WOLFRAM WHITE PAPER

Heterogeneous Computing in *Mathematica*® 8

WOLFRAM

Introduction

The past few years have seen an increase in the number of cores on both the CPU and GPU. Despite this, developers have not been able to fully utilize the parallel frameworks. As developers began to target the CPU and GPU frameworks, they realized that some algorithms map nicely onto the CPU while others work best with the GPU. As a result, much research has been done to develop algorithms that exploit both the CPU's and GPU's capabilities. The research culminated in a heterogeneous school of thought, with both the CPU and GPU collaborating to maximize the accuracy and speed of the user's program. In such methodology, the GPU is utilized in parts where it excels, as is the CPU. Yet while heterogeneous algorithms are ideal, no system has exposed built-in access to both the CPU and the GPU with a concise and easy-to-use syntax until *Mathematica* 8 which makes heterogeneous computing a reality.

By providing an environment where programs can be run on either the CPU or GPU, *Mathematica* is the only computational system that realizes the heterogeneous message. Coupled with *Mathematica*'s language, comprehensive import/export support, symbolic computation, extensive field coverage, state-of-the-art visualization features, platform neutrality, and ease of use, *Mathematica* is ideal for heterogeneous algorithm development.

This white paper is divided as follows: first, we briefly discuss what *Mathematica* is and why you should use it for heterogeneous computing, then we look at some of *Mathematica*'s multicore and GPU features, develop a dozen applications along the way, and finally discuss why *Mathematica* has an advantage over other systems.

A Brief Introduction to *Mathematica*

Mathematica is a development environment that combines a flexible programming language with a wide range of symbolic and numeric computational capabilities, production of high-quality visualizations, built-in application area packages, and a range of immediate deployment options. Combined with integration of dynamic libraries, automatic interface construction, and C code generation, *Mathematica* is the most sophisticated build-to-deploy environment on the market today.

Mathematica 8 introduces the ability to perform computations on the GPU and thus facilitates heterogeneous computing. For developers, this new integration means native access to *Mathematica*'s computing abilities—creating hybrid algorithms that combine the CPU and the GPU. Some of *Mathematica*'s features include:

Free-form linguistic input

Mathematica's free-form linguistic input is the ability to interpret English text as *Mathematica* code. It is a breakthrough in usability, making development intuitive and simple.

Multiparadigm programming language

Mathematica provides its own highly declarative functional language, as well as several different programming paradigms, such as procedural and rule-based programming. Programmers can choose their own style for writing code with minimal effort. Along with comprehensive documentation and resources, *Mathematica*'s flexibility greatly reduces the cost of entry for new users.

Symbolic-numeric hybrid system

The principle behind *Mathematica* is full integration of symbolic and numeric computing capabilities. Through its full automation and preprocessing mechanisms, users reap the power of a hybrid computing system without needing knowledge of specific methodologies and algorithms.

Scientific and technical area coverage

Mathematica provides thousands of built-in functions and packages that cover a broad range of scientific and technical computing areas, such as statistics, control systems, data visualization, and image processing. All functions are carefully designed and tightly integrated with the core system.

Unified data representation

At the core of *Mathematica* is the foundational idea that everything—data, programs, formulas, graphics, documents—can be represented as symbolic entities, called *expressions*. This unified representation makes *Mathematica*'s language and functions extremely flexible, streamlined, and consistent.

Data access and connectivity

Mathematica natively supports hundreds of formats for importing and exporting, as well as real-time access to data from Wolfram|Alpha®, Wolfram Research's computational knowledge engine™. It also provides APIs for accessing many programming languages and databases, such as C/C++, Java, .NET, MySQL, and Oracle.

Full-featured, unified development environment

Through its unique interface and integrated features for computation, development, and deployment, *Mathematica* provides a streamlined workflow. Wolfram Research also offers Wolfram *Workbench*™, a state-of-the-art integrated development engine based on the Eclipse platform.

High-performance computing

Mathematica has built-in support for multicore systems, utilizing all cores on the system for optimal performance. Many functions automatically utilize the power of multicore processors, and built-in parallel constructs make high-performance programming easy.

Platform-independent deployment options

Through its interactive notebooks, Wolfram *CDF Player*™, and browser plugins, *Mathematica* provides a wide range of options for deployment. Built-in code generation functionality can be used to create standalone programs for independent distribution.

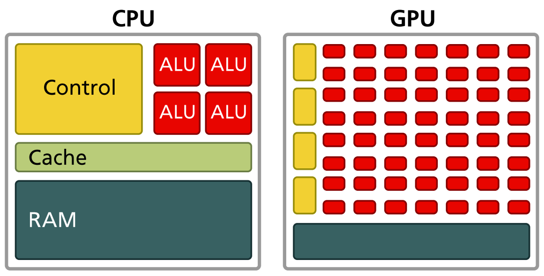
Scalability

Wolfram Research's grid*Mathematica* ™ allows *Mathematica* programs to be parallelized on many machines in cluster or grid configuration. Also availableis web*Mathematica* ™ which allows *Mathematica* programs to be run on a web server.

Motivations for Heterogeneous Computing in *Mathematica*

Over the past few years, multicore systems have transitioned from being found only on specialty devices to commodity devices. With CPUs routinely being 2-, 4-, or 8-cores, software has not been able to exploit the now-common multicore features.

The GPU has also transitioned from being used only for graphics and gaming to a device capable of performing general computation. The GPU has quickly surpassed the CPU in terms of performance, with a modest GPU able to perform over 20 times more computations per second than a CPU. This is because of the GPU architecture, which reduces cache and RAM in favor of arithmetic logical units (ALU).



Unlike Moore's law for the CPU—which states that the number of transistors that can be placed inexpensively on an integrated circuit doubles approximately every 18 months—GPUs have been quadrupling the number of transistors every 18 months. With an architecture that facilitates the addition of more ALUs, the GPU has quickly surpassed the CPU in terms of raw computational power, making the GPU ideal for certain forms of computation.

Yet as algorithms were implemented for the GPU, it became apparent that the lack of cache and control makes some algorithms either difficult or inefficient on the GPU. Hence the popularization of heterogeneous computing, which uses the CPU in cases where the GPU is inefficient and the GPU where speedups are desired.

Mathematica 7 introduced multicore programming in *Mathematica*, and *Mathematica* 8 made many algorithms run on multicore systems in addition to introducing GPU programming. By providing an interface for running programs on both the CPU and the GPU, *Mathematica* embodies the heterogeneous computing philosophy, allowing users to choose either the CPU or GPU based on the program's performance.

CPU Multicore Integration in *Mathematica*

Most of the built-in functions, like linear algebra, image processing, and wavelets, already make use of all cores on the system. *Mathematica* also exposes to the user a few ways to accelerate programs using multiple cores. Those functions can be further extended to run on a cluster of machines with the use of grid*Mathematica*, as well as be used in conjunction with the GPU capabilities in *Mathematica* 8.

Compile

Mathematica's `Compile` command takes a sequence of inputs and expressions, and, based on the input usage, performs type inferencing to construct a byte code representation of the program. The byte code can be targeted to *Mathematica*'s internal virtual machine (WVM) or C. `Compile` is used internally by many *Mathematica* functions and, when it makes sense, some expressions are automatically compiled for the user—when constructing a list using a function, for example, the function is automatically created to speed up the construction.

`Compile` Command	`Compile` Instructions	`Compile` Targets

`Compile [{x},`
`  Sin[x] + x^2 - 1/(x^2)`
`]`
	`Instruction[Sin, r1, a1]`
	`Instruction[Power, r2, a1]`
	`Instruction[Recip, r3, r2]`
	`Instruction[Sub, {a1, r3}]`
	`Instruction[Plus, res, {r1, r2}]`
		`Compile[{x},`
		`  Sin[x] + x^2 - 1/(x^2)`
		`  CompilationTarget->"C"`
		`]`

*Mathematica*s **`Compile`** command translates *Mathematica* expressions into a sequence of instructions. The compiler performs type inferencing, type coorsions, common subexpression elimination, and dead code removal. Since **`Compile`** translates expressions into byte code close to machine instructions, it works with machine precisions and tracks when overflows and underflows occur.	The *Mathematica* byte code is a machine-independent representation of both arithmetic scalar instructions as well as list manipulation instructions. The list instructions represent common function such as **`Map`** and **`Fold`**. The instructions are simple, either unary, binary, or ternary instructions that are inert, allowing the user to further manipulate the symbolically.	Code can be written to target the *Mathematica* instruction set to the Wolfram Virtual Machine (an internal virtual machine bundled with *Mathematica*), C, LLVM IR, or CUDA and OpenCL *Mathematica* 8 has built-in support to target either the Wolfram Virtual Machine or C, allowing the user to write code in a high-level language that is capable of running at the same speed as C.

`Compile` works only on a subset of the *Mathematica* language, and although it sacrifices some *Mathematica* features, it does afford the user significant speed improvements.

