# Navigating Stormy Waters: Crises, Selection and Productivity Dynamics Under Financial Frictions

**Ufuk Akcigit**
UChicago

**Harun Alp**
FRB

**Ege Y. Ercan**
Stanford GSB

CEBRA 2022 Annual Meeting

---

The views expressed here are those of the authors and do not necessarily reflect those of the Board of Governors or the Federal Reserve System.

# Motivation

- Firms grow by borrowing and investing in risky projects to enhance their productivity.
- Research Questions
  - How do financial conditions affect firm dynamics and productivity growth?
  - How do firm heterogeneities interact with financials?

# What We Do

- Build a GE model of firm dynamics with
  - Endogenous productivity growth
  - Rich firm heterogeneity (types)
    - Crucial to match both size and age moments
  - Firms relying borrowing under financial frictions
    - default decision
    - endogenous interest rate
    - endogenous borrowing constraints

# What We Do

- Build a GE model of firm dynamics with
  - Endogenous productivity growth
  - Rich firm heterogeneity (types)
    - Crucial to match both size and age moments
  - Firms relying borrowing under financial frictions
    - default decision
    - endogenous interest rate
    - endogenous borrowing constraints
- Calibrate the model with micro data on US firms.

# What We Do

- Build a GE model of firm dynamics with
  - Endogenous productivity growth
  - Rich firm heterogeneity (types)
    - Crucial to match both size and age moments
  - Firms relying borrowing under financial frictions
    - default decision
    - endogenous interest rate
    - endogenous borrowing constraints
- Calibrate the model with micro data on US firms.
- Study productivity dynamics under a "crisis" shock.

# Preview of Results

- Firm heterogeneity is important in evolution of crises.
  - Crises hit harder but recovery is faster under heterogeneity.
  - It determines the extend of reallocation through bankruptcy.
- Efficiency of the bankruptcy process is also critical in crises.
  - If the bankruptcy process did not result in reallocation, the economy would not bounce back from the crisis.
- Uniform stabilization policies can depress long-term recovery.

# MODEL

# Market Structure

- The household consumes some final good, $Y$, aggregated with technology:

$$\ln(Y) = \int_{\mathcal{N}} \ln(y_j) dj, \tag{1}$$

- Each intermediate good is produced by one firm.

$$y_j = q_j l_j,$$

where $q_j$ is the productivity and $l_j$ is the labor hired.

# Market Structure

- The household consumes some final good, $Y$, aggregated with technology:

$$\ln(Y) = \int_{\mathcal{N}} \ln(y_j) dj, \tag{1}$$

- Each intermediate good is produced by one firm.

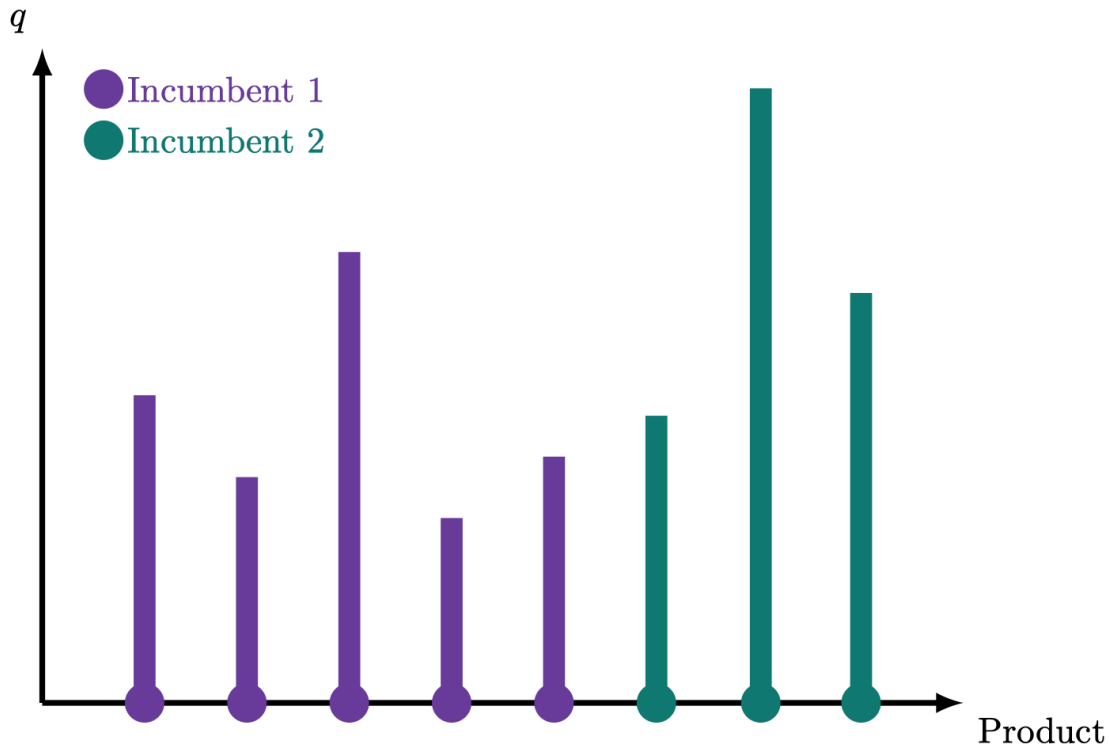
$$y_j = q_j l_j,$$

  where $q_j$ is the productivity and $l_j$ is the labor hired.

- A firm is a collection of production lines
  - Investment to capture other product lines by improving their productivity.
  - Shrinks due to other firms' investment.

# Snapshot of Competition I

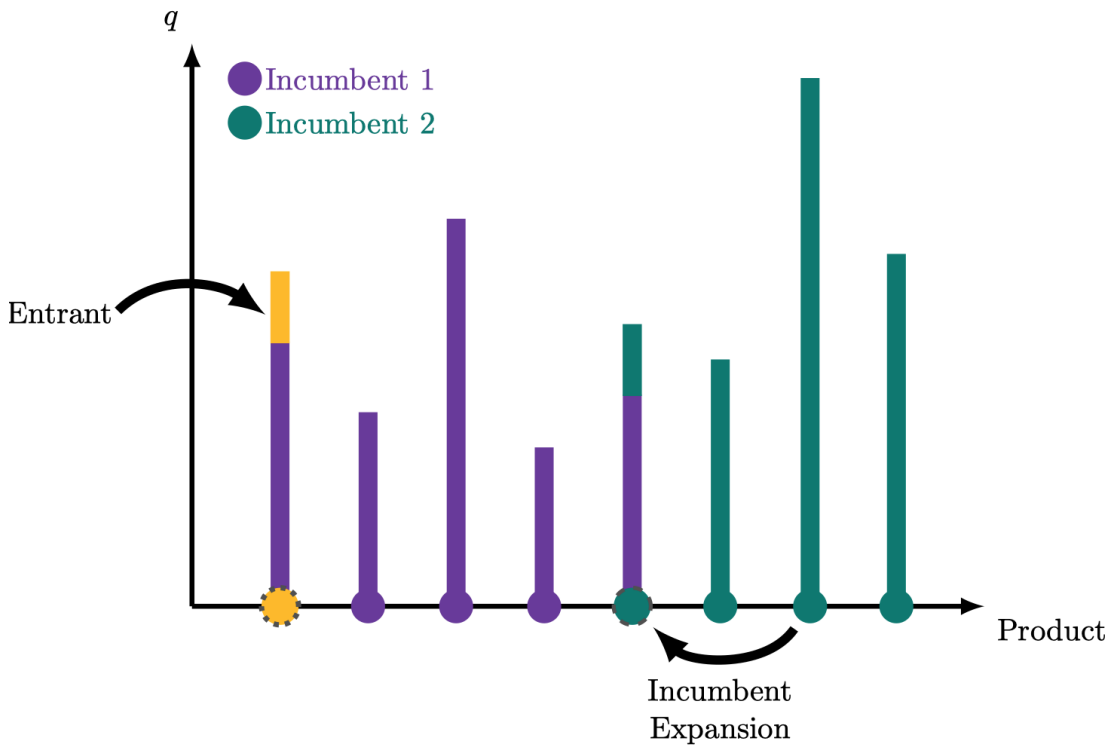
Figure 1: Before




- Two incumbents with cutting edge technology in 8 products.

