Particle Quantization and its Application to Nucleons, Mesons, and Baryons

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Abstract
The rules of particle quantization (PQ) are enunciated and examined. They are applied to nucleons, the muon, and many mesons and baryons in order to explain their masses, electric charges, and spin states. For particles with a stable state, this property is explained. For particles with a magnetic moment, an explanation is proposed. The rules of particle disintegration are drawn and applied to many stable and unstable particles.

Key words: particle quantization, mass, charge, magnetic moment, stability and spin of particles, particle decays

1. INTRODUCTION

The rules of particle quantization (PQ) appeared long ago.[1-3] As the measurements and other data about particles have become more numerous and accurate, these rules have become more precise.[3] More recently, the work has been published in *Physics Essays*.[4] References 1 to 4 apply PQ to explain measurements of the mass and magnetic moment of the neutron (n^0) and proton (p); no other model can do this at this time.

Several other papers[5] concern the application of PQ to the mass, electric charge, and decay modes of the muon (μ) and the light stable mesons π^0, π^-, K^0, K^-.

Recent improvements in mass measurements have been used to extend PQ to all the heavy mesons having a stable state: D^0, D^-, D_s^-, B^-. PQ is also extended to many unstable mesons: η_{548}, ρ^0, ρ^+, ρ^-, ω, η'_{958}, ϕ_{1020}, K^{*0}, K^{*+}, K^{*-}. Another paper[6] concerns all the hyperons Λ^0, Σ^+, Σ^0, Σ^-, Ξ^0, Ξ^-, Ω^-.

Measurements of masses and magnetic moments are taken into account here; moreover, PQ is also applied to the heavy stable baryons Λ_c^+, Σ_c^+, Σ_c^{++}, Σ_c^0.

Over time, much of the data have increased in accuracy while many new particles have appeared. Now many measurements (although not all) have reached a satisfactory accuracy and await an explanation that would be just as satisfactory. In this respect, we first examine the physics of particles.

We agree with the experimental evidence that when a particle is charged, its charge is e or $-e$ or some integral multiple thereof. For macroscopic objects, it is well known that the charge is due to an excess or a lack of electrons; the same is true for the atomic ions. But particle physics does not identify the origin of particle charge. Nobody would pretend that a charged particle has one electron more or less than its neutral homologue. For example, some particles like μ^-, Ω^-, and D_s^- have no neutral homologue while Λ^0 has no charged homologue; besides $m(\pi^-)$ or $m(\pi^+)$ is greater than $m(\pi^0)$; $m(K^-)$ or $m(K^+)$ is lower than $m(K^0)$; $m(\rho^-) = m(\rho^+) = m(\rho^0)$; $m(\Sigma^+) < m(\Sigma^0) < m(\Sigma^-)$, and so on. None of this is explained classically. Some neutral particles decay by generating two neutral particles or two charged particles with opposite signs or by both modes; the conservation of charge is always obeyed in particle decay, but the question of the origin of charge is not answered. PQ proposes an answer.

It is well known that particles have spin-1/2, 1, or 0 and that spin is conserved in any particle interaction or particle decay. What is the origin of the spinning property 0, 1/2, 1 (expressed in the unit $h/2\pi$)? Such a question is fundamental for particle physics. It distinguishes mesons from baryons by saying that a baryon has a half-integer spin and a meson has an integer spin. There is no classical explanation of this. PQ identifies the spin of a particle as a peculiarity in its structure. The same is true for the electric state. There is in any particle something ("some things") responsible for its electric states and something else responsible for its spinning state. This is quite new.

PQ has a general design: any nucleon, meson, or baryon derives "from" a determinate association of quantum layers. The mass of the quantum (qm) is a known physical mass, which is the electron rest mass m_e. One has therefore 1 qm = 0.510 999 06(15) MeV. Each layer is characterized by a quantum number that can take the value $n = 1, 2, 3, 4, 5, 6, 7$. The layer of rank n contains $16n^2$ qm, so the basic mass of a particle is equal to $16\Sigma n^2$ qm, where the summation extends to the specific numbers, n, of the considered particle. However, this quantized association does not coincide with the particle itself, for we shall see in the following sections that each physical property of a particle is characterized by some specific mass deviation. It is only the basic association from which the considered particle derives with its own attributes. Among them, we consider the following:

(1) The spin state corresponds to a determinate deviation, and only the particles that bear that kind of deviation have the

spin property: this is an integer number of qm additional to its layers and is not borne by any layer.
(2) The electric state is due to some deviations; each deviation is a determined integer number of qm. It can be positive or negative, and it is borne by a quantized layer. The numerical structure of the particle is the basic structure modified by the specific deviations. The values of these deviations are discussed in Sec. 3.

The mass of a particle is roughly determined by its basic structure and better determined by its numerical structure. Some particles have a stable state, but most are unstable, their mass being measured from the products of their decay. The stable state of a particle is the result of a mass defect due to a binding between its layers. This defect expressed in qm obeys the law $-\Delta = -(M/784)$ qm, where M is a specific part of the numerical mass of the considered particle. The true mass of a stable particle is its numerical mass diminished by Δ. In the specific mass M before, the deviations that correspond to a physical property may not contribute to M, since they contain a number of quanta that must remain integers. Obviously, the deviations that correspond to a "negative" number of qm do not participate in M since they are missing qm, but the layers that bear them can participate in M ("can" means a possibility since M is a specific mass).

The nucleons, mesons, and baryons are considered to be a family of particles governed by some common rules, but each particle obeys rules that agree with its own individuality, that is, its own physical attributes and interactions. So while PQ is a unitary theory since all the particles obey the same ensemble of rules, it is also an individualist theory, owing to the choice that it allows to each particle. Such an organization allows a family formed by numerous other particles as it is really; however, the number of particles, although large, is limited because the choices are limited.

PQ identifies the mass quantum with the existing mass m_e. It further uses some physical quantities like the charge e, the spin expressed in the unit $h/2\pi$ of angular momentum, the magnetic moment expressed in the usual unit μ_N, and the binding energy expressed in the unit m_ec^2 or the equivalent mass m_e. PQ does not use the hyperquantum numbers. We recognize that these allow an easy classification of the particles, but being pure numbers, they have a mathematical meaning and no good physical meaning. Their explanatory capacity about the masses, electric charge, and spin states, and magnetic moment of the particles is poor. PQ does not exclude them, but it has no need for them. The aim of this paper is not to compare the standard model with PQ but to present to the reader an alternative model that seems to explain the origin of particles, their mass, electric charge, spin, magnetic moment, and decay modes.

2. THE RULES OF PARTICLE QUANTIZATION

The rules of PQ have already been published,[3,4] but are reiterated briefly here. However, the earlier work only concerned the application of PQ to n^0 and p. Because this paper also concerns the muon and many mesons and baryons, some rules, although enunciated in Refs. 1 to 4, require more details.

2.1 The Basic Structure of a Particle

Particles are assumed to be composed of an association of quantum layers characterized by the set of corresponding values of n. For the lightest particles other than the electron, one has the following binary associations:

μ^-: $n = 2, n = 3$; $(64 + 144) = 208$ qm;

π^-: $n = 1, n = 4$; $(16 + 256) = 272$ qm;

K: $n = 5, n = 6$; $(400 + 576) = 976$ qm;

η_{548}: $n = 4, n = 7$; $(256 + 784) = 1040$ qm.

Note: in this paper a particle is indifferently denoted by its usual symbol or as its substituent qm layers enclosed in parentheses.

These simple results strongly suggest that the existence of quantized layers according to the law $16n^2$ qm is experimentally well-grounded.

For the other particles the number of various layers increases, and, for heavier particles, some layers can even be repeated, that is, one layer can be present several times.

2.2 The Deviations

As said in the Introduction, the quantized structure of particles exhibits two kinds of deviations corresponding to their spin and electric charge.

2.2.1 *The Rule of Spin*

A particle has spin-1/2 if its structure contains one additional central qm. "Central" has no special geometrical meaning; it only means that this quantum does not belong to any particular layer. This is the case for μ, n^0, p,[1–4] and all baryons. Now we add that a particle has spin-1 if it has two central qm with parallel spins-1/2. A particle has spin-0 in two cases:

(1) it has no central qm; this is the case for many mesons;
(2) it has two central qm with antiparallel spins-1/2, $-1/2$; this is the case for some mesons, although case (2) is rarer than case (1).

This rule specifies why a baryon has the half-integer spin-1/2, while a meson has the integer spin-0 or 1. It is also new in particle physics.

2.2.2 *The Deviations Corresponding to the Electric State*

A layer of a particle is charged if it bears one of the following deviations: $+16$ or -16 qm, $+1$ or -1 qm, or $+3$, or -3 qm. Each of these deviations confers to the layer that bears it the charge $-e$ or $+e$ according to the considered particle. One must not confuse the sign of a deviation with the sign of the corresponding charge. A layer of a particle is neutral in four cases:

(1) it bears no deviation;
(2) it bears the neutral deviation $+2$ or -2 qm;

(3) it bears one of the four possible "mixed" deviations $|\pm 16 \pm 1|$; they can be $+16+1$, $+16-1$, $-16+1$, $-16-1$;
(4) it bears one of the four possible "mixed" deviations $|\pm 16 \pm 3|$; they can be $+16+3$, $+16-3$, $-16+3$, $-16-3$.

Note: In this paper a mixed derivation is denoted by a sum enclosed by straight lines.

Rules (3) and (4) express that the charged deviation ± 16 is neutralized by the charged deviation ± 1 or ± 3. As already shown,[3,4] these rules explain the electric charge of particles. On the contrary, the mixed deviation $\pm 16 \pm 2$ remains charged because the charged deviation ± 16 is not neutralized by the neutral deviation ± 2.

In Refs. 3 and 4 it was said that another deviation exists but is not present in the n^0 and p. *However, it exists in the muon and in most mesons and baryons.* We call it deviation of type 8 qm. It can exist in the form $|\pm 8|$ or in the form of the mixed deviations $|\pm 8 \pm 1|$, $|\pm 8 \pm 2|$ or $|\pm 8 \pm 3|$. Whatever it is, this deviation is always neutral. Thus it modifies the mass of the particle without modifying its eventual charge due to the charged deviations. Like the deviations of type 16 or type 1 (i.e., 1 or 2 or 3), *a deviation of type 8 is always borne by a layer of the considered particle. The mixed deviations $|\pm 16 \pm 8|$ do not exist.* A layer can bear a deviation $|\pm 16|$, and another layer can bear a deviation $|\pm 8|$, but the mixed deviation $|\pm 16 \pm 8|$ cannot be borne by a single layer.

Remarks: (1) The deviations related to the charge or neutrality of the particles are numerous. However, for any particle their number is limited and often small. This is certainly true for light particles and even for heavy ones because many layers bear no deviation. (2) One must not confuse the neutral deviation $|16 \pm 1|$ borne by a layer with the charged layer (16 ± 1) where 16 is the layer $n = 1$.

In short, the charged deviations are denoted by the symbols $|16|$, $|1|$, $|3|$, $|16{,}2|$ qm, while the neutral deviations are denoted by $|2|$, $|16{,}1|$, $|16{,}3|$, $|8|$, $|8{,}1|$, $|8{,}2|$, $|8{,}3|$.

The existence of the deviations of type 8 is supported by experimental evidence. Indeed, one has

$$m(\pi^-) - m(\pi^0) = 4.594 \pm 0.0005 \text{ MeV} = 8.99 \pm 0.001 \text{ qm} \cong (8+1) \text{ qm},$$

$$m(K^0) - m(K^-) = 4.02 \pm 0.035 \text{ MeV} = 7.87 \pm 0.07 \text{ qm} \cong 8 \text{ qm},$$

$$m(K^{*-}) - m(K^{*0}) = 4.3 \pm 0.5 \text{ MeV} = 8.4 \pm 1 \text{ qm} \cong (8+1) \text{ or } 8 \text{ qm},$$

$$m(\Sigma^0) - m(\Sigma^+) = 3.2 \pm 0.17 \text{ MeV} = 6.2 \pm 0.4 \text{ qm} \cong (8-1) \text{ qm},$$

$$m(\Sigma^-) - m(\Sigma^0) = 4.89 \pm 0.08 \text{ MeV} = 9.59 \pm 0.16 \text{ qm} \cong (8+1) \text{ qm},$$

$$m(D^0) - m(D^-) = 4.8 \pm 0.25 \text{ MeV} = 9.3 \pm 0.5 \text{ qm} \cong (8+1) \text{ qm}.$$

In these differential measurements the sign $\cong$ corresponds to some lack of accuracy in the measurements.

2.3 The Rule of Stability

As enunciated in Refs. 1 to 4 and recalled in the Introduction, if $-\Delta < 0$, the particle is stable, where $\Delta = (M/784)$ qm.

2.4 The Decay Rules of the Mesons

Excepting e and p, all particles decay. The only particle that has a single decay mode is the n^0. All mesons have several, say N, decay modes. Classically, this complexity remains largely unexplained. PQ brings a more complete understanding thanks to the following rules, which express that *the decay modes of any particle only depend on its quantized structure*. These rules are in Refs. 5 to 7, but they are repeated here.

(1) If one considers a meson X with N decay modes, these modes are produced respectively by N kinds of isobaric structures of X, each being responsible for one of the N modes. The common mass of the N isobars of X is the numerical structure of X. As a result, the branching ratio of the decay modes which the Table of Particles expresses in percent is the respective population of the N isobars expressed in a percent of the total population of the mesons X. Two cases may be distinguished: (a) If all the isobaric structures of X contain all the layers of high quantum rank n of the numerical structure of X, so that they only differ from one other by some deviations or by some layers of low quantum rank, the isobars of X are said to be "non-degenerate." (b) If, in some isobars of X, some layers of high quantum rank are missing and are replaced by an equivalent set of layers of lower quantum rank, these isobars of X are said to be "degenerate." The degeneracy of an isobar increases with the number of high quantum rank layers of the numerical structure of X that are missing in its structure.
(2) Let us now consider some various mesons X, Y, Z that decay by emitting the same particle A. We interpret this fact in claiming that the corresponding isobars of X, Y, Z have in their structures what is necessary for emitting A, what we call "the generating association of A" [in short, GA(A)]. As a consequence, if an isobar of X emits the particles A, B, C, its structure contains GA(A), GA(B), and GA(C).
(3) The GA of a meson is identical to the numerical structure of this meson, except for $GA(\pi^\pm)$ and $GA(\pi^0)$. [The particular cases of $GA(\pi^\pm)$ and $GA(\pi^0)$ are examined in Sec. 4.]
(4) The isobar that decays in A, B, C thus contains in its structure GA(A), GA(B), and GA(C). Because the emission of A, B, C requires momentum and energy transfer, some layers of the considered isobar must vanish in energy emission. It is well known that the decay of a particle obeys conservation laws of energy, linear momentum, charge, and angular momentum. Such laws are experimentally

established, so we accept them.

(5) Some isobars of mesons decay by emitting either a γ photon and a particle or 2γ photons. In these cases the spin rule plays an important part, which will be specified.

In the application of these decay rules of mesons, it appears that the deviations play a main part because, without them, the number of isobaric structures of a meson would be small; consequently, the number of decay modes of the meson would also be small in contradiction to experimental evidence.

In relating the decay modes of mesons to their quantized structure only and in introducing the concepts of isobaric structures and generating associations, PQ clears the apparent complexity of the decay modes of the mesons. In addition, the decay rules that apply to mesons (see Sec. 4) can be extended to the decay of baryons (Sec. 5).

2.5 The Generating Associations of Mesons

In the review of the decay modes of all mesons the following emitted particles are observed: e^- or e^+ accompanied by a neutrino; a pair e^-e^+; a meson μ^- (or the conjugate) accompanied by a muon neutrino; a pair of muons; a γ photon accompanied by a meson; a meson π^- or π^0 or some of them; a meson K^0_S or K^0_L or both of them; a meson K^- (or its conjugate) or both K^- K^+; an unstable meson like η, ρ^- (or ρ^+) or ρ^0; ω; η'; ϕ; K^{*-} (or its conjugate); K^*_0, and so on. Again, the meson that emits one of those particles is an isobar that contains, in its structure, the GA of the emitted particle. Again, the GA of a particle is identical to the numerical structure of this particle unless the emitted particle is a π meson. The numerical structures of the mesons are given in Sec. 4 together with their GA; in the same section GA($\pi^\pm$) and GA(π^0) are established. In this section we only consider the GA of some peculiar particles.

2.5.1 *GA($e^-\nu_e$) (or its conjugate)*

The emission $e^-\nu_e$ can be observed when the isobar of a meson is charged or neutral. If it is charged, the $e^-\nu_e$ emission must be accompanied by the emission of a neutral particle; for example, one has $K^- \to e^-\bar{\nu}_e\pi^0$. If it is neutral, the emission of $e^-\nu_e$ must be accompanied by a charged particle; for example, one has $K^0 \to e^-\bar{\nu}_e\pi^+$.

We claim that the neutrino ν_e that accompanies e^- is emitted by the process $8 \to \nu_e + |8 - \nu_e|$, the residue $|8 - \nu_e|$ resulting in the emission energy of e^- and ν_e and also of the particle that accompanies them. So the emission of the pair $e^-\nu_e$ requires that the isobar that emits them contains the deviation 8 qm from which ν_e is created and also the charged deviation +1 qm that is emitted in the form e^-. In summary, the generating association that emits $e^-\nu_e$ is GA($e^-\nu_e$) = |1 + 8| qm. Sections 3, 4, and 5 have several examples.

2.5.2 *GA($\mu^-\nu_\mu$) (or its conjugate)*

The muon neutrino ν_μ appears at the birth of a μ^- and also at the decay of a μ^+. For example, one has $\pi^- \to \mu^-\bar{\nu}_\mu$ and $\mu^+ \to e^+\bar{\nu}_e\nu_\mu$. Classical particle physics explains how in such processes the conservation of spin is satisfied. But it does not explain how ν_μ is generated. We claim that while ν_e is created from a deviation +8 qm, ν_μ is created from a deviation 1 qm by the process $1 \to \nu_\mu + |1 - \nu_\mu|$, the residue $|1 - \nu_\mu|$ also being transformed in energy emission. That rule will be detailed in Sec. 4.2.