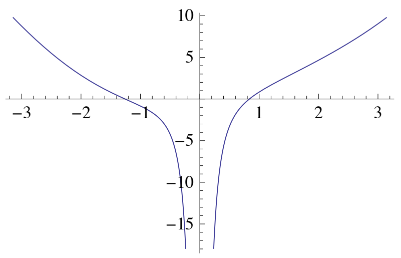
To use the `Compile` command, the user passes in the input variables along with a sequence of *Mathematica* expressions. `Compile` will automatically perform code optimization—the following code, for example, performs common subexpression elimination evaluating x^2 once.

$$\mathbf{cf = Compile\left[\{x\},\ Sin[x] + x^2 - \frac{1}{x^2}\right]}$$

$$\mathtt{CompiledFunction}\left[\{x\},\ \mathtt{Block}\left[\{\mathtt{Compile`\$2}\},\ \mathtt{Compile`\$2} = x^2;\ \mathtt{Sin}[x] + \mathtt{Compile`\$2} - \frac{1}{\mathtt{Compile`\$2}}\right],\ \text{-CompiledCode-}\right]$$

The `CompiledFunction` returned behaves the same as any *Mathematica* function. It can be visualized as follows.

```
Plot[cf[x], {x, -π, π}]
```



The `Compile` statement generates byte code from the *Mathematica* program input by the user. The byte code can be printed to understand what code is being generated.

```
Needs["CCodeGenerator`"]
CompilePrint[cf]

        1 argument
        5 Real registers
        Underflow checking off
        Overflow checking off
        Integer overflow checking on
        RuntimeAttributes -> {}

        R0 = A1
        Result = R2

1     R1 = Square[ R0]
2     R2 = Sin[ R0]
3     R3 = Reciprocal[ R1]
4     R4 = - R3
5     R2 = R2 + R1 + R4
6     Return
```

The byte code generated can be used to target the Wolfram Virtual Machine (WVM), C, LLVM IR, CUDA, OpenCL, or JVM with both WVM and C built into *Mathematica* 8.

Compilation Target

By default, `Compile` targets the WVM, but if `CompilationTarget->"C"` is set, then `Compile` will generate a C version of the *Mathematica* program, compile it, and run it. This allows *Mathematica* to run at the same speed as C, without going through low-level C programming.

```
cf = Compile[{x}, Sin[x] + x^2 - 1/x^2, CompilationTarget → "C"]

CompiledFunction[{x}, Block[{Compile`$3},
  Compile`$3 = x^2; Sin[x] + Compile`$3 - 1/Compile`$3], -CompiledCode-]
```

Since *Mathematica* is a highly expressive language, programs that take dozens or hundreds of lines in C code can be written in two or three lines in *Mathematica* and have similar performance. The following, for example, uses `Compile` to generate a multi-threaded C program to compute the Mandelbrot set. The code is executed from within *Mathematica*.

```
compileMandelbrot = Compile[{{c, _Complex}},
   Boole[Norm[FixedPoint[#^2 + c &, 0, 1000, SameTest → (Norm[#] ≥ 4 &)]] ≥ 4],
   CompilationTarget → "C", RuntimeAttributes → Listable
  ];
```

The following sets the evaluation points for the Mandelbrot function.

```
evaluationPoints = Table[a + I b, {b, -1, 1, 0.005}, {a, -1.5, 0.5, 0.005}];
```

And finally, we invoke the function with the evaluation points and plot the result.

```
ArrayPlot[compileMandelbrot[evaluationPoints]]
```

Written in C, the above would have taken many hundreds of lines of code, required project setup, and might not have been portable across systems.

Automatic vectorization

Mathematica 8 added the ability to automatically parallelize `Compile` statements. Making a `CompiledFunction` run in parallel is simple—the user only has to pass the option `RuntimeAttributes->"Listable"`. From there, `Compile` will run in as many threads as there are on the system.

As an example, we implement a basic ray tracer. The ray tracer takes a list of sphere centers, sphere radii, sphere colors, and a ray origin position. It then shines a parallel ray from the ray origin and records which sphere intersects the ray and associates a color with the intersection. We will compile the code into `"C"` by passing the `CompilationTarget->"C"` option, as we will use `RuntimeAttributes->"Listable"` to make the code run in parallel.

```
raySpheresIntersectionColor = Compile[{{centers, _Real, 2},
    {radii, _Real, 1}, {colors, _Real, 2}, {x, _Real}, {y, _Real}},
   Module[{ray = {x, y, 0.0}, v, rad, center, res = {0., 0., 0.}},
    Do[
     rad = radii[[ii]]^2;
     center = centers[[ii]];
     v = Norm[ray - center]^2;
     If[v < rad,
                           rad - v
      res = colors[[ii]] ---------;
                             rad
     ], {ii, Length[centers]}
    ];
    res
   ], CompilationTarget → "C", RuntimeAttributes → Listable
  ];
```

Here, we define a *Mathematica* function that calls the above compiled function.

```
rayTraceCompile[centers_, radii_, colors_, x_, y_] :=
 Module[{ xx, yy},
  xx = Transpose[ConstantArray[x, Length[y]]];
  yy = ConstantArray[y, Length[x]];
  raySpheresIntersectionColor[centers, radii, colors, xx, yy]
 ]
```

This constructs the scene, generating 100 spheres in random positions and creating the ray origins. We use the *Mathematica* symbol `ColorData` to get colors that follow the `"DarkBands"` color scheme.

```
width = height = 300;
nx = ny = 300;
numSpheres = 100;
centers = Table[{RandomReal[{-width/2, width/2}],
    RandomReal[{-height/2, height/2}], RandomReal[100]}, {numSpheres}];
radii = RandomReal[{5, 30}, numSpheres];
colors = ReplaceAll[
   ColorData["DarkBands"] /@ Range[0, 1, 1 / numSpheres], RGBColor → List];
x = width * (N[Range[0, nx]] / nx - .5);
y = height * (N[Range[0, ny]] / ny - .5);
```

This runs the ray tracer and visualizes the result.

```
imgc = rayTraceCompile[centers, radii, colors, x, y];
Image[imgc]
```

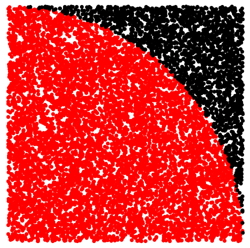
Again, the speed of this program is comparable to a program written in C, but unlike the C version, our code is around 20 lines long.

Parallel Computing

Built into the *Mathematica* language since Version 7 is the capability for multicore computing. *Mathematica*'s parallel tools allow users to make use of all cores on their system, developing more sophisticated projects that execute at a fraction of the time. *Mathematica*'s parallel infrastructure is also set up to allow seamless scaling to networks, clusters, and grids.

The most basic parallel program you can write is a Monte Carlo integrator for approximating $\frac{\pi}{4}$. This is done by generating uniform random points in the $[0,1]\times[0,1]$ region. The proportion of points with a norm less than 1 approximates $\frac{\pi}{4}$. The following shows uniform points in the $[0,1]\times[0,1]$ region with red points having a norm less than 1. As can be seen, those points approximately cover a quarter of a unit circle.

```
pts = RandomReal[1, {10 000, 2}];
Graphics[{AbsolutePointSize[2], {Red, Point[Select[pts, Norm[#] ≤ 1 &]]},
  {Black, Point[Select[pts, Norm[#] > 1 &]]}}]
```



The *Mathematica* language has extensive support for list processing. This makes writing the above Monte Carlo π-approximation trivial.

```
n = 1 000 000;
Mean[Table[If[Norm[RandomReal[1, {2}]] ≤ 1, 1.0, 0.0], {n}]]

0.784838
```

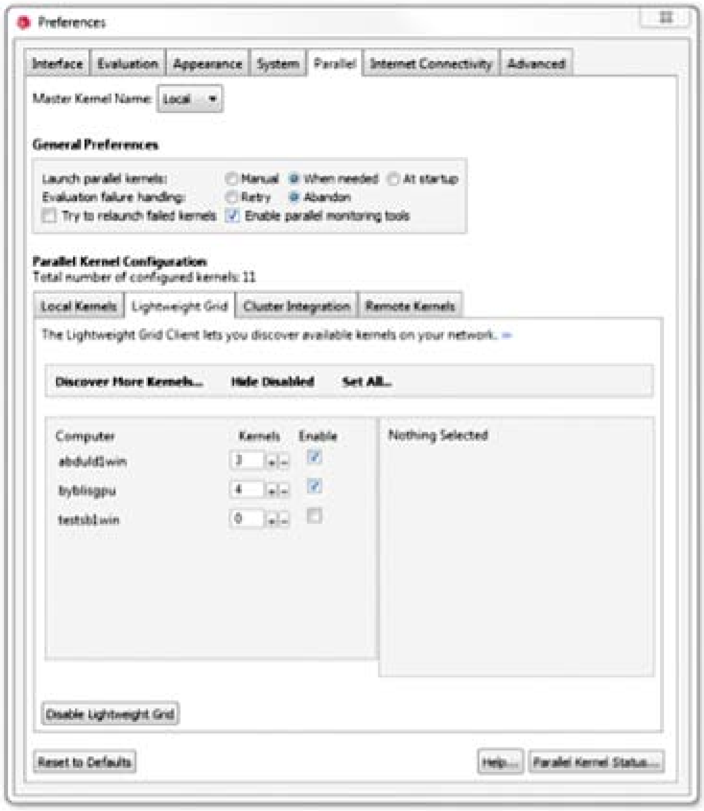
Mathematica 7 introduced parallel primitives such as `ParallelMap` and `ParallelTable`. The parallelization has been further simplified in *Mathematica* 8 by introducing `Parallelize`, which performs automatic parallelization. So, to make the above program run on multiple cores, the user just has to place `Parallelize` around `Table`, and it is automatically run in parallel.

```
n = 1 000 000;
Mean[
 Parallelize[
  Table[If[Norm[RandomReal[1, {2}]] ≤ 1, 1.0, 0.0], {n}]
 ]
]

0.784934
```

Parallel tools allow fine-grained control on how to split the parallel tasks and how many parallel kernels to run. A queuing mechanism is also available to saturate the CPU usage and achieve the best speedup.

grid*Mathematica*

grid*Mathematica* extends *Mathematica*'s parallel functionality to run on homogeneous and heterogeneous networks and clusters. By installing the grid*Mathematica* server on a machine, the machine will broadcast its availability to the master machine.