# Snapshot of Competition II

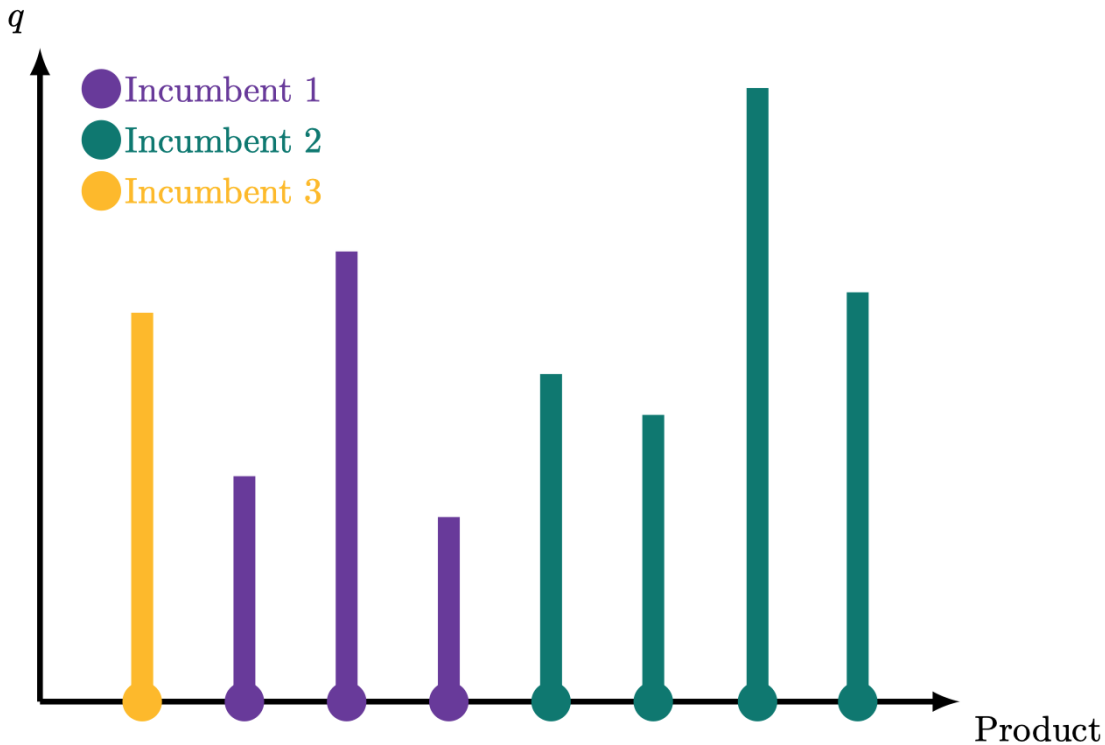
Figure 2: During




- Both **entrant** and **incumbent 2** innovate over **incumbent 1**.
- That is, **incumbent 1** loses two product lines.
- New technologies in these lines are **more productive**.

# Snapshot of Competition III

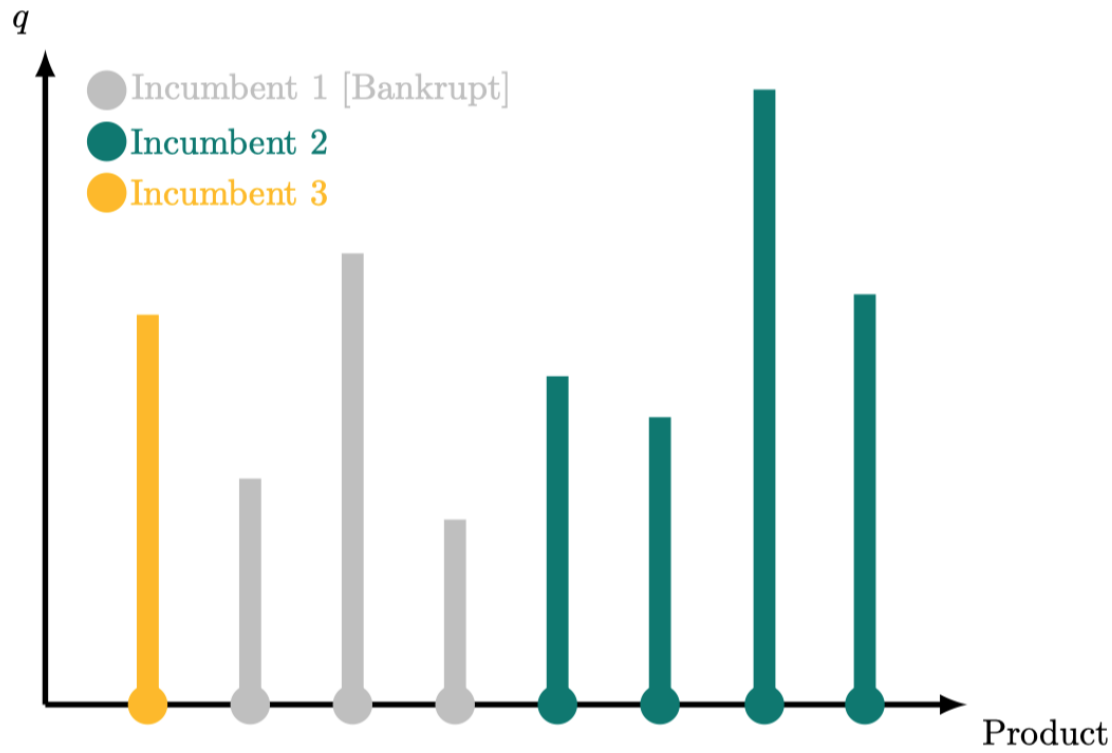
Figure 3: After (Solvent)




- After **turnover**, there are **three incumbents**.

# Snapshot of Competition IV

Figure 4: After (Insolvent)




- Sudden contraction can lead **incumbent 1** to **bankruptcy**.
- **Reallocation** of **bankrupt technologies** will be important.

# Incumbent Heterogeneity

**1) Size, $n \in \mathbb{N}$ (i.e., number of product lines)**

- Grows with firm innovation.
- Shrinks with the market's innovation $\tau \geq 0$.

# Incumbent Heterogeneity

**1) Size, $n \in \mathbb{N}$ (i.e., number of product lines)**

- Grows with firm innovation.
- Shrinks with the market's innovation $\tau \geq 0$.

**2) Investment Efficiency Type, $k \in H, L$ (i.e, high/low type)**

- Determines the firm's investment productivity (growth potential) $\theta_k > 0$.

# Incumbent Heterogeneity

**1) Size, $n \in \mathbb{N}$ (i.e., number of product lines)**

- Grows with firm innovation.
- Shrinks with the market's innovation $\tau \geq 0$.

**2) Investment Efficiency Type, $k \in H, L$ (i.e, high/low type)**

- Determines the firm's investment productivity (growth potential) $\theta_k > 0$.

**3) Debt $D \in \mathbb{R}$:**

- Financing with defaultable short-term debt instrument
- Heterogenous endogenous debt limits and interest rate spreads.

# Investments and Productivity

**Investment Efficiency Type, $k \in H, L$ (i.e, high/low type)**

- Determines the firm's innovation productivity, $\theta_k > 0$.
- $\alpha \in (0, 1)$ portion of firms are born as high types.
- High types transition into low types with arrival $\phi_{H \to L} > 0$.

# Investments and Productivity

**Investment Efficiency Type, $k \in H, L$ (i.e, high/low type)**

- Determines the firm's innovation productivity, $\theta_k > 0$.
- $\alpha \in (0, 1)$ portion of firms are born as high types.
- High types transition into low types with arrival $\phi_{H \to L} > 0$.

**Investments Cost Function:**

- Productivity flow rate $xn \geq 0$ costs

$$C(x) = \frac{1}{\theta_k} x^{\eta} n Y. \tag{2}$$

  where $\eta > 1$ is the inverse innovation elasticity.
- Undirected search for better technology.
- W/ arrival rate $xn \geq 0$, firm discovers a better technology.