2.5.3 *Emission of a γ Photon*

An isobar of several mesons can decay by emitting a γ photon accompanied by a particle. This happens if the isobar satisfies two conditions: if it has spin-1 and if its structure contains the GA of the considered particle.

2.5.4 *Emission of $\gamma\gamma$*

Such an emission needs two conditions to be fulfilled: the isobar has spin-0 due to the presence, in its structure, of two central qm with antiparallel spins-1/2 or due to the absence of central qm, and its structure contains no GA of a particle.

3. APPLICATION OF PARTICLE QUANTIZATION TO NUCLEONS

3.1 Remark

This paper only considers the particles whose masses $m(1 \pm \varepsilon)$ are known with a relative accuracy ε better than 10^{-3} in order to submit PQ to an available test. More than 40 particles are concerned.

The best measurements are m(p), $m(n^0)$, m(e). For example, the measurements of the ratio m(p)/m(e) = 1836.152 701(100)[4] precisely express the proton mass in qm with seven exact figures. We have recovered that value and similar ones concerning the nucleons in the frame of the model of PQ.[4] The reader may refer to that paper for more details. However, to help in reading Secs. 4 and 5, which respectively concern the mesons and baryons, we condense some results that will be used in these sections.

The basic structure of n^0 is written as

$$n = 1,2,3,4,6,7\text{: } [16 + 64 + 144 + 256 + 576 + 784] = 1840 \text{ qm.} \quad (1)$$

The numerical structure of n^0 is

$$1 + 16 + 64 + (144 + 16)^- + (256 - 16)^+ + 576 + 784 = 1841 \text{ qm.} \quad (2)$$

So n^0 is a spinning electric doublet, the layer $n = 4$ having transferred a group of 16 qm to the layer $n = 3$.

The mass defect of n^0 has the value[4]

$$-\Delta(n^0) = -(1816/784) = -2.316\,327 \text{ qm.} \quad (3)$$

n^0 contains a magnetic group formed by the central qm and the odd layers $n = 1, 3, 7$ of (2). Its negative charge is responsible for the magnetic moment of n^0. The numerical mass of this group is equal to $[1 + 16 + (144 + 16)^- + 784] = 961^-$ qm. Its true mass is the preceding diminished by the part of $-\Delta(n^0)$ that this group bears. The exact value of the magnetic moment of n^0 follows.[3,4]

We also recall the decay process $n^0 \to p + e^- + \nu_e + W$ because it plays an important part in Secs. 4 and 5. Analyzed from PQ, it is shown how n^0 decays by means of some internal transfers of qm in such a way that a particle (here, n^0) is transformed in another structure (here, that of p).

At the end of its lifetime the neutron is transformed as follows:

(1) The neutronic doublet disappears; the layers $n = 3$, $n = 4$ join together in forming a layer $n = 5$; this process leads to the association

$$1 + 16 + 64 + 400 + 576 + 784 = 1841 \text{ qm}. \quad (4)$$

At the same time another doublet appears, the layer $n = 5$ transferring a group of 16 qm to the layer $n = 6$. One gets the structure

$$\mathrm{GA(p)} = 1 + 16 + 64 + (400 - 16)^+ + (576 + 16)^- + 784 = 1841 \text{ qm}, \quad (5)$$

which coincides with the generating association of p.

(2) Still at the same instant, the layer $n = 5$ loses three qm and is neutralized in the form $(400 - |16 + 3|)$; one of the lost qm is emitted in the form e^-. Thus, since a negative charge has appeared, a positive charge must also appear: it is the second lost qm that is transferred to the layer $n = 2$ that becomes charged in the form $(64 + 1)^+$. The third lost qm is transferred to the layer $n = 6$ that is neutralized in the form $(576 + |16 + 1|)$. So the numerical structure of p is

$$1 + 16 + (64 + 1)^+ + (400 - |16 + 3|) + (576 + |16 + 1|) + 784 = 1840^+ \text{ qm}. \quad (6)$$

So we have our first example showing that the charge of a particle is connected with some suitable deviations. More examples will be given in Secs. 4 and 5.

(3) The association (4) has inherited from n^0 the mass defect $-\Delta(n^0)$; thus the structure (6) of p also inherits it. Simultaneously, there appears in the structure of p another mass defect

$$-\delta = -1200/784 = -1.530\,612 \text{ qm} \quad (7)$$

(see Ref. 4 for more details). Particle p is the only one that has two negative defects, $-\Delta(n^0)$ and $-\delta$, and this circumstance confers to p a rigorous stability in the free state. The mass 1200 qm before exactly corresponds to the odd layers $n = 1, 5, 7$ of (4). The magnetic moment of p is explained in the same way as that of n^0.[3,4]

A final remark about the nucleons must be made: Eq. (5) yields the association of layers that creates p from n^0, that is, its generating association GA(p):

$$\mathrm{GA(p)} = 1 + 16 + 64 + (400 - 16)^+ + (576 + 16)^- + 784 = 1841 \text{ qm}. \quad (8)$$

Let us now consider a light baryon that decays, for example, Λ^0. It can generate p or n^0. When it emits p, it contains GA(p), that is, the structure (8). When it emits n^0, it contains GA(n^0), that is, (2). So Λ^0 has two isobaric structures: one emitting p and the other emitting n^0. If we consider N individuals of this baryon, we have $N = N_1 + N_2$, N_1 being the number of individuals that contain GA(p) and N_2 being the number of individuals that contain GA(n^0). According to experiment, the ratio N_1/N_2 varies according to the baryon that decays. If we now consider a heavy baryon, it decays by generating a lighter one under the form of two isobars with their abundance in the same proportion.

Moreover, since the final term of the decay chain of a baryon always contains a nucleon and a meson, it also contains the GA of the emitted meson.

4. APPLICATION OF PARTICLE QUANTIZATION TO MESONS

4.1 Stable and Unstable Mesons

This section deals with the muon, the mesons π^0, π^-, K^0_S, K^0_L, K^-, D^-, D^0, D^-_S, and B^- having a stable state and some unstable mesons such as η_{548}, ρ^0, ρ^-, ω, η'_{958}, ϕ_{1020}, K^{*-}_{892}, and K^{*0}_{892}.

The concerned data are their masses, their spin, their electric charge, and their various decay modes. We have intentionally restricted ourselves to the mesons whose masses are known to six significant figures for the lightest particles μ^-, π^-, π^0, K^-, K^0 and to three figures for the heavy ones. Our goal is to show that PQ satisfactorily explains such data.

4.2 The Muon

4.2.1 *Numerical Structure*

Data: charge $\pm e$, spin-1/2.

The mass is $m(\mu^-) = m(\mu^+) = (105.658\,387 \pm 0.000\,034)$ MeV. Expressed in the unit qm, one has (precision 6×10^{-7})

$$m(\mu^-) = (206.768\,26 \pm 0.000\,13) \text{ qm}.$$

When a muon is emitted from another particle, a muon neutrino is simultaneously emitted; for example, in the process $\pi^- \to \mu^-\bar{\nu}_\mu$ conservation of the spin-0 of π^- is secured by the emission of μ^- and $\bar{\nu}_\mu$ with antiparallel spins-1/2.

μ^- has a stable state [mean lifetime $(2.197\,03 \pm 0.000\,04) \times 10^{-6}$ s]. It decays according to the dominant process

$$\mu^- \to e^-\bar{\nu}_e\nu_\mu \; (\approx 100\%),$$

which obeys the conservation of spin-1/2 of μ^-: e^- and ν_e are emitted with antiparallel spins-1/2 and ν_μ with the spin-1/2. So a muon neutrino accompanies μ^- at its decay as well as at its birth. Two other decay modes have been observed:

$$\mu^- \to e^-\bar{\nu}_e\nu_\mu\gamma (1.4 \pm 0.4)\%$$

$$\mu^- \to e^-e^+\bar{\nu}_e\nu_\mu e^- (3.4 \pm 0.4) \times 10^{-5}.$$

4.2.2 *Numerical Structure and Numerical Mass of* μ^-

Both μ^- and ν_μ derive from the basic association $n = 2$, $n = 3$ (mass 208 qm) as follows. Since μ^- and ν_μ appear simultaneously, it is logical to admit that both come from the basic association (208 qm) which contains two components: one is charged and has the numerical mass 207 qm; indeed, a numerical mass, expressed in qm, is always an integer slightly larger than the true mass; since this is equal to $m(\mu^-) \approx 206.7$ qm, the integer 207 qm is suitable for the numerical mass of μ^-. The other component is 1 qm and this generates ν_μ; in fact, the Table of Particle Properties gives $m(\nu_\mu) < 0.27$ MeV, that is, $m(\nu_\mu) < 0.53$ qm; thus 1 qm suffices to generate ν_μ by the process $1 \rightarrow \nu_\mu + |1 - \nu_\mu|$, the residue $|1 - \nu_\mu|$ annihilating in the emission energy of both μ^- and ν_μ.

In applying the rules of PQ, we analyze the process from which the component 207 qm becomes the charged 207^- component and generates the numerical structure of μ^-. We conjecture that the basic associations $n = 2$, $n = 3$ (208 qm) undergoes three internal transfers of quanta that determine the numerical structure of μ^- and cause the emission of ν_μ.

(1) The layer $n = 2$ loses a group of 16 qm and becomes charged in the form $(64 - 16)^-$ (cf. the rule of charge: a deviation ± 16 qm charges the layer that bears it); the lost group of 16 qm remains in the numerical structure of μ^- in the form of a neutral regular layer $n = 1$.
(2) At the same time, the layer $n = 2$ loses two supplementary qm and remains charged in the form $(64 - |16 + 2|)^-$ (cf. the rule of charge: a deviation ± 2 does not neutralize a deviation ± 16). One of the lost quanta remains in the numerical structure of μ^- in the form of a central quantum associated with the spin-1/2 of μ^-; the other is truly lost by μ^-: it is the quantum from which the muon neutrino ν_μ is emitted by the process before. Owing to (1) and (2), the basic mass 208 qm becomes $[207]^- + [1]$ qm.
(3) Still at the same time, the layer $n = 3$ loses a group of 8 qm and becomes the neutral $(144 - 8)$ layer (a deviation of type 8 is neutral); the 8 qm is lost by the layer $n = 3$ and remains in the structure of μ^- in the form of a deviation $+8$ borne by the layer $n = 1$. So the complete numerical structure of μ^- is

$$1 + (16 + 8) + (64 - |16 + 2|)^- + (144 - 8) = 207^- \text{ qm}. \quad (9)$$

The hypothesis of the internal transfers explains the numerical mass 207 qm of μ^- (it approaches the experimental mass with three significant figures), its charge, its spin-1/2, and the joint appearance of the pair $\mu^-\nu_\mu$. Below we justify explaining the exact mass of μ^- and its decay modes.

4.2.3 *The Mass of the Muon*

Being stable, μ^- has a mass defect $-\Delta(\mu^-) = -M/784$ qm, where M is some numerical mass. In order to determine M in agreement with the quantization rules, we recall that the spin and charge deviations are integers (expressed in the unit qm). They may not contribute to M; this is the case for the central qm responsible for the spin-1/2 of μ^-, for the deviation -16 qm present in the layer $n = 2$ and that has become the layer $n = 1$, and for the group of 8 qm lost by the layer $n = 3$ that has become the deviation borne by the layer $n = 1$.

Thus one has

$$M = 207 - (1 + 16 + 8) = 182 \text{ qm}, \quad (10)$$

$$-\Delta(\mu^-) = -182/784 = -0.232\,143 \text{ qm},$$

$$m(\mu^-) = 207 - \Delta(\mu^-) = 206.767\,857 \text{ qm}, \quad (11)$$

which approaches the measured mass to six significant figures.

4.2.4 *The Decay Modes of* μ^-

The isobar responsible for the dominant mode $\mu^- \rightarrow e^-\bar{\nu}_e\nu_\mu$ has the numerical structure (9); its decay process is as follows.

It contains $GA(e^-\bar{\nu}_e) = (1 + 8)$ qm, the central quantum receiving the charge $-e$ lost by the layer $n = 2$, which becomes neutral in the way $(64 - |16 + 3|)$ (cf. the rule of charge: a deviation ± 3 neutralizes a deviation ± 16). The charged central quantum is emitted in the form e^-. Simultaneously, the neutrino is generated by the process (Sec. 2.5.1)

$$8 \rightarrow \nu_e + |8 - \nu_e|. \quad (12)$$

The muon neutrino ν_μ is emitted from the quantum lost by the layer $n = 2$ according to the process (Sec. 2.5.2)

$$1 \rightarrow \nu_\mu + |1 - \nu_\mu|. \quad (13)$$

This is the same process that causes the emission of ν_μ at the birth of μ^-; so one understands how a muon neutrino is emitted when a muon appears or when a muon disappears in decaying.

The isobar for the mode $e^-\bar{\nu}_e\nu_\mu\gamma$ also has the structure (9), the difference being that the spin-1/2 of μ^- is conserved by the emission of e^- and ν_e with parallel spins-1/2 counterbalanced by the antiparallel spin-1 of the photon γ, ν_μ being emitted with spin-1/2 as for the dominant isobar.

Both isobars for the mode $e^-\bar{\nu}_e\nu_\mu e^+e^-$ before may only emit one electron because their structures (9) contain only one deviation $+1$ able to generate one electron. The present mode corresponds to a rare isobar that contains two supplementary deviations $+1$ qm; its structure still contains the central quantum 1 and the deviation $+8$ borne by the layer $n = 1$ which form together $GA(e,\nu_e)$. We conjecture that this isobar contains the charged layer $n = 2(64 - |16 + 2|)^-$; so the only possibility that this isobar contains two supplementary deviations $+1$ is that the layer $n = 3$, that is, $(144 - 8)$, is missing and is replaced by some layers $n \leq 2$ adding up to 136 qm. The numerical structure of this isobar is most probably as follows:

$$1 + (16 + 8) + (64 - |16 + 2|)^- + (64 + 1)^+ + (16 + 1)^- + (16 + |8 - 2|) + 16 + 16 = 207^- \text{ qm}, \quad (14)$$

the last five layers being equivalent to 136 qm. It is not a surprise that both layers $(64 + 1)^+$ and $(16 + 1)^-$, which have the same deviation $+1$, may bear charges of opposite signs, for the charge rules dictate the sign of a charged deviation.

This isobar of μ^- is about 10^5 times less abundant than the dominant one. However, it is interesting in showing that in a muon a quantum layer of rank 3 may degenerate into other layers of lower ranks. This isobar of μ^- is a first example of such a possibility. More generally it can be shown that many abundant isobars of mesons may present such a degeneracy, which is one of many causes of the numerous decay modes of mesons.

So the rules of PQ applied to $\mu^\pm$ explain its mass, spin, charge, stable state, emission of ν_μ at its birth and at its decay, and the mechanism of its dominant decay mode and of rarer modes. Moreover, it is shown that μ^-, a lepton, nevertheless, obeys the rules of PQ.

PQ also identifies the mass quantum as the mass of the electron, this also being a lepton. It also explains how the neutrino ν_e and the muon neutrino ν_μ, which also are leptons, are produced. This means that PQ is applicable to leptons as well as hadrons: nucleons, mesons, baryons. It is also applicable to the photon when some particles decay by emitting one or two of them.

4.3 The π^- Meson (or π^+)

4.3.1 *Numerical Structure*

$$m(\pi^-) = (139.5675 \pm 0.0004)\ \text{MeV}$$
$$= (273.1267 \pm 0.0008)\ \text{qm (precision } 3 \times 10^{-6}).$$

The basic layers are $n = 1$, $n = 4$,

$$(16 + 256) = 272\ \text{qm}. \quad (15)$$

The numerical structure of π^- results from the fact that both layers join in a single charged layer:

$$(256 + 16)^- = 272^-\ \text{qm}. \quad (16)$$

Remark: π^- being stable, it must possess a negative mass defect. Therefore, it seems rather paradoxical that its measured mass is greater than its numerical mass. The solution to that puzzle may be found in the simultaneous existence in $\pi^\pm$ of a negative mass defect, $-\Delta_2$, and a positive mass defect, $+\Delta_1$, the origin of which appears to be structural, as detailed below.

4.3.2 *The Positive Mass Defect of π^-*

We first recall how the neutron generates the proton in the decay $n^0 \to p + e^- + \nu_e$. The neutronic doublet

$$[1 + 16 + 64 + (144 + 16)^- + (256 - 16)^+ + 576 + 784] = 1841\ \text{qm} \quad (17)$$

spontaneously generates a preprotonic doublet at the end of its life; this doublet is written as

$$\text{GA(p)} = [1 + 16 + 64 + (400 - 16)^+ + (576 + 16)^- + 784] = 1841\ \text{qm}. \quad (18)$$

It generates p in losing one quantum under the form e^-.

We recall that each particle that can generate p contains necessarily, in its numerical structure, GA(p). Consider, for example, the decay mode of the hyperon $\Lambda^0 \to p\pi^-$. We may say that the corresponding isobar Λ^0 contains GA(p) in its numerical structure. The numerical structure of Λ^0 is easily found to be a little greater than its measured mass $m(\Lambda^0) = (1115.67 \pm 0.05)$ MeV $= (2183.31 \pm 0.10)$ qm; so one has the numerical mass of Λ^0:

$$[1841\ (= \text{GA(p)}] + [256 + 64] + [16 + |8 + 1|] = 2186\ \text{qm}. \quad (19)$$

We admit that the first term is responsible for the emission of p from Λ^0 while the second term emits π. The third term annihilates in emission energy. Thus the numerical association that generates π, in the decay of Λ^0, is the association $n = 2$, $n = 4$, that is, $(256 + 64)$. In order to discover the exact form of GA(π), we must specify how the charge zero of Λ^0 is conserved throughout the emission process and how the mass defect is transmitted.

