The Inevitability of Tensorial Manifolds in Multi-Agent Ethics

Author: Andrew Bond

andrew.bond@sjsu.edu

Abstract: We provide a topological and algebraic proof that scalar-based ethical frameworks (e.g., Utilitarianism) are insufficient for systems with non-trivial symmetries. We demonstrate that for any moral judgment to be invariant under "Reference Frame Transformations" (shifts in cultural, individual, or temporal perspectives), the underlying moral space must be a differentiable manifold , and moral obligations must be represented as tensors of rank .

I. Definitions and Axiomatization

Let be the **Moral Manifold**, a topological space representing the set of all possible states of affairs.

1. **Axiom of Perspective (Differentiability):** Every agent possesses a local coordