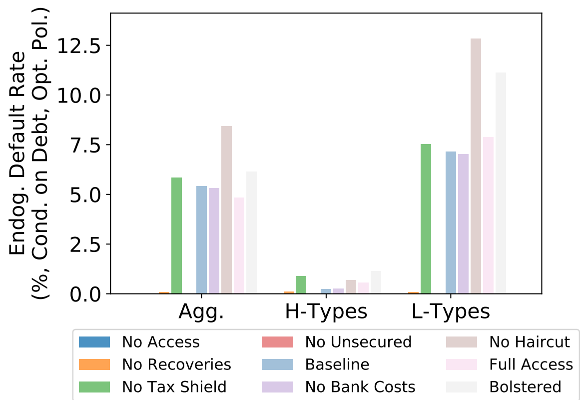


Figure 20: Selection Before Policy

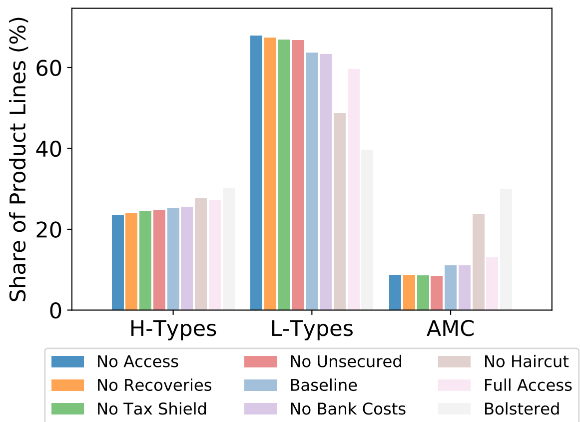


Figure 21: Selection After Policy

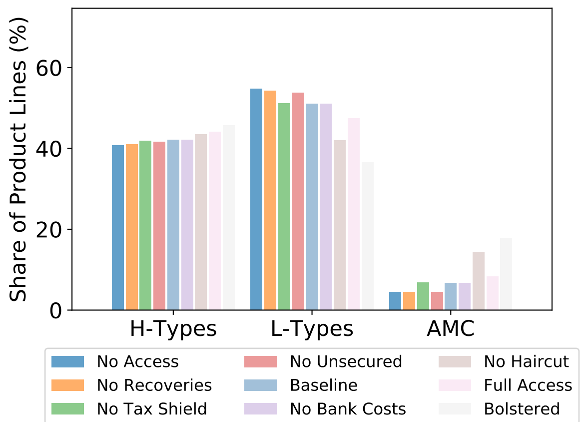


Table 7: Equilibrium Objects Across Frictions (with Optimal Policy)

#	Statistics	No Access	No Recoveries	No Tax Shield	No Unsecured	Baseline	No Bank Costs	No Haircut	Full Access	Bolstered
1	H-Type Share of Product Lines	40.8%	41.1%	41.9%	41.7%	42.2%	42.2%	43.6%	44.2%	45.7%
2	L-Type Share of Product Lines	54.74%	54.33%	51.23%	53.86%	51.12%	51.11%	42.09%	47.44%	36.55%
3	AMC Share of Product Lines	4.5%	4.57%	6.83%	4.48%	6.72%	6.7%	14.36%	8.37%	17.73%
4	Default Rate (%, H, Cond. on Debt, Endog.)	-	0.1%	0.89%	0.0%	0.25%	0.26%	0.69%	0.57%	1.16%
5	Default Rate (%, L, Cond. on Debt, Endog.)	-	0.09%	7.54%	0.0%	7.17%	7.03%	12.84%	7.88%	11.12%
6	H-Type Share of Debt	-	42.7%	44.4%	29.4%	39.6%	38.9%	48.5%	44.9%	53.2%
7	H-Type Share of Debt Increase	-	91.3%	76.9%	70.6%	79.3%	79.0%	80.9%	80.9%	82.9%
8	H-Type Share of Incumbents	30.64%	30.85%	35.98%	31.39%	35.85%	35.84%	42.41%	41.68%	50.39%
9	Agg. Growth	4.27%	4.27%	4.29%	4.3%	4.3%	4.3%	4.25%	4.43%	4.39%
10	Agg. Creative Destruction	0.209	0.2089	0.2101	0.2106	0.2104	0.2105	0.2082	0.2167	0.2148
11	Incumbent Creative Destruction	0.1894	0.1893	0.1893	0.1896	0.1893	0.1893	0.1866	0.19	0.1871
12	Entrant Creative Destruction	0.0195	0.0195	0.0207	0.0209	0.021	0.0212	0.0215	0.0267	0.0277
13	Avg. (Incumbent) Innov. Intensity	0.1983	0.1984	0.2032	0.1985	0.2029	0.2029	0.218	0.2073	0.2274
14	Avg. Innov. Intensity (H)	0.3543	0.3546	0.3542	0.354	0.3541	0.354	0.3526	0.3521	0.3514
15	Avg. Innov. Intensity (L)	0.0822	0.0802	0.0795	0.0782	0.0782	0.0781	0.0786	0.0725	0.0724
16	Avg. Innov. Intensity (H & Access)	-	0.3266	0.3263	0.3258	0.3275	0.3279	0.3277	0.3521	0.3514
17	Avg. Innov. Intensity (H & No Access)	0.3543	0.4576	0.4526	0.452	0.4479	0.4459	0.4421	-	-
18	Avg. Innov. Intensity (L & Access)	-	0.0821	0.081	0.0793	0.0796	0.0795	0.0807	0.0725	0.0724
19	Avg. Innov. Intensity (L & No Access)	0.0822	0.0668	0.0707	0.0708	0.0701	0.0697	0.0689	-	-
20	Debtor Share of Incumbents	-	57.37%	50.3%	56.83%	50.57%	50.53%	40.93%	99.47%	98.78%
21	Debtor Share of Incumbents (H)	-	35.64%	35.49%	35.58%	35.45%	35.36%	34.94%	98.97%	98.04%
22	Debtor Share of Incumbents (L)	-	67.06%	58.62%	66.55%	59.02%	59.01%	45.34%	99.84%	99.54%
23	Avg. Debtor Leverage	-	14.22%	12.49%	12.66%	23.74%	25.42%	42.64%	27.48%	45.62%
24	Avg. Debtor Leverage (H)	-	15.01%	12.97%	9.1%	21.99%	23.11%	41.72%	25.59%	43.76%
25	Avg. Debtor Leverage (L)	-	13.69%	12.13%	15.12%	25.04%	27.15%	43.55%	29.25%	47.94%
26	Avg. Spread	-	3.12%	1.24%	1.01%	2.51%	1.59%	2.15%	2.77%	1.66%
27	Avg. Spread (H)	-	2.15%	1.21%	1.01%	1.49%	0.52%	1.13%	1.53%	0.45%
28	Avg. Spread (L)	-	3.84%	1.26%	1.01%	3.18%	2.27%	3.11%	3.78%	3.04%

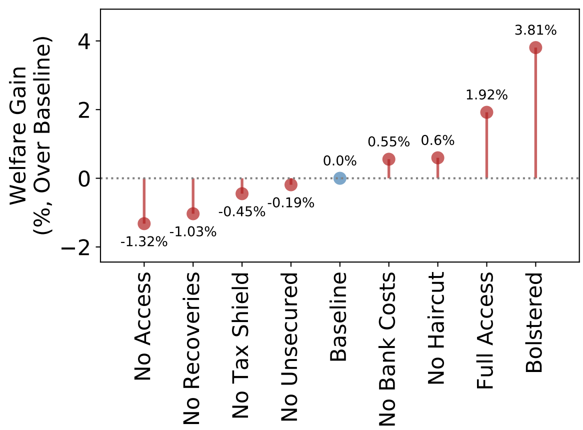
*Each row is colored to represent the relative magnitude of each statistic across frictions (i.e., colors are comparable only within each row).

Boosted growths come at a cost: endogenous defaults. With market turnover increasing significantly, endogenous default rates increase for both types (see Figures 18 and 19). Despite the general increase, low types continue to have dramatically larger endogenous default rates. Interestingly, despite the dramatic increase in endogenous defaults, average interest spreads remain similar before and after policy (see rows 26 to 28 in Tables 6 and 7).

Unsurprisingly, the combination of endogenous defaults and increased incentives for innovative firms to invest, high types are strongly selected across all frictions after optimal subsidies are implemented (see Figures 20 and 21). High type share of the economy increases by around 20 percentage points across all frictions. However, the trend of financial frictions creating negative selection remains true even after optimal subsidies.

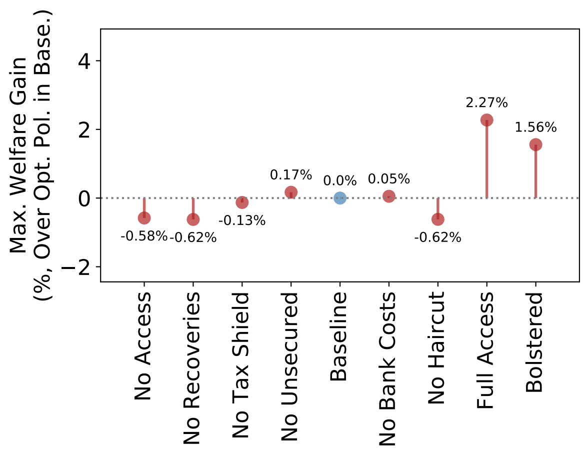
5.1.1 Welfare Comparisons

Figure 22: Welfare Before Policy



*Steady state welfare comparisons.

Figure 23: Welfare After Policy



*Steady state welfare comparisons.

Optimal subsidies reduce welfare gaps among different frictions (see Figures 22 and 23). For example, the welfare improvement of bolstered access over the baseline economy is reduced by 60% after optimal policies are instituted for both the baseline and bolstered economies. The implication is that there is greater room for subsidy policies when financial frictions are greater. Though subsidies improve welfare for all frictional economies, the added benefits for more highly frictional economies outstrip less frictional economies'.

While optimal policies close welfare gaps among financial frictions, they generally do not reverse the trend that frictions hurt welfare except for two important scenarios: no unsecured debt and no haircut. Though the no unsecured debt economy is more frictional than the baseline, after optimal policies, the more frictional economy is better off than the less frictional one. Similarly, though the no haircut economy is less frictional than the baseline, optimal policies, once again, cause the more frictional economy to be better off than the less frictional one. These important deviations from the pre-policy trend is caused by the cost of endogenous defaults on society.

Subsidies put creative destruction on overdrive, dramatically increasing market turnover and thus endogenous defaults. Endogenous defaults help in selection but at the expense of diminishing the mass of product lines belonging to innovating incumbents. Hence, endogenous defaults may counteract the benefits of increased creative destruction. Because the no unsecured debt economy precludes endogenous defaults, baseline economy ends up having worse welfare after optimal policies are implemented. Similarly, because the no haircut economy promotes endogenous defaults even without subsidies, baseline economy ends up having better welfare results.

6 Conclusion

We build a firm dynamics model set in an economy burdened by financial frictions. Then, we study the interaction of innovation with financial margins such as default, firm-specific interest rates, and debt limits. We find that frictions can cause major distortions on innovation.

Our model suggests that these distortions are especially emblematic in heterogeneous economies where firms have varying degrees of innovation capacity. Our calibration suggests that an economy with a homogeneous innovation type would be considerably less sensitive to varying the degree of financial frictions. We see that financial frictions strongly select against high-type (high innovation capacity) firms. The presence of frictions lower market turnover and save low-type (low innovation capacity) firms from endogenously defaulting.