(1) When the GA(p) present in the numerical structure (19) of Λ^0 emits p, it loses one quantum which is not emitted under the form of an electron but is transferred to the association of layers $[256 + 64]$; this becomes the charged association

$$\text{GA}(\pi^-) = [256 + (64 + 1)]^-, \quad (20)$$

which emits π^- by the process

$$[256 + (64 + 1)]^- \to \underline{(256 + 16)}^- + (16 + |8 + 1|) + (16 + 8). \quad (21)$$

The neutral residual layers $n = 1$, that is, $(16 + |8 + 1|)$ and $(16 + 8)$, annihilate in emission energy of p and π^-. So the charge zero of Λ^0 is conserved by the emission of p. The underlined association of layers in (21) represents the numerical structure of π^-. Hence we arrive at this important conclusion: when Λ^0, or any baryon, generates $p\pi^-$, the π^- meson whose numerical structure contains the association of layers $n = 1$, $n = 4$ results from the charged association (20) of layers $n = 2$, $n = 4$.

(2) When GA(p) is present in the structure of n^0 that decays into p, it loses a quantum that is emitted under the form e^-; it also loses a mass defect $+\delta = (1200/784)$ qm $= 1.530\,612$ qm which corresponds to the emission energy of the trio (p, e^-, ν_e). However, when Λ^0 decays in $p\pi^-$, no electron is emitted. Thus the quantum lost by GA(p) is simply transferred to the association $[256 + 64]$ that becomes the charged GA(π^-) (20). We admit that the mass

defect δ is not emitted but is also transferred to (20), which becomes

$$\mathrm{GA}(\pi^-) = [256 + (64 + 1)]^- + \delta. \tag{22}$$

The generation of π^- in the Λ^0 decay occurs, therefore, in agreement with

$$[256 + (64 + 1)]^- + \delta \rightarrow \underline{[256 + 16]}^- + \delta + (16 + |8 + 1|) + (16 + 8) \tag{23}$$

instead of (21). This establishes the existence of a positive mass defect $+\delta$ in the structure of π^- when it is emitted from a baryon, and, as announced before, this defect $+\delta$ has a structural origin. Therefore, it has no incidence on particle stability. Moreover, it is remarkable that this defect is also present in π^- if it is generated differently, for example, in a meson decay. This is experimentally confirmed by the coincidence of the measured mass of π^- reaching the precision 3×10^{-6} whatever the particle that emits it.

This does not mean, however, that the expression $\mathrm{GA}(\pi^-)$ given by (22) in the case of a baryon disintegration is also valid in the case of a meson disintegration. This point will be clarified in Sec. 4.3.10.

4.3.3 *The Negative Mass Defect $-\Delta_2$ of π^-*

The $\mathrm{GA}(\pi^-)$ given by (22) must also contain a negative mass defect $-\Delta_2$ in agreement with the rule of stability. This defect is given by the formula

$$-\Delta_2 = -(M/784)\ \mathrm{qm}, \tag{24}$$

where M is an integer mass $M \leq \mathrm{GA}(\pi^-)$. This defect is transmitted from $\mathrm{GA}(\pi^-)$ to π^-. In $\mathrm{GA}(\pi^-)$ we note that the deviation $+1$ must remain an integer and may therefore not contribute to M; one thus has $M \leq 320$ qm. We conjecture that both layers $n = 4$, $n = 2$ contribute to M except for two quanta in each layer. So we find so

$$M = (256 - 2) + (64 - 2) = 316\ \mathrm{qm}, \tag{25}$$

$$-\Delta_2 = -316/784 = -0.403\,061\ \mathrm{qm}. \tag{26}$$

4.3.4 *The Mass of π^-*

We find

$$m(\pi^-) = (256 + 16)^- + \delta - \Delta_2 = 272 + 1200/784 - 316/784 = 273.127\,551\ \mathrm{qm} \tag{27}$$

in agreement with the experimental mass.

Remark: The reader may perhaps ask why the mass defect $-\Delta_2(\pi^-)$ is determined from the numerical mass $M = [(256 - 2) + (64 - 2)] = 316$ qm and not from the numerical mass $(256 + 16)^-$ of π^-. Our answer is this: with the numerical mass $M = 316$ qm, one agrees with the measurement $m(\pi^-)$, the precision of which is 3×10^{-6}. If one considered the numerical mass $(256 + 16)^-$ of π^-, the specific value of M would be ≤ 256 qm, because the deviation $+16$ must remain an integer; so one should have $-\Delta_2(\pi^-) = -256/784 = -0.327$ qm and $m(\pi^-) = 272 + \delta - \Delta_2 = 273.204$ qm, which deviates from the measurement by the proportion 3×10^{-4}, that is, 100 times the error of the measured value.

Moreover, it has been shown that when a baryon, for example, Λ^0, emits a meson π^-, $\mathrm{GA}(\pi^-)$ is the association $[256 + (64 + 1)]^-$. So it is established that π^-, formed from the association of both layers $n = 4$, $n = 1$, is generated from a determined association (21) of both layers $n = 4$, $n = 2$, which transmits its mass defect to π^-.

A similar remark concerns π^0. The preceding is not a violation of the quantization rules, because the rule of stability says that the mass defect of a stable particle is determined from a "specific" numerical mass M. For the proton, neutron, muon, and many other stable particles the value of M is effectively a part of the numerical mass of the considered particle. For the π meson it is not so because π inherits its mass defect of another quantized association, that is, $\mathrm{GA}(\pi)$.

4.3.5 *The π^0 Meson*

Data:

$$m(\pi^0) = (134.9739 \pm 0.0006)\ \mathrm{MeV} = (264.1373 \pm 0.0012)\ \mathrm{qm}\ (\text{precision } 5 \times 10^{-6}).$$

The differential measurement $[m(\pi^-) - m(\pi^0)]$ is a little more accurate:

$$m(\pi^-) - m(\pi^0) = (4.5935 \pm 0.0005)\ \mathrm{MeV} = (8.9893 \pm 0.0010)\ \mathrm{qm}.$$

4.3.6 *Numerical Structure of π^0*

The numerical structure of π^0 looks like that of π^- except that the layer $n = 1$, which plays the part of a charged $+16$ deviation in π^-, is replaced in π^0 by a neutral deviation $+ |8 - 1|$. Thus the numerical structure of π^0 is

$$(256 + |8 - 1|) = 263\ \mathrm{qm}. \tag{28}$$

The short lifetime of π^0 ($\approx 10^{-16}$ s) is compared to that of π^- ($\approx 10^{-8}$ s); it is possible that this difference results from the fact that π^- is the association of two regular layers $n = 4$, $n = 1$ while it is not the case for π^0.

4.3.7 *The Mass of π^0*

We allege that π^0, like π^-, contains the positive mass defect $+\delta = +1.530\,612$ qm but has a negative mass defect $-\Delta_2'$ that differs slightly from the negative defect $-\Delta_2$ of π^-. Again, the meson π^- is formed by the association $(256 + 16)^-$ of both layers $n = 4$, $n = 1$ that bears the inherited negative mass defect $-\Delta_2 = -M/784$ with $M = (256 - 2) + (64 - 2) =$

316 qm. In the same way, π^0 is generated from the association of the layers $n = 4$, $n = 2$ and the negative mass defect $-\Delta'_2$ is due to the specific mass

$$M' = (256 - |8-2|) + (64 - |8-2|) = 308 \text{ qm}. \quad (29)$$

Thus one has for π^0

$$-\Delta'_2 = -308/784 = -0.392\,857 \text{ qm} \quad (30)$$

and therefore

$$\begin{aligned} m(\pi^0) &= (256 + |8-1|) + \delta - \Delta'_2 \\ &= 263 + 1.530\,612 - 0.392\,857 \\ &= 264.137\,755 \text{ qm}, \end{aligned} \quad (31)$$

in agreement with measurement.

The difference between the "theoretical" masses of π^- and π^0 equal to 8.989 796 qm also agrees with the differential measurement above.

4.3.8 *The Decay Modes of* π^-

The dominant mode is $\pi^- \to \mu^-\bar{\nu}_\mu(99.988\%)$. Very rare modes like $e^-\bar{\nu}_e$, $e^-\bar{\nu}_e\pi^0$, $e^-\bar{\nu}_e\gamma$ also exist. The dominant mode corresponds to the following process:

$$(256 + 16)^- \to \underline{[207^-] + [1]} + 64,$$

where $[207^-] + [1]$ coincides with $GA(\mu^-\nu_\mu)$. The emission energy is furnished by the annihilation of the layer 64 and by the residue $|1 - \nu_\mu|$ qm, account being taken of the mass defects of π^- and μ^-. The spin-0 of π^- is conserved since μ^- and ν_μ are emitted with antiparallel spins-1/2. One understands why the dominant isobar is unable to generate the pair $e^-\nu_e$ since it does not exhibit $GA(e^-\nu_e) = |1 + 8|$ qm. However, this mode $\pi^- \to e^-\bar{\nu}_e$ is really observed and PQ explains it by considering the following strongly degenerate isobar of π^-:

$$\begin{aligned} &(64 + \underline{1})^- + (64 + \underline{8}) + (64 - 8) \\ &\qquad + (64 - |16 + 1|) + 16 + 16 = 272^- \text{ qm}. \end{aligned}$$

The underlined terms correspond to the expected $GA(e^-\nu_e)$. Other strongly degenerate isobars are responsible for rare decays which are not considered here for the sake of brevity.

4.3.9 *The Decay Modes of* π^0

We restrict ourselves to three modes for the same sake of brevity.

(1) $\pi^0 \to \gamma\gamma(98.8\%)$. This dominant isobar of π^0 has the numerical structure of π^0 $(256 + |8 - 1|)$, which cannot emit an electron because no deviation +1 is present in it. Thus π^0 may not emit μ^- or μ^+. The only remaining possibility is the decay $\gamma\gamma$, with both photons being emitted with antiparallel spins-1 in order to conserve the zero spin of π^0.

(2) $\pi^0 \to e^-e^+\gamma(1.2\%)$. The isobar corresponding to this mode exhibits two deviations +1 responsible for the emission e^-, e^+ with parallel spins-1/2 opposite to that of γ; the structure of this isobar is thus

$$1 + 1 + (256 + |8 - 3|) = 263 \text{ qm}.$$

For the same reason as before, the only possibility for $(256 + |8 - 3|)$ is to generate a photon and furnish the emission energy of the pair (e^+,e^-).

(3) $\pi^0 \to e^-e^+e^-e^+(3 \times 10^{-5})$. The corresponding isobar has a structure sufficiently degenerate to exhibit four deviations +1. In detail, it is written as

$$\begin{aligned} &(64 + 1)^+ + (16 + 1)^+ + (64 + 1)^- + (16 + 1)^- \\ &\qquad + (64 + |16 + 3|) + 16 = 263 \text{ qm}. \end{aligned}$$

The spin-0 of π^0 is conserved in an obvious way (no central qm). *Like π^-, π^0 possesses rare isobars with strongly degenerate numerical structures.*

Note: the decay mode $\pi^- \to \pi^0e^-\bar{\nu}_e$, which obeys the conservation of charge and spin-0 of π, is extremely rare (frequency 10^{-8}). PQ explains why below.

The numerical structure $(256 + 16)^-$ of π^- contains no $GA(e^-\nu_e) = (1 + 8)$ qm. However, it is possible that the structure of π^- contains $GA(e^-\nu_e)$ in the degenerate structure

$$(64 + \underline{1})^- + (64 + \underline{8}) + 64 + (64 + |8 - 1|) = 272^- \text{ qm}.$$

Such a structure can emit $e^-\nu_e$, but how can it emit π^0, the structure of which is $(256 + |8 - 1|)$? It is possible under the condition that the four layers $n = 2$ join together in forming $(256 + |8 - 1|)$, while the deviations $(1 + 8)$ underlined emit $e^-\nu_e$. But such a process has a very small probability of happening; indeed, two conditions must be satisfied:

(1) the structure of π^- must be degenerate in order to contain $GA(e^-\nu_e)$ as shown before;

(2) to produce π^0, it is necessary that the layers $n = 2$ join together in regenerating the structure $(256 + |8 - 1|)$ of π^0. Thus the structure of π^- must first degenerate and afterwards be regenerated for producing π^0. One understands that the probability of occurrence of such opposite transformations is very small.

A similar remark concerns the process $K^0 \to e^+\nu_eK^-$, which satisfies the conservation of charge and spin. The structure of K^0 contains no $GA(e^+\nu_e)$ unless it is degenerate. But for forming K^-, this degenerate structure of K^0 must be regenerated; this is why only the process $K^0 \to K^-e^+\nu_e$ is extremely rare, with the frequency of occurring of 10^{-8}.

4.3.10 *Generating Associations of* π^- *and* π^0

Many mesons or baryons decay in generating π^- or π^0. Their numerical structure must therefore exhibit $GA(\pi^-)$, $GA(\pi^0)$, or both. We have shown before that in a baryon (b) decay, the structures of $GA^{(b)}(\pi^-)$ and $GA^{(b)}(\pi^0)$ are respectively written as

$$GA^{(b)}(\pi^-) = [256 + (64 + 1)]^- = 321^- \text{ qm}, \quad (32)$$

$$GA^{(b)}(\pi^0) = [256 + 64] = 320 \text{ qm}. \quad (33)$$

We shall show below (Sec. 5) that in a meson (m) decay, $GA^{(m)}(\pi^-)$ and $GA^{(m)}(\pi^0)$ are not the same as in a baryon decay. We take as an example the meson K^- or K^0; K^- has a decay mode $\pi^- \pi^+ \pi^-$ and K^0 has a mode $\pi^0 \pi^0 \pi^0$. The proof is that PQ forbids that the numerical structure of K^- contains three charged $GA^{(b)}(\pi)$, each of mass 321 qm, and that the numerical structure of K^0 forbids the presence of three $GA^{(b)}(\pi^0)$. The demonstration is delayed to Sec. 5.7. In waiting, we conjecture that $GA^{(m)}(\pi^-)$ and $GA^{(m)}(\pi^0)$ are respectively given by

$$GA^{(m)}(\pi^-) = [256 + (64 - 16)]^- = 304^- \text{ qm}, \quad (34)$$

$$GA^{(m)}(\pi^0) = [256 + (64 - |16 + 1|)] = 303 \text{ qm}. \quad (35)$$

We shall show (Sec. 5) that with these $GA^{(m)}(\pi)$, the process $K^- \rightarrow \pi^-\pi^+\pi^-$ or $K^0 \rightarrow \pi^0\pi^0\pi^0$ can be explained from the quantization rules. The processes by which $GA^{(m)}(\pi^-)$ generates π^- and $GA^{(m)}(\pi^0)$ generates π^0 are as follows:

$$[256 + (64 - 16)]^- \rightarrow \underline{(256 + 16)}^- + 16 + 16, \quad (36)$$

$$[256 + (64 - |16 + 1|)] \rightarrow \underline{(256 + |8 - 1|)} + (16 + 8) + 16, \quad (37)$$

where the underlined associations are the numerical structures of π^- and π^0.

Compared with $GA^{(b)}(\pi^-)$ and $GA^{(b)}(\pi^0)$, one has

$$[256 + (64 + 1)]^- \rightarrow \underline{(256 + 16)}^- + (16 + |8 + 1|) + (16 + 8)$$

$$[256 + 64] \rightarrow \underline{(256 + |8 - 1|)} + (16 + |8 + 1|) + 16 + 16$$

4.4 The Stable K Mesons

4.4.1 *Experimental Data*

Stable K mesons are neutral (K^0) or charged ($K^\pm$). The observation of a large population of neutral K^0's reveals that they are made up of two equal classes of particles denoted K^0_L and K^0_S with a different mean life. The masses of the K mesons have recently been measured with increased accuracy in comparison with the data available when the authors considered the problem in a previous paper. One has

$$m(K^\pm) = (493.646 \pm 0.009) \text{ MeV} = (966.041 \pm 0.018) \text{ qm (accuracy } 1.9 \times 10^{-5})$$

$$m(K^0) = (497.671 \pm 0.031) \text{ MeV} = (973.918 \pm 0.062) \text{ qm (accuracy } 6.4 \times 10^{-5}).$$

The very slight difference between K^0_L and K^0_S is

$$m(K^0_L) - m(K^0_S) = (3.522 \pm 0.016) \times 10^{-12} \text{ MeV} = (6.892 \pm 0.032) \times 10^{-12} \text{ qm}$$

and escapes at present the rules of PQ; it is not considered here.

4.4.2 *Basic Association of K^0 and K^-*

The basic association of K^0 and K^- results from the association of the layers $n = 5$ and $n = 6$: $400 + 576 = 976$ qm.

4.4.3 *Numerical Association of K^-*

The numerical association of K^- is given by the following structure:

$$[(400 - 16)^- + (576 + |8 - 1|)] = 967^- \text{ qm}, \quad (38)$$

which is near the measured value (accuracy 10^{-3}).

The spin-0 results from the lack of central qm.

4.4.4 *Mass of K^-*

Since K^- is stable, its mass is equal to its numerical mass, diminished by the stability mass defect $-\Delta(K^-) = -(M/784)$ qm. The mass M is an integer equal to 752 qm in order to fit the experimental datum. That means that the layer $n = 6$ contributes to M, but not the deviation $|8 - 1|$, which must remain an integer, and the layer $n = 5$ partly contributes to M for an amount of $(144 + 16 + 16)$ qm. So one has

$$-\Delta(K^-) = -752/784 = -0.959 \quad (39)$$

and

$$m(K^-) = 967 - \Delta(K^-) = 966.041 \text{ qm}. \quad (40)$$

4.4.5 *Numerical Association of K^0*

The numerical association of K^0 is similar to that of K^- except for two neutral deviations:

$$[(400 - |8 + 1|) + (576 + 8)] = 975 \text{ qm}.$$

4.4.6 *Mass of K^0*

The stability mass defect is again given by $-\Delta(K^0) = -(M/784)$. We conjecture that the layer $n = 6$ contributes to M (except the deviation $+8$, which must remain an integer) and that the layer $n = 5$ contributes partly to M by an amount of $(256 + 16)$; so one finds

$$-\Delta(K^0) = -848/784 = -1.082 \text{ qm}, \quad (41)$$

$$m(K^0) = 975 - \Delta(K^0) = 973.918 \text{ qm}. \quad (42)$$

These models of K^- and K^0 also accurately account for the difference of mass between K^0 and K^-:

$$m(K^0) - m(K^-) = 7.877 \text{ qm} \quad (43)$$

experimentally, (7.875 ± 0.063) qm.