Master Kernel Name: Local

General Preferences

Launch parallel kernels: Manual [✓ When needed] At startup
Evaluation failure handling: Retry [✓ Abandon]
☐ Try to relaunch failed kernels ✓ Enable parallel monitoring tools

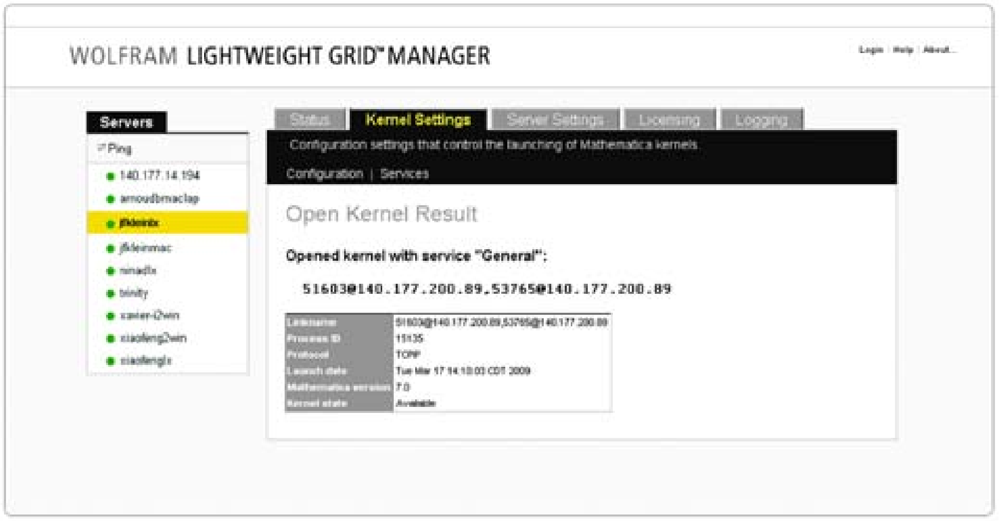
Parallel Kernel Configuration
Total number of configured kernels: 11

The Lightweight Grid Client lets you discover available kernels on your network. »

Computer	Kernels	Enable
abduld1win	3	✓
byblisgpu	4	✓
testsb1win	0	☐

Nothing Selected

Similar to *Mathematica*'s parallel tools, grid*Mathematica* allows for fine-grained control over the scheduling of tasks, giving priority to certain kernels. grid*Mathematica* also interfaces with existing grid management software from Microsoft, Sun, LFS, PBS, etc. It also contains its own management software called Wolfram Lightweight Grid™ Manager.



Open Kernel Result

Opened kernel with service "General":

```
51603@140.177.200.89,53765@140.177.200.89
```

Linkname	51603@140.177.200.89,53765@140.177.200.89
Process ID	15135
Protocol	TCPIP
Launch date	Tue Mar 17 14:10:03 CDT 2009
Mathematica version	7.0
Kernel state	Available

Programs written using *Mathematica*'s parallel tools are valid grid*Mathematica* programs, and thus *Mathematica* is easily scalable from single multicore systems to clusters.

GPU Integration in *Mathematica*

CUDALink and *OpenCLLink* offer a high-level interface to the GPU built on top of *Mathematica*'s language and development technologies. They allow users to execute code on their GPU with minimal effort. By fully integrating and automating the GPU's capabilities using *Mathematica,* hiding unnecessary complexity associated with GPU programming, and lowering the learning curve for GPU computation, users experience a more productive and efficient development cycle.

CUDALink and *OpenCLLink* Support

CUDALink is supported on all CUDA-enabled NVIDIA hardware. *Mathematica* is bundled with the CUDA Toolkit. This makes the NVIDIA driver and a supported C compiler the only requirements for *CUDALink* programming.

OpenCLLink supports both NVIDIA and AMD GPUs, with only the NVIDIA driver being needed for NVIDIA GPUs. Both the AMD video driver and AMD APP SDK are needed for AMD. *OpenCLLink* also supports CPU implementations of OpenCL provided by either AMD or Intel.

GPUs that have CUDA or OpenCL support are supported by *CUDALink* or *OpenCLLink* with *Mathematica* automatically determining the precision of the card and executing the appropriate function based on the maximal floating point precision. *CUDALink* and *OpenCLLink* are supported on all platforms supported by *Mathematica* 8—Windows, Mac OS X, and Linux, both 32- and 64-bit.

Making GPU Programming Easy

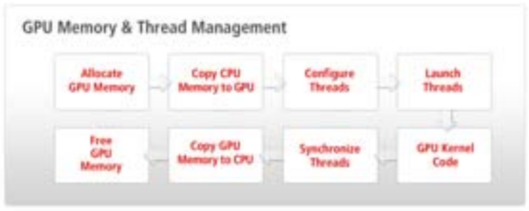
By removing repetitive and low-level GPU related tasks, *Mathematica* makes GPU programming simple.

Automation of development project management

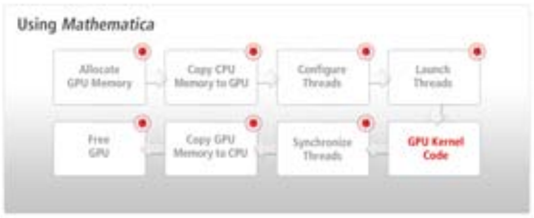
Unlike other development frameworks that require the user to manage project setup, platform dependencies, and device configuration, *Mathematica* makes the process of GPU programming transparent and automated.

Automated GPU memory and thread management

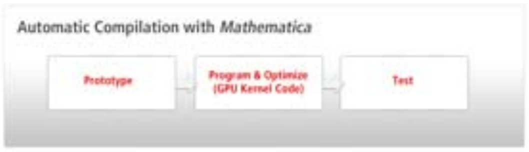
A GPU program written from scratch delegates memory and thread management to the programmer. This bookkeeping is required in addition to the need to write the GPU kernel.



With *Mathematica*, memory and thread management are automatically handled for the user. The *Mathematica* memory manager handles memory transfers intelligently in the background. Memory, for example, is not copied to the GPU until computation is needed and is flushed out when the GPU memory gets full.



As a result of hiding the bookkeeping and repetitive tasks found in GPU programming, *Mathematica* streamlines the whole programming process, allowing for simpler code, shorter development cycle, and better performance.


Automatic Compilation with Mathematica
Prototype
Program & Optimize (GPU Kernel Code)
Test

Integration with *Mathematica*'s built-in capabilities

Mathematica's GPU integration provides full access to *Mathematica*'s native language and built-in functions. Users can write hybrid algorithms that use the CPU and GPU depending on the efficiency of each algorithm.

Ready-to-use applications

Mathematica provides several ready-to-use GPU functions that cover a broad range of topics such as computational mathematics, image processing, financial engineering, and more.

Zero device configuration

Mathematica automatically finds, configures, and makes GPU devices available to users. *Mathematica* will automatically detect and configure the required tools to make it simple to use the GPU.

Through *Mathematica*'s built-in parallel programming support, users can launch GPU programs on different devices. Users can also scale the setup across machines and networks using grid*Mathematica*.

Web deployment

Mathematica can be deployed onto a web server using web*Mathematica*. This means that GPU computation can be initiated from devices that do not have a GPU. This is ideal for the classroom, where the instructor may want to control the development environment, or mobile devices, which do not have GPUs.

Mathematica's OpenCLLink: Integrated GPU Programming

OpenCLLink is a built-in *Mathematica* application that provides an interface for using OpenCL within *Mathematica*. Through *OpenCLLink*, users can execute OpenCL programs from *Mathematica* with little effort. Because *Mathematica* caches program compilation and makes intelligent choices about GPU memory transfer, users experience better execution speed versus handwritten OpenCL programs.