# Firm Financials

## 1) Law of Motion for Debt:

$$\dot{D} = \varphi(p) - (1 - \tau_{tax}) \underbrace{[\pi n - c(x, \theta_k) - R(D, n, k)D]}_{\text{Before-Tax Net Income}}, \quad (3)$$

where

$$R(D, n, k) = r + \underbrace{\delta(D, n, k)}_{\text{spread}} \quad (4)$$

coming from the bank's problem and $p$ is payout.

# Firm Financials

## 1) Law of Motion for Debt:

$$\dot{D} = \varphi(p) - (1 - \tau_{tax}) \underbrace{[\pi n - c(x, \theta_k) - R(D, n, k)D]}_{\text{Before-Tax Net Income}}, \quad (3)$$

where

$$R(D, n, k) = r + \underbrace{\delta(D, n, k)}_{\text{spread}} \quad (4)$$

coming from the bank's problem and $p$ is payout.

## 2) Exit:

- The firm can voluntarily shutdown and default on the outstanding debt

$$V(\cdot) = \max\{0, V_{\text{Cont.}}(\cdot)\}. \quad (5)$$

# Firm Financials

## 1) Law of Motion for Debt:

$$\dot{D} = \varphi(p) - (1 - \tau_{tax}) \underbrace{[\pi n - c(x, \theta_k) - R(D, n, k)D]}_{\text{Before-Tax Net Income}}, \quad (3)$$

where

$$R(D, n, k) = r + \underbrace{\delta(D, n, k)}_{\text{spread}} \quad (4)$$

coming from the bank's problem and $p$ is payout.

## 2) Exit:

- The firm can voluntarily shutdown and default on the outstanding debt

$$V(\cdot) = \max\{0, V_{\text{Cont.}}(\cdot)\}. \quad (5)$$

- Firm exits when it loses all its product line (competition).
- At the rate $\varepsilon > 0$, the firm exits exogenously.

# Bank's Problem

- Competitive banks offering firm-specific loan menus:
  - Endogenous firm-specific interest spreads, $\delta(D, n, k)$.
  - Endogenous firm-specific debt limits, $\bar{D}(n, k)$.
- Debtor firms may utilize limited liability and default.

# Bank's Problem

- Competitive banks offering firm-specific loan menus:
  - Endogenous firm-specific interest spreads, $\delta(D, n, k)$.
  - Endogenous firm-specific debt limits, $\bar{D}(n, k)$.
- Debtor firms may utilize limited liability and default.
- Firms in default are resold through a bankruptcy process:
  - Firms have value due to their leading technology.
  - Banks resell them at a discount.
  - Bankruptcy process is resolved after some delay ($\psi$).

# Bank's Problem

- Competitive banks offering firm-specific loan menus:
  - Endogenous firm-specific interest spreads, $\delta(D, n, k)$.
  - Endogenous firm-specific debt limits, $\bar{D}(n, k)$.
- Debtor firms may utilize limited liability and default.
- Firms in default are resold through a bankruptcy process:
  - Firms have value due to their leading technology.
  - Banks resell them at a discount.
  - Bankruptcy process is resolved after some delay ($\psi$).
  - **Source of reallocation**
    - New firm has a lower debt.
    - New firm may have a higher growth potential ($\theta_k$).

# Incumbent's Value Function

$$
\begin{aligned}
rV(D,n,k) = \max\Bigg\{0, \max_{x,p}\Bigg\{ & pnY \\
& + \frac{\partial V}{\partial D}(D,n,k)\dot{D}(x,p;D,n,k) \\
& + xn\left[V(D,n+1,k) - V(D,n,k)\right] \\
& + \tau n\left[V(D,n-1,k) - V(D,n,k)\right] \\
& + 1_{k=H}\phi_{H\to L}\left[V(D,n,L) - V(D,n,H)\right] \\
& - \varepsilon V(D,n,k) \\
& + \partial_t[V(D,n,k)]\Bigg\}\Bigg\},
\end{aligned}
$$

s.t.

$$
\bar{D}(n,k) \geq D.
$$

# CALIBRATION

# Set-Up

- We calibrate the model to the US.
- 12 internally calibrated parameters with 17 moments calculated with "Federal Reserve Y-14" data and one moment calculated with Compustat.

## Loss Function

$$\sum_{i=1}^{12} \frac{w(i)|\,\text{model}(i) - \text{data}(i)|}{\frac{1}{2}|\,\text{model}(i)| + \frac{1}{2}|\,\text{data}(i)|}, \tag{6}$$

where $w(i)$ is the $i$th moment's weight.

# Data

- Supervisory FR Y-14 dataset (2013-2019)
  - Financial information for US firms
    - Not only public firms but also a large number of small and mid-sized private businesses.
  - More than 600,000 firm×year observations, around 190,000 unique firms.
  - Represent almost 80 percent of the US aggregate output (public firms account only around 40 percent).
- Compustat.
  - Payout information for large firms.

# Fit

## Table 1: Moments

| # | Moment | Data | Model |
|---|---|---|---|
| 1 | Payout Mean (+, Very Large) | 0.081 | 0.061 |
| 2 | Interest Spread (Small Size-Low Leverage) | 0.014 | 0.012 |
| 3 | Interest Spread (Small Size-High Leverage) | 0.018 | 0.023 |
| 4 | Interest Spread (Large Size-Low Leverage) | 0.013 | 0.012 |
| 5 | Interest Spread (Large Size-High Leverage) | 0.015 | 0.018 |
| 6 | Default Rate (Small Size-Low Leverage) | 0.026 | 0.047 |
| 7 | Default Rate (Small Size-High Leverage) | 0.049 | 0.061 |
| 8 | Default Rate (Large Size-Low Leverage) | 0.018 | 0.007 |
| 9 | Default Rate (Large Size-High Leverage) | 0.030 | 0.049 |
| 10 | Leverage (Small Size-Low Leverage) | 0.090 | 0.130 |
| 11 | Leverage (Small Size-High Leverage) | 1.540 | 1.502 |
| 12 | Leverage (Large Size-Low Leverage) | 0.097 | 0.161 |
| 13 | Leverage (Large Size-High Leverage) | 0.997 | 0.945 |
| 14 | Growth (Small Size-Low Leverage) | 0.050 | 0.048 |
| 15 | Growth (Small Size-High Leverage) | 0.148 | 0.116 |
| 16 | Growth (Large Size-Low Leverage) | -0.000 | -0.001 |
| 17 | Growth (Large Size-High Leverage) | 0.019 | 0.009 |
| 18 | Aggregate Growth | 0.022 | 0.022 |

*Large/Small firms are divided w.r.t. the median size.
**High/Low leverage firms are divided w.r.t. the median leverage, defined as debt to sales ratio.
***Very Large firms are the top 10% by size.

# Parameters

## Table 2: External Calibration

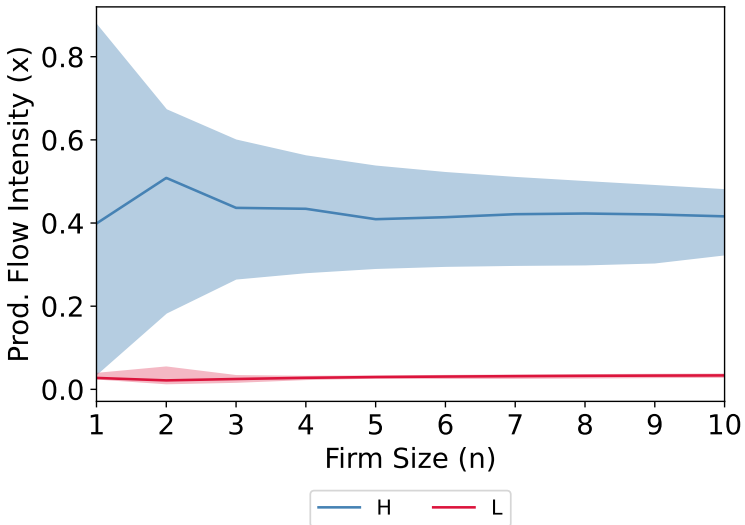
| # | Parameter | Description | Value |
|---|---|---|---|
| 1 | $\rho$ | Discount Rate | 0.02 |
| 2 | $\vartheta$ | Inverse Intertemp. Elas. of Subs. | 2.00 |
| 3 | $\eta$ | Inverse Innovation Elasticity | 2.00 |
| 4 | $\tau_{tax}$ | Tax Rate | 0.35 |
| 5 | $\psi$ | Default Resolution Rate | 0.46 |

## Table 3: Internal Calibration

| # | Parameter | Description | Value |
|---|---|---|---|
| 1 | $\kappa$ | Payout Penalty Scale | 3.605 |
| 2 | $\chi$ | Fixed Cost | 0.246 |
| 3 | $\mu_0$ | Fixed Recovery | 2.735 |
| 4 | $\xi$ | Buyer's Leverage | 0.247 |
| 5 | $\nu_0$ | Banking Costs | 0.012 |
| 6 | $\theta_H$ | H-Type Innovation Productivity | 0.387 |
| 7 | $\theta_L$ | L-Type Innovation Productivity | 0.032 |
| 8 | $\theta_E$ | Entrant Innovation Productivity | 0.015 |
| 9 | $\lambda$ | Innovation Step Size | 1.287 |
| 10 | $\alpha$ | Share of H-Types in Entry | 0.500 |
| 11 | $\phi_{H\to L}$ | Transition Rate into L-Type | 0.267 |
| 12 | $\varepsilon$ | Death Shock | 0.002 |

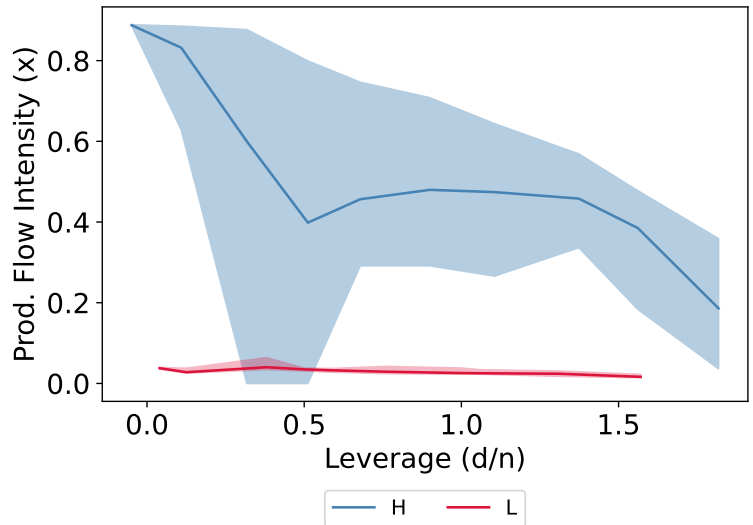
# Investment by Firm Size and Leverage

(a) Investment Across Size




*Shaded regions represent middle 90% of distributions

(b) Investment Across Leverage




*Shaded regions represent middle 90% of distributions

- Healthy heterogeneity in investments across size, type, and leverage.
- With debt accumulation, investments start falling again (debt overhang).

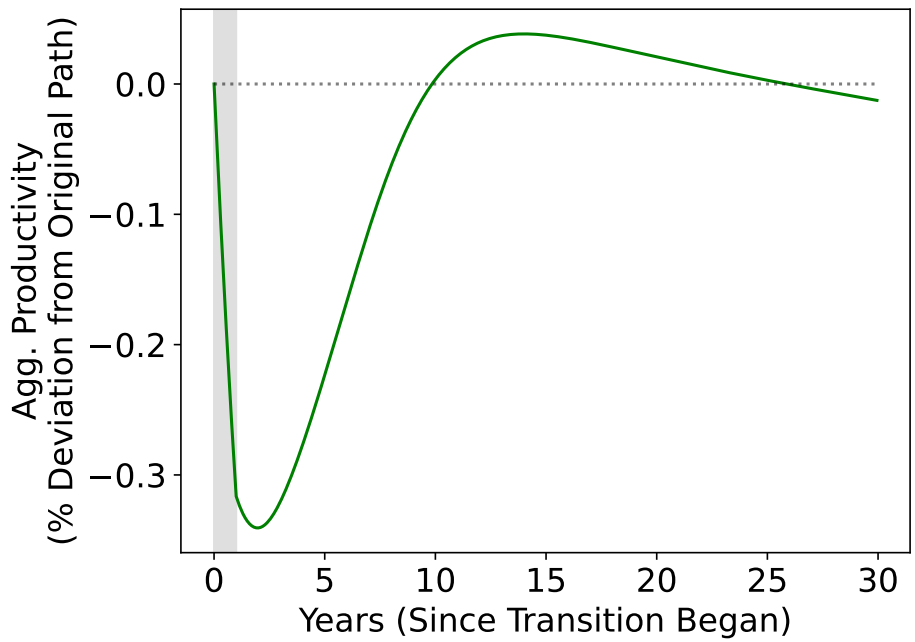
# CRISIS

# Crisis

- **Unanticipated Shock**: A zero probability event that occurs unexpectedly, but once it occurs, transition evolves deterministically.
- **Duration and Magnitude:** 33% decrease in gross profit margin for one year.
- **What we study:**
  - Effects of Bankruptcy on Reallocation
  - Effects of Firm Heterogeneity
  - Effects of Stabilization Policies

# Productivity Loss

Figure 6: Deviation from Original Path




- Decline in productivity growth attributable to:
  - Mass bankruptcies.
  - Difficulty in expending resources on investments during crises.
  - Diminished value of expanding to new products.

# Survival and Leverage

Figure 7: Survival

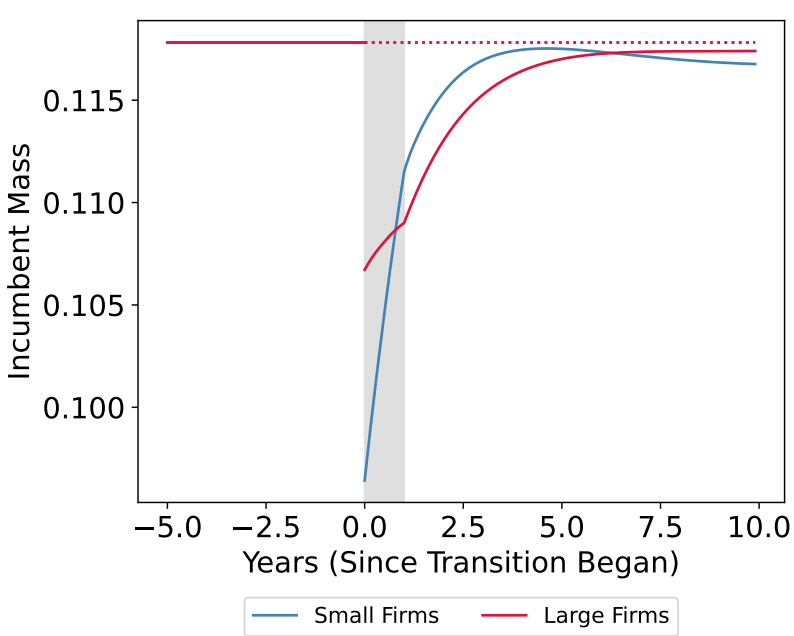
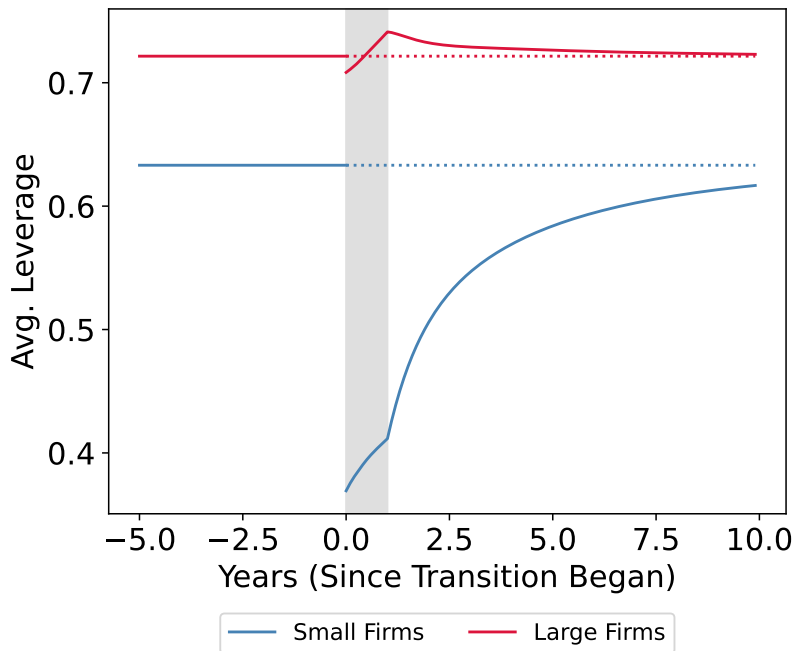



Figure 8: Leverage




- Small firms are driven into bankruptcy in large volumes.
- In the wake of bankruptcies, surviving small firms are less levered due to rising interest rates and decreasing debt limits.