Our policy exercises suggest that optimal subsidies counteract financial frictions, closing welfare gaps among different levels of frictions. Equivalently, subsidy policies are more effective when frictions are stronger. Interestingly, financial mechanisms grant an important instrument for selection to policy makers. As subsidies promote market turnover, low type firms start defaulting in large numbers.

Underpinning the social cost of default can be very important for results of structural models that investigate innovation and financial frictions. A future venue of research is empirically studying the consequences of innovative firms's defaulting.

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Appendices

A Derivations

A.1 Labor Market

Total labor demand is

$$\begin{aligned} L_t^D &= \int_{i\in\Phi} l_i di, \\ &= \int_{i\in\Phi} \frac{Y}{\alpha W} di, \\ &= \frac{||\Phi||}{\omega\alpha}. \end{aligned}$$

where $\omega = \frac{W}{Y}$. Suppose there is some $L^S > 0$ mass of labor supply. Labor market clearing, $L^S = L^D$, implies that

$$L^S = \frac{||\Phi||}{\omega\alpha}. \tag{32}$$

A.2 Final Good Production

We can solve for the (log) final good production as,

$$\begin{aligned} \ln Y &= \int_{i\in\Phi} \ln y_i di, \\ &= \int_{i\in\Phi} \ln\left(\frac{q_i Y}{\alpha w}\right) di, \\ &= \int_{i\in\Phi} \ln\left(\frac{q_i}{\alpha\omega}\right) di, \\ &= \int_{i\in\Phi} \ln(q_i)\, di + \int_{i\in\Phi} \ln\left(\frac{1}{\alpha\omega}\right) di, \\ &= \int_{i\in\Phi} \ln(q_i)\, di + \ln\left(\frac{1}{\alpha\omega}\right)\Phi, \\ &= \int_{i\in\Phi} \ln(q_i)\, di + \ln\left(\frac{L^S}{\Phi}\right)\Phi, \\ &= \ln Q + \ln\left[\left(\frac{L^S}{\Phi}\right)^{\Phi}\right]. \end{aligned} \tag{33}$$

Then, the level of the final good production is

$$Y = Q\left(\frac{L^S}{\Phi}\right)^{\Phi}, \tag{34}$$

where $\ln(Q) = \int_{\Phi_t} \ln(q_j)dj$.

A.3 Firm Value

The unnormalized value function of incumbent intermediate good producers can be written recursively as

$$\begin{aligned} r_f V(D,n,k,a) - \partial_t[V(D,n,k,a)] = \max\Bigg\{0, \max_{x,p}\Bigg\{ & pnY \\ & + \frac{\partial V}{\partial D}(D,n,k,a)\dot{D}(x,p;D,n,k,a) \\ & + xn\left[V(D,n+1,k,a) - V(D,n,k,a)\right] \\ & + \tau n\left[V(D,n-1,k,a) - V(D,n,k,a)\right] \\ & + 1_{k=H}\phi_{H\to L}\left[V(D,n,L,a) - V(D,n,H,a)\right] \\ & + 1_{a=N}\lambda(n)\left[V(D,n,k,A) - V(D,n,k,N)\right] \\ & - \varepsilon V(D,n,k,a)\Bigg\}\Bigg\}, \end{aligned} \tag{35}$$

Solving for the balanced growth path, we normalize the firm's value by the final good, Y. Let lower case variables represent the normalizations. Observe that, $\frac{rV(D,n,k,a)-\partial[V(D,n,k,a)]}{Y}$ equals the following:

$$\begin{aligned} \frac{r_f V(D,n,k,a) - \partial_t[V(D,n,k,a)]}{Y} &= rv(d,n,k,a)Y - \partial_t[v(d,n,k,a)Y], \\ &= (r_f - g)v(d,n,k,a). \end{aligned} \tag{36}$$