Remark: We have shown that stable mesons present a negative mass defect, responsible for their stability. However, there is no evident connection between the value of the mass defect and the mean life of the particle. An example is furnished by K^0_S and K^0_L, which have the same mass defect but distinct lifetimes. Similarly, π^- has a mass defect of about -0.403 qm, and it is 120 times more short-lived than μ^-, which has a mass defect of about -0.232 qm.

4.4.7 *Remark about GA(π)*

In Sec. 4.3.10 we made a distinction between

$$GA^{(b)}(\pi^-) = [256 + (64 + 1)]^- = 321^- \text{ qm}$$

and

$$GA^{(m)}(\pi^-) = [256 + (64 - 16)]^- = 304^- \text{ qm},$$

depending upon whether π^- is generated from a baryon or a meson. The proof that $GA^{(m)}(\pi^-)$ may exist in the form $[256 + (64 - 16)]^-$ and may not exist in the form $[256 + (64 + 1)]^-$ is made here by exclusion. Let us first consider K^- (numerical mass 967^- qm). One of its decay modes is $K^- \to \pi^-\pi^+\pi^-$. If $GA^{(m)}(\pi^\pm)$ was $[256 + (64 + 1)]^\pm$, the three charged π's would have together a numerical mass of 963 qm; it would thus remain in the numerical structure of K^- 4 qm, that is, two neutral deviations each equal to $+2$ qm; this is impossible because a deviation must be borne by a disposable layer and no disposable layer exists. On the contrary, $GA^{(m)}(\pi^-)$ is equal to 304^- qm; so the three charged $GA^{(m)}(\pi^\pm)$ together have the numerical mass 912 qm. The remaining 55 qm in the structure of K^- would coincide with the neutral layer $(64 - |8 + 1|) =$ 55 qm or, equivalently, to the three neutral layers: 16, 16 and $(16 + |8 - 1|)$.

Similarly, K^0 has the decay mode $K^0 \to \pi^0\pi^0\pi^0$. If $GA(\pi^0)$ were $[256 + 64] = 320$ qm, the three $GA(\pi^0)$ would together have the numerical mass 960 qm and there would remain 15 qm in the structure of K^0, for its numerical mass is 975 qm. Now 15 qm may be the neutral deviation $|16 - 1|$, but that is impossible since there is no disposable layer to bear it. Fifteen qm may also be a charged layer $(16 - 1)^-$, but that is impossible since K^0 is neutral. However, $GA(\pi^0)$ may be $[256 + (64 - |16 + 1|)] = 303$ qm so that the three $GA(\pi^0)$ together have the numerical mass 909 qm. There remains, in the structure of K^0, $(975 - 909) = 66$ qm; that is, the neutral layer $(64 + 2)$ or, equivalently,

$$(16 + |8 + 1|) + (16 + |8 + 1|) + 16 = 66 \text{ qm}.$$

This proof confirms that the rules of quantization allow some quantized structures and forbid others.

4.4.8 *Another Mode of Generation of $\pi^\pm$ and π^0*

Some mesons that decay into π^- or π^+ contain no $GA^{(m)}(\pi)$ in their structure. However, such mesons contain a layer $n = 7$, $n = 6$, or $n = 5$ in their structure, and at the end of their lifetime this layer generates $GA^{(m)}(\pi)$. Some examples follow:

$$(784 + 1)^- \to [\underline{|256 + (64 - 16)}]^- + 256 + 144 + 64 + \text{deviation } |16 + 1|$$

$$(576 + 16)^+ \to [\underline{|256 + (64 - 16)}]^+ + 256 + 16 + 16$$

$$(400 - 3)^- \to [\underline{|256 + (64 - 16)}]^- + (64 + |16 - 3|) + 16$$

$$784 \to [\underline{|256 + (64 - |16 + 1|)}] + 256 + 144 + 64 + \text{deviation } |16 + 1|$$

$$(576 + 8) \to [\underline{|256 + (64 - |16 + 1|)}] + 256 + 16 + \text{deviation } |8 + 1|$$

$$(400 - 8) \to [\underline{|256 + (64 - |16 + 1|)}] + 64 + 16 + \text{deviation } |8 + 1|$$

Remark: When a meson has in its numerical structure both layers $n = 5$ and $n = 6$, it always generates a K meson; if a heavy meson has in its numerical structure two layers $n = 5$ and two layers $n = 6$, it always emits two K mesons.

4.4.9 *The Decay Modes of K^0_S*

K^0_S and K^0_L differ in their mean lifetimes and their decay modes. K^0_S has three modes: $\pi^-\pi^+$ ($\approx 68.61\%$), $\pi^0\pi^0$ ($\approx 31.39\%$), $\pi^-\pi^+\gamma$ ($\approx 1.85\%$), and several rare modes that we neglect.

(1) $K^0_S \to \pi^-\pi^+$. A question must be asked: how can the neutral K^0_S generate two charged pions? PQ explains the mechanism by which two opposite charges appear in a neutral particle, that is, how K^0_S is transformed in a doublet. The layer $n = 5$ loses $|8 - 1|$ qm and becomes $(400 - 16)^-$, while the layer $n = 6$ loses $|8 + 1|$ qm and becomes $(576 - 1)^+$. Together the layers have lost 16 qm that form a layer $n = 1$. So this isobar of K^0_S has the structure $(400 - 16)^- + (576 - 1)^+ + 16 = 975$ qm. So, one sees how both charges can appear by means of an internal displacement of qm.

At the end of the lifetime of this isobar, the layers $n = 5$ and $n = 6$ decay "separately":

$$(400 - 16)^- \to [\underline{|256 + (64 - 16)}]^- + 64 + 16,$$

$$(576 - 1)^+ \to [\underline{|256 + (64 - 16)}]^+ + (256 + |16 - 1|),$$

where one recognizes $GA^{(m)}(\pi^-)$ and $GA^{(m)}(\pi^+)$.

(2) $K^0_S \to \pi^0\pi^0$. This isobar of K^0_S has the numerical structure

$$(400 - |8 + 1|) + (576 + 8) = 975 \text{ qm}.$$

Once again, the layers decay "separately"

$$(400 - |8 + 1|) \to [\underline{|256 + (64 - |16 + 1|)}] + (64 + 8) + 16$$

$$(576 + 8) \to [\underline{[256 + (64 - |16 + 1|)]} + 256 + (16 + |8 + 1|)]$$

where one recognizes 2 $GA^{(m)}(\pi^0)$.

Remark: There is no drawback in writing $(256 + |8 + 1| + 16)$ instead of $256 + (16 + |8 + 1|)$, for these residual layers vanish in energy emission.

(3) $K_S^0 \to \pi^-\pi^+\gamma$. The preceding isobars (1) and (2) of K_S^0 have the spin-0 (no central qm). Since this isobar emits γ, the conservation of spin requires that it has spin-1, due to the presence in its structure of two central qm of parallel spins-1/2. One notes that thanks to the existence of various isobaric structures, PQ solves, in an elegant way, the problem settled by those particles that decay into more than one spin state. The structure of this isobar of K_S^0 is, therefore,

$$1 + 1 + (400 - |16 + 2|)^- + (576 - 1)^+ + 16 = 975 \text{ qm}.$$

The layer $n = 5$ decays in the following way:

$$(400 - |16 + 2|)^- \to [\underline{(256 + (64 - 16)}]^- + 64 + 16 + \text{deviation } |-2|.$$

The deviation is borne by one of the layers $n = 2$, $n = 1$ that participate in the energy emission. The layer $n = 6$ decays as in (1).

In the decay modes of K_S^0 the deviations play a necessary role.

4.4.10 *The Decay Modes of K_L^0*

The decay modes of K_L^0 are

(1) $\pi^0\pi^0\pi^0$ (21.6%),
(2) $\pi^-\pi^+\pi^0$ (12.38%),
(3) $\pi^-\mu^+\bar{\nu}_\mu$ (and its conjugate) together (27.0%),
(4) $\pi^-e^+\bar{\nu}_e$ (and its conjugate) together (38.7%),
(5) $\pi^-\pi^+$ ($\approx 2.03 \times 10^{-3}$),
(6) $\pi^0\pi^0$ ($\approx 9.09 \times 10^{-4}$),

and many other very rare modes.

While the numerical structure of K_S^0 contains the layers $n = 5$ and $n = 6$, the various decay modes of K_L^0 suggest that the structure of its isobars may be more or less degenerate ($n \leq 4$) in order to contain the GA of various particles.

(1) $K_L^0 \to \pi^0\pi^0\pi^0$. This isobar contains three $GA^{(m)}(\pi^0)$; its structure is

$$3 \times [256 + (64 - |16 + 1|)] + (64 + 2) = 975 \text{ qm}.$$

Unlike K_S^0, where the layers $n = 5$ and $n = 6$ are present and decay separately, in K_L^0 this isobar is degenerate and all its layers decay together.

(2) $K_L^0 \to \pi^-\pi^+\pi^0$. Similarly, this isobar has the degenerate structure

$$\underline{[256 + (64 - 16)]}^- + \underline{[256 + (64 - 16)]}^+ + \underline{[256 + (64 - |16 + 1|)]} + 64.$$

(3) $K_L^0 \to \pi^-\mu^+\nu_\mu$. This isobar contains $GA^{(m)}(\pi^-)$ and $GA(\mu^+\nu_\mu)$. In this isobar the layers $n = 5$ and $n = 6$ degenerate "separately":

$n = 5$:

$$(400 - |8 + 1|) = \underline{[207]^+ + [1]} + 2 \times 64 + (64 - |8 + 1|)$$

$n = 6$:

$$(576 + 8) = \underline{[256 + (64 - 16)]}^- + 4 \times 64 + (16 + 8).$$

There is no drawback in writing 256 instead of 4×64, these layers vanishing in energy emission.

(4) $K_L^0 \to e^-\bar{\nu}_e\pi^+$. Similarly one has

$n = 5$:

$$(64 + \underline{1})^- + (16 + \underline{8}) + 4 \times 64 + (16 + |8 - 1|) + (16 + |8 - 1|),$$

where one recognizes $GA(e^-\nu_e) = [1 + 8]$ underlined.

$n = 6$: as in (3).

(5) $K_L^0 \to \pi^-\pi^+$. This isobar of K_L^0 has the same decay mode as K_S^0 but it is less abundant. We ascribe this difference to the fact that in this isobar of K_L^0, the layers $n = 5$, $n = 6$ are degenerate:

$n = 5$:

$$(400 - |8 + 1|) = \underline{[256 + (64 - 16)]}^- + 64 + (16 + |8 - 1|)$$

$n = 6$:

$$(576 + 8) = \underline{[256 + (64 - 16)]}^+ + 4 \times 64 + (16 + 8)$$

No supplementary pion can be emitted by the remaining layers $(256) + (16 + 8)$ because $GA^{(m)}(\pi^0)$ contains 303 qm.

In short, K_S^0 has three isobars containing the layers $n = 5$ and $n = 6$, each of them generating a π meson. On the contrary, the isobars of K_L^0 exhibit degenerate structures that contain the GA of various particles. These structures are possible thanks to the deviations that PQ authorizes.

4.4.11 *The Decay Modes of* K^- *(id.* K^+*)*

They are

(1) $\pi^-\pi^+\pi^-$ ($\approx$5.59%),
(2) $\pi^-\pi^0\pi^0$ (1.7%),
(3) $\pi^-\pi^0$ (21%),
(4) $\mu^-\bar{\nu}_\mu$ (68.59%),
(5) $\pi^0\mu^-\bar{\nu}_\mu$ (3.18%),
(6) $\pi^0 e^-\bar{\nu}_e$ (1.82%),

and many rare ones that we neglect.

(1) $\pi^-\pi^+\pi^-$. The layers $n = 5$ and $n = 6$ are degenerate in such a way that "together" they form three $GA^{(m)}(\pi)$ that are charged:

$$\underline{[256 + (64 - 16)]}^- + \underline{[256 + (64 - 16)]}^+ + \underline{[256 + (64 - 16)]}^- + (64 - |8 + 1|) = 967^- \text{ qm}.$$

(2) $\pi^-\pi^0\pi^0$. Similarly,

$$\underline{[256 + (64 - 16)]}^- + \underline{[256 + (64 - |16 + 1|)]} + \underline{[256 + (64 - |16 + 1|)]} + (64 - |8 - 1|)$$

(3) $\pi^-\pi^0$. In this isobar of K^- the layers $n = 5$ and $n = 6$ are separately degenerate:

$$(400 - 16)^- \rightarrow \underline{[256 + (64 - 16)]}^- + 64 + 16$$

$$(576 + |8 - 1|) \rightarrow \underline{[256 + (64 - |16 + 1|)]} + 4 \times 64 + (16 + 8),$$

where 4×64 may be replaced by 256.

(4) $\mu^-\bar{\nu}_\mu$. This dominant isobar of K^- is characterized by the fact that the layer $n = 5$ is degenerate in such a way that it contains $GA(\mu^-\nu_\mu)$, and the layer $n = 6$ is degenerate in such a way that it contains no GA of a particle:

$$(400 - 16)^- \rightarrow \underline{[207]}^- + \underline{[1]} + 2 \times 64 + 3 \times 16,$$

$$(576 + |8 - 1|) \rightarrow (8 \times 64) + (64 + |8 - 1|);$$

there is no drawback in writing 2×256 instead of 8×64.

(5) $\mu^-\bar{\nu}_\mu\pi^0$. The layer $n = 5$ is degenerate as in (4), but the layer $n = 6$ is degenerate in such a way that it contains $GA^{(m)}(\pi^0)$:

$$(576 + |8 - 1|) \rightarrow \underline{[256 + (64 - |16 + 1|)]} + 4 \times 64 + (16 + 8)$$

(6) $e^-\bar{\nu}_e\pi^0$. In this isobar of K^- the layer $n = 5$ is degenerate in such a way that it contains $GA(e^-\nu_e) = |1 + 8|$ while the layer $n = 6$ contains $GA(\pi^0)$:

$$(400 - 16)^- \rightarrow (64 + \underline{1}) + (64 + \underline{8}) + 3 \times 64 + (64 - |8 + 1|)$$

$$(576 + |8 + 1|) \rightarrow \text{as in (5)}.$$

4.5 The Mesons D^0 and D^-

4.5.1 *Numerical Structure*

$$m(D^0) = (1864.5 \pm 0.5)\text{ MeV} = (3648.7 \pm 1.0)\text{ qm}$$

$$m(D^-) = (1869.3 \pm 0.4)\text{ MeV} = (3658.1 \pm 0.8)\text{ qm}.$$

The differential measurement is

$$m(D^-) - m(D^0) = (4.77 \pm 0.27)\text{ MeV} = (9.33 \pm 0.53)\text{ qm}.$$

This datum shows that the difference of mass between D^- and D^0 is probably equal to $|8 + 1|$ qm. One recalls two other differential measurements:

$$m(\pi^-) - m(\pi^0) = |8 + 1|\text{ qm and } m(K^0) - m(K^-) = 8\text{ qm}.$$

A similar deviation of 8 qm also appears in the numerical structures of the hyperons. So the mass deviation of type 8, which PQ foresees, is experimentally well confirmed.

4.5.2 *Mass Defect*

The mesons D^0 and D^- have a lifetime of about 10^{-13} s, like the other heavy stable mesons D^-_S, B^0, B^-. Does the stability of the heavy mesons correspond with the presence of a mass defect in their structure? If the answer is yes, then it means that their numerical mass is a little larger than their measured mass. However, one might impart their stability to a peculiar numerical structure that authorizes them to have a short lifetime without a mass defect; in such a case both numerical and measured masses would be equal. The relative inaccuracy in the mass measurements of D^0 and D^- does not allow us to decide. However, we may consider that if a mass defect exists for these heavy mesons, it is a small part of their numerical mass. It is on such grounds that we temporarily assign to D^0 and D^- the numerical masses

$$m(D^0) = 3648\text{ qm and } m(D^-) = 3657^-\text{ qm}.$$

4.5.3 *Basic and Numerical Mass of* D^0

We claim that the numerical structure of D^0 contains two truncated nucleon structures where the layer $n = 1$ is missing:

D^0:

$$[64 + 400 + 576 + 784] + [64 + 400 + 576 + 784] = 1824 + 1824 = 3648\text{ qm}. \quad (44)$$

4.5.4 *Numerical Structure of* D^-

The numerical structure of D^- is similar except that in one component a deviation $|8 + 1|$ exists, which confers to D^- a numerical mass of 3657 qm:

D^-:

$$[64 + 400 + 576 + 784 + |8 + 1|] + [64 + 400 + 576 + 784] = 1833 + 1824 = 3657 \text{ qm}.$$

According to PQ, the deviation $+|8 + 1|$ belongs to a layer. We have no way of knowing which, but this is not important, for it suffices to know that it exists whatever the layer that bears it. The former structure is the basic association of D^-, which determines its mass but not its numerical association since D^- is charged.

According to the rule of charge, if a layer of D^- bears the deviation -16 qm, the loss of mass will be counterbalanced by the presence of a layer $n = 1$. One finds

D^-:

$$[64 + 400 + 576 + 784 + |8 + 1|] + [16 + 64 + (400 - 16)^- + 576 + 784] = 1833 + 1824^- = 3657^- \text{ qm}. \quad (45)$$

From the quantization rules one gets an explanation of the masses of D^0 and D^-. Although the meson D is correlated to the nucleon, D^0 and D^- may not generate a nucleon because each component of D has a mass lower than that of the nucleon.

4.5.5 *The Meson $D^*(2010)^0$*

$$m[D^*(2010)^0] = (2007.1 \pm 1.4) \text{ MeV} = (3927.8 \pm 2.8) \text{ qm}$$

$$m[D^*(2010)^0] - m(D^0) = (142.5 \pm 1.3) \text{ MeV} = (278.9 \pm 2.6) \text{ qm}.$$

This last measurement suggests that the structure of $D^*(2010)^0$ includes that of D^0 with the additional layers 256 + 16 + 8; the deviation +8 is uncertain owing to the lack of accuracy. The structure of this meson is therefore

$D^*(2010)^0$:

$$[64 + 400 + 576 + 784] + [64 + 400 + 576 + 784] + 256 + 16 + \text{deviation } |+8|, \quad (46)$$

with the reservation that the deviation +8 may be borne by the layer 256 or 16.

