# LINEAR ALGEBRA

Sonu Memon's NoteBook

ABDUL HASEEB MEMON

Core Mathematical Concepts for ML, DL, Agentic AI & Robotics

**------ Robotics & Agentic Ai Enthusiast ------**

# LINEAR ALGEBRA

Linear Algebra is the branch of mathematics that studies vectors, matrices, linear equations, and linear transformations, and how these objects behave in vector spaces.

**Linear** → Straight

**Algebra** → Restoration / Completion (Arabic Word الجبر)

**Algebra:** In Simple Terms the combination of Symbols, Letters, and numbers that find Unknown Values Called Algebra as Algebra itself a Process of completing what is Missing and Incomplete So, the technique itself Sum up the whole Concept of Algebra.

So, Linear means something that moves Linearly in a Straight way, and the behavior of that thing should be Linear & Directed.

## Basic Characteristics

**Expression:** A combination of numbers, variables, and operations without an equals sign.

**Variable:** A symbol (like x or y) that represents an unknown or changeable value.

**Coefficient:** The numerical factor multiplied by a variable (e.g., in 5x, the coefficient is 5).

**Constant:** A fixed value that does not change (e.g., 3, –7, 10).

**Equation:** A mathematical statement that shows two expressions are equal using an equals sign (=).

## Basic Terms

**Coordinates:** A pair of numbers that locates a point on a plane (like (x, y)).

**Graph:** A visual representation of data, equations, or relationships on axes.

**Points:** Exact locations in space with no size or dimension.

**Line:** A straight path extending forever in both directions. **OR (**distance b/w two points**)**

**Shape:** A closed figure formed by points and lines (like triangle, square).

**Axis:** A reference line used to plot points (x-axis, y-axis).

**Plane:** A flat surface extending infinitely in all directions.

**Origin:** The point (0, 0) where x-axis and y-axis meet.

**Slope:** A number showing how steep a line is.

**Distance:** The length between two points.

**Midpoint:** The exact middle point between two points.

**Angle:** The space between two rays/lines meeting at a point.

**Ray:** A line that starts at one point and extends forever in one direction.

# Linear Equation

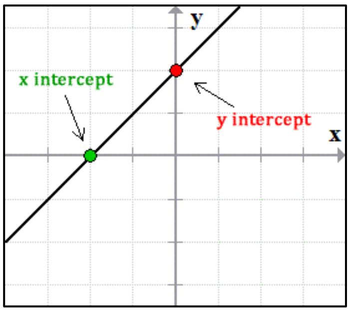
Linear Equation is the type of equation who has highest degree/power/index/exponent is equal to 1.

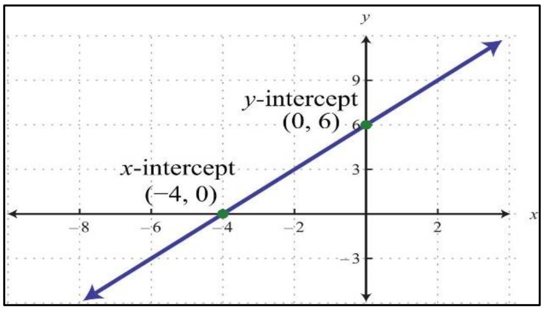
**Eg:** $x^1 + 2 = 5$ therefore, x (degree = 1). So, its Linearly behaves.

## Representation of Linear Equation on a Graph:

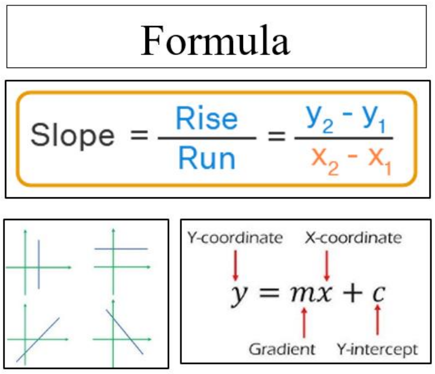
- **Slope Intercept Form: y = mx + c**
  - It is used to represent the straight line but cannot represent a Vector Line.
  - It is Preferred for Graphing only, not for Accurate or Specified Calculation.
  - It does not Extend to higher dimension from lower (like 2D to 3D).
  - Tilt ($0^o$ → Horizontal Line, ($0^o$ to $90^o$ but not = = 90) → Normal Slanted Tilt).