Setting up *OpenCLLink*

OpenCLLink supplies functions that query the system's GPU hardware. To use *OpenCLLink* operations, users have to first load the *OpenCLLink* application.

```
Needs["OpenCLLink`"]
```

`OpenCLQ` tells whether the current hardware and system configuration support *OpenCLLink*:.

```
OpenCLQ[]

True
```

`OpenCLInformation` gives information on the available OpenCL hardware. Here, we query information about the first platform and device installed on the system.

```
OpenCLInformation[1, 1]

{Type → GPU, Name → Tesla C2050 / C2070, Version → OpenCL 1.0 CUDA,
 Extensions → {cl_khr_byte_addressable_store, cl_khr_icd, cl_khr_gl_sharing,
   cl_nv_d3d9_sharing, cl_nv_d3d10_sharing, cl_khr_d3d10_sharing, cl_nv_d3d11_sharing,
   cl_nv_compiler_options, cl_nv_device_attribute_query, cl_nv_pragma_unroll,
   cl_khr_global_int32_base_atomics, cl_khr_global_int32_extended_atomics,
   cl_khr_local_int32_base_atomics, cl_khr_local_int32_extended_atomics, cl_khr_fp64},
 Driver Version → 270.81, Vendor → NVIDIA Corporation, Profile → FULL_PROFILE,
 Vendor ID → 4318, Compute Units → 14, Core Count → 448,
 Maximum Work Item Dimensions → 3, Maximum Work Item Sizes → {1024, 1024, 64},
 Maximum Work Group Size → 1024, Preferred Vector Width Character → 1,
 Preferred Vector Width Short → 1, Preferred Vector Width Integer → 1,
 Preferred Vector Width Long → 1, Preferred Vector Width Float → 1,
 Preferred Vector Width Double → 1, Maximum Clock Frequency → 1147,
 Address Bits → 32, Maximum Memory Allocation Size → 695 091 200, Image Support → True,
 Maximum Read Image Arguments → 128, Maximum Write Image Arguments → 8,
 Maximum Image2D Width → 4096, Maximum Image2D Height → 32 768,
 Maximum Image3D Width → 2048, Maximum Image3D Height → 2048,
 Maximum Image3D Depth → 2048, Maximum Samplers → 16, Maximum Parameter Size → 4352,
 Memory Base Address Align → 4096, Memory Data Type Align Size → 128,
 Floating Point Precision Configuration → {Denorms, Infinity, NaNs,
   Round to Nearest, Round to Infinity, Round to Zero, IEEE754-2008 Fused MAD},
 Global Memory Cache Type → Read Write, Global Memory Cache Line Size → 128,
 Global Memory Cache Size → 229 376, Global Memory Size → 2 780 364 800,
 Maximum Constant Buffer Size → 65 536, Maximum Constant Arguments → 9,
 Local Memory Type → Local, Local Memory Size → 49 152, Error Correction Support → True,
 Profiling Timer Resolution → 1000, Endian Little → True, Available → True,
 Compiler Available → True, Execution Capabilities → {Kernel Execution},
 Command Queue Properties → {Out of Order Execution, Profiling Enabled}}
```

Example of a report generated by `OpenCLInformation`*.*

OpenCLLink Programming

Programming the GPU in *Mathematica* is straightforward. It begins with writing an OpenCL program. The following OpenCL program negates colors of a multichannel image.

```
src = "
__kernel void openclColorNegate(__global
    mint *img, __global mint *dim, mint channels) {
    int width = dim[0], height = dim[1];
    int xIndex = get_global_id(0), yIndex = get_global_id(1);
    int index = channels * (xIndex + yIndex*width);
    if (xIndex < width && yIndex < height) {
        for (int c = 0; c < channels; c++)
            img[index + c] = 255 - img[index + c];
    }
}";
```

The source code is passed to `OpenCLFunctionLoad` and the user gets a *Mathematica* function as output.

```
OpenCLColorNegate = OpenCLFunctionLoad[src, "openclColorNegate",
  {{_Integer, "InputOutput"}, {_Integer, "Input"}, _Integer}, {16, 16}]
```

```
OpenCLFunction[<>, cudaColorNegate,
 {{_Integer, InputOutput}, {_Integer, Input}, _Integer}]
```

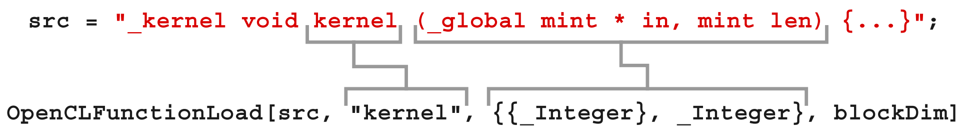
Now you can apply this new OpenCL function to an image.

img = ;

```
OpenCLColorNegate[img, ImageDimensions[img], ImageChannels[img]]
```

{ }

`OpenCLFunctionLoad` follows a simple syntax, where the first argument is the OpenCL source, the second argument is the function name to be invoked, the third argument is a list of function parameter types, and the final argument is the work group size (block dimension).



```
src = "_kernel void kernel (_global mint * in, mint len) {...}";

OpenCLFunctionLoad[src, "kernel", {{_Integer}, _Integer}, blockDim]
```

Several things need to happen behind the scenes at this stage to load the OpenCL code into *Mathematica* efficiently.

First, we need to check to see if the arguments to `OpenCLFunctionLoad` are valid. Since this is the most common source of errors, we have to catch it early. Second, we compile the OpenCL function and cache it. The output from this step is an `OpenCLFunction` that behaves like any *Mathematica* function.

When an `OpenCLFunction` is invoked, we perform input checking to make sure it matches the input specification. We next load the data onto the GPU (memory copies are performed in a lazy fashion to minimize memory copy overhead). We then invoke a synchronization to make sure all computation has been performed, and get the results from the GPU.

OpenCLLink Applications in *Mathematica*

In this section we discuss some applications that run on the GPU using *OpenCLLink*. In *Mathematica* you can perform sophisticated heterogeneous computation easily by leveraging a variety of built-in features.

Black–Scholes Equation

The Black–Scholes equation is the basis of computational finance. It states that the call of a European style option can be modeled by a formula implemented by the following OpenCL program.

```
code = "
#ifdef USING_DOUBLE_PRECISIONQ
#ifdef OPENCLLINK_USING_NVIDIA
#pragma OPENCL EXTENSION cl_khr_fp64 : enable
#else /* OPENCLLINK_USING_NVIDIA */
#pragma OPENCL EXTENSION cl_amd_fp64 : enable
#endif /* OPENCLLINK_USING_NVIDIA */
#endif /* USING_DOUBLE_PRECISIONQ */

#define N(x)       (erf((x)/sqrt(2.0))/2+0.5)

__kernel void blackScholes(__global Real_t * call, __global Real_t * S,
    __global Real_t * X, __global Real_t * T, __global Real_t *
    R, __global Real_t * Q, __global Real_t * V, mint length) {
    int ii = get_global_id(0);
    if (ii < length) {
        Real_t d1 =
    (log(S[ii]/X[ii])+(R[ii]-Q[ii]+(pow(V[ii],(Real_t)2.0)/2)*T[ii]))/(V
    [ii]*sqrt(T[ii]));
        Real_t d2 = d1 - V[ii]*sqrt(T[ii]);
        call[ii] =
    S[ii]*exp(-Q[ii]*T[ii])*N(d1) - X[ii]*exp(-R[ii]*T[ii])*N(d2);
    }
}";
```

`Real_t` is a type defined by *Mathematica* that maps to the highest precision of the OpenCL device. This ensures that users are getting the best accuracy when computing.

The following loads the above OpenCL code into *Mathematica*.

```
OpenCLBlackScholes = OpenCLFunctionLoad[code, "blackScholes",
  {{_Real}, {_Real, "Input"}, {_Real, "Input"}, {_Real, "Input"},
   {_Real, "Input"}, {_Real, "Input"}, {_Real, "Input"}, _Integer}, 128]
```

```
OpenCLFunction[<>, blackScholes,
 {{_Real}, {_Real, Input}, {_Real, Input}, {_Real, Input},
  {_Real, Input}, {_Real, Input}, {_Real, Input}, _Integer}]
```

This gets the stock price for the S&P 500 from the beginning of 2010 to March 2011. This data is curated by Wolfram Research and accessible via a web connection.

```
data = Transpose[FinancialData["SP500", {{2010, 0, 0}, {2011, 4, 0}}]];
```

This uses the S&P 500 data for the spot price and the dates for the expiration values. The rest of the data is randomly generated.