4.5.6 *The Meson $D_S^\pm$*

One has

$$m(D_S^-) = m(D_S^+) = (1968.8 \pm 0.7) \text{ MeV} = (3852.8 \pm 1.4) \text{ qm}.$$

The meson D^-_S is the lightest member of the D_S family. The 1992 Table of Particle Properties shows that D^-_S does not decay by generating D. Indeed, such a mode is unlikely to account for the difference $m(D_S) - m(D)$, which is equal to (99.5 ± 0.6) MeV = (194.7 ± 1.2) qm. The difference is insufficient for generating a K, π, or $\mu^-\bar{\nu}_\mu$ meson; the only possibility would be $D^-_S \rightarrow D_0 e^- \bar{\nu}_e$, which does not appear in the table; moreover, the absence of the emission $e^-\nu_e$ probably results from the absence in the numerical structure of D^-_S of an isolated deviation of +1 qm, which would be necessary for emitting an e^-, and from the absence of a deviation of +8 qm, which would be necessary for emitting ν_e. One may thus consider that D_S and D constitute two different classes of particles. While we consider D an association of two truncated nucleonic components, we consider D_S an association of two nucleonic structures of mass 1840 qm but with some additional layers. So the basic structure of D_S which determines its mass is

$$[16 + 64 + 400 + 576 + 784] + [16 + 64 + 400 + 576 + 784] + 64 + 64 + 16 + 16 + \text{deviations } (|8 - 1| + |8 - 2|) = 3853 \text{ qm}. \quad (47)$$

However, since D_S is charged, its numerical structure is the following:

D^-_S:

$$[16 + 64 + 400 + 576 + 784] + [16 + 64 + 400 + 576 + 784] + (64 + 16)^- + 64 + 16 + \text{deviations } (|8 - 1| + |8 - 2|) = 3853^- \text{ qm}. \quad (48)$$

In D^- the charged layer belongs to a nucleonic component; while in D^-_S the charge is borne by a layer that is additional to both nucleonic components.

4.5.7 *The B Mesons*

$$m(B^-) = m(B^+) = (5277.6 \pm 1.4) \text{ MeV} = (10328.0 \pm 2.8) \text{ qm}$$

$$m(B^0) = (5279.4 \pm 1.5) \text{ MeV} = (10331.5 \pm 3.0) \text{ qm}.$$

We interpret these data as follows: the numerical associations of B are made up of five nucleonic structures of mass 1840 qm plus some additional layers:

B^0:

$$5 \times 1840 + 784 + 256 + 64 + 16 + |8 + 3| = 10331 \text{ qm}, \quad (49)$$

B^-:

$$5 \times 1840 + 784 + (256 - 16)^- + 64 + 16 + 16 + 8 = 10328 \text{ qm}, \quad (50)$$

where the deviation of type +8 is borne by an additional layer; its exact value requires better measurements.

Remarks: (1) B^0 and B^- may decay into D_S or D_0; the corresponding isobars thus contain the numerical structures of

D_S or D_0. (2) The Table of Particle Properties shows that B may emit p, $p\bar{p}$, or other baryons belonging to the classes called N, Δ, Λ; it shows also that B emits the meson D or D(2010) plus some other mesons π, ρ, ..., etc.

Experimentally, independent of any theory, a meson is a particle that eventually decays into an electron e^- or e^+ and a neutrino or γ photons; a baryon is a particle that eventually decays into a proton and electrons or γ photons. The particle B presents both kinds of decays.

4.6 The Unstable Meson η_{548}

$m(\eta) = (540.8 \pm 0.6)$ MeV $= (1074.0 \pm 1.2)$ qm is neutral and possesses some neutral and charged decay modes:

$\eta \to \gamma\gamma$ $(38.9 \pm 0.5)\%$,

$\pi^0\pi^0\pi^0$ $(31.9 \pm 0.4)\%$,

$\pi^-\pi^+\pi^0$ $(23.6 \pm 1.2)\%$,

$\pi^-\pi^+\gamma$ $(4.88 \pm 0.15)\%$,

$\pi^0\gamma\gamma$ $(7.1 \pm 1.4) \times 10^{-3}$,

$e^-e^+\gamma$ $(5.0 \pm 1.2) \times 10^{-4}$,

and other rare modes that are neglected.

The numerical structure and numerical mass of η are written as

η:

$$(784 + |16 + 1|) + (256 + |16 + 1|) = 1074 \text{ qm},$$

formed by the association of the layers $n = 7$ and $n = 4$, each of them bearing the mixed neutral deviation $+|16 + 1|$.

Remark. The mass measurement of η is ancient and fairly inaccurate. According to PQ, a particle with a stable state has a mass defect $-\Delta = -(M/784)$ qm, M being a numerical mass that is a part of the numerical mass of the considered particle (exception made for π). The error for $m(\eta)$ is too large to be sure that η has a mass defect obeying this rule. We need a better measurement. The same remark holds for many unstable particles.

The list of the various decay modes indicates the existence of 0-spin decays and 1-spin decays. How do we conciliate these facts with the spin conservation law? We shall see that PQ solves the puzzle by considering various η isobars of spins-0 and 1, respectively.

η being neutral, we do not exclude the possibility that some isobars may contain two charged GA with opposite signs if their layers bear some adequate deviations. We analyze the main decay modes.

(1) Mode $\eta \to \gamma\gamma$. This is the most frequent. The spin of this isobar is equal to 0, being due to the presence in its structure of two central qm with antiparallel spins-1/2; it is conserved by the emission of two γ photons with antiparallel spins-1. This isobar contains no GA of a particle e, ν_e, π^-, π^0, K^0, K^-. Its numerical structure is degenerate (no layer $n = 7$):

$$1 + 1 + 4 \times 256 + 3 \times 16 = 1074 \text{ qm}. \qquad (51)$$

In this isobar of η, the two deviations $+1$ that are present in its numerical structure have become two central qm.

(2) $\eta \to \pi^0\pi^0\pi^0$. As in (1), this isobar also has zero spin but this time due to the absence of a central qm. Its structure is also degenerate:

$$\underline{[256 + (64 - |16 + 1|)]} + \underline{[256 + (64 - |16 + 1|)]} + \underline{[256 + (64 - |16 + 1|)]} + 144 + 16 + |8 - 3|.$$

The deviation $+|8 - 3|$ is borne by one of the two residual layers. Note that the deviation $+|8 - 3|$ results from the two deviations $+1$ present in the numerical structure of η and from the three deviations -1 that appear in the three GA(π^0) present in this isobar.

(3) $\eta \to \pi^-\pi^+\pi^0$. Similarly, this isobar of η has the structure

$$\underline{[256 + (64 - 16)]}^- + \underline{[256 + (64 - 16)]}^+ + \underline{[256 + (64 - |16 + 1|)]} + (144 + |16 + 3|).$$

Note that the deviation $+3$ comes from the vanishing of the two deviations $+1$ of the numerical structure of η and from the appearance of the deviation -1 present in GA(π^0).

(4) $\eta \to \pi^-\pi^+\gamma$. Emitting γ, this isobar of η has spin-1 due to the presence in its structure of two central qm with parallel spins-1/2. Its structure is

$$1 + 1 + \underline{[256 + (64 - 16)]}^- + \underline{[256 + (64 - 16)]}^+ + 7 \times 64 + 16.$$

There is no drawback in replacing 4×64 by 256 since these layers vanish in energy. Note that in this isobar of η the two deviations $+1$ present in the numerical structure have become the two central qm responsible for the spin-1 of this isobar.

(5) $n \to \pi^0\gamma\gamma$. This isobar has spin-0 due to two central qm with antiparallel spins-1/2; it is conserved by the emission of two γ photons with antiparallel spins 1. Its structure is

$$1 + 1 + \underline{[256 + (64 - |16 + 1|)]} + 2 \times 256 + 3 \times 64 + 3 \times 16 + |16 + 1|.$$

The deviation $|16 + 1|$ is borne by a layer $n = 4$ or $n = 2$; one of the layers $n = 4$ can also be degenerate as 4×64. The central qm comes from the two deviations $+1$ present in the numerical structure of η.

(6) $\eta \to e^-e^+\gamma$. This rare isobar of η also has spin-1 as in (4)

and probably has a nondegenerate structure (the layers $n = 7$ and $n = 4$ are present), that is,

$$1 + 1 + (784 + \underline{1})^- + (256 + \underline{1})^+ + (16 - 1)^+ + (16 - 1)^- = 1074 \text{ qm}$$

where the two underlined +1s form GA(e^-e^+). The two deviations of +1 present in the numerical structure of η are replaced in this isobar by four deviations of +1 and two deviations of −1. Note that a deviation +1 (or −1) may correspond to a positive or negative charge, since it is in agreement with the charge rule: the sign of a charged deviation is specific to the particle.

In conclusion, the numerical structure of η which contains the layers $n = 7$, $n = 4$ determines some isobaric structures that are degenerate, at least for the layer $n = 7$. In the η meson the two deviations +1 present in its numerical structure play a main role in determining at once the spin-0 or 1 of its isobars and the particles, charged or neutral, that they emit.

The η meson is a good example of the role that the deviations play in the masses of particles, their electric charge and spin states, and their various decay modes, all in agreement with the rules of PQ as they have been formerly enunciated.

4.7 The Meson ρ_{770}

This unstable meson results from the interaction $\pi^-p \to \rho^0n^0$ or ρ^-p (or the conjugates). It has charge 0, −e or +e and mass

$$m(\rho^0) = m(\rho^+) = m(\rho^-) = (768.3 \pm 0.5) \text{ MeV} = (1503.5 \pm 1) \text{ qm}.$$

We identify its numerical structure and numerical mass as follows:

$$\rho^0\text{: } 784 + 576 + 144 = 1504 \text{ qm} \tag{52}$$

$$\rho^\pm\text{: } (784 - 16)^\pm + 576 + 144 + 16 = 1504^\pm \text{ qm}. \tag{53}$$

Particle quantization explains very simply the equality $m(\rho^0) = m(\rho^-)$, the charged deviation −16 borne by the layer $n = 7$ of ρ^- being counterbalanced by the layer $n = 1$.

ρ^0 decays into $\pi^0\pi^0$ or $\pi^-\pi^+$ (near 100%) and also in $\pi^-\pi^+\gamma$ (1.11 ± 0.10)%. ρ^- decays into $\pi^-\pi^0$ (≈100%) except three rare isobars that we neglect. We explain these modes as follows:

(1) $\rho^0 \to \pi^0\pi^0$. The layers of this isobar decay separately, each leading to the underlined GA$^{(m)}(\pi^0)$:

$$784 \to \underline{[256 + (64 - |16 + 1|)]} + 256 + 144 + (64 + |16 + 1|)$$

$$576 \to \underline{[256 + (64 - |16 + 1|)]} + (256 + |16 + 1|).$$

(2) $\rho^0 \to \pi^-\pi^+$. In this isobar of ρ^0 the layer $n = 7$ has transferred a group of 16 qm to the layer $n = 6$, so these layers become $(784 - 16)^-$ and $(576 + 16)^+$. These layers decay separately:

$$(784 - 16)^- \to \underline{[256 + (64 - 16)]}^- + 256 + 144 + 64$$

$$(576 + 16)^+ \to \underline{[256 + (64 - 16)]}^+ + 256 + 16 + 16.$$

For both (1) and (2) there is no possibility that the residual layers produce more than one pion for they cannot form GA$^{(m)}(\pi)$. In (2) the origin of the emission of two charged pions by the neutral ρ^0 is clear thanks to the exchange of 16 qm between the layers $n = 7$ and $n = 6$.

(3) $\rho^0 \to \pi^-\pi^+\gamma$. Emitting γ, this isobar has spin-1 (two central qm with parallel spins-1/2). Its structure is thus

$$1 + 1 + (784 - |16 + 2|)^- + (576 + 16)^+ + 144 = 1504 \text{ qm}.$$

Both layers $n = 7$ and $n = 6$ still decay separately:

$$(784 - |16 + 2|)^- \to \underline{[256 + (64 - 16)]}^- + 256 + 144 + 64 + \text{deviation } |-2|,$$

where the deviation is borne by one of the three residual layers: $(576 + 16)^+ \to$ as in (2).

(4) $\rho^- \to \pi^-\pi^0$. We refer now to the numerical structure (53) of ρ^-. In this isobar of ρ^- the charged layer $n = 7$ and the neutral layer still decay separately:

$$(400 - 16)^- \to \underline{[256 + (64 - 16)]}^- + 64 + 16$$

$$576 \to \underline{[256 + (64 - |16 + 1|)]} + (256 + |16 + 1|).$$

The neutral layer $(256 + |16 + 1|)$ cannot emit a supplementary π^0 because it is not GA$^{(m)}(\pi^0)$.

In conclusion, the isobars of ρ are nondegenerate for they contain the layers $n = 7$ and $n = 6$ of the numerical structure of ρ^0 or ρ^-; ρ^0 or ρ^- decay into 2π's because the layers $n = 7$ and $n = 6$ each decay separately by generating one π. The various isobars of ρ only differ from one other by some deviations.

4.8 The Meson ω

ω is produced in the $p\bar{p}$ interaction $p\bar{p} \to \omega n^0$. ω is neutral and unstable. Its mass is

$$m(\omega) = (781.95 \pm 0.14) \text{ MeV} = (1530.24 \pm 0.28) \text{ qm}.$$

It has three decay modes:

$$\omega \to \pi^-\pi^+\pi^0 \ (88.8 \pm 0.8)\%,\ \pi^0\gamma \ (6.55 \pm 0.8)\%,$$

$$\pi^-\pi^+ \ (2.21 \pm 0.3)\%.$$

The masses of ρ and ω are similar to each other, and they share the same decay modes 3π, 2π, and π with various frequencies, which is why we consider them two matched mesons. This does not mean that their numerical structures are the same. While ρ^0 has a numerical structure that is formed by the layers $n = 7$, $n = 6$, and $n = 3$, we conjecture that the numerical structure of ω is formed by the layers $n = 7$, $n = 6$ and $n \leq 2$:

ω:

$$784 + 576 + 64 + 64 + 16 + 16 + \text{deviation } |8 + 2| = 1530 \text{ qm}, \quad (54)$$

this deviation being borne by one of the layers $n \leq 2$.

However, the isobars of ρ and those of ω must differ from each other, since ρ has four $2\pi^-$ decay modes which correspond to four isobars where the layers $n = 7$ and $n = 6$ decay separately, with each layer producing one π. Since the dominant mode of ω is 3π, it is impossible that its layers decay separately, since the three $GA^{(m)}(\pi)$ that are necessary cannot come from either layer $n = 7$ or $n = 6$. Thus at least the dominant mode 3π of ω comes from a degenerate structure in such a way that this isobar can emit three π. We identify this structure as follows:

$$\underline{[256 + (64 - 16)]}^- + \underline{[256 + (64 - 16)]}^+ + \underline{[256 + (64 - |16 + 1|)]} + 2 \times 256 + 64 + 2 \times 16 + \text{deviation } |8 + 3| = 1530 \text{ qm}, \quad (55)$$

the deviation $|8 + 3|$ being borne by one of the disposable layers.

We conjecture that the two other isobars of ω are also degenerate:

$\omega \rightarrow \pi^-\pi^+$:

$$\underline{[256 + (64 - 16)]}^- + \underline{[256 + (64 - 16)]}^+ + 3 \times 256 + 2 \times 64 + 16 + \text{deviation } |8 + 2|.$$

As for the isobar $\omega \rightarrow \pi^0\gamma$, it has spin-1 (two central qm with parallel spins-1/2) and its structure is more degenerate:

$$1 + 1 + \underline{[256 + (64 - |16 + 1|)]}^- + 4 \times 256 + 3 \times 64 + \text{deviation } |8 + 1|.$$

The preceding shows that PQ explains simply the masses of ρ^0, $\rho^\pm$, and ω and also how the isobars of ρ and ω differ, those of ρ being not degenerate, while those of ω are degenerate. The same is true for K^0_S compared to K^0_L. Moreover, although the ρ and ω mesons are matched, they have their specificity, nature having refused to allow different mesons the same decay modes.

4.9 The Meson η'_{957}

$$m(\eta') = (957.50 \pm 0.24) \text{ MeV} = (1873.81 \pm 0.47) \text{ qm}$$

η' is unstable and neutral. The decay modes are

(1) $\pi^-\pi^+\eta$ $(44.2 \pm 0.27)\%$,
(2) $\rho^0\gamma$ $(30.0 \pm 1.6)\%$,
(3) $\pi^0\pi^0\eta$ $(20.5 \pm 1.3)\%$,
(4) $\omega\gamma$ $(3.00 \pm 0.37)\%$,
(5) $\gamma\gamma$ $(2.16 \pm 0.17)\%$,
(6) $\pi^0\pi^0\pi^0$ $(1.53 \pm 0.26) \times 10^{-3}$.

We identify the numerical structure and mass of η'_{957} as follows:

η'_{957}:

$$784 + 576 + 256 + 144 + 64 + 3 \times 16 + \text{deviation } |+2| = 1874 \text{ qm}. \quad (56)$$

This association differs from n^0 by some additional layers; moreover it has no spin. Some isobars of η'_{957} are more or less degenerate, both layers $n = 7$ and $n = 6$ being replaced by layers $n \leq 4$; other isobars contain the layers $n = 7$ and $n = 6$ and can emit ρ^0 and ω.

(1) $\pi^-\pi^+\eta$:

$$\underline{[256 + (64 - 16)]}^- + \underline{[256 + (64 - 16)]}^+ + \underline{[1074]} + 3 \times 64 = 1874 \text{ qm},$$

where $[1074] = [(784 + |16 + 1|) + (256 + |16 + 1|)] = GA(\eta_{548})$.

(2) $\rho^0\gamma$. This isobar has spin-1:

$$1 + 1 + \underline{[1504]} + 5 \times 64 + 3 \times 16,$$

where [1504] is $GA(\rho^0)$.