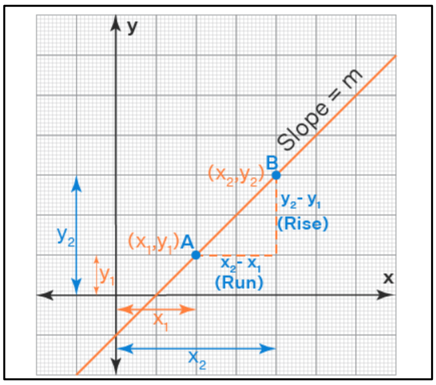
1. **Slope X- Intercept:** y = 0 for all x - axis values.
2. **Slope Y- Intercept:** x = 0 for all y – axis values.




- **Slope/ Gradient:** rise over run of a graph (how much graph is steeper or gradient).




- **Properties of m or slope:**
  - m > 0 → line go upward.
  - m < 0 → line go downward
  - m = 0 → line is a flat
  - m is large → line is steep
  - m is small → line is gentle

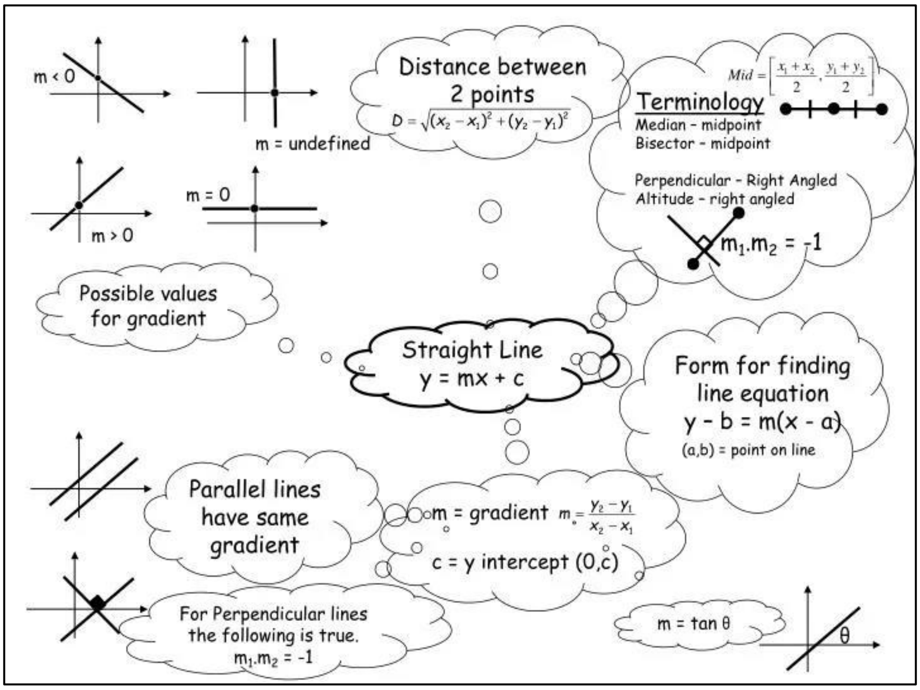
- **General Form: $ax + by + c = 0$**
    - It is used to represent the straight line as well as a Vector Line.
    - It is Preferred for higher Calculations + Graphing
    - It extends to higher Dimensions to Perform Kernel transformations.
    - It measures 2D, 3D, Arbitrary Dimension (Gaussian Surface) & Vertical Lines. Also, for SVM Classifier boundaries, Linear Layers in Neural Networks.

## CONCEPTUAL FRAGMENTS

Coordinates ≠ Object / Points Coordinates(x,y,z) → Dimensions, object → Locations

**Eg:**

| HOUSE A | | HOUSE B |
|---|---|---|
| Street= 05<br>City= Karachi<br>Country= Pakistan | ———— | Street= 04<br>City= Nawabshah<br>Country= Pakistan |



## COORDINATE GEOMETRY

Coordinate geometry, also known as analytic geometry or Cartesian geometry, is a branch of mathematics that links geometry and algebra by using a coordinate system to describe the position of points and shapes numerically. This allows for geometric problems to be solved using algebraic equations and formulas.