Also note that the debt drift $\dot{D}(x,p;D,n,k,a)$'s normalization is

$$\dot{D} = \frac{d(Yd)}{dt}, \tag{37}$$

$$\dot{D} = Y\dot{d} + \dot{Y}d, \tag{38}$$

$$\frac{\dot{D}}{Y} = \dot{d} + d\frac{\dot{Y}}{Y}, \tag{39}$$

$$\frac{\dot{D}}{Y} = \dot{d} + dg, \tag{40}$$

$$\dot{d} = \frac{\dot{D}}{Y} - dg, \tag{41}$$

$$\dot{d} = \frac{-[\pi n - \chi Y - c(x) - r(D,n,k)D] + \varphi(p)}{Y} - dg, \tag{42}$$

$$\dot{d} = -[\pi n \tfrac{1}{Y} - \chi - c(x)\tfrac{1}{Y} - (r(d,n,k)(1-\tau_{tax}) - g)d] + \varphi(p)\tfrac{1}{Y}. \tag{43}$$

Finally note that $\frac{\partial V}{\partial D}(D,n,k,a)\dot{D}(x,p;D,n,k,a)$ equals the following:

$$\begin{aligned} \frac{\partial V}{\partial D}(D,n,k,a)\dot{D}(x,p;D,n,k,a) &= \frac{\partial (vY)}{\partial D}\frac{\partial D}{\partial d}\frac{\partial d}{\partial t}, \\ &= Y\frac{\partial v}{\partial d}\dot{d}(x,p;d,n,k,a). \end{aligned} \tag{44}$$

Hence, normalizing firm value by Y, we get

$$\begin{aligned} (r_f - g_Y)v(d,n,k,a) = \max\Bigg\{0, \max_{x,p}\Bigg\{ & pn \\ & + \frac{\partial v}{\partial d}(d,n,k,a)\dot{d}(x,p;d,n,k,a) \\ & + xn\left[v(d,n+1,k,a) - v(d,n,k,a)\right] \\ & + \tau n\left[v(d,n-1,k,a) - v(d,n,k,a)\right] \\ & + 1_{k=H}\phi_{H\to L}\left[v(D,n,L,a) - v(D,n,H,a)\right] \\ & + 1_{a=N}\lambda(n)\left[v(D,n,k,A) - v(D,n,k,N)\right] \\ & - \varepsilon v(d,n,k,a)\Bigg\}\Bigg\}. \end{aligned} \tag{45}$$

s.t.

$$0 \geq d \tag{46}$$

for $a = N$, and

$$\bar{d}(n,k) \geq d \tag{47}$$

for $a = A$, and $\dot{d}$ as in (43).

A.4 Firm Optimal Decisions

Let ψ be the Lagrange multiplier ensuring that debt amount obeys the upper bounds. First Order Condition for payout intensity (p) is are such that

$$0 = n + \left[\frac{\partial v}{\partial d}(d, n, k, a) + \psi\right][1 + 2\kappa p]\, n, \tag{48}$$

$$-\frac{1}{\frac{\partial v}{\partial d}(d, n, k, a) + \psi} = 1 + 2\kappa p, \tag{49}$$

$$p^* = -\frac{1}{2\kappa}\left[1 + \frac{1}{\frac{\partial v}{\partial d}(d, n, k, a) + \psi}\right]. \tag{50}$$

First Order Conditions for R&D arrival (x) are such that

$$0 = \left[\frac{\partial v}{\partial d}(d, n, k, a) + \psi\right]\eta\frac{1}{\theta_k}x^{\eta-1}n + n\left[v(d, n+1, k, a) - v(d, n, k, a)\right], \tag{51}$$

$$v(d, n+1, k, a) - v(d, n, k, a) = -\left[\frac{\partial v}{\partial d}(d, n, k, a) + \psi\right]\eta\frac{1}{\theta_k}x^{\eta-1}, \tag{52}$$

$$x^* = \left(-\frac{\theta_k}{\eta}\frac{v(d, n+1, k, a) - v(d, n, k, a)}{\frac{\partial v}{\partial d}(d, n, k, a) + \psi}\right)^{\frac{1}{\eta-1}}. \tag{53}$$