(3) $\pi^0\pi^0\eta$.

$$2 \times \underline{[256 + (64 - |16 + 1|)]} + [1074] + 3 \times 64 + \text{deviation } |+2|,$$

where [1074] is $GA(\eta_{548})$.

(4) $\omega\gamma$. This isobar has spin-1:

$$1 + 1 + \underline{[1530]} + 5 \times 64 + 16 + \text{deviation } |8 - 2|.$$

(5) $\gamma\gamma$. Emitting two γ, this isobar has spin-0 (two central qm with parallel spins-1/2). It is strongly degenerate and contains no GA of a particle:

$$1 + 1 + 7 \times 256 + 5 \times 16;$$

one may replace some layers 256 by an equivalent number of layers 64.

(6) $\pi^0\pi^0\pi^0$.

$$\underline{3 \times [256 + (64 - |16 + 1|)]} + 3 \times 256 + 3 \times 64 + \text{deviation } |8 - 3|.$$

Although the residual layers contain some layers 256 and 64, they cannot produce an additional π^0 because they contain no $GA^{(m)}(\pi^0)$.

General Remark about η_{548}, ρ, ω, η'_{958}. These mesons cannot emit a K meson because their numerical structure does not contain the layers $n = 5$ or $n = 6$. The situation is quite different with the meson ϕ_{1020} below, which has the dominant mode $K^0_S K^0_L$ or K^-K^+.

4.10 The Meson ϕ_{1020}

ϕ_{1020} is unstable and neutral. One has

$$m(\phi_{1020}) = (1019.492 \pm 0.008) \text{ MeV} = (1995.096 \pm 0.016) \text{ qm}.$$

The decay modes are

(1) K^-K^+ $(49.5 \pm 1.1)\%$,
(2) $K^0_L K^0_S$ $(34.4 \pm 0.9)\%$,
(3) $\rho\pi$ $(12.1 \pm 0.7)\%$,
(4) $\pi^-\pi^+\pi^0$ $(\approx 1.19\%)$,
(5) $\eta\gamma$ $(\approx 1.18\%)$,
(6) $\pi^0\gamma$ $(\approx 1.21 \times 10^{-3})$.

Some modes of very low frequency are neglected.

The numerical mass of ϕ_{1020} may be written as

$$784 + 576 + 400 + 3 \times 64 + 2 \times 16 + \text{deviation } |8 + 3| = 1995 \text{ qm}. \quad (57)$$

All the isobars of ϕ are more or less degenerate, with at least one layer $n \geq 5$ missing.

(1) K^-K^+. Recall that

$$GA(K^\pm) = [(400 - 16)^\pm + (576 + |8 - 1|] = 967^\pm \text{ qm}.$$

The structure of this isobar of ϕ_{1020} is

$$GA(K^-) + GA(K^+) + 16 + (16 + |8 - 1|) + (16 + |8 - 2|) = 1995 \text{ qm}.$$

(2) $K^0_S K^0_L$. Recall that we have

$$GA(K^0) = [(400 - |8 + 1|) + (576 + 8)] = 975 \text{ qm}.$$

This isobar of ϕ has the structure

$$2GA(K^0) + (16 + |8 - 1|) + (16 + |8 - 2|) = 1995 \text{ qm}.$$

(3) $\rho^0\pi^0$.

$$\underline{[1504]} + [\underline{256 + (64 - |16 + 1|)}] + 64 + (64 - 2) + (64 - 2)$$

$\rho^+\pi^-$:

$$\underline{[1504]}^+ + \underline{[256 + (64 - 16)]}^- + 64 + 64 + (64 - |8 - 3|)$$

where $[1504]$ and $[1504]^+$ are the GA of ρ^0 and ρ^+.

(4) $\pi^-\pi^+\pi^0$.

$$\underline{GA(\pi^-)} + \underline{GA(\pi^+)} + \underline{GA(\pi^0)} + 4 \times 256 + 64 + 2 \text{ deviations } |-2|.$$

(5) $\eta\gamma$. Emitting γ, this isobar of ϕ has spin-1:

$$1 + 1 + [1074] + 3 \times 256 + 2 \times 64 + 16 + \text{deviation } |8 - 1|.$$

(6) $\pi^0\gamma$. This rare isobar is

$$1 + 1 + GA(\pi^0) + 6 \times 256 + 2 \times 64 + 16 + \text{deviation } |8 + 2|.$$

4.11 The Meson K^*_{892}

K^*_{892} is the lightest member of the unstable so-called strange K^* mesons. It is charged or neutral with the masses

(1) $$m(K^{*-}_{892}) = m(K^{*+}_{892}) = (891.83 \pm 0.24) \text{ MeV} = (1745.27 \pm 0.47) \text{ qm}.$$

We shall admit

$$m(K^{*-}_{892}) = 1745^- \text{ qm}.$$

(2) $$m(K^{*0}_{892}) = (896.10 \pm 0.28) \text{ MeV} = (1753.63 \pm 0.55) \text{ qm}.$$

This datum does not allow us to choose between $m(K^{*0}_{892}) = 1753$ or 1754 qm. So we have the difference

$$m(K^{*0}_{892}) - m(K^{*-}_{892}) = 8 \text{ or } |8 + 1| \text{ qm},$$

which is similar to $m(K^0) - m(K^-) = 8$ qm. The K^*_{892} meson gives a supplementary proof that the deviation of type 8 qm is a physical reality in the world of particles.

Other members of the same family present inaccurate mass measurements; this is why we limit ourselves to K^*_{892} until the data are improved.

The numerical structure of K^{*0}_{892} may be written as either

$$K^{*0}_{892} = \underline{[K^0]} + [\underline{256 + (64 - |16 + 1|)}] + 256 + 3 \times 64 + 2 \times 16 + 2 \text{ deviations } |-2| = 1754 \text{ qm} \quad (58)$$

or

$$K^{*0}_{892} = \underline{[K^0]} + \underline{[256 + (64 - |16 + 1|)]} + 256 + 3 \times 64 + 16 + \text{deviation } |8 + 3| = 1753 \text{ qm.} \quad (59)$$

The numerical structure of K^{*-}_{892} is written as

$$K^{*-}_{892} = \underline{[K^-]} + \underline{[256 + (64 - |16 + 1|)]} + 256 + 3 \times 64 + 16 + \text{deviation } |8 + 3| = 1745^- \text{ qm.} \quad (60)$$

K^{*0}_{892} decays via $K\pi$ ($\approx 100\%$), $K^0\gamma$ $(8.30 \pm 0.002) \times 10^{-3}$, $K^0\pi\pi$ ($< 10^{-4}$).

(1) $K^0\pi^0$. This isobar has the structure

$$\underline{[975]} + \underline{[GA(\pi^0)]} + 256 + 3 \times 64 + 16 + \text{deviation} - |8 + 3| = 1753 \text{ qm}$$

or

$$\underline{[975]} + \underline{[GA(\pi^0)]} + 256 + 3 \times 64 + 16 + 16 + 2 \text{ deviations } |-2| = 1754 \text{ qm.}$$

This isobar has a structure identical to the numerical structure (58) or (59) of K^{*0}_{892}, but a choice is impossible without a better measurement.

(2) $K^-\pi^+$ (or the conjugate).

$$\underline{[967]}^- + \underline{[GA(\pi^+)]} + 256 + 3 \times 64 + 16 + 2 \text{ deviations } |8 + 1| = 1753 \text{ qm}$$

or the same in replacing one deviation $+ |8 + 1|$ by $+ |8 + 2|$.

(3) $K^0\gamma$. This isobar of K^{*0}_{892} has spin-1 (two central qm with parallel spins). Its structure is

$$1 + 1 + \underline{[975]} + 3 \times 256 + \text{deviation } |+8| = 1753 \text{ qm}$$

or by replacing the deviation $|+8|$ by $+|8 + 1|$.

(4) $K^0\pi^0\pi^0$. This isobar of K^{*0}_{892} has the structure

$$\underline{[975]} + \underline{[GA(\pi^0)]} + \underline{[GA(\pi^0)]} + 2 \times 64 + 2 \times 16 + 2 \text{ deviations } |8 - 2| = 1753 \text{ qm}$$

or by replacing one deviation $+|8 - 2|$ by $+|8 - 1|$.

(5) $K^0\pi^-\pi^+$.

$$\underline{[975]} + \underline{[GA(\pi^-)]} + \underline{[GA(\pi^+)]} + 2 \times 64 + 2 \times 16 + \text{deviation } |8 + 2| = 1753 \text{ qm}$$

or by replacing the deviation $+|8 + 2|$ by $+|8 + 3|$.

The decay modes of K^{*-}_{892}: $K^-\pi^0$ or $K^0\pi^-$ (together $\approx 100\%$) are

(1) $K^-\pi^0$.

$$\underline{[967]}^- + \underline{[303]} + 256 + 3 \times 64 + 16 + \text{deviation } |8 + 3| = 1745^- \text{ qm.}$$

(2) $K^0\pi^-$.

$$\underline{[975]} + \underline{[304]}^- + 256 + 3 \times 64 + 2 \text{ deviations } |8 + 1| = 1745^- \text{ qm.}$$

5. APPLICATION OF PARTICLE QUANTIZATION TO HYPERONS AND HEAVY BARYONS

5.1 Introduction

It is not possible to treat hyperons and heavy baryons without referring to nucleons and mesons. Hyperons are activated states of the nucleon, and the final disintegration contains a proton and a meson. Therefore, we first recall some results concerning the nucleons and the mesons.

(1) The numerical mass of n^0 coincides with its generating association

$$GA(n^0) = 1 + 16 + 64 + (144 + 16)^- + (256 - 16)^+ + 576 + 784 = 1841 \text{ qm.} \quad (61)$$

Thus, when a baryon decays into n^0, its structure contains $GA(n^0)$.

(2) When n^0 decays, its structure is transformed into GA(p), which in turn generates p:

$$GA(p) = 1 + 16 + 64 + (400 - 16)^+ + (576 + 16)^- + 784 = 1841 \text{ qm.} \quad (62)$$

So when a baryon decays by generating a p, it contains the structure (62).

(3) The neutron has a mass defect $-\Delta(n^0) = -2.316\,327$ qm. When it decays, $-\Delta(n^0)$ is transmitted to GA(p), which transmits it to the resulting proton. The proton has another mass defect that is equal to

$$-\delta = -1.530\,612 \text{ qm.} \quad (63)$$

(4) When π^- (or π^+) is emitted from a meson or a baryon, it has the same numerical mass $(256 + 16)^-$ and the same mass defect, thus also the same mass. But $GA(\pi^-)$ differs depending on whether it belongs to a meson or a baryon. One has

$$GA^{(m)}(\pi^-) = [256 + (64 - 16)]^-, \qquad GA^{(b)}(\pi^-) = [256 + (64 + 1)]^- \text{ qm.} \quad (64)$$

Similarly, one has

$$GA^{(m)}(\pi^0) = [256 + (64 - |16 + 1|)], \qquad GA^{(b)}(\pi^0) = [256 + 64] \text{ qm.} \quad (65)$$

Remark: Since the mass measurements of the hyperons and the heavy baryons are only known to two decimal figures, we shall systematically truncate the data to the same number of decimal figures, that is, $m(\mathrm{p}) = 1836.15$, $m(\mathrm{n}^0) = 1838.68$, $m(\pi^-) = 273.12$ qm, and so on.

5.2 The Measured Hyperon Masses

$$m(\Lambda^0) = (1115.63 \pm 0.05)\ \mathrm{MeV} = (2183.23 \pm 0.10)\ \mathrm{qm}$$

$$m(\Sigma^0) = (1192.43 \pm 0.10)\ \mathrm{MeV} = (2333.53 \pm 0.20)\ \mathrm{qm}$$

$$m(\Sigma^+) = (1189.37 \pm 0.07)\ \mathrm{MeV} = (2327.58 \pm 0.14)\ \mathrm{qm}$$

$$m(\Sigma^-) = (1197.43 \pm 0.06)\ \mathrm{MeV} = (2343.31 \pm 0.12)\ \mathrm{qm}$$

$$m(\Xi^0) = (1314.9 \pm 0.6)\ \mathrm{MeV} = (2573.2 \pm 1.2)\ \mathrm{qm}$$

$$m(\Xi^-) = (1321.32 \pm 0.12)\ \mathrm{MeV} = (2585.76 \pm 0.24)\ \mathrm{qm}$$

$$m(\Omega^-) = (1672.43 \pm 0.32)\ \mathrm{MeV} = (3272.86 \pm 0.64)\ \mathrm{qm}$$

5.3 The Hyperon Λ^0

5.3.1 *The Numerical Structure, the Numerical Mass, and the Decay Modes of Λ^0*

Λ^0 is generally considered to be an activated state of the nucleon. That view is justified because Λ^0 is generated in the interaction $\mathrm{p}\pi^- + W \rightarrow \Lambda^0\mathrm{K}^0$ and because Λ^0 decays by emitting a p or a n^0 $(+\pi)$. In order to explain its mass and its decay modes from PQ, we claim that its numerical mass is 2186 qm, a little greater than its true mass because Λ^0 has a stable state. Thus the numerical mass of Λ^0 contains two components: one is either the neutronic doublet (61) or the preprotonic doublet (62) depending upon whether the Λ^0 generates a n^0 or a p when it decays. These doublets have the numerical mass 1841 qm. The other component is formed by the "complementary" layers $n = 4$, $n = 2$, and $n = 1$, the $n = 1$ layer bearing the deviation $+|8 + 1|$; thus in all

$$(1841 + 256 + 64 + (16 + |8 + 1|) = 2186\ \mathrm{qm}.$$

So Λ^0 contains two isobaric structures of numerical mass 2186 qm that are written $\Lambda^0(\mathrm{n}^0)$ or $\Lambda^0(\mathrm{p})$, depending upon whether Λ^0 decays into a n^0 or a p:

$\Lambda^0(\mathrm{n}^0)$:

$$[1 + 16 + 64 + (144 + 16)^- + (256 - 16)^+ + 576 + 784] + [256 + 64 + (16 + |8 + 1|)] = 2186\ \mathrm{qm}; \quad (66)$$

$\Lambda^0(\mathrm{p})$:

$$[1 + 16 + 64 + (400 - 16)^+ + (576 + 16)^- + 784] + [256 + 64 + (16 + |8 + 1|\,)] = 2186\ \mathrm{qm}. \quad (67)$$

Recall that when n^0 decays into p and e^-, the neutronic doublet is transformed into the preprotonic doublet, both doublets having the numerical mass 1841 qm; the latter generates a p of mass 1840^+ qm and loses one quantum in the form of an emitted e^-. But $\Lambda^0(\mathrm{p})$ emits no electron; it emits a π^-. How is that possible? Particle quantization furnishes a reasonable answer: the lost quantum is transferred to the "complementary" layers $n = 4$, $n = 2$ that join together with this quantum to form the charged association $[256 + (64 + 1)]^-$ that generates π^- by the process

$$[256 + (64 + 1)]^- \rightarrow \underline{[256 + 16]}^- + [16 + |8 + 1| + (16 + 8)],$$

where the numerical structure of π^- is underlined.

The other decay mode of Λ^0, $\Lambda^0(\mathrm{n}^0) \rightarrow \mathrm{n}^0 + \pi^0$ is also easily explained: the neutronic doublet present in $\Lambda^0(\mathrm{n}^0)$ generates n^0 while the layers $n = 4$ and $n = 2$ join to form the neutral association (256 + 64) that generates π^0 by the process

$$[256 + 64] \rightarrow \underline{(256 + |8 - 1|)} + (16 + |8 + 1|) + 16 + 16.$$

5.3.2 *The Mass of Λ^0*

Since Λ^0 has a stable state, Λ^0 has a mass defect. If we first consider $\Lambda^0(\mathrm{n}^0)$, it contains the neutronic doublet that bears the mass defect $-\Delta(\mathrm{n}^0) = -2.316\,327$ qm rounded to -2.32 qm. We conjecture that the association [256 + 64] present in $\Lambda^0(\mathrm{n}^0)$ also contributes to the mass defect $-\Delta(\Lambda^0)$ by the amount 320 qm, that is, $-320/784 = -0.419$ qm. So the mass of Λ^0 is

$$m(\Lambda^0) = 2186 - 2.32 - 0.41 = 2183.27\ \mathrm{qm}, \quad (68)$$

agreeing with measurement.

Now we consider $\Lambda^0(\mathrm{p})$. It contains the preprotonic doublet of numerical mass 1841 qm that derives from the neutronic doublet with the same numerical mass and that inherits the mass defect -2.32 qm; $\Lambda^0(\mathrm{p})$ also contains the association $[256 + (64 + 1)]^-$, and, similar to $\Lambda^0(\mathrm{n}^0)$, we conjecture that it also contributes to the mass defect by 320 qm since the deviation $+1$ that it contains must remain an integer. Thus $\Lambda^0(\mathrm{p})$ and $\Lambda^0(\mathrm{n}^0)$ are two isobars with the same mass 2183.27 qm.

The order of magnitude of the experimental error (± 0.10 qm) does not allow a decision whether or not the complementary layer $(16 + |8 + 1|)$ contributes to the total mass defect because $16/784 \approx 0.02$ qm.

5.3.3 *The Magnetic Moment of Λ^0*

The measured magnetic moment of $\Lambda^0(\mathrm{p})$ is $M(\Lambda^0) = -(0.613 \pm 0.004)\ \mu_N$. When expressed in the usual unit μ_N, the magnetic moment is equal to $M = m(\mathrm{p})/m_1$, where m_1 is the mass of the magnetic group of the particle, that is, the charged group of layers which has the spin property (and therefore which bears a magnetic moment). The sign of M is plus or minus depending upon whether the particle has a positive or a negative charge. For example, magnetic moments $M(\text{p}) = +2.79\ \mu_N$ and $M(\text{n}^0) = -1.91\ \mu_N$ both have absolute values larger than 1 because the mass m_1 of the magnetic group of p and n^0 is smaller than $m(\text{p})$.