**I. Distance b/w two Points:**

By Pythagoras Theorem

$H^2 = B^2 + P^2$

$H = \sqrt{(B^2 + P^2)}$

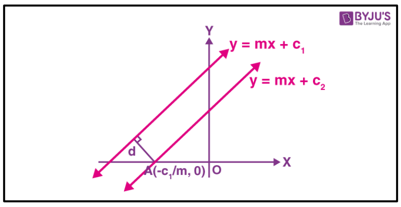
$H = \sqrt{((x_2 - x_1)^2 + (y_2 - y_1)^2)}$

**II. Slope b/w two Lines:**

**a.** If Lines are **Parallel:** here m (slope will be same just intercepts changes)

$Y = mx + c_1$

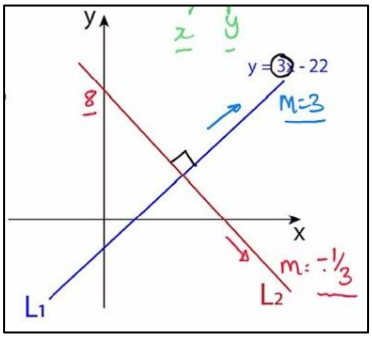
$Y = mx + c_2$




**b.** If Lines are **Perpendicular:** Here m & C both changes.

$Y = mx_1 + c_1$

$Y = mx_2 + c_2$



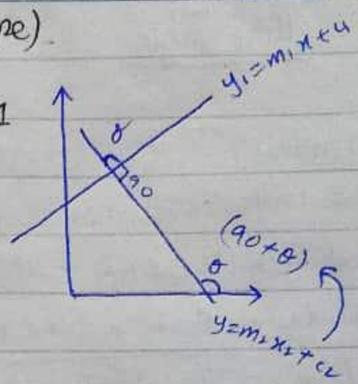
**Prove $m_1 . m_2 = -1$** from y=mx+c equation **(Hassi – ML journey)**

Two Perpendicular lines (slope)

↳ $m_1 \cdot m_2 = -1$ (Proof)

$y = m_1 x_1 + c_1$

$y = m_2 x_1 + c_2$




$\tan \theta = m_1$

$\tan 90 + \theta = m_2$

$$\tan 90 + \theta = \frac{\sin 90 + \theta}{\cos 90 + \theta}$$

Hassi - ML

$\therefore \sin(90+\theta) = \cos\theta$

$\therefore \cos(90+\theta) = -\sin\theta$

Putting values

$$\tan 90 + \theta = \frac{\cos\theta}{-\sin\theta}$$

$$\therefore \frac{1}{\tan} = \cot\theta$$

$\tan 90 + \theta = -\cot\theta$

↓

$m_2 = -\cot\theta$

$$m_2 = -\frac{1}{\tan\theta}$$

as $\tan\theta = m_1$

so,

$$m_2 = -\frac{1}{m_1} = \boxed{m_2 \cdot m_1 = -1} \rightarrow \text{Proof} ✓$$

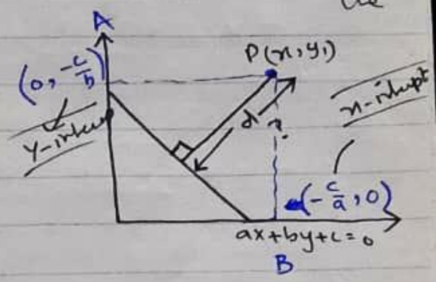
**Prove distance (perpendicular) from a point** **(Hassi – ML journey)**

$$d = \frac{ax + by + c}{\sqrt{a^2 + b^2}}$$ → **ax+by+c=0**

$$d = \frac{|mx_0 - y_0 + c|}{\sqrt{m^2 + 1}}$$ → **y= mx+c**

Distance b/w two Perpendicular lines or point & perp: line

Formula: $\boxed{d = \frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}}$




Derivation:

Here we calculate Area of triangle so,

$\Delta A = \frac{1}{2}(\text{Base} \times \text{Height})$

Height $= d$

Base $= A \to B$ so,

↳ $AB = \sqrt{\left(\frac{c}{b}\right)^2 + \left(\frac{c}{a}\right)^2}$

$= \frac{c^2}{b^2} + \frac{c^2}{a^2}$

$= \sqrt{c^2\left(\frac{a^2 + b^2}{a^2 b^2}\right)}$

$\boxed{AB = c\,\frac{\sqrt{a^2+b^2}}{ab}}$ → Base

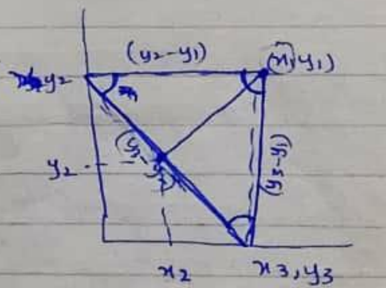
$\sqrt{y^2 + x^2}$

$\sqrt{\left(\frac{c}{b}\right)^2 + \left(\frac{c}{a}\right)^2}$

$\sqrt{(y_2-y_1)^2 + (x_2-x_1)^2}$

$\left(\frac{c}{b} - 0\right)^2 + \left(0 + \frac{c}{a}\right)^2$

$\sqrt{\left(\frac{c}{b}\right)^2 + \left(\frac{c}{a}\right)^2}$




$= \frac{1}{2}\left(d \times \frac{c\sqrt{a^2+b^2}}{ab}\right)$

$\Delta = \frac{1}{2}\left|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right|$

$\cancel{\tfrac{1}{2}}\, d \times \frac{c\sqrt{a^2+b^2}}{ab} = \cancel{\tfrac{1}{2}}\left|x_1\left(-\frac{c}{b} - 0\right) + 0(x) + \left(-\frac{c}{a}\left(y_1 - \left(-\frac{c}{b}\right)\right)\right)\right|$

$\frac{d \times c\sqrt{a^2+b^2}}{ab} = -\frac{cx_1}{b} - \frac{cy_1}{a} - \frac{c^2}{ab}$

$$d \times \frac{c\sqrt{a^2+b^2}}{ab} = \frac{-cx_1}{b} - \frac{cy_1}{a} - \frac{c^2}{ab}$$

$$d \times \frac{c\sqrt{a^2+b^2}}{ab} = \frac{-acx_1 - bcy_1}{ab} - \frac{c^2}{ab}$$

$$= \frac{-c(ax_1+by_1)}{ab} - \frac{c^2}{ab}$$

$$d \times \frac{c\sqrt{a^2+b^2}}{ab} = \frac{-c(ax_1+by_1) - c^2}{ab}$$

$$d \times \frac{\cancel{c}\sqrt{a^2+b^2}}{ab} = -\cancel{c}\left(\frac{1(ax_1+by_1)+c}{ab}\right)$$

$$d \times \sqrt{a^2+b^2} = \frac{ax_1+by_1+c}{\cancel{ab}} \times \cancel{ab}$$

$$\boxed{d = \frac{ax_1+by_1+c}{\sqrt{a^2+b^2}}}$$

→ Derived (Proof)!!

**Prove distance b/w two parallel Lines** **(Hassi – ML journey)**

$$d = \frac{|c_2 - c_1|}{\sqrt{1+m^2}}$$ → **Y= mx+c**

$$d = \frac{|c_1 - c_2|}{\sqrt{a^2+b^2}}$$ → **ax+by+c=0**

$y = mx_0 + c_1$

$y = mx_0 + c_2$

$y_2 - y_1 = (mx_0 + c_2) - (mx_0 + c_1)$

$y_2 - y_1 = \cancel{mx_0} + c_2 - \cancel{mx_0} + c_1$

$y_2 - y_1 = c_2 - c_1$

$\frac{P}{H} = \sin\theta$

$\sin 90 - \theta = \frac{d}{|c_2 - c_1|}$

$\cos\theta = \frac{d}{|c_2 - c_1|}$

$\tan\theta = m = \frac{\sin}{\cos}$

$\left(\frac{\sqrt{1-\cos^2\theta}}{\cos\theta}\right)^2 = (m)^2$

$1 - \cos^2\theta = m^2 \cos^2\theta$

$m^2\cos^2\theta + \cos^2\theta - 1 = 0$

$\cos^2\theta(m^2+1) - 1 = 0$

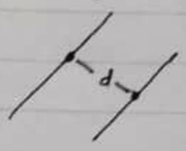
$\cos^2\theta = m^2+1 = 1$

$\cos^2\theta = \frac{1}{1+m^2}$

$\sqrt{\cos^2\theta} = \sqrt{\frac{1}{1+m^2}}$

$\boxed{\cos\theta = \frac{1}{\sqrt{1+m^2}}}$

Parallel Line
(Distance)




(Hassi – ML – journey)