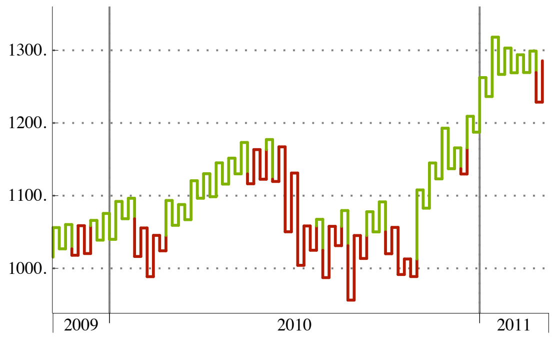
```
Needs["Calendar`"]
numberOfOptions = Length[data[[1]]];
call = ConstantArray[0.0, numberOfOptions];
S = data[[2]];
X = 1.1 * data[[2]];
T = (DaysBetween[#, {2011, 4, 20}] & /@ data[[1]]) / 365.;
R = RandomReal[{0.03, 0.07}, numberOfOptions];
Q = RandomReal[{0.01, 0.04}, numberOfOptions];
V = RandomReal[{0.10, 0.50}, numberOfOptions];
```

The following runs the computation on the OpenCL device.

```
res = OpenCLBlackScholes[call, S, X, T, R, Q, V, numberOfOptions];
```

We visualize the result as a Kagi chart. This visualization function is part of *Mathematica's* comprehensive support for visualization and charting, which includes, plotting surfaces, computing bar and pie charts, and interacting with trading charts.

```
KagiChart[Transpose[{data[[1]], res[[1]]}]]
```



Computing with Data from an Excel File

Mathematica supports many import and exports formats. One such format is Excel 2007 (XLSX), which we use in this example as source for our data when calculating the one-touch option.

```
code = "
#define N(x)       (erf((x)/sqrt(2.0))/2+0.5)
__kernel void onetouch(__global Real_t * call,
    __global Real_t * put, __global Real_t * S, __global
    Real_t * X, __global Real_t * T, __global Real_t * R,
    __global Real_t * D, __global Real_t * V, mint length) {
    Real_t tmp, d1, d5, power;
    int ii = get_global_id(0);
    if (ii < length) {
        d1 = (log(S[ii]/X[ii]) + (R[ii] - D[ii] +
    0.5f * V[ii] * V[ii]) * T[ii]) / (V[ii] * sqrt(T[ii]));
        d5 = (log(S[ii]/X[ii]) - (R[ii] - D[ii] + 0.5f *
    V[ii] * V[ii]) * T[ii]) / (V[ii] * sqrt(T[ii]));
        power = pow(X[ii]/S[ii], 2*R[ii]/(V[ii]*V[ii]));
        call[ii] = S[ii] <
    X[ii] ? power * N(d5) + (S[ii]/X[ii])*N(d1) : 1.0;
        put[ii] = S[ii] > X [ii] ? power * N(-d5)
    + (S[ii]/X[ii])*N(-d1) : 1.0;
    }
}";
```

This loads the OpenCL function into *Mathematica* in single precision mode.

```
OpenCLOneTouchOption =
  OpenCLFunctionLoad[code, "onetouch", {{_Real, "Output"},
    {_Real, "Output"}, {_Real, "Input"}, {_Real, "Input"},
    {_Real, "Input"}, {_Real, "Input"}, {_Real, "Input"},
    {_Real, "Input"}, _Integer}, 128, "TargetPrecision" → "Single"];
```

This imports the data from an Excel file and stores it in a *Mathematica* table.

```
rawData = First[Import["dataset.xlsx", "Data"]];
Do[data[First[row]] = Drop[row, 1], {row, Transpose[rawData]}];
numberOfOptions = Length[data["Spot Price"]];
```

This allocates memory for both the call and put result. We allocate the data as a `"float"`.

```
call = OpenCLMemoryAllocate["Float", numberOfOptions];
put = OpenCLMemoryAllocate["Float", numberOfOptions];
```

This calls the function.

```
OpenCLOneTouchOption[call, put, data["Spot Price"],
 data["Strike Price"], data["Expiration"], data["Interest"],
 data["Dividend"], data["Volatility"], numberOfOptions]

{OpenCLMemory[<29650>, Float], OpenCLMemory[<29636>, Float]}
```

This retrieves the result for the call option.

```
OpenCLMemoryGet[call]

{1., 1., 0.93962, 1.63052, 0.895195, 1., 1., 1., 1., 0.940062,
 0.719849, 0.844359, 0.95156, 1., 1., 1., 0.933998, 1., 1.00517, 1.,
 0.113093, 0.34359, 1., 1., 1., 1., 1., 0.865092, 0.913892, 6.13967,
 1., 1.38213, 1., 0.939484, 1., 1., 1., 1.0914, 1., 1.0485, 0.889275,
 0.991108, 0.35102, 0.689786, 1., 1., 1., 1., 0.982171, 3.07142, 1., 1.,
 0.999357, 0.850779, 1., 2.53988, 1., 1., 1., 1., 1., 0.852829, 1., 1.}
```

Conway's Game of Life

Conway's Game of Life is an example of a simple two-dimensional cellular automaton. From simple rules that look only at the eight neighbors, it gives rise to complicated patterns. Here is a basic OpenCL program that implements the Game of Life.

```
src = "
__kernel void gol_kernel(__global mint *
    prev, __global mint * nxt, mint width, mint height) {
  int xIndex = get_global_id(0), yIndex = get_global_id(1);
  int index = xIndex + yIndex*width;
  int ii, jj, curr, neighbrs;
  if (xIndex < width && yIndex < height) {
    curr = prev[index];
    neighbrs = 0;
    for (ii = -1, neighbrs = -curr; ii <= 1; ii++) {
      if (xIndex + ii >= 0 && xIndex+ii < width) {
        for (jj = -1; jj <= 1; jj++) {
          if (yIndex+jj >= 0 && yIndex+jj < height)
            neighbrs += prev[xIndex + ii + (yIndex+jj)*width];
        }
      }
    }
    if (curr == 1)
      nxt[index] = (neighbrs == 2 || neighbrs == 3) ? 1 : 0;
    else
      nxt[index] = (neighbrs == 3) ? 1 : 0;
  }
}";
```

This loads the function using `OpenCLFunctionLoad`. We set the work group size to 16×16.

```
OpenCLGameOfLife = OpenCLFunctionLoad[src, "gol_kernel",
  {{_Integer, "Input"}, {_Integer, "Output"}, _Integer, _Integer}, {16, 16}]
```

```
OpenCLFunction[<>, gol_kernel,
 {{_Integer, Input}, {_Integer, Output}, _Integer, _Integer}]
```

We set the initial state using random choice, making 70% of the 512×512 initial state set to zero while the others are set to 1. We set the output state to all zeros.

```
initialState = RandomChoice[{0.7, 0.3} → {0, 1}, {512, 512}];
outputState = ConstantArray[0, {512, 512}];
```

This uses `Dynamic` to animate the result at 60 frames per second.

```
Dynamic[
 Refresh[
  initialState =
   First[OpenCLGameOfLife[initialState, outputState, 512, 512]];
  ArrayPlot[initialState, ImageSize → Medium],
  UpdateInterval → 1 / 60
 ]
]
```

Many-Body Physical Systems

The N-body simulation is a classic Newtonian problem. The OpenCL implementation is included as part of the *Mathematica* distribution.

```
srcf = FileNameJoin[{$OpenCLLinkPath, "SupportFiles", "NBody.cl"}];
```

This loads `OpenCLFunction`. Note that you can pass the vector type `"float4"` into the OpenCL program and *Mathematica* handles the conversion.

```
NBody = OpenCLFunctionLoad[{srcf}, "nbody_sim",
  {{"Float[4]", "Input"}, {"Float[4]", "Input"}, _Integer, "Float", "Float",
   {"Local", "Float"}, {"Float[4]", "Output"}, {"Float[4]", "Output"}}, 256]
```

```
OpenCLFunction[<>, nbody_sim,
 {{Float[4], _, Input}, {Float[4], _, Input}, _Integer, Float, Float,
  {Local, Float}, {Float[4], _, Output}, {Float[4], _, Output}}]
```

The number of particles, time step, and epsilon distance are chosen.

```
numParticles = 1024;
deltaT = 0.05;
epsSqrt = 50.0;
```

This sets the input and output memories.