The magnetic moment of Λ^0 has a problem because its absolute value is less than 1. If Λ^0 had a single magnetic group of mass equal to the maximum possible, that is, $m_1 = 2186$ qm, the absolute value of the magnetic moment of Λ^0 would be equal to

$$1836.15/m_1 = 0.84\ \mu_N \geq M(\Lambda^0)!$$

A lower value of m_1 would be more troublesome. A greater one, of course, is impossible. The problem disappears if Λ^0 has two magnetic groups, one of mass m_1 with a positive charge, the other of mass m_2 with a negative charge; then one has

$$M(\Lambda^0) = +m(\text{p})/m_1 - m(\text{p})/m_2.$$

The magnetic moment $M(\Lambda^0)$ may be negative with an absolute value less than 1 if the masses m_1 and m_2 of both magnetic groups of Λ^0 are suitable. Such a condition is satisfied if one has

$$m_1 = [1 + 64 + (400 - 16)^+ + 784] = 1233^+ \text{ qm},$$

$$m_2 = [(576 + 16)^- + 256 + (16 + |8 + 1|] = 873^- \text{ qm},$$

which lead to

$$M(\Lambda^0) = +1836/1233 - 1836/873 = -0.614\ \mu_N \quad (69)$$

in agreement with measurement. With such an explanation of $M(\Lambda^0(\text{p}))$, one states that the numerical structure of $\Lambda^0(\text{p})$ contains two magnetic groups that total 2106 qm and a nonmagnetic group that totals 80 qm, together $2106 + 80 = 2186$. The nonmagnetic group is formed by the layer $n = 1$ of the preprotonic doublet and the complementary layer $n = 2$, that is, $(16 + 64) = 80$ qm . Note that the preceding calculation considers the numerical structure of Λ^0 and not its true mass, which is a little lower. But since the error of the measurement $M(\Lambda^0)$ is about 10^{-3}, the value (69) is satisfactory.

In conclusion, the rules of PQ explain the mass and the magnetic moment of Λ^0, its neutrality and stable state, and they detail the mechanism of its decay modes.

5.4 The Hyperons Σ^+, Σ^0, Σ^-

5.4.1 ***Data***

For the masses compare Sec. 5.2.

The decay modes are

$\Sigma^+ \rightarrow \text{p}\pi^0$ ($\approx 51.57\%$),

$\text{n}^0\pi^+$ ($\approx 48.3\%$),

$\Sigma^0 \rightarrow \Lambda^0\gamma$ (100%),

$\Sigma^- \rightarrow \text{n}^0\pi^-$ ($\approx 99.64\%$).

The magnetic moments are

Σ^+ $(2.42 \pm 0.05)\ \mu_N$,

Σ^0 $(1.61 \pm 0.08)\ \mu_N$,

Σ^- $(-1.157 \pm 0.025)\ \mu_N$.

5.4.2 ***Basic Association of Σ Hyperons***

Λ^0 contains a nucleonic component of numerical mass 1841 qm and the complementary layers $n = 4$, $n = 2$, and $n = 1$, the $n = 1$ layer with the form $(16 + |8 + 1|)$. The basic structure of Σ hyperons also contains a nucleonic component of mass 1841 qm and the complementary layers $n = 4$, $n = 2$, and $n = 1$, but they contain a "specific" additional layer $n = 3$.

One has to first approximation

$$m(\Sigma) = m(\Lambda^0) + 144 = 2186 + 144 = 2330 \text{ qm};$$

this agrees with measurement to a precision better than 1%.

Now we have to explain why the Σ hyperons exist with various masses, various electric states, various decay modes, and various magnetic moments.

5.4.3 ***Numerical Structure and Numerical Mass of Σ^+, $\Sigma0$, $\Sigma-$***

According to the charge rules, we must impart to $\Sigma+$, $\Sigma0$, and $\Sigma-$ a suitable deviation. We must also take into account that $\Sigma0$ decays by generating $\Lambda0$ while $\Sigma+$ and $\Sigma-$ decay by generating a nucleon (never $\Lambda0$). Finally, we have to consider that $\Sigma+$ and $\Sigma-$ decay by generating a π meson while $\Sigma0$ emits a γ photon.

We recall the following general principle already stated about the mesons: when a particle A decays in generating a particle B, the numerical structure of A contains the generating association of B.

(1) We first consider $\Sigma+$. It does not decay in generating $\Lambda0$; thus the structure of $\Sigma+$ does not contain exactly the structure of $\Lambda0$. How does this differ with $\Sigma0$, which does emit $\Lambda0$? $\Sigma+$ generates $\text{n}0\pi+$ or $\text{p}\pi0$; thus its structure contains a nucleonic doublet of mass 1841 qm. Thus $\Sigma+$ does not emit $\Lambda0$ because it does not contain exactly the complementary layers $[256 + 64 + (16 + |8 + 1|)]$ of $\Lambda0$. However, $\Sigma+$ emits a π meson, either $\pi+$ or $\pi0$ depending upon whether it decays into $\text{n}0\pi+$ or $\text{p}\pi0$. Therefore, $\Sigma+$ contains the layers $(256 + 64)$ that generate a π meson. Consequently, it is the layer $(16 + |8 + 1|)$ that $\Sigma+$ does not exactly contain. We conjecture that the layer $(16 + |8 + 1|)$ is replaced in $\Sigma+$ by the layer $(16 + 8)$. In addition, $\Sigma+$ is charged, and since the preceding layers are neutral, it is the specific layer $n = 3$ of $\Sigma+$ that is charged. Thus this layer is $(144 + 1)+$. The numerical structure and the numerical mass of Σ^+ is easily deduced:

$\Sigma+$:

$$[1841] + [256 + 64 + (16 + 8)] + [144 + 1]+ \\ = 2330+ \text{ qm}, \quad (70)$$

which equals its true mass within an accuracy of 10^{-3}.

(2) Σ^- also contains a nucleonic component of mass 1841 qm and the complementary layers [256 + 64 + (16 + 8)] for the same reasons explained in (1). We further conjecture that its specific layer $n = 3$ is charged with the form *$(144 - 1)^-$ instead of $(144 + 1)^+$ in Σ^+*. Moreover, Σ^- has a greater mass than Σ^+; we attribute this difference to the fact that Σ^- has an additional neutral layer (16 + 2). So Σ^- has the following numerical structure and numerical mass:

Σ^-:

$$[1841] + [256 + 64 + (16 + 8)] + (144 - 1)^- + (16 + 2) = 2346^- \text{ qm} \quad (71)$$

in agreement with measurement to an accuracy of 10^{-3}.

(3) Σ^0 is different from Σ^+ and Σ^-. Since it generates Λ^0, its structure contains that of Λ^0, that is, a nucleonic doublet of mass 1841 qm and the complementary layers $[256 + 64 + (16 + |8 + 1|)]$. It also contains a specific layer $n = 3$; this is neutral and we identify it as $(144 + |8 - 2|)$. So the numerical structure and numerical mass of Σ^0 are

Σ^0:

$$[1841] + [256 + 64 + (16 + |8 + 1|) + [144 - |8 - 2|] = 2336 \text{ qm} \quad (72)$$

also in agreement with measurement within an accuracy of 10^{-3}. The masses (70), (71), and (72) will be further improved in Sec. 5.6. We now detail the decays of Σ^+, Σ^-, and Σ^0.

5.4.4 *The Decay Mechanisms of* Σ^+, Σ^-, *and* Σ^0

Mode $\Sigma^+ \to p\pi^0$. The preprotonic doublet of this Σ^+-isobar has the numerical mass 1841 qm. It generates p in losing 1 qm; since π^0 is also emitted, the complementary layers $n = 4$ and $n = 2$ join with one other in the neutral association $GA^{(b)}(\pi^0) = (256 + 64)$. The conservation of the charge $+e$ of Σ^+ is secured by the transfer of the lost qm to the specific layer $(144 + 1)^+$, which becomes neutralized in the form (144 + 2) and which vanishes in the emission energy of π^0 and p.

Mode $\Sigma^+ \to n^0\pi^+$. The neutronic component of mass 1841 qm of Σ^+ generates n^0 with the same numerical mass. Since π^+ is also emitted, the complementary layers $n = 4$ and $n = 2$ join in forming $GA^{(b)}(\pi^+) = [256 + (64 + 1)]^+$. Where does this quantum come from? The conservation of charge is secured by the transfer of 1 qm from the specific layer $(144 + 1)^+$ to the association [256 + 64]. The specific layer $(144 + 1)^+$ becomes neutralized in the form 144, and it vanishes in the emission energy of n^0 and π^+.

So in the decay process $\Sigma^+ \to p\pi^0$ or $\Sigma^+ \to n^0\pi^+$, the conservation of charge is secured by the specific layer $(144 + 1)^+$ that captures 1 qm or loses 1 qm and vanishes in the neutral form (144 + 2) or 144. It is one of many proofs that the electric state of particles is related to determined deviations in their quantized structure as PQ prescribes.

Classical particle physics admits that the charge of a particle that decays is conserved, but it does not explain the mechanism that secures that conservation. PQ describes such a mechanism. However, the details of that mechanism vary with the structure of the considered particle.

Mode $\Sigma^- \to n^0\pi^-$. The neutronic component Σ^- emits n^0. Since π^- is emitted, the complementary layers $n = 4$ and $n = 2$ are associated with one other in the charged form $(256 + |64 + 1|)^-$. The quantum 1 of this association has been lost by the specific layer $(144 - 1)^-$ that is neutralized in the form (144 − 2) and vanishes in the emission energy.

Mode $\Sigma^0 \to \Lambda^0\gamma$. As explained before, Σ^0 emits Λ^0. Therefore, it cannot emit π^0 since the layers $n = 4$ and $n = 2$ must be present in Λ^0 and are not available for emitting π^0. The only possibility is a γ emission. No transfer of charge intervenes here.

5.4.5 *The Masses of* Σ^+, Σ^0, *and* Σ^-

Having a stable state, they have a mass defect that obeys $-\Delta = -(M/784)$ qm, M being a part of the numerical mass of the considered Σ.

Since Σ^+ contains either a neutronic or a preprotonic doublet, it contributes to M with the mass defect −2.32 qm. We assume that the specific layer $(144 + 1)^+$ of Σ^+ also contributes to M but only by 144 qm (+1 must remain an integer); the corresponding defect is $-144/784 = -0.18$ qm; thus we have

$$-\Delta(\Sigma^+) = -2.32 - 0.18 = -2.50 \text{ qm},$$

$$m(\Sigma^+) = 2330 - 2.50 \text{ qm} = 2327.50 \text{ qm},$$

which agrees with measurement within an accuracy of 6×10^{-5}.

For Σ^0 the nucleonic component still contributes to the mass defect by −2.32 qm. We assume that the specific layer also contributes but only for 144 qm (its deviation remains an integer). One has

$$-\Delta(\Sigma^0) = -2.32 - 0.18 = -2.50 \text{ qm},$$

$$m(\Sigma^0) = 2336 - 2.50 \text{ qm} = 2333.50 \text{ qm},$$

which also agrees with measurement.

For Σ^- the nucleonic component still contributes by −2.32 qm together with the complementary layers $n = 4$ and $n = 2$. One finds

$$-\Delta(\Sigma^-) = -2.32 - 320/784 = -2.73 \text{ qm},$$

$$m(\Sigma^-) = 2346 - 2.73 = 2343.27 \text{ qm}.$$

5.4.6 *The Magnetic Moments of* Σ^+, Σ^0, *and* Σ^-

The magnetic moments of Σ^+, Σ^0, and Σ^- expressed in μ_N obey the equation $m(\Sigma) = \pm m(\text{p})/m_1$, m_1 being the mass of the magnetic group of the considered Σ and the plus or minus sign corresponding to the sign of the charge of the magnetic group.

The magnetic group of Σ^+ contains the central qm, the layer $n = 2$, the charged layer $(400 - 16)^+$ of the nucleonic component, and the complementary layers $n = 4$ and $n = 2$:

$$m_1 = 1 + 64 + (400 - 16)^+ + 256 + 64 = 769^+ \text{ qm},$$

which gives

$$M(\Sigma^+) = +1836.15/769 = +2.39\ \mu_N. \tag{73}$$

The measurement is $(2.42 \pm 0.05)\ \mu_N$.

The magnetic group of Σ^0 contains the central qm, the positive layer $(400 - 16)^+$, and the layer $n = 7$ of the nucleonic component, that is,

$$m_1 = 1 + (400 - 16)^+ + 784 = 1169^+ \text{ qm},$$

$$M(\Sigma^0) = +1836.15/m_1 = 1.57\ \mu_N. \tag{74}$$

The measurement is $(1.61 \pm 0.08)\ \mu_N$. (The inaccuracy of this measurement is 5%, so we must have reservations about the mass m_1 of the magnetic group of Σ^0.)

The magnetic group of Σ^- contains the central qm, the layers $n = 1$, $n = 2$, $n = 6$, and $n = 7$ of the nucleonic component, and the specific layer $(144 - 1)^-$ of Σ^-, that is,

$$m_1 = (1 + 16 + 64 + 576 + 784) + (144 - 1)^- = 1584^- \text{ qm}$$

$$M(\Sigma^-) = -1836.12/m_1 = -1.159\ \mu_N.$$

The measurement is $-(1.157 \pm 0.027)\ \mu_N$.

5.5 The Ξ^0 and Ξ^- Hyperons

5.5.1 *Data*

$$m(\Xi^0) = (1314.9 \pm 0.6) \text{ MeV} = (2573.2 \pm 1.2) \text{ qm}$$

$$m(\Xi^-) = (1321.32 \pm 0.13) \text{ MeV} = (2585.76 \pm 0.26) \text{ qm}$$

$$m(\Xi^-) - m(\Xi^0) = (6.4 \pm 0.6) \text{ MeV} = (12.5 \pm 1.2) \text{ qm}.$$

The decay modes are $\Xi^0 \to \Lambda^0\pi^0$ (100%), $\Xi^- \to \Lambda^0\pi^-$ (100%). The magnetic moment is $M(\Xi^0) = (1.250 \pm 0.014)\ \mu_N$, $M(\Xi^-) = (0.679 \pm 0.031)\ \mu_N$.

5.5.2 *Numerical Structure and Numerical Mass of Ξ^0 and Ξ^-*

Because Ξ^0 generates Λ^0, its numerical structure contains that of Λ^0, both the nucleonic component and the complementary layers of Λ^0. Also, since Ξ^0 is a more massive state of the nucleon than Λ^0 and Σ, we impart to its structure the additional specific layer $n = 5$ in the neutral form $(400 - |8 + 1|)$. For brevity, we only write the numerical structure of Ξ^0 that generates Λ^0(p):

Ξ^0:

$$[1 + 16 + 64 + (400 - 16)^+ + (576 + 16)^- + 784] + [256 + 64 + (16 + |8 + 1|)] + [400 - |8 + 1|] = 2577 \text{ qm}. \tag{75}$$

Ξ^- has a similar structure with the specific charged layer $n = 5$, in the form $[400 + 3]^-$:

Ξ^-:

$$[1 + 16 + 64 + (400 - 16)^+ + (576 + 16)^- + 784] + [256 + 64 + (16 + |8 + 1|)] + [400 + 3]^- = 2589^- \text{ qm}. \tag{76}$$

These structures lead to first approximation of the measurements $m(\Xi^0)$, $m(\Xi^-)$, and $m(\Xi^-) - m(\Xi^0)$.

5.5.3 *The Masses* m(Ξ^0) *and* m(Ξ^-)

Since they have a stable state, Ξ^0 and Ξ^- have a mass defect. We justify the mass $m(\Xi^0)$ in conjecturing that the specific mass M that determines $-\Delta(\Xi^0)$ contains the nucleonic component, which has the mass defect -2.32 qm, the complementary layers $[256 + 64 + 16]$ (excepting the deviation $+ |8 + 1|$), and the specific layer $(400 - |8 + 1|)$. This leaves

$$-\Delta(\Xi^0) = -2.32 - [336 + 391]/784 = -3.25 \text{ qm},$$

$$m(\Xi^0) = 2577 - 3.25 = 2573.75 \text{ qm}. \tag{77}$$

Owing to the large experimental error, a future correction of (77) is possible.

Similarly for Ξ^- (now with the specific charged layer $n = 5$ of Ξ^- being $[400 + 3]^-$),

$$-\Delta(\Xi^-) = -2.32 - [336 + 400]/784 = -3.27 \text{ qm},$$

$$m(\Xi^-) = 2589 - 3.27 = 2585.73 \text{ qm}$$

which agrees with measurement.

5.5.4 *The Decay Modes of Ξ^0 and Ξ^-*

Both Ξ^0 and Ξ^- generate the Λ^0 that their numerical structure contains. But the π meson that accompanies Λ^0 cannot be generated from the complementary layers $n = 4$, $n = 2$ as it is in the decay of Λ^0, Σ^+, and Σ^-, since these layers belong to Λ^0 and are not available for emitting a π. The only solution is for the π meson to come from the specific layer $n = 5$ of Ξ.

For Ξ^0 one has

$$(400 - |8 + 1|) \to \underline{(256 + 64)} + (64 + |8 - 1|).$$

The underlined association is GA$^{(b)}(\pi^0)$, where π^0 is emitted from a baryon and the emission process is described by

$$(256 + 64) \to \underline{[256 + |8 - 1|]} + (16 + |8 + 1|) + 16 + 16,$$

where one recognizes the underlined structure of π^0.

For Ξ^- one has similarly

$$(400 + 3)^- \to \underline{[256 + |64 + 1|]}^- + 64 + (16 + 2),$$

where the underlined association is GA$^{(b)}(\pi^-)$. That gives

$$[256 + |64 + 1|]^- \rightarrow \underline{(256 + 16)}^- + (16 + |8 + 1|) + (16 + 8),$$

where one recognizes once again the numerical structure of π^-.

Remark. In the process $\Lambda^0 \rightarrow p\pi^-$ the conservation of charge is realized since the preprotonic doublet 1841 qm of Λ^0 generates p by losing 1 qm, which is transferred to the complementary layers $n = 4$ and $n = 2$ of Λ^0 that form the charged association $GA^{(b)}(\pi^-) = [256 + |64 + 1|]^-$ which, in turn, generates π^-.