☺

Now

$\cos\theta = \frac{d}{|c_2 - c_1|}$

$d = \cos\theta \times c_2 - c_1$

$d = \frac{1}{\sqrt{1+m^2}} \times |c_2 - c_1|$

$\boxed{d = \frac{|c_2 - c_1|}{\sqrt{1+m^2}}}$

Ans

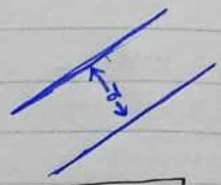
| Form | Looks Like | Good For | Bad For |
|---|---|---|---|
| **y = mx + c** | slope + intercept | school math, drawing | fails for vertical lines, hard formulas, not usable in ML |
| **ax + by + c = 0** | general form | ML, geometry, SVM, higher dimensions, perpendicular distance | a little less intuitive |

| Case | Object 1 | Object 2 | Do They Intersect? | Perpendicular Distance Exists? | Distance |
|---|---|---|---|---|---|
| **1** | Line | Line | No (parallel) | Yes | **Non-zero** |
| **2** | Point | Line | Depends on | Yes | **Non-zero** |
| **3** | Line | Line | **Yes (perpendicular)** | No (because they meet at a point) | **Zero** |

**In a Nut Shell : On a portrait of a Frame** **(Hassi – ML journey)**

# Concept : Build ML = Basic

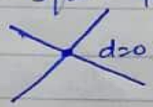
* For two Parallel Lines:
  - $y = mx + c \longrightarrow \boxed{d = \frac{|c_2 - c_1|}{\sqrt{1+m^2}}}$
  - $ax + by + c = 0 \longrightarrow \boxed{d = \frac{|c_1 - c_2|}{\sqrt{a^2+b^2}}}$



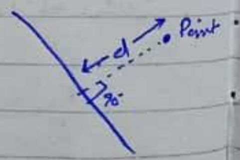

$\boxed{\text{Margin} = d}$

* For two Perpendicular Lines:

$\boxed{d=0}$ because there is no any gap b/w them as two lines are intersecting so there's no need to Calculate the distance b/w two lines as they are crossing. ($\perp$)

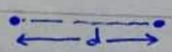



* For Perpendicular distance from a Point:
  - $y = mx + c \longrightarrow \boxed{d = \frac{|mx_0 - y_0 + c|}{\sqrt{m^2+1}}}$
  - $ax + by + c = 0 \longrightarrow \boxed{d = \frac{ax_1 + by_1 + c}{\sqrt{a^2+b^2}}}$




* Distance b/w two Points:

$\boxed{d = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}}$




* Distance b/w ~~three~~ two Points:

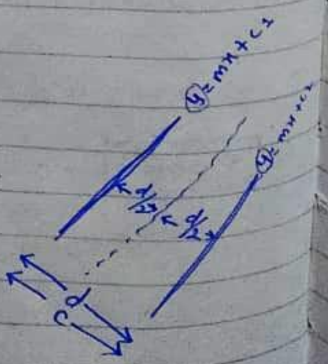
$\boxed{d = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)^2}}$

* Slope / Gradient :

$\boxed{m = \frac{y_2 - y_1}{x_2 - x_1}}$

* ~~Margin~~ HyperPlane:

$\boxed{y = mx + \frac{c_1 + c_2}{2}}$



# MATRIX

It is a 2D set of numbers enclosed by two large array brackets. And the numbers are arranged on the basis of Rows & Columns.

**Represented by:**

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}$$

**Size / Order of Matrix:** no of Rows x no of Columns.

r x c

2 x 3

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$$

**Equality of Matrix:** same order + same elements in two matrix than it is said to be equality of matrix.

C = D

$$C = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}, \quad D = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