# Incumbent Investments

Figure 9: Incumbent Investment

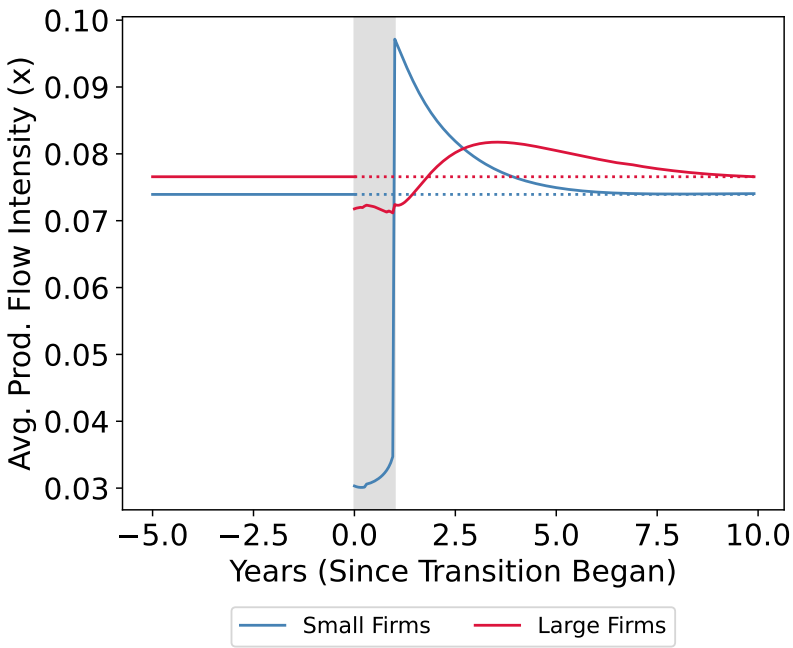
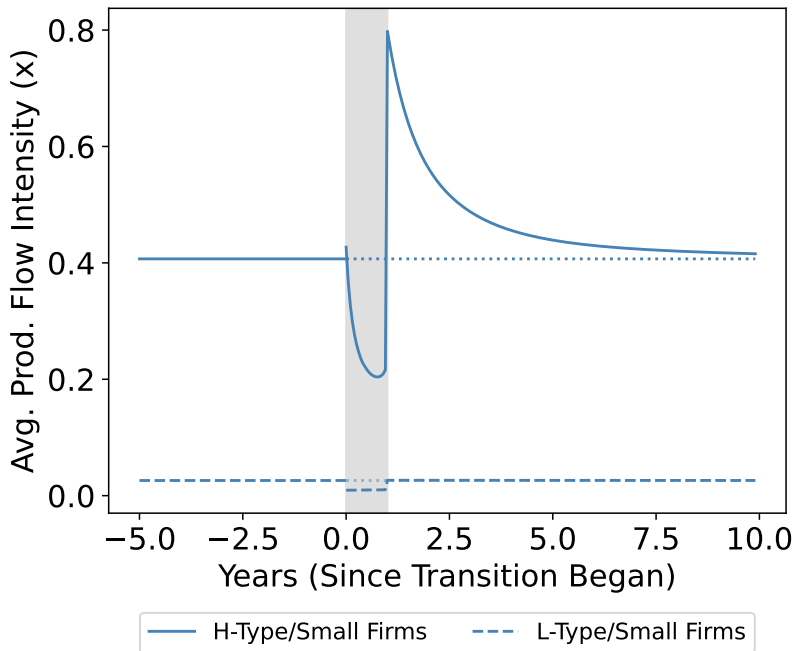



Figure 10: By Type




L Reduction in small firm's investments cause the initial dip.

L Composition Effect: Small low types are acquired by new high-type entrepreneurs on average. Recovery is fueled by small firms.

R Within small firms, high-type firms bolster growth.

# Bankruptcies and Rebound

Figure 11: Growth Decomposition

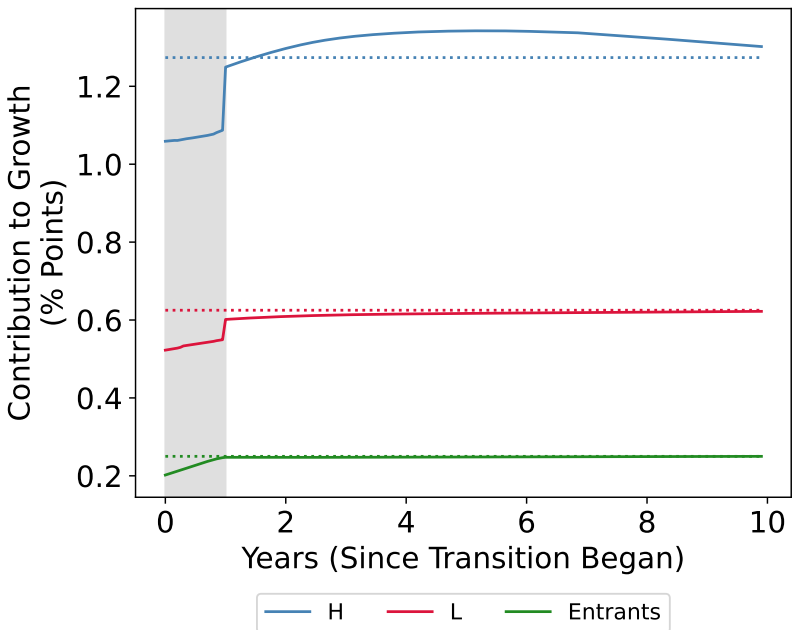
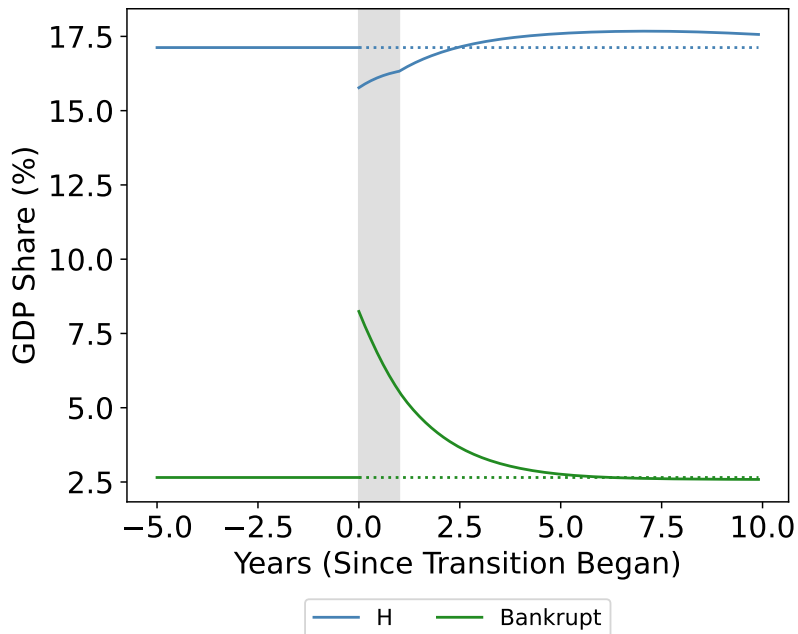