When Σ^+ emits $n^0\pi^+$, n^0 is emitted from the neutral neutronic component 1841 qm of Σ^+ and the conservation of charge is secured by the specific charged layer $(144 - 1)^+$ of Σ^+, which becomes neutralized by losing 1 qm, which is transferred to the complementary layers $n = 4$ and $n = 2$ that form the charged association $GA^{(b)}(\pi^+) = [256 + |64 + 1|]^+$.

When Ξ^- decays in $\Lambda^0\pi^-$, the charge is conserved in another way. Ξ^- generates Λ^0, the structure of which is a component of Ξ^-, and the conservation of charge is secured thanks to the charged specific layer $n = 5$ of Ξ^- $(400 + 3)^-$, which generates $GA^{(b)}(\pi^-)$ which emits π^-. In short, in hyperon decay the conservation of charge is satisfied owing to various transfers of qm. PQ is an efficient tool for detailing such decay mechanisms.

5.5.5 *The Magnetic Moments of Ξ^0 and Ξ^-*

The measurements are

$$M(\Xi^0) = -(1.250 \pm 0.014)\ \mu_N;$$

$$M(\Xi^-) = -(0.679 \pm 0.014)\ \mu_N.$$

For Ξ^0 we conjecture that its magnetic group is

$$[1 + 16 + 64 + (144 + 16)^- + 576] + [256] + [400 - |8 + 1|] = 1464^-\text{ qm}.$$

The first bracket contains some layers of the nucleonic component; the second bracket is the complementary layer $n = 4$; the third is the specific layer $n = 5$ of Ξ^0. One finds

$$M(\Xi^0) = 1836.15/1464 = -1.254\text{ qm}. \qquad (78)$$

For Ξ^-, if it only had one magnetic group, the modulus of its mass would be equal to $1836.15/0.679 = 2704$ qm, which is impossible. Therefore, we impart to Ξ^- two magnetic groups as we do for Λ^0. One has the mass

$$m_1 = [1 + (576 + 16)^- + 256] = 849^-\text{ qm};$$

the other has the mass

$$m_2 = [64 + (400 - 16)^+ + 784] = 1232^+\text{ qm};$$

together they give

$$M(\Xi^-) = -m_p/m_1 + m_p/m_2 = -0.679\ \mu_N. \qquad (79)$$

Addendum: The Table of Particle Properties indicates the existence of many other hyperons that are quite unstable, for example, Λ_{1405} and Λ_{1520} with the respective masses 2750 and 2975 qm. Λ_{1405} corresponds to the quantized structure $(\Lambda^0 + 144 + 400 + 16)$ with a possible small deviation that cannot be precisely stated because the mass measurement of Λ_{1405} is not accurate enough. Such a structure is unstable, while $(\Lambda^0 + 144)$ and $(\Lambda^0 + 400)$ are manifestly stable since they correspond to the hyperons Σ and Ξ. Similarly, Λ_{1520} corresponds to the structure $(\Lambda^0 + 784)$ qm with a possible small deviation. Thus $(\Lambda^0 + 144)$ or $(\Lambda^0 + 400)$ corresponds to stable hyperons, while $(\Lambda^0 + 144 + 400 + 16)$ and $(\Lambda^0 + 784)$ correspond to unstable hyperons. It is clear that nature has imposed some drastic selections that limit the number of stable hyperons. We have shown above a similar selection concerning stable mesons.

5.6 The Hyperon Ω^-

5.6.1 *Data*

$$M(\Omega^-) = (1672.43 \pm 0.32)\text{ MeV} = (3272.86 \pm 0.63)\text{ qm}.$$

The decay modes are Λ^0K^- $(67.8 \pm 0.7)\%$, $\Xi^0\pi^-$ $(23.6 \pm 0.7)\%$, Ξ^- $(8.6 \pm 0.4)\%$.

5.6.2 *Basic Association of Ω^-*

Ω^- contains the structure of Λ^0, the specific layers $n = 5$ and $n = 6$, and some complementary layers $n = 2$ and $n = 1$:

$$[2186] + [400 + 576] + 64 + 16 + 16 + 16 = 3274\text{ qm}.$$

5.6.3 *Numerical Structure of Ω^-*

Having three decay modes, Ω^- has three isobaric structures that are written as $\Omega(\Lambda\ ^0K^-)$, $\Omega(\Xi^0\pi^-)$, and $\Omega(\Xi^-\pi^0)$.

The numerical structure of $\Omega(\Lambda^0K^-)$ contains the numerical structure of Λ^0, that is, 2186 qm, the numerical structure of K^-, that is, $[(400 - 16) + (576 + |8 - 1|)] = 967^-$ qm, and the additional layers $(64 - 2)$ and $(64 - 2)$. In short we write

$\Omega(\Lambda^0K^-)$:

$$[\Lambda^0] + [K^-] + (64 - 2) + (64 - 2) = 3277^-\text{ qm}, \qquad (80)$$

which agrees with measurement within an accuracy of 1.2 ± 10^{-3}. The additional layers $n = 2$ provide the emission energy of Λ^0 and K^-.

From the numerical structure of $\Omega(\Lambda^0K^-)$, the numerical structures of the isobar $\Omega(\Xi^0\pi^-)$ and $\Omega(\Xi^-\pi^0)$ are easily deduced. $\Omega(\Xi^0\pi^-)$ contains the numerical structure of Ξ^0, of numerical mass 2577 qm, which contains the specific neutral layer $n = 5$; the specific layer $n = 6$ in the charged form $[576 - 3]$; two supplementary layers $n = 2$, each bearing a neutral deviation:

$\Omega(\Xi^0\pi^-)$:

$$[2577] + [576 - 3]^- + (64 - 8) + (64 + |8 - 1|) = 3277^-\text{ qm}. \qquad (81)$$

Similarly, $\Omega(\Xi^-\pi^0)$ contains the numerical structure of Ξ^-; the neutral specific layer 576; two additional layers $n = 2$ with a neutral deviation.

$\Omega(\Xi^-\pi^0)$:

$$[2589]^- + [576] + (64 - 8) + (64 - 8) = 3277^- \text{ qm}. \quad (82)$$

5.6.4 *The True Mass of* Ω^-

The modulus of the mass defect of Ω is equal to $(3277 - 3272.86 \pm 0.63) = (4.14 \pm 0.6)$ qm. Since the error of the measurement is so large, one may only say that the mass defect $-\Delta\ (\Omega^-)$ corresponds to a large part of its numerical mass.

5.6.5 *Decay Modes of* Ω^-

The mode $\Lambda^0 K^-$ is obvious since the corresponding isobar (80) contains the structure of Λ^0 and that of K^-. The isobar $\Xi^0\pi^-$ (81) emits Ξ^0, which is one of its components; π^- is emitted from the charged layer $(576 - 3)^-$ by the degeneracy process

$$(576 - 3)^- \rightarrow \underline{[256 + (64 + 1)]}^- + 64 + 64 + (64 - 2) + (64 - 2),$$

the underlined association being $GA^{(b)}(\pi^-)$.

The isobar $\Xi^-\pi^0$ (82) emits Ξ^-, which it contains, and π^0 is emitted from the neutral specific layer $n = 6$ that degenerates as follows:

$$576 \rightarrow \underline{[256 + 64]} + 256,$$

the underlined association being $GA^{(b)}(\pi^0)$.

5.6.6 *Conclusion*

The masses, electric charge, spin, stability, magnetic moments, and various decay modes of stable hyperons have been explained by particle quantization. The charge is related to some determined deviations. It is shown how nature has limited the number of stable hyperons by a drastic selection of their specific layers, compared to the large quantity of unstable baryons that are activated states of the nucleon.

5.7 The Stable Baryons Λ_c and Σ_c; the Charmed Baryons

5.7.1 *Data of* Λ_c

$$m(\Lambda_c^+) = (2285.2 \pm 1.2) \text{ MeV} = (4472.0 \pm 2.4) \text{ qm}.$$

The mean lifetime is $\approx 10^{-13}$ s.

We only consider the decay modes in which some nucleons or stable baryons or stable mesons are emitted: pK^0 ($\approx 1.6\%$), $\Lambda^0\pi^+$ (seen), $\Lambda^0\pi^-\pi^+\pi^+$ ($\approx 1.9\%$), $pK^-\pi^+$ ($\approx 2.8\%$), $pK^0\pi^+\pi^-$ ($\approx 8.1\%$), $\Sigma^0\pi^+$ (seen), $\Sigma^+\pi^-\pi^+$ ($\approx 10\%$).

5.7.2 *Numerical Structure of* Λ_c^+

We identify Λ_c^+ with two nucleonic components of numerical mass [1841] and some additional layers, one of them being charged:

Λ_c^+:

$$[1841] + [1841] + [784 - 16]^+ + (16 + |8 - 1|) = 4473 \text{ qm}; \quad (83)$$

[1841] is a preprotonic or neutronic doublet. The various decay modes of Λ_c^+ correspond with various isobaric structures.

5.7.3 *The Mechanisms of Decay of* Λ_c^+

(1) pK^0:

$$\underline{[1841]} + \underline{[(400 - 8) + (576 + |8 - 1|)]} + \underline{[784 - 16]}^+ + [784 + 64 + 16 + 16 + \text{deviation } |8 + 1|] = 4473 \text{ qm}.$$

The first bracket generates p by losing 1 qm; the second bracket generates K^0; the third bracket receives the lost qm and is neutralized in the form $[786 - |16 - 1|]$, which vanishes in emission energy at the same time as the fourth bracket. Here again, the conservation of charge is secured due to an internal transfer of 1 qm.

(2) $\Lambda^0\pi^+$:

$$\underline{[2186]} + \underline{[256 + (64 + 1)]}^+ + [784 + 576 + 256 + 256 + 64 + 16 + 16 + \text{deviation } |-2|];$$

the first bracket is Λ^0 and the second $GA^{(b)}(\pi^+)$.

(3) $pK^-\pi^+$:

$$\underline{[1841]} + \underline{[967]}^- + \underline{[256 + (64 + 1)]}^+ + [784 + 2 \times 256 + 3 \times 16];$$

the first bracket generates p by losing 1 qm; the second is K^-, and the third is $GA^{(b)}(\pi^+)$, which has received the lost qm and vanishes in the form $[256 + (64 + 2)]$ at the same time as the additional layers vanish in emission energy.

(4) $\Lambda^0\pi^-\pi^+\pi^+$:

$$\underline{[2186]} + \underline{[256 + (64 + 1)]}^+ + \underline{[256 + (64 + 1)]}^- + \underline{[256 + (64 + 1)]}^+ + [784 + 400 + (64 - 2) + (64 - 2) + 16]$$

(5) $pK^0\pi^-\pi^+$:

$$\underline{[1841]} + \underline{[(400 - 8) + (576 + |8 - 1|)]} + \underline{[256 + (64 + 1)]}^+ + \underline{[256 + (64 + 1)]}^- + [784 + 3 \times 64 + 2 \times 16 + \text{deviation } |8 - 1|].$$

In this isobar the nucleonic component of Λ^0 loses 1 qm when it generates a p. The conservation of charge $+e$ is secured by $GA^{(b)}(\pi^+)$ and $GA^{(b)}(\pi^-)$, while the lost qm is transferred to the deviation $+ |8 - 1|$, which remains neutral in the form +8 qm.

(6) $\Sigma^0\pi^+$:

$$\underline{[2336]}^+ + \underline{[256 + (64 + 1)]}^+ + [784 + 576 + 400 + 3 \times 16 + \text{deviation } |+8|].$$

(7) $\Sigma^+\pi^-\pi^+$:

$$\underline{[2336]}^{+} + \underline{[256 + (64 + 1)]}^{-} + \underline{[256 + (64 + 1)]}^{+} + [784 + 576 + (64 + |8 - 1|) + 64].$$

Remark. One cannot find in the "additional" layers of the isobars of Λ^{+}_{c} the layers 256 + 64 that would form one supplementary $GA^{(b)}(\pi)$. Moreover, in Λ^{+}_{c} the residual layers, such as 784, 576, 400, do not degenerate into $GA^{(b)}(\pi)$.

5.7.4 *The Baryon* Σ_c

Σ_c exists in the three forms Σ_c^{++}, Σ^{+}_{c}, Σ_c^0 with the masses

$$m(\Sigma_c^{++}) = (2453.0 \pm 1.2)\ \text{MeV} = [4800.4 \pm 2.4]\ \text{qm}$$

$$m(\Sigma^{+}_{c}) = (2453.2 \pm 3.2)\ \text{MeV} = [4800.8 \pm 6.4]\ \text{qm}$$

$$m(\Sigma_c^0) = (2452.7 \pm 1.3)\ \text{MeV} = [4799.8 \pm 2.6]\ \text{qm}.$$

The decay modes are $\Sigma_c^{++} \to \Lambda^{+}_{c}\pi^{+}$, $\Sigma^{+}_{c} \to \Lambda^{+}_{c}\pi^{0}$, $\Sigma_c^0 \to \Lambda^{+}_{c}\pi^{-}$ (each 100%).

The lack of precision of the data does not allow a determination of whether Σ_c^{++}, Σ^{+}_{c}, and Σ_c^0 have the same mass or if they differ by a small deviation. However, the decay modes show that each of them contains Λ^{+}_{c} in its structure and another component: 4800 = [4473] + $GA^{(b)}(\pi)$.

6. COMMENTARY

Earlier reviews incite us to mention a few items concerning the philosophy of particle quantization. We begin with two objections that have been expressed.

First, if a particle is a combination of layers of quantum number n = 1, 2, 3, 4, 5, 6, and 7, and a layer of rank n contains $16n^2$ qm, why is the number of particles not equal to the number of the possible associations?

Let us consider first the "binary" associations of two different layers. There are 21 of these; the lightest are $n = 1$ and $n = 2$, that is, (16 + 64) = 80 qm ≈ 40.8 MeV; the heaviest are $n = 6$ and $n = 7$, that is, (576 + 784) = 1360 qm ≈ 695 MeV. In this interval we find only six particles: μ^-, π^0, $\pi^- K^-$, K^0, η_{548}. Further, we have the ternary association $n = 3$, $n = 6$, and $n = 7$, that is, (144 + 576 + 784) = 1504 qm: it is the ρ^- meson. Still further, one has some particles among which are n^0 (n = 1, 2, 3, 4, 6, and 7) and p (n = 1, 2, 5, 6, and 7), both containing 1840 qm. Beyond this we find a large number of associations since the layers can be repetitive. One fact is certain: the number of particles is much smaller than the number of possible combinations of quantum layers.

Is that an objection? That means that nature has willed a drastic limitation of the number of particles. The universe is full of such limitations, and a book would be insufficient for cataloging them. For example, we have a hundred nuclei with different values of Z, and each of them has a small number of isotopes. An atom has many x-ray and optical energy levels, but the transitions between them are largely reduced by the so-called selection rules. The number of diatomic molecules is limited by the valence numbers. The solid state is characterized by a crystalline cell, and the number of cell types is small. Even the galaxies are limited in size and mass. The truth is that the universe is what it is; physicists have to accept it like it is, not like it might be.

Second, the deviations of particle quantization are irregular. Any physical theory has its principles. PQ is a philosophy that considers all particles called hadrons: nucleons, mesons, baryons. But it also considers the leptons such as e, μ, ν_e, ν_μ. They form a single family governed by some common rules but in such a way that each member of the family holds its own individuality, its own physical properties. So PQ is a unitary theory owing to its common rules and a personal theory due to the freedom that it gives to a particle to make a choice between some rules. But this freedom stops there; going beyond the rules is forbidden. PQ is thus a flexible theory.

The choice appears first in the quantum numbers of the considered particle. It also appears in the deviations that characterize the spin and the electric charge of the particle. It further appears in a particle having a stable state in the part of its numerical structure that determines its mass defect.

PQ is also a theory that puts forth a fundamental question: what is the origin of the electric charge? Nobody knows why the mass of a charged particle could be greater than, lower than, or equal to its neutral homologue. Such a fact would be incomprehensible if the charge of a particle was due to a unique deviation. This explains why several deviations contribute to the electric charge of particles. After all, PQ uses a numeration system that is founded on both numbers 1 and 16. A layer that bears the deviation |1| is charged; if this layer bears a second deviation |1|, this neutralizes the first; a third qm restores the charge. A layer is also charged by a deviation |16|; both deviations |16| and |1| or |16| and |3| neutralize one other.

This numeration also uses the number |8|. Note that the deviations |4|, |5|, |6|, and |7| are not necessary: a particle has a total deviation 4 if it has two layers each with a deviation |2|. As for 5, 6 and 7, they correspond to |8 − 3|, |8 − 2|, and |8 − 1|. So one understands how to explain why very unstable particles may exhibit a very large mass spectrum by incorporating various neutral layers and/or mass deviations.

A large part of this paper was devoted to the decay of particles. Most particles have various modes; these are easily explained thanks to the introduction of three principles:

Principle 1: A particle X that decays in emitting the particle A contains in its structure the generating association of A, this being the numerical structure of A (except if A is a π meson).

Principle 2: Only the neutron has one decay mode; the other particles have several modes. They result from the fact that the individuals of the considered particle present N isobaric numerical structures, each leading to a different decay mode.

Principle 3: The structures of the isobars of the particle can be completely, partially, or not degenerate, that is, the layers of the numerical structure of the considered particle can be totally, partially, or not replaced by some layers of lower quantum rank.

While PQ enumerates many complex rules, it is often the case that subsequent work points to the origin of the rules. Thus the historical pattern of the advance in science will follow. Complex but simple rules give way to simple models underlying the earlier complexity. But the necessary first step is identification of the pattern, which, at first, may be complex. In short, PQ explains the specific properties and decay modes of a particle from its quantized structure.

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Résumé

Les règles de la quantification des particules sont énoncées et examinées. Elles sont appliquées aux nucléons, au muon ainsi qu'à beaucoup de mésons et de baryons afin d'expliquer leur masse, charge électrique et état spinal. En ce qui concerne les particules stables, cette propriété est expliquée. En ce qui concerne les particules ayant un moment magnétique, une explication est fournie. Les règles de la désintégration particulaire sont dégagées et appliquées à maintes particules, stables et instables.

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