**Row Matrix:** a matrix contains only one row.

$$\begin{bmatrix} 1 & 2 & 3 \end{bmatrix}$$

**Column Matrix:** a matrix contains only one Column.

$$\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

**Square Matrix:** a matrix whose rows = columns.

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

**Rectangular Matrix:** a matrix whose rows ≠ columns.

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$$

**Zero (Null) Matrix:** a matrix whose all elements should be zero.

$$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

**Diagonal Matrix:** Non-diagonal elements 0.

$$\begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$$

**Scalar Matrix:** All diagonal elements should be equal. Diagonal=all equal.

$$\begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$$

**Identity Matrix:** diagonal element is equal to 1, rest 0. Diagonal = all 1.

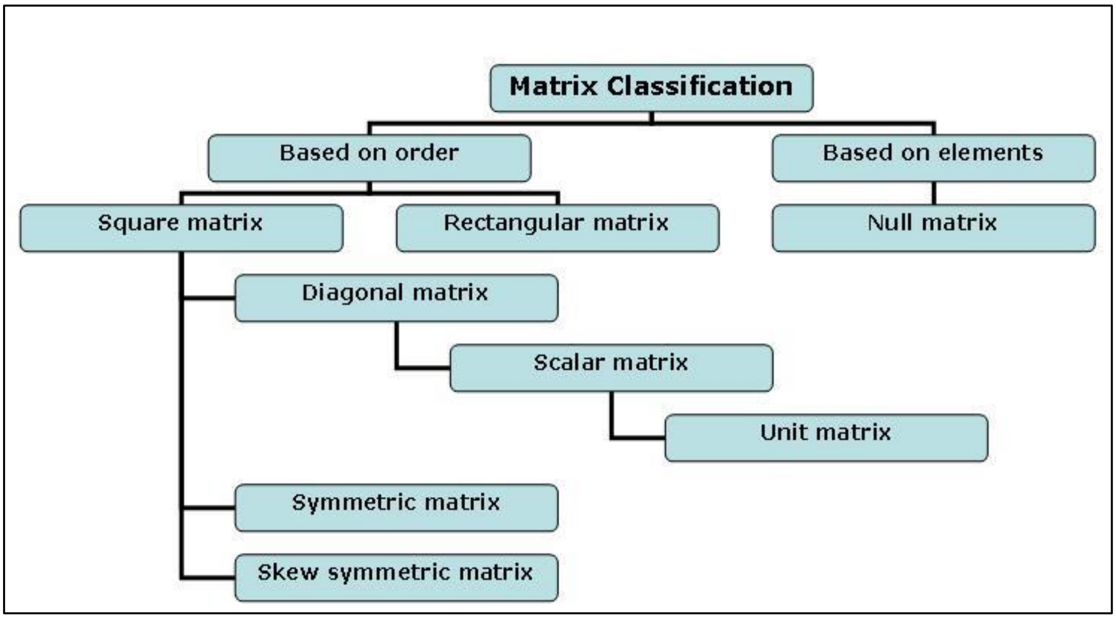
$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

**Symmetric Matrix:** elements of rows = element of column (order wise). $A = A^t$.

$$\begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}$$

**Skew – Symmetric:** $A = -A^t$ .

$$\begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix}$$



## OPERATIONS ON MATRICES:

**a)** **Addition of two matrix:** when two matrices are added, their respective element will be added. Make sure the order should be same of both matrices.

$$A = \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 7 \\ 6 & 2 \end{bmatrix}$$

$$A + B = \begin{bmatrix} 2+1 & 3+7 \\ 4+6 & 5+2 \end{bmatrix} = \begin{bmatrix} 3 & 10 \\ 10 & 7 \end{bmatrix}$$

**b)** **Subtraction of two matrix:** when two matrices are subtracted, their respective element will be subtracted. Make sure the order should be same of both matrices.

$$A = \begin{bmatrix} 5 & 8 \\ 4 & 7 \end{bmatrix}, \quad B = \begin{bmatrix} 2 & 3 \\ 1 & 6 \end{bmatrix}$$

$$A - B = \begin{bmatrix} 5-2 & 8-3 \\ 4-1 & 7-6 \end{bmatrix} = \begin{bmatrix} 3 & 5 \\ 3 & 1 \end{bmatrix}$$

**c)** **Multiplication of two matrix:** Matrix multiplication is possible only when **(Columns of A = rows of B)**. Each element is computed by **row × column** multiplication.

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 1 & 3 \end{bmatrix}$$

$$AB = \begin{bmatrix} (1\cdot 2 + 2\cdot 1) & (1\cdot 0 + 2\cdot 3) \\ (3\cdot 2 + 4\cdot 1) & (3\cdot 0 + 4\cdot 3) \end{bmatrix}$$

$$AB = \begin{bmatrix} 4 & 6 \\ 10 & 12 \end{bmatrix}$$

d) **Multiplication of matrix with Scalar:** When a matrix is multiplied by a number (scalar), you multiply each element of the matrix by that number.

$$kA = [k \cdot a_{ij}]$$

$$A = \begin{bmatrix} 2 & 4 \\ 3 & 5 \end{bmatrix}, \quad k = 3$$

$$3A = \begin{bmatrix} 3\cdot 2 & 3\cdot 4 \\ 3\cdot 3 & 3\cdot 5 \end{bmatrix} = \begin{bmatrix} 6 & 12 \\ 9 & 15 \end{bmatrix}$$

## Determinant of Matrix:

Single Value that tells how much a matrix scales space (area/volume).