Figure 12: Selection

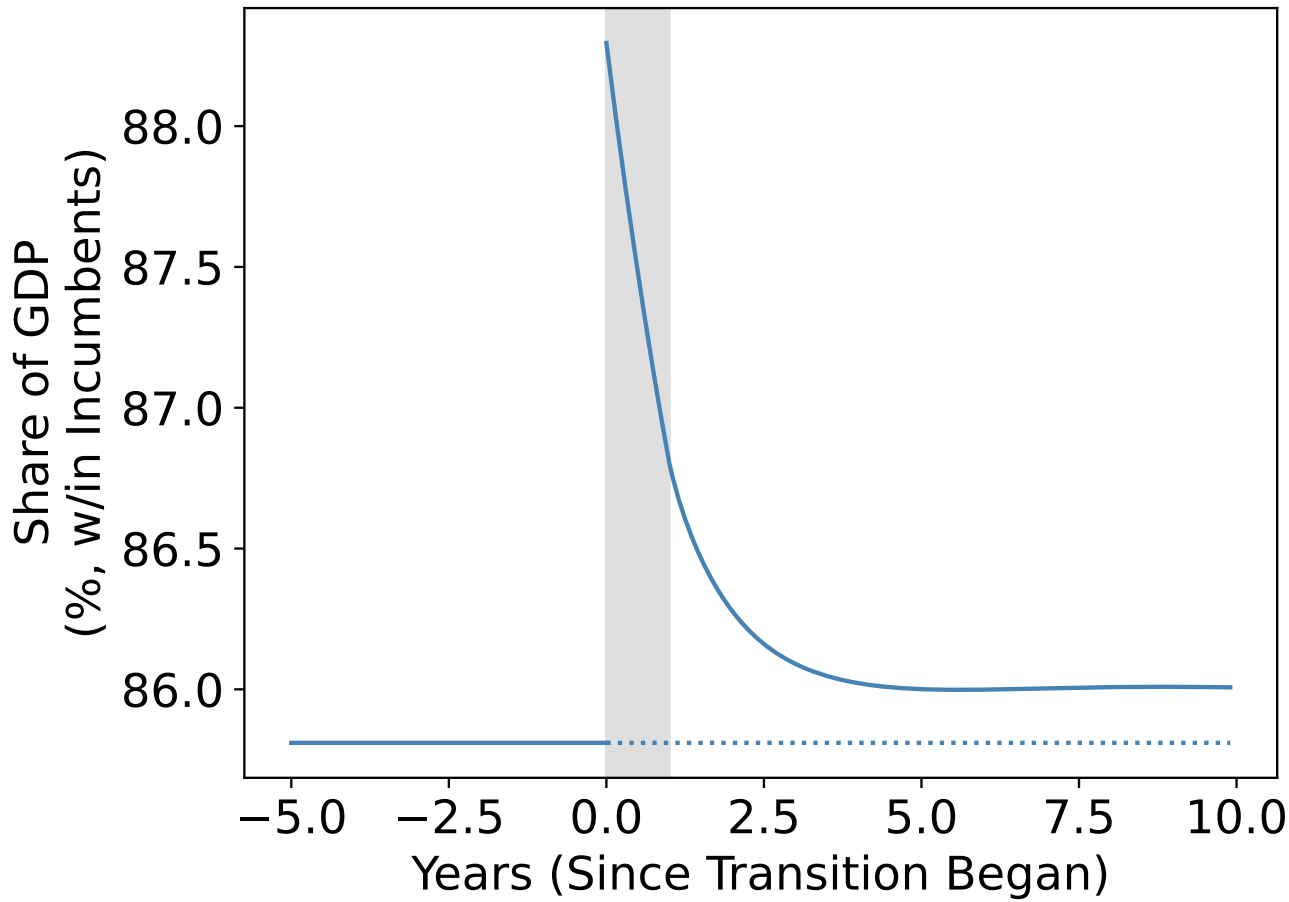



L Low-types tend to go bankrupt more.

R Recovery is driven by high-types born out of bankruptcies.

# Concentration

Figure 13: Top 50%

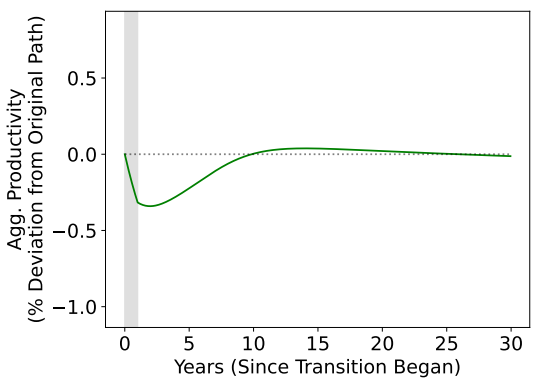



- Default of small firms lead to greater market concentration.

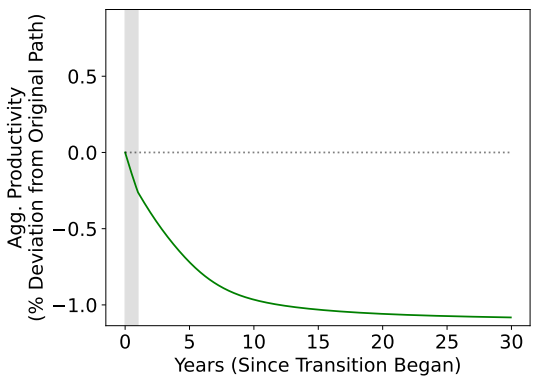
# Bankruptcy Reallocation I

Figure 14: Productivity and Reallocation

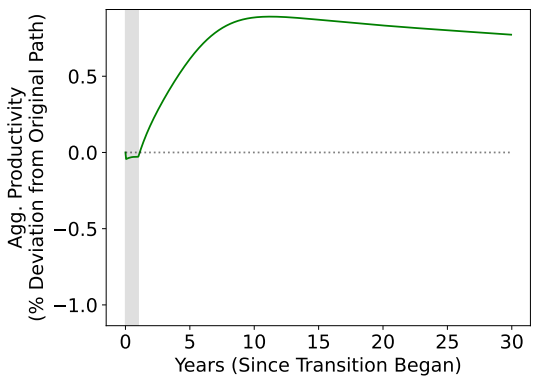
(a) Baseline




(b) No Resolution ($\psi = 0$)




(c) Instant Resolution ($\psi = \infty$)

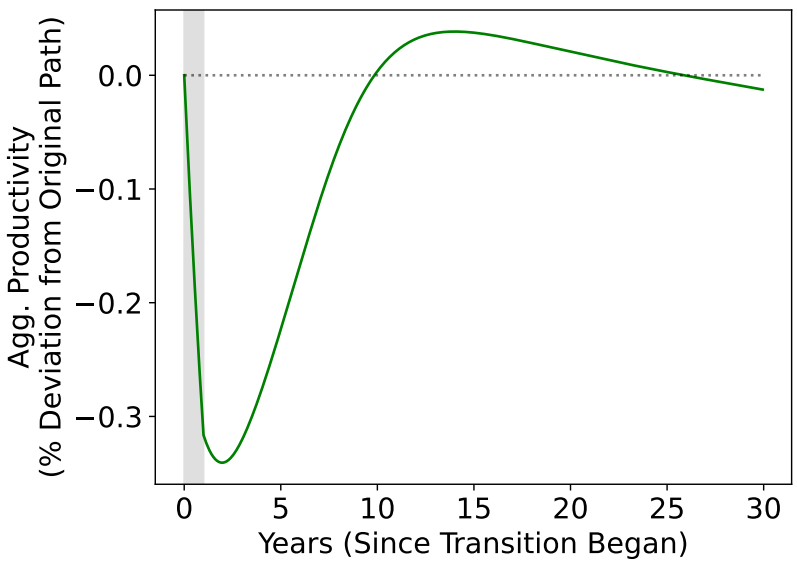



- W/out active reallocation, productivity is permanently slashed.
- Speed of resolution can have profound effects during crises.