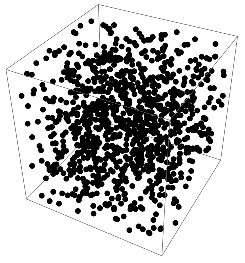
```
pos = OpenCLMemoryLoad[RandomReal[512, {numParticles, 4}], "Float[4]"];
vel = OpenCLMemoryLoad[RandomReal[1, {numParticles, 4}], "Float[4]"];
newPos = OpenCLMemoryAllocate["Float[4]", {numParticles, 4}];
newVel = OpenCLMemoryAllocate["Float[4]", {numParticles, 4}];
```

This calls the `NBody` function.

```
NBody[pos, vel, numParticles, deltaT, epsSqrt, 256 * 4, newPos, newVel, 1024];
NBody[newPos, newVel, numParticles, deltaT, epsSqrt, 256 * 4, pos, vel, 1024];
```

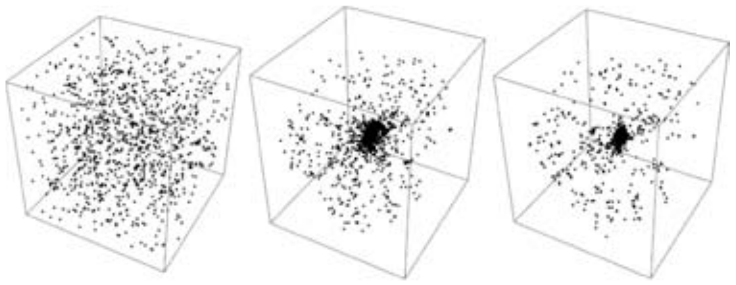
This plots the body points.

```
Graphics3D[Point[Take[#, 3] & /@ OpenCLMemoryGet[pos]]]
```



This animates the result using `Dynamic.`

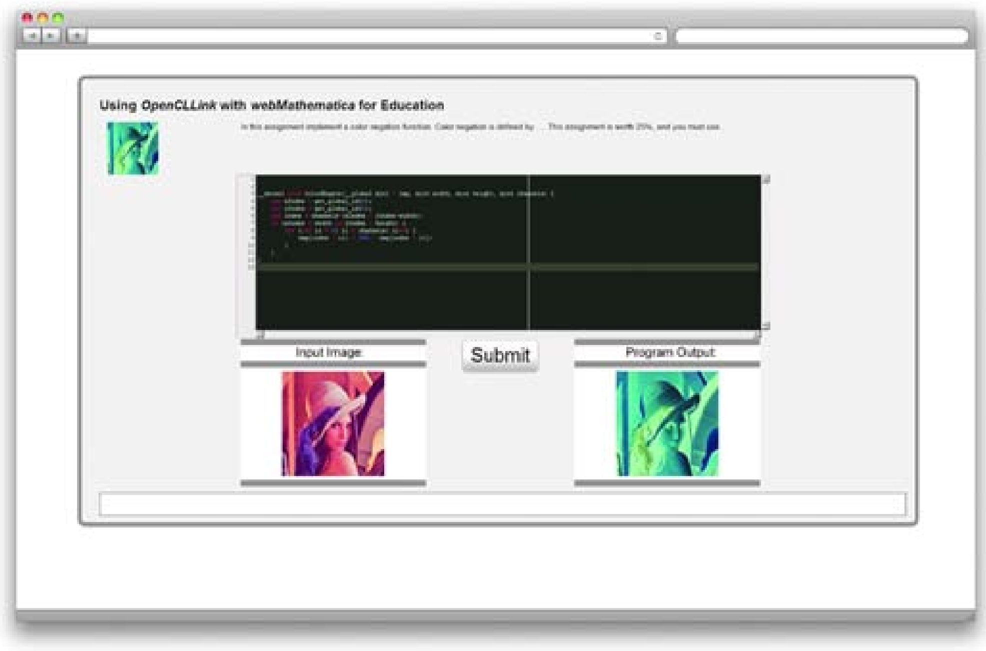
```
Graphics3D[Point[
  Dynamic[Refresh[
    NBody[pos, vel, numParticles,
     deltaT, epsSqrt, 256 * 4, newPos, newVel, 1024];
    NBody[newPos, newVel, numParticles, deltaT,
     epsSqrt, 256 * 4, pos, vel, 1024];
    Take[#, 3] & /@ OpenCLMemoryGet[pos], UpdateInterval → 0]]]]
```



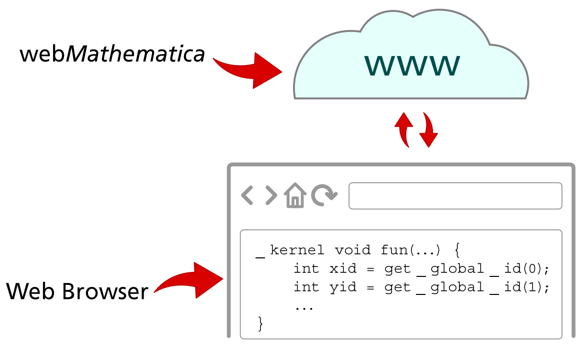
Real-time animation of the N-body simulation.

OpenCL on the Web with web*Mathematica*

Wolfram Research also offers web*Mathematica*, which allows you to deploy *Mathematica* programs on the web by embedding them in JavaServer Pages (JSP). This allows heterogeneous computation to be performed on the server from within a client's web browser.



There are many possible applications for this. The above shows a teaching module developed to enable students to program an OpenCL kernel without being exposed to either *Mathematica* syntax or host side programming. When a user clicks submit, the OpenCL kernel is compiled, an `OpenCLFunction` is generated, and the function is applied to an image. The computed image is then displayed on the screen for the user.



```
_kernel void fun(...) {
    int xid = get_global_id(0);
    int yid = get_global_id(1);
    ...
}
```

Aside from the academic applications, in some cases it is desirable to have a powerful workstation where users can invoke OpenCL computation from within the browser or mobile devices—invoking a financial computation using the latest stock data from a smart phone, for example. web*Mathematica* is a solution for such scenarios.

Mathematica's *CUDALink* Applications

In addition to OpenCL support, *Mathematica* provides CUDA support. By providing the same usage syntax as *OpenCLLink*, *Mathematica* is unique in enabling easy porting of CUDA applications to OpenCL and vice versa. In this section, we show some of the built-in functionality in *CUDALink* as well as how to use *CUDALink* to program the GPU from within *Mathematica*.

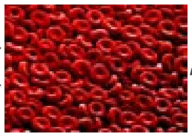
Image Processing

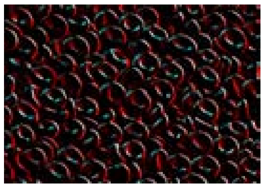
CUDALink's image processing capabilities can be classified into three categories. The first is convolution, which is optimized for CUDA. The second is morphology, which contains abilities such as erosion, dilation, opening, and closing. Finally, there are the pixel operators. These are the image multiplication, division, subtraction, and addition.

To use *CUDALink*, users must first load it.

```
Needs["CUDALink`"]
```

CUDALink's convolution is similar to *Mathematica*'s `ListConvolve` and `ImageConvolve` functions. It will operate on images, lists, or CUDA memory references, and it can use *Mathematica*'s built-in filters as the kernel.

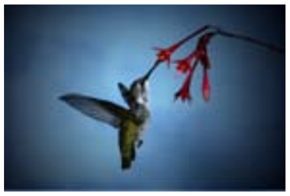
CUDAImageConvolve[, $\begin{pmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{pmatrix}$]



Convolving a microscopic image with a Sobel mask to detect edges.

CUDALink supports pixel operations on one or two images, such as adding or multiplying pixel values from two images.

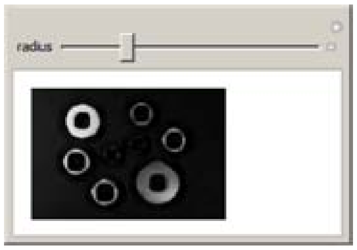
CUDAImageMultiply[,]



Multiplication of two images.

Finally, morphology operations are supported. Here we use the `Manipulate` function, which makes creating GUI interfaces simple.

```
Manipulate[CUDAErosion[            , radius], {radius, 0, 9}]
```



Construction of an interface that performs a morphological operation on an image with varying radii.

Fast Fourier Transforms and Linear Algebra

Users can perform Fourier transforms and various linear algebra operations with *CUDALink.* Methods such as matrix-matrix multiplication, matrix-vector multiplication, finding minimum and maximum elements, and transposing matrices are all accelerated to use the GPU.

Here, we multiply two matrices together.

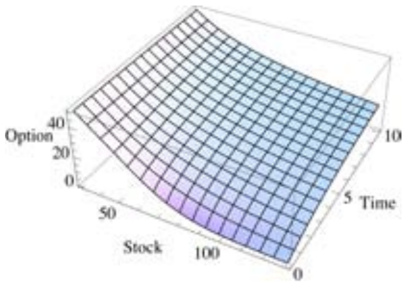
```
         ( 1 1 1 1 1 )   ( 1 1 1 1 1 )
         ( 2 2 2 2 2 )   ( 2 2 2 2 2 )
CUDADot[ ( 3 3 3 3 3 ) , ( 3 3 3 3 3 ) ] // MatrixForm
         ( 4 4 4 4 4 )   ( 4 4 4 4 4 )
         ( 5 5 5 5 5 )   ( 5 5 5 5 5 )

( 15  15  15  15  15 )
( 30  30  30  30  30 )
( 45  45  45  45  45 )
( 60  60  60  60  60 )
( 75  75  75  75  75 )
```

Performing matrix multiplication.