- Determinant exists only for square matrices.
- If determinant = 0, matrix is singular (no inverse).
- If determinant ≠ 0, matrix is invertible.
- Determinant gives area (2×2) and volume (3×3) scaling factor.

1. **2 x 2 Matrix:**

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$\det(A) = ad - bc$$

$$A = \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}$$

$$\det(A) = (2)(5) - (3)(4) = 10 - 12 = -2$$

2. **3 x 3 Matrix:**

$$A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$$

$$\det(A) = a(ei - fh) - b(di - fg) + c(dh - eg)$$

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

$$\det(A) = 1(5 \cdot 9 - 6 \cdot 8) - 2(4 \cdot 9 - 6 \cdot 7) + 3(4 \cdot 8 - 5 \cdot 7)$$

$$= 1(45 - 48) - 2(36 - 42) + 3(32 - 35)$$

$$= (-3) - 2(-6) + 3(-3)$$

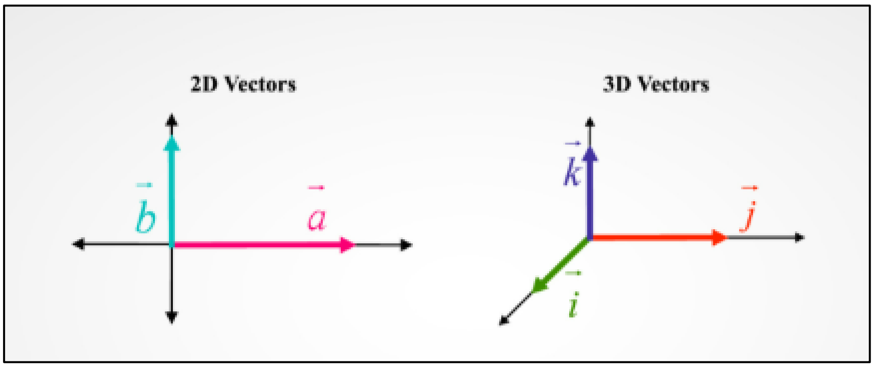
$$= -3 + 12 - 9 = 0$$

# VECTORS:

Vectors are those who has magnitude + Direction.

**Magnitude** = Mass / Value

**Direction** = Dimension (where its directed).

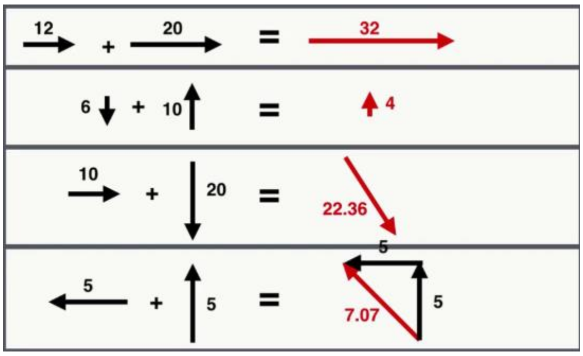



- **Addition of two Vectors:** Two vectors can be added only if they have the same dimension, by adding their corresponding components.

$$\vec{u} = \begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix}, \quad \vec{v} = \begin{bmatrix} 1 \\ 3 \\ 5 \end{bmatrix}$$

$$\vec{u} + \vec{v} = \begin{bmatrix} 2+1 \\ 4+3 \\ 6+5 \end{bmatrix} = \begin{bmatrix} 3 \\ 7 \\ 11 \end{bmatrix}$$

$$[3 \;\; 5 \;\; 7] + [1 \;\; 2 \;\; 4] = [3+1 \;\; 5+2 \;\; 7+4] = [4 \;\; 7 \;\; 11]$$



- **Product of two Vectors:**

  **a. Dot Product:** Dot product gives a scalar (number) by multiplying corresponding components and adding them.

  - If result > **0** → angle < 90° (acute)
  - If result = **0** → vectors are perpendicular
  - If result < **0** → angle > 90° (obtuse)