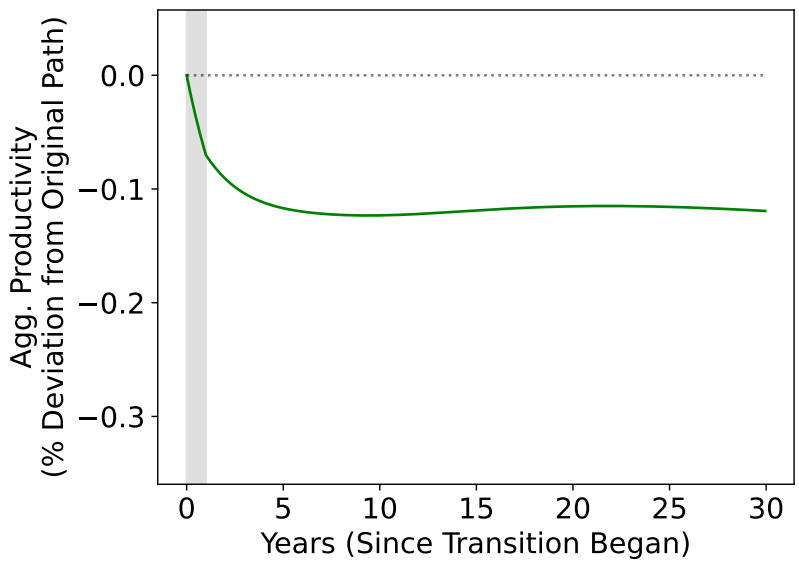
# Stabilization Policies I

Figure 16: Productivity and Interest Rate Spreads

(a) No Intervention



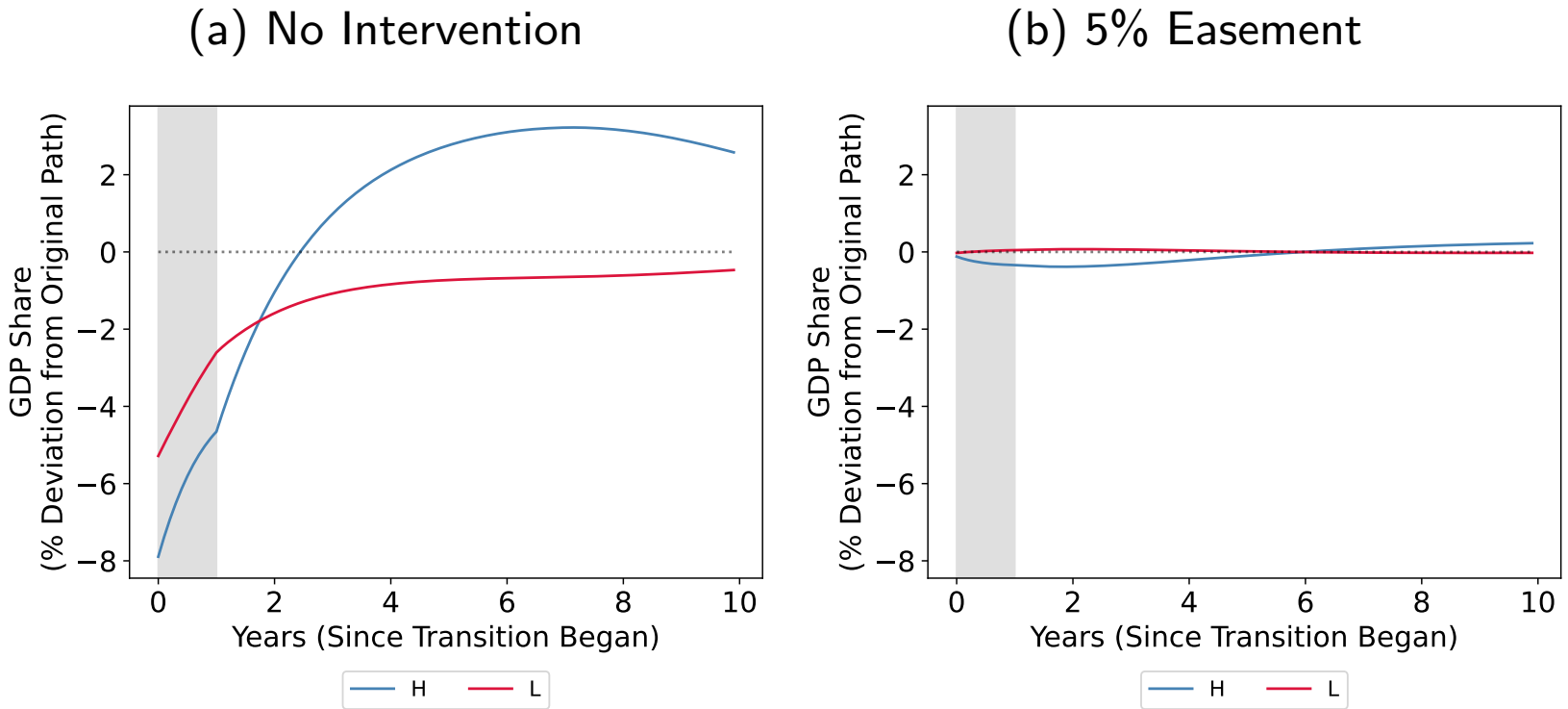

(b) 5% Easement




- Stabilization can couch the economy in the short-run.
- But, it may rob the economy of its robust recovery.

# Stabilization Policies II

Figure 18: Selection and Interest Rate Spreads




- Over stabilization kills positive selection by preventing the bankruptcy of weaker (low-type) firms.

# Conclusion

- Heterogeneity of the economy is important during crises.
- Efficiency of the bankruptcy process is also critical.
- Debt can cause mass bankruptcies.
- Stabilization policies can depress long-term recoveries.
- Stabilization is more important when the bankruptcy process is ineffective.

# APPENDIX

# Competition

- Within each product-line, firms compete à la Bertrand.
- Price competition eliminates all but the most productive firm.
- Price is set at the second-most productive firm's marginal cost (limit pricing).
- Holding the leading technology generates the following profit:

$$\Pi = \pi Y = \frac{\lambda - 1}{\lambda} Y, \tag{7}$$

where $\lambda > 1$ represents how much more productive the leading technology is compared to its closest challenger.

# Payouts to Shareholders

- The firm's payouts to its shareholders drives its value.
- In determining payout policy, the firm faces financial rigidities.

- Based on equity adjustment costs in Jermann and Quadrini (2012), to issue payout $pnY \in \mathbb{R}$, the firm expends

$$\varphi(p) = pnY + \underbrace{\kappa n p^2 Y}_{\text{Rigidity}}, \tag{8}$$

where $\kappa > 0$ is the penalty scale.

# Interest Rate Menu

- The bank is risk-neutral and prices its loans competitively.
- The equilibrium firm-specific interest rate spread is

$$\delta^*(D,n,k) = \underbrace{\nu}_{\substack{\text{Banking}\\ \text{Costs}}} + \underbrace{\mathbb{E}\left[\Phi_D(D,n,k)\right]}_{\substack{\text{Expected Losses}\\ \text{from Bankruptcy}}}. \tag{9}$$

- However, if the firm defaults with certainty, the bank is unwilling extend a loan at any premium, implying limit

$$\bar{D}(n,k) = \max\{D : V(D,n,k) \geq 0\}. \tag{10}$$

Bank's Profits

# Household

- The representative household has CRRA preferences over consumption:

$$U_0 = \int_0^\infty e^{-\rho t} \frac{C_t^{1-\vartheta} - 1}{1-\vartheta} dt, \tag{11}$$

  where $\rho$ is the discount factor and $C$ is consumption.
- The standard Euler equation determines the risk-free interest rate, $r$,

$$\frac{\dot{C}}{C} = \frac{r-\rho}{\vartheta}. \tag{12}$$

*All normalized by $Y$.