Financial Engineering

CUDALink has built-in financial options pricing capabilities, which use either the analytic solution, the binomial solution, or Monte Carlo methods depending on the type of option selected. The following shows the American put option’s surface plot as the spot price and expiry vary.

```
ListPlot3D[ParallelMap[CUDAFinancialDerivative[{"American", "Put"},
     {"StrikePrice" → 80., "Barriers" → 100, "Expiration" → #},
     {"CurrentPrice" → Range[30., 130., 1], "InterestRate" → 0.06,
      "Volatility" → 0.45, "Dividend" → 0.02, "Rebate" → 5.}] &,
   Range[0.2, 10, 0.2]], DataRange → {{30, 130}, {0.2, 10}},
  AxesLabel → {"Stock", "Time", "Option"}]
```



A three-dimensional plot of the CUDA-evaluated American put. In this case, we utilize parallel programming over CPUs in addition to that provided by the GPU.

Complex Dynamics

CUDALink enables you to easily investigate computationally intensive complex dynamics structures. We will compute the Julia set, which is a generalization of the Mandelbrot set. The following implements the CUDA kernel.

```
code = "
__global__ void julia_kernel(Real_t *
     set, int width, int height, Real_t cx, Real_t cy) {
   int xIndex = threadIdx.x + blockIdx.x*blockDim.x;
   int yIndex = threadIdx.y + blockIdx.y*blockDim.y;
   int ii;

   Real_t x = ZOOM_LEVEL*(width/2 - xIndex);
   Real_t y = ZOOM_LEVEL*(height/2 - yIndex);
   Real_t tmp;
   Real_t c;

   if (xIndex < width && yIndex < height) {
       for (ii = 0; ii <
     MAX_ITERATIONS && x*x + y*y < BAILOUT; ii++) {
             tmp = x*x - y*y + cx;
             y = 2*x*y + cy;
             x = tmp;
        }
        c = log(0.1f + sqrt(x*x + y*y));
        set[xIndex + yIndex*width] = c;
     }
}";
```

This loads the `CUDAFunction`. Notice that the syntax is the same as `OpenCLFunctionLoad`. While we did not show macros being used in *OpenCLLink*, macros are used here to allow the compiler to further optimize the code.

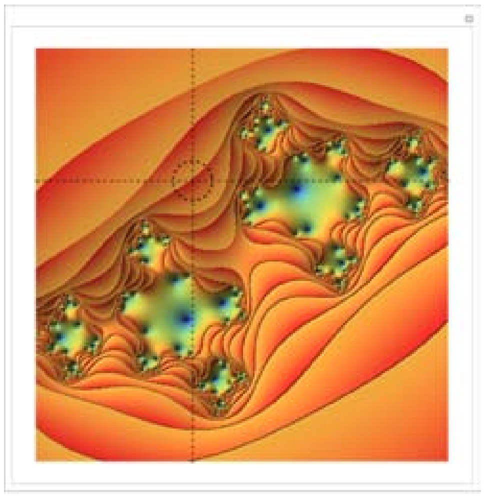
```
JuliaCalculate = CUDAFunctionLoad[code, "julia_kernel", {{_Real, "Output"},
     _Integer, _Integer, _Real, _Real}, {16, 16}, "Defines" →
     {"MAX_ITERATIONS" → 10, "ZOOM_LEVEL" → "0.0050", "BAILOUT" → "4.0"}];
```

The width and height are set and the output memory is allocated.

```
{width, height} = {512, 512};
jset = CUDAMemoryAllocate[Real, {height, width}];
```

This creates an interface using `Manipulate` and `ReliefPlot` where you can adjust the value of the constant *c* interactively.

```
Manipulate[
 JuliaCalculate[jset, width, height, c[[1]], c[[2]], {width, height}];
 ReliefPlot[Reverse@CUDAMemoryGet[jset], ColorFunction → "Rainbow",
  DataRange → {{-2.0, 2.0}, {-2.0, 2.0}}, ImageSize → 512,
  Frame → None, Epilog → {Opacity[.5], Dashed, Thick,
    Line[{{{c[[1]], -2}, {c[[1]], 2}}, {{-2, c[[2]]}, {2, c[[2]]}}}]}],
 {{c, {0, 1}}, {-2, -2}, {2, 2}, Locator, Appearance →
   Graphics[{Thick, Dashed, Opacity[.75], Circle[]}, ImageSize → 50]}]
```



Interactive computation and rendering of a Julia set.

Brownian Motion

Brownian motion is a very important concept in many scientific fields. It is used in computational chemistry, physics, and finance. In the following code, we use the CURAND kernel library to generate random numbers for computing sample paths of Brownian motion.

```
code = "
#include \"curand_kernel.h\"
extern \"C\" __global__ void
    brownianMotion(Real_t *out, mint pathLen, mint pathN) {
    curandState rngState;
    Real_t sum = 0;
    int index = threadIdx.x + blockIdx.x*blockDim.x;
    curand_init(1234, index, 0, &rngState);
    if (index < pathN) {
        out[index] = sum;
        for (int ii = 1; ii < pathLen; ii++) {
            sum += curand_normal(&rngState);
            out[ii*pathN + index] = sum;
        }
    }
}";
```

The following loads the above CUDA code into *Mathematica*.

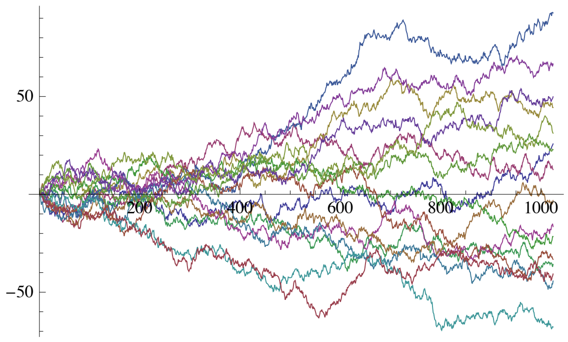
```
cudaBM = CUDAFunctionLoad[code, "brownianMotion",
   {{_Real, "Output"}, _Integer, _Integer}, 64, "UnmangleCode" → False];
```

The following sets the function parameters. We use a low path length and path number to make it easy to see the motion path.

```
pathLen = 1024;
pathN = 16;
out = ConstantArray[0, {pathLen, pathN}];
```

The following visualizes the result.

```
res = Transpose[First[cudaBM[out, pathLen, pathN, pathN]]];
ListLinePlot[res]
```



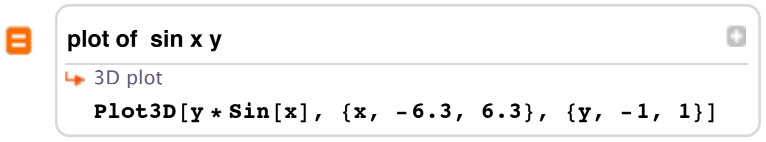
The possibilities are open for more complicated and broader applications of the GPU capabilities in *Mathematica*. And, with little effort, programs can be written so that they execute on either the CPU or GPU depending on the detected hardware.

The *Mathematica* Advantage

If you were to combine the performance computing aspects in *Mathematica* with the following *Mathematica* features, you could develop non-trivial heterogeneous programs intuitively.

Free-Form Linguistic Input

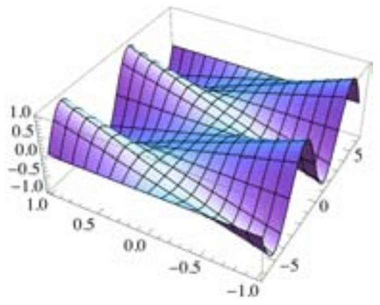
Mathematica is unique in providing an avenue for users to write programs in plain English. *Mathematica* uses Wolfram|Alpha to interpret the result, giving the output and the interpreted *Mathematica* program to get the output.



plot of sin x y

3D plot

```
Plot3D[y * Sin[x], {x, -6.3, 6.3}, {y, -1, 1}]
```

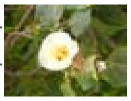


Simple Interface Creation

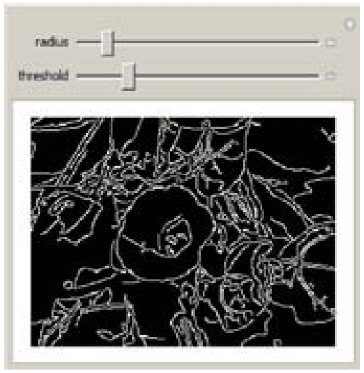
Mathematica makes it simple to create interactive user interfaces. The interfaces can be used to experiment with parameter values, as teaching modules, or deployed using Wolfram's Computable Document Format™.

The following creates an interface that allows users to adjust the radius and threshold parameters for the Canny edge detector.

```
Manipulate[EdgeDetect[
```



```
, r, t],
 {{r, 2, "radius"}, 1, 10}, {{t, .1, "threshold"}, 0, .5}]
```

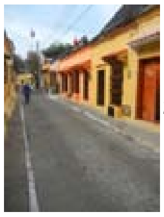


Broad Field Coverage

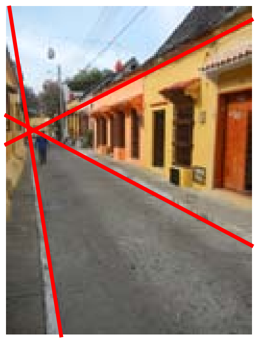
By using both the CPU and the GPU, and making them available to the user, *Mathematica* embodies the heterogeneous message. Users have written code that uses both the CPU and GPU concurrently on multiple machines to solve tasks in computer vision, medical imaging, mathematics, and physics.