  **Formula:**

$$\vec{a} = (a_1, a_2, a_3), \quad \vec{b} = (b_1, b_2, b_3)$$
$$\vec{a} \cdot \vec{b} = a_1b_1 + a_2b_2 + a_3b_3$$

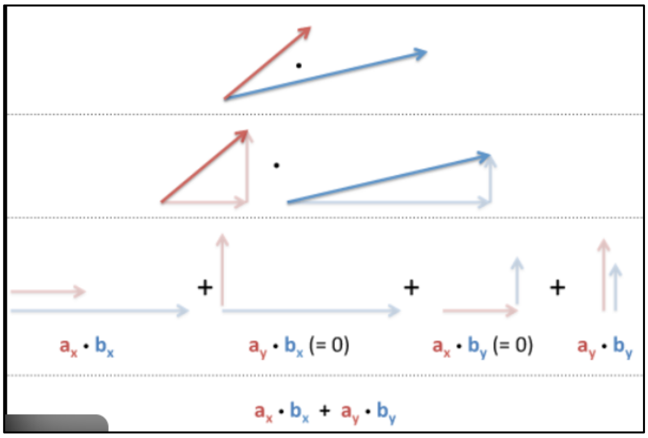
  **Geometric Formula:**

$$\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}| \cos\theta$$

  **Example:**

$$\vec{a} = (2, 3, 1), \quad \vec{b} = (1, 4, 2)$$
$$\vec{a} \cdot \vec{b} = (2)(1) + (3)(4) + (1)(2)$$
$$= 2 + 12 + 2 = 16$$

  **Pictorial (VIEW):**



**b. <u>Cross Product:</u>** Cross product gives a **new vector** that is **perpendicular** to both vectors.

**<u>Formula / Concept:</u>**

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

**<u>Geometrical Formula:</u>**

$$|\vec{a} \times \vec{b}| = |\vec{a}||\vec{b}| \sin\theta$$

**<u>Example:</u>**

$$\vec{a} = (2, 3, 1), \quad \vec{b} = (1, 4, 2)$$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 1 \\ 1 & 4 & 2 \end{vmatrix}$$

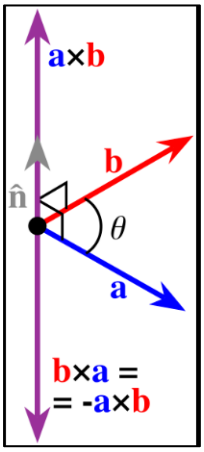
$$\hat{i}(3 \cdot 2 - 1 \cdot 4) - \hat{j}(2 \cdot 2 - 1 \cdot 1) + \hat{k}(2 \cdot 4 - 3 \cdot 1)$$

$$= \hat{i}(6 - 4) - \hat{j}(4 - 1) + \hat{k}(8 - 3)$$

$$= 2\hat{i} - 3\hat{j} + 5\hat{k}$$

$$\boxed{(2,\ -3,\ 5)}$$

**<u>Pictorial (VIEW):</u>**



## Eigen Vectors:

It tells how much a vector is stretched or shrunk & in this basically a matrix is multiplied with a Vector. Direction does not changes.

**One Dimensional Matrix = (Eigen Vector)**

# AX=λX

AX - λX = 0

X [A – λ] = 0

λI = identity x λ

then Find Determinant & then multiply X(vector) with above formula.

**Example:**

$$A = \begin{bmatrix} 2 & 3 \\ 4 & 3 \end{bmatrix}$$

$$\lambda_1 = 6, \quad \lambda_2 = -1$$

$$A - 6I = \begin{bmatrix} 2-6 & 3 \\ 4 & 3-6 \end{bmatrix} = \begin{bmatrix} -4 & 3 \\ 4 & -3 \end{bmatrix}$$

$$\begin{bmatrix} -4 & 3 \\ 4 & -3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$-4x + 3y = 0$$

$$3y = 4x \quad \Rightarrow \quad y = \frac{4}{3}x$$

$$A - (-1)I = A + I = \begin{bmatrix} 2+1 & 3 \\ 4 & 3+1 \end{bmatrix} = \begin{bmatrix} 3 & 3 \\ 4 & 4 \end{bmatrix}$$

$$3x + 3y = 0$$

$$x + y = 0 \quad \Rightarrow \quad y = -x$$

$$\lambda_1 = 6, \quad \vec{v}_1 = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$$

$$\lambda_2 = -1, \quad \vec{v}_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$