# Entrant's Problem

- Mass one of homogeneous potential entrants.
- Upon entry, the entrant has no debt, holds one product line, and draws a type.
- Chooses innovation $z \geq 0$ to maximizes its expected value:

$$z\left[m_{E,H}V(0,1,H) + (1-m_{E,H})V(0,1,L)\right] - \frac{1}{\theta_E}z^{\eta}Y. \qquad (13)$$

Optimal Decision

# Optimal Firm Policy

## 1) Optimal Innovation Rate:

$$x^* = \left( \frac{\theta_k}{\eta} \frac{w(n+1,k) - w(n,k)}{\psi_2} \right)^{\frac{1}{\eta-1}}. \tag{14}$$

## 2) Optimal Payout:

$$p^* = -\frac{1}{2\kappa} \left[ 1 + \frac{1}{\psi_2} \right]. \tag{15}$$

*All normalized by $Y$.

Back to Firm Value

# Optimal Entrant Policy

- The innovation rate, $z \geq 0$, that maximizes entrant value,

$$z\left[m_{E,H}W(0,1,H) + (1-m_{E,H})W(0,1,L)\right] - \frac{1}{\theta_E}z^{\eta}Y, \quad (16)$$

- is

$$z^* = \left(\frac{\theta_E}{\eta}\left[m_{E,H}W(0,1,H) + (1-m_{E,H})W(0,1,L)\right]\frac{1}{Y}\right)^{\frac{1}{\eta-1}}. \quad (17)$$

Back to the Entrant's Problem

# Why Might Incumbents Want Debt?

- Equity Rigidity
- Tax Benefit

Back to Incumbent and Debt

# Bank's Profits

- $E[\Phi_D(D,n,k)] \geq 0$ represents the expected ratio of unrecovered funds over the loan's principal.
- The bank expends $\nu > 0$ proportional to the loans it extends.
- The expected profit from loaning $D$ to firm $(n,k)$ is

$$\begin{aligned}\pi_B(D,n,k) = & \underbrace{D(1+r+\delta(D,n,k))}_{\text{Debtor Firm's Obligations}} \\ & - \underbrace{D\mathbb{E}[\Phi_D(D,n,k)]}_{\text{Expected Unrecovered Bankruptcies}} \\ & - \underbrace{D(1+r)}_{\text{Bank's Obligations}} - \underbrace{D\nu}_{\text{Banking Costs}} .\end{aligned} \tag{18}$$

Back to the Interest Rate Menu

# Bankruptcy Recoveries

- Let $1_\phi$ indicate whether the firm defaults due to a type shock
- Let $1_\tau$ indicate whether the firm defaults due to creative destruction.
- Then, the ratio of unrecovered funds is

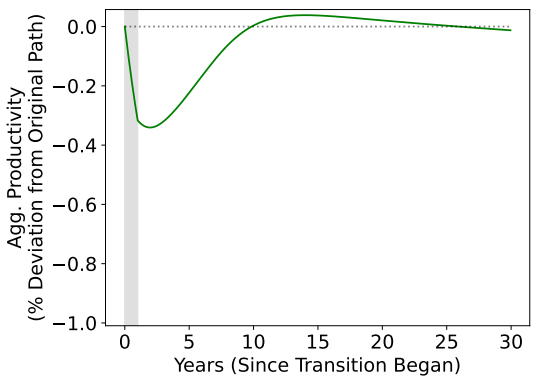
$$\mathbb{E}[\Phi_D(D,n,k)] = (\varepsilon + \phi_{H\to L}1_\phi)\underbrace{\left[\frac{D-\max\{\mu_0, V_R(n)\}}{D}\right]^+}_{\text{Losses at Size } n} + \tau n 1_\tau \underbrace{\left[\frac{D-\max\{\mu_0, V_R(n-1)\}}{D}\right]^+}_{\text{Losses at Size } n-1}, \tag{19}$$

where $[]^+$ stands for the positive part of the expression.

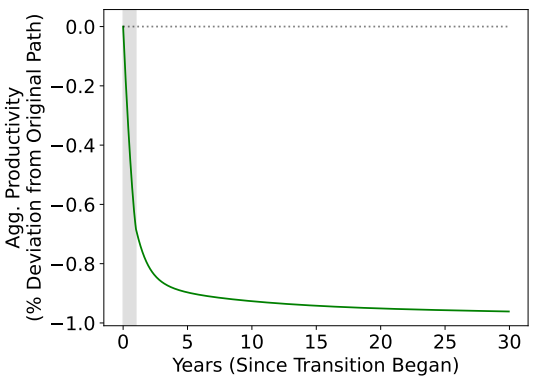
# Types I

Figure 20: Productivity and Types

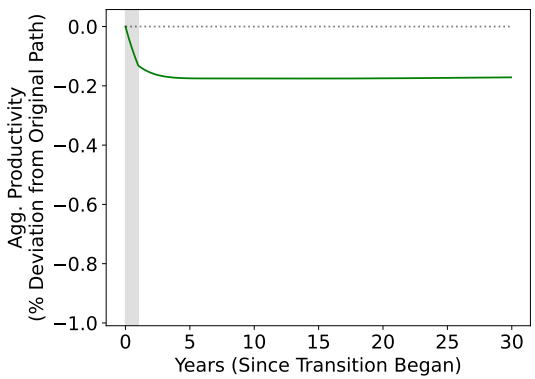
(a) Heterogeneous




(b) H-Only




(c) L-Only

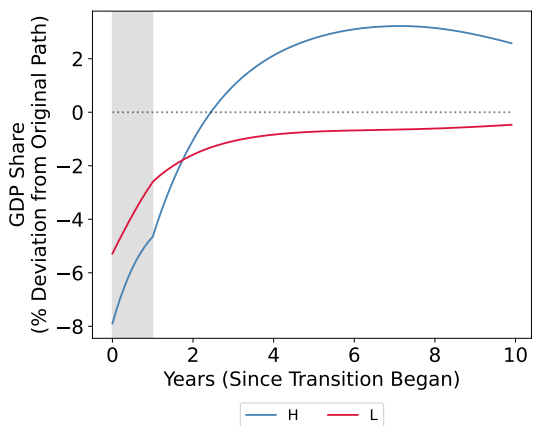



- Homogenous economies are permanently hurt.
- Type reallocation in bankruptcy is critical for recovery.

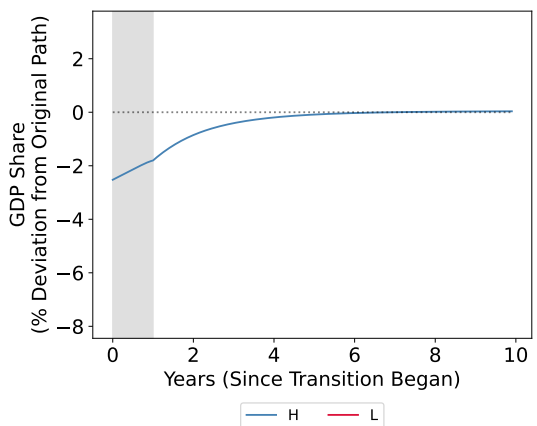
# Types II

Figure 22: Selection and Types

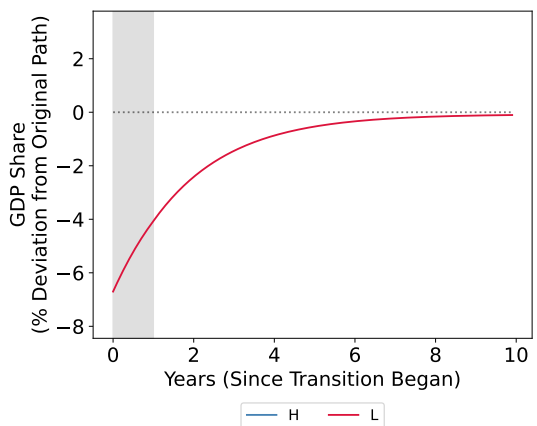
(a) Heterogeneous




(b) H-Only




(c) L-Only



# Short-Term v Medium-Term

- **Bankruptcies are socially costly in the short-run:**
  - Firms are sidelined despite their cutting-edge technology.
  - Firms do not contribute to productivity during bankruptcy.

- **Bankruptcies can improve composition in the medium-run:**
  - Lower type and higher leverage firms tend to default.
  - Bankrupt resources can be reallocated to better firms.
  - **Speed** and **effectiveness** of bankruptcy process is critical.