B Financial Frictions

Table 1: Equilibrium Objects Across Frictions (High-Type Only)

#	Statistics	No Access	No Recoveries	No Tax Shield	No Unsecured	Baseline	No Bank Costs	No Haircut	Full Access	Bolstered
1	H-Type Share of Product Lines	94.3%	94.3%	94.3%	94.4%	94.1%	93.9%	92.7%	93.9%	91.1%
2	L-Type Share of Product Lines	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
3	AMC Share of Product Lines	5.66%	5.68%	5.66%	5.65%	5.93%	6.08%	7.33%	6.1%	8.86%
4	Default Rate (%, H, Cond. on Debt, Endog.)	-	0.03%	-	0.0%	0.63%	1.02%	2.12%	0.51%	1.66%
5	Default Rate (%, L, Cond. on Debt, Endog.)	-	-	-	-	-	-	-	-	-
6	H-Type Share of Debt	-	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%
7	H-Type Share of Debt Increase	-	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%
8	H-Type Share of Incumbents	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%
9	Agg. Growth	3.35%	3.36%	3.35%	3.37%	3.38%	3.41%	3.41%	3.41%	3.49%
10	Agg. Creative Destruction	0.164	0.1644	0.1642	0.165	0.1655	0.1669	0.1668	0.1669	0.1708
11	Incumbent Creative Destruction	0.1531	0.1533	0.151	0.1516	0.152	0.1532	0.1534	0.1448	0.1477
12	Entrant Creative Destruction	0.0109	0.011	0.0131	0.0133	0.0134	0.0136	0.0134	0.0222	0.0231
13	Avg. (Incumbent) Innov. Intensity	0.1623	0.1626	0.1601	0.1607	0.1616	0.1631	0.1655	0.1542	0.162
14	Avg. Innov. Intensity (H)	0.1623	0.1626	0.1601	0.1607	0.1616	0.1631	0.1655	0.1542	0.162
15	Avg. Innov. Intensity (L)	-	-	-	-	-	-	-	-	-
16	Avg. Innov. Intensity (H & Access)	-	0.1651	0.1627	0.1633	0.1643	0.1659	0.1686	0.1542	0.162
17	Avg. Innov. Intensity (H & No Access)	0.1623	0.1345	0.1363	0.1368	0.1373	0.1379	0.1385	-	-
18	Avg. Innov. Intensity (L & Access)	-	-	-	-	-	-	-	-	-
19	Avg. Innov. Intensity (L & No Access)	-	-	-	-	-	-	-	-	-
20	Debtor Share of Incumbents	-	59.86%	49.15%	57.98%	56.48%	55.61%	50.39%	99.6%	98.8%
21	Debtor Share of Incumbents (H)	-	59.86%	49.15%	57.98%	56.48%	55.61%	50.39%	99.6%	98.8%
22	Debtor Share of Incumbents (L)	-	-	-	-	-	-	-	-	-
23	Avg. Debtor Leverage	-	19.02%	7.56%	12.95%	29.03%	31.53%	57.98%	33.51%	60.22%
24	Avg. Debtor Leverage (H)	-	19.02%	7.56%	12.95%	29.03%	31.53%	57.98%	33.51%	60.22%
25	Avg. Debtor Leverage (L)	-	-	-	-	-	-	-	-	-
26	Avg. Spread	-	2.6%	1.03%	1.01%	1.77%	0.88%	1.39%	2.08%	1.07%
27	Avg. Spread (H)	-	2.6%	1.03%	1.01%	1.77%	0.88%	1.39%	2.08%	1.07%
28	Avg. Spread (L)	-	-	-	-	-	-	-	-	-

*Each row is colored to represent the relative magnitude of each statistic across frictions (i.e., colors are comparable only within each row).