Since *Mathematica* has broad field coverage, a reference implementation is likely to exist. This makes benchmarking and testing simple. This, for example, finds all lines in an input image.

```
img =
```



```
;

lines = ImageLines[EdgeDetect[img], .28, .06];
Show[img, Graphics[{Thick, Red, Line /@ lines}]]
```

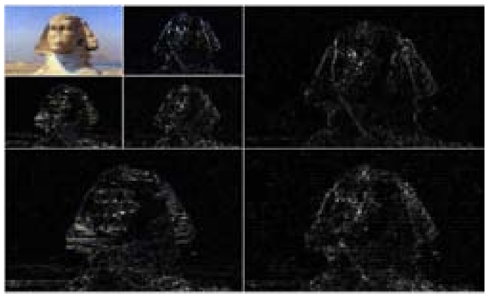


Here is another example that computes the discrete wavelet transform of an image.

`dwd = DiscreteWaveletTransform[` `, Automatic, 2];`

We can plot the wavelet decomposition as an image pyramid.

```
WaveletImagePlot[dwd]
```



Import/Export

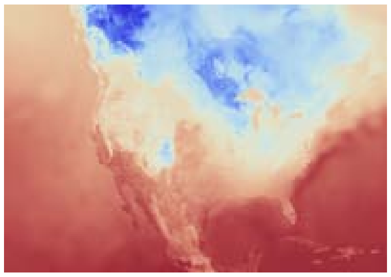
Mathematica has extensive support for importing and exporting data from hundreds of formats. These formats include PNG and JPEG for images, LaTeX and EPS for typesetting, and XLS and CSV for spreadsheet data.

For example, the following imports the dataset from a GRIB file. This file format is common in meteorology to store historical and forecast weather data.

```
data =
  Import["ExampleData/temperature.grb", {"Datasets", "Temperature", 1}];
```

This renders the dataset as an image.

```
Colorize[ImageAdjust[Image[Reverse[data]]],
 ColorFunction → "ThermometerColors"]
```



C Code Generation

Mathematica 8 introduces the ability to export expressions written using `Compile` to a C file. The C file can then be compiled and run either as a *Mathematica* command (for native speed), or be integrated with an external application using the Wolfram Runtime Library.

LibraryLink

LibraryLink allows you to load C functions as *Mathematica* functions. It is similar in purpose to *MathLink*®, but by running in the same process as the *Mathematica* kernel, it avoids the memory transfer cost associated with *MathLink*. This loads a C function from a library; the function adds one to a given integer.

```
addOne = LibraryFunctionLoad["demo", "demo_I_I", {Integer}, Integer]
```

```
LibraryFunction[<>, demo_I_I, {Integer}, Integer]
```

The library function is run with the same syntax as any other function.

```
addOne[3]
```

```
4
```

CUDALink and *OpenCLLink* are written using *LibraryLink* and thus are prime examples of *LibraryLink's* capabilities.

Symbolic C Code

Using *Mathematica*'s symbolic capabilities, users can generate C programs within *Mathematica*. The example presented here creates macros for common math constants and manipulates the expression to convert the macros to constant declarations. To use *Mathematica*'s symbolic C code generation capabilities, you first need to import the SymbolicC package.

```
Needs["SymbolicC`"]
```

This gets all constants in the *Mathematica* system context and uses SymbolicC's `CDefine` to declare a C macro.

```
s = Map[CDefine[ToString[#], N[#]] &, Map[ToExpression,
   Select[Names["System`*"], MemberQ[Attributes[#], Constant] &]]]
```

```
{CDefine[Catalan, 0.915966],
 CDefine[Degree, 0.0174533], CDefine[E, 2.71828],
 CDefine[EulerGamma, 0.577216], CDefine[Glaisher, 1.28243],
 CDefine[GoldenRatio, 1.61803], CDefine[Khinchin, 2.68545],
 CDefine[MachinePrecision, 15.9546], CDefine[Pi, 3.14159]}
```

The symbolic expression can be converted to a C string using the `ToCCodeString` function.

```
ToCCodeString[s]
```

```
#define Catalan  0.915965594177219
#define Degree  0.017453292519943295
#define E  2.718281828459045
#define EulerGamma  0.5772156649015329
#define Glaisher  1.2824271291006226
#define GoldenRatio  1.618033988749895
#define Khinchin  2.6854520010653062
#define MachinePrecision  15.954589770191003
#define Pi  3.141592653589793
```

By representing the C program symbolically, users can manipulate it using standard *Mathematica* techniques. Here, we convert all the macros to constant values.

```
s = ReplaceAll[s,
  CDefine[name_, val_] → CDeclare[{"const", "double"}, CAssign[name, val]]]
```

```
{CDeclare[{const, double}, CAssign[Catalan, 0.915966]],
 CDeclare[{const, double}, CAssign[Degree, 0.0174533]],
 CDeclare[{const, double}, CAssign[E, 2.71828]],
 CDeclare[{const, double}, CAssign[EulerGamma, 0.577216]],
 CDeclare[{const, double}, CAssign[Glaisher, 1.28243]],
 CDeclare[{const, double}, CAssign[GoldenRatio, 1.61803]],
 CDeclare[{const, double}, CAssign[Khinchin, 2.68545]],
 CDeclare[{const, double}, CAssign[MachinePrecision, 15.9546]],
 CDeclare[{const, double}, CAssign[Pi, 3.14159]]}
```

Again, the code can be converted to a C string using `ToCCodeString`.

```
ToCCodeString[s]
```

```
const double Catalan = 0.915965594177219;
const double Degree = 0.017453292519943295;
const double E = 2.718281828459045;
const double EulerGamma = 0.5772156649015329;
const double Glaisher = 1.2824271291006226;
const double GoldenRatio = 1.618033988749895;
const double Khinchin = 2.6854520010653062;
const double MachinePrecision = 15.954589770191003;
const double Pi = 3.141592653589793;
```

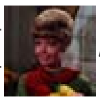
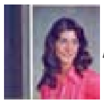
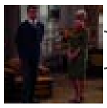
Using *Mathematica*'s symbolic code generation tools, you can easily write domain-specific languages that facilitate meta-programming—programs that write other programs.

Scalability

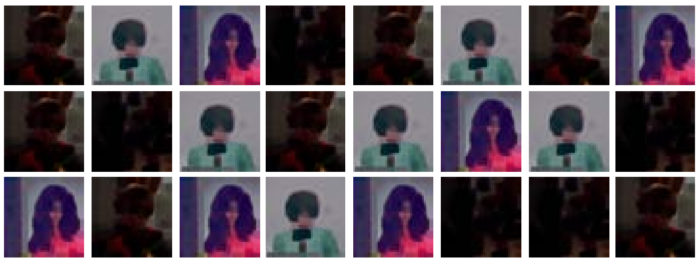
Mathematica programs from low-end netbooks to high-end workstations and clusters. Through our support of all GPU cards and automatic floating point precision detection, *Mathematica* facilitates scalable GPU programming.

Multi-GPU programming, for example, is as simple as wrapping `Parallelize` around a GPU function. The following performs an image morphological operation on the GPU using all four GPU cards installed on a system.

```
GraphicsGrid[Partition[Parallelize[

 Table[CUDAErosion[img, 5, "Device" → $KernelID], {img,

  Flatten[Permutations[{
```

,
,
,


```
}, 2]]}]], 8]]
```



Wolfram Research also has many technological offerings that make scaling upward and downward possible. The *Mathematica* licensing is also adaptive, allowing users to choose the most convenient cost-effective plan for their needs.

Summary

Mathematica provides several key built-in technologies that allow for easy transitioning to using heterogeneous-based computing. As proof of the simplicity, in the past few pages we wrote a dozen heterogeneous programs in diverse fields that would have been difficult to do in any other system.

Mathematica's advantage lies in being able to provide all these features built into the product, having them be portable across operating systems, providing an intuitive interface for their use through careful functionality design, and making them scalable to low-end and high-end systems.

Pricing and Licensing Information

Wolfram Research offers many flexible licensing options for both organizations and individuals. You can choose a convenient, cost-effective plan for your workgroup, department, directorate, university, or just yourself, including network licensing for groups.

Visit us online for more information:
www.wolfram.com/mathematica-purchase

Recommended Next Steps

Try *Mathematica* for free:
www.wolfram.com/mathematica/trial

Schedule a technical demo:
www.wolfram.com/mathematica-demo

Learn more about OpenCL programming in *Mathematica*:

U.S. and Canada:
1-800-WOLFRAM (965-3726)
info@wolfram.com

Europe:
+44-(0)1993-883400
info@wolfram.co.uk

Outside U.S. and Canada (except Europe and Asia):
+1-217-398-0700
info@wolfram.com

Asia:
+81-(0)3-3518-2880
info@wolfram.co.jp