



$$\textcircled{1} \sum F_{x,m_1} = m_1 \ddot{x} = (-k_1 x - \sin(\theta) F_1 - \cos(\theta) F_2) \hat{i}$$

$$\textcircled{2} \sum F_{y,m_1} = m_1 \ddot{y} = (-k_1 y + \cos(\theta) F_1 - \sin(\theta) F_2) \hat{j}$$

$$\textcircled{3} \sum F_{r,m_2} = m_2 a_{G/O} \hat{r} = -F_1 \hat{r}$$

$$\textcircled{4} \sum F_{\theta,m_2} = m_2 a_{G/O} \hat{\theta} = -F_2 \hat{\theta}$$

$$\textcircled{5} \sum \tau_{/G} = \frac{dH_{/G}}{dt} = [-k_2 \theta + F_2 d] \hat{k} = I_{zz} \ddot{\theta} \hat{k}$$

$$\begin{aligned} a_{G/O} &= a_{A/O} + \underbrace{\omega_{/O} \times r_{G/A}}_{\ddot{x} \hat{i} + \ddot{y} \hat{j}} + \underbrace{\omega_{/O} \times (\omega_{/O} \times r_{G/A})}_{\ddot{\theta} d \hat{\theta} - \dot{\theta}^2 d \hat{r}} + 2 \underbrace{\omega_{/O} \times v_{G/A}}_{\dot{\theta} d \hat{\theta}} \\ &= \ddot{x} \hat{i} + \ddot{y} \hat{j} + \ddot{\theta} d \hat{\theta} - \dot{\theta}^2 d \hat{r} \\ &= (\sin \theta \ddot{x} + \cos \theta \ddot{y} - \dot{\theta}^2 d) \hat{r} + (-\cos \theta \ddot{x} - \sin \theta \ddot{y} + \ddot{\theta} d) \hat{\theta} \end{aligned}$$

$$\textcircled{3} \quad m_2 [-\sin(\theta)\ddot{x} + \cos(\theta)\ddot{y} - \dot{\theta}^2 d] = -F_1$$

$$\rightarrow F_1 = m_2 [\sin(\theta)\ddot{x} - \cos(\theta)\ddot{y} + \dot{\theta}^2 d]$$

$$\textcircled{4} \quad m_2 [-\cos(\theta)\ddot{x} - \sin(\theta)\ddot{y} + \dot{\theta}^2 d] = -F_2$$

$$F_2 = m_2 [\cos(\theta)\ddot{x} + \sin(\theta)\ddot{y} - \dot{\theta}^2 d]$$

$$\textcircled{5} \quad -k_2 \theta + [m_2 (\cos(\theta)\ddot{x} + \sin(\theta)\ddot{y} - \dot{\theta}^2 d)] d = I_{zz} \ddot{\theta}$$

$$(I_{zz} + m_2 d^2) \ddot{\theta} - m_2 \cos(\theta) \ddot{x} - m_2 \sin(\theta) \ddot{y} + k_2 \theta = 0$$

$$\textcircled{1} \quad m_1 \ddot{x} + k_1 x + \sin(\theta) F_1 + \cos(\theta) F_2 = 0$$

$$m_1 \ddot{x} + k_1 x + \sin(\theta) m_2 [\sin(\theta) \ddot{x} - \cos(\theta) \ddot{y} + \dot{\theta}^2 d] \\ + \cos(\theta) m_2 [\cos(\theta) \ddot{x} + \sin(\theta) \ddot{y} - \dot{\theta}^2 d]$$

$$= m_1 \ddot{x} + k_1 x + m_2 [\sin^2(\theta) \ddot{x} - \cancel{\sin(\theta)\cos(\theta)} \ddot{y} + \sin(\theta) \dot{\theta}^2 d] \\ + m_2 [\cos^2(\theta) \ddot{x} + \cancel{\cos(\theta)\sin(\theta)} \ddot{y} - \cos(\theta) \dot{\theta}^2 d]$$

$$\text{with } \sin^2(\theta) + \cos^2(\theta) = 1$$

$$= m_1 \ddot{x} + k_1 x + m_2 (\ddot{x} + \sin(\theta) \dot{\theta}^2 d - \cos(\theta) \dot{\theta}^2 d)$$

$$= (m_1 + m_2) \ddot{x} - \cos(\theta) d \dot{\theta}^2 + \sin(\theta) d \dot{\theta}^2 + k_1 x = 0$$

$$(2) \quad m_1 \ddot{y} + k_1 y - \cos(\theta) F_1 + \sin(\theta) F_2 = 0$$

$$= m_1 \ddot{y} + k_1 y - \cos(\theta) m_2 [\cancel{\sin(\theta) \ddot{x}} - \cos(\theta) \ddot{y} + \dot{\theta}^2 d] \\ + \sin(\theta) m_2 [\cancel{\cos(\theta) \ddot{x}} + \sin(\theta) \ddot{y} - \ddot{\theta} d]$$

$$= m_1 \ddot{y} + k_1 y + m_2 [\cos^2(\theta) \ddot{y} + \cos(\theta) \dot{\theta}^2 d + \sin^2(\theta) \ddot{y} - \sin(\theta) \ddot{\theta} d]$$

$$\text{with } \sin^2(\theta) + \cos^2(\theta) = 1$$

$$\rightarrow m_1 \ddot{y} + k_1 y + m_2 \ddot{y} + m_2 \cos(\theta) \dot{\theta}^2 d - m_2 \sin(\theta) \ddot{\theta} d$$

$$(m_1 + m_2) \ddot{y} - m_2 \sin(\theta) d \ddot{\theta} + m_2 \cos(\theta) d \dot{\theta}^2 + k_1 y = 0$$