Table 2: Equilibrium Objects Across Frictions (Low-Type Only)

#	Statistics	No Access	No Recoveries	No Tax Shield	No Unsecured	Baseline	No Bank Costs	No Haircut	Full Access	Bolstered
1	H-Type Share of Product Lines	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
2	L-Type Share of Product Lines	88.98%	88.98%	89.04%	89.07%	88.93%	88.03%	79.36%	89.12%	72.0%
3	AMC Share of Product Lines	11.02%	11.02%	10.96%	10.93%	11.07%	11.97%	20.64%	10.88%	28.0%
4	Default Rate (%, H, Cond. on Debt, Endog.)	-	-	-	-	-	-	-	-	-
5	Default Rate (%, L, Cond. on Debt, Endog.)	-	0.0%	-	0.0%	0.09%	0.58%	4.11%	0.04%	3.41%
6	H-Type Share of Debt	-	-	-	-	-	-	-	-	-
7	H-Type Share of Debt Increase	-	-	-	-	-	-	-	-	-
8	H-Type Share of Incumbents	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
9	Agg. Growth	1.63%	1.63%	1.63%	1.64%	1.65%	1.67%	1.64%	1.68%	1.72%
10	Agg. Creative Destruction	0.0796	0.0796	0.08	0.0805	0.0807	0.0819	0.0805	0.0824	0.0843
11	Incumbent Creative Destruction	0.0626	0.0623	0.0604	0.0604	0.0605	0.0612	0.058	0.0551	0.0511
12	Entrant Creative Destruction	0.0169	0.0172	0.0196	0.02	0.0201	0.0206	0.0226	0.0273	0.0331
13	Avg. (Incumbent) Innov. Intensity	0.0703	0.0701	0.0678	0.0679	0.068	0.0695	0.0731	0.0618	0.071
14	Avg. Innov. Intensity (H)	-	-	-	-	-	-	-	-	-
15	Avg. Innov. Intensity (L)	0.0703	0.0701	0.0678	0.0679	0.068	0.0695	0.0731	0.0618	0.071
16	Avg. Innov. Intensity (H & Access)	-	-	-	-	-	-	-	-	-
17	Avg. Innov. Intensity (H & No Access)	-	-	-	-	-	-	-	-	-
18	Avg. Innov. Intensity (L & Access)	-	0.0725	0.0701	0.0702	0.0704	0.0723	0.0765	0.0618	0.071
19	Avg. Innov. Intensity (L & No Access)	0.0703	0.0587	0.0585	0.0587	0.0588	0.0591	0.0634	-	-
20	Debtor Share of Incumbents	-	59.27%	53.07%	58.41%	57.98%	55.86%	43.52%	99.62%	99.0%
21	Debtor Share of Incumbents (H)	-	-	-	-	-	-	-	-	-
22	Debtor Share of Incumbents (L)	-	59.27%	53.07%	58.41%	57.98%	55.86%	43.52%	99.62%	99.0%
23	Avg. Debtor Leverage	-	20.56%	24.0%	23.79%	40.41%	57.46%	96.11%	49.11%	99.57%
24	Avg. Debtor Leverage (H)	-	-	-	-	-	-	-	-	-
25	Avg. Debtor Leverage (L)	-	20.56%	24.0%	23.79%	40.41%	57.46%	96.11%	49.11%	99.57%
26	Avg. Spread	-	3.7%	1.13%	1.01%	1.78%	1.25%	2.25%	2.22%	2.51%
27	Avg. Spread (H)	-	-	-	-	-	-	-	-	-
28	Avg. Spread (L)	-	3.7%	1.13%	1.01%	1.78%	1.25%	2.25%	2.22%	2.51%

*Each row is colored to represent the relative magnitude of each statistic across frictions (i.e., colors are comparable only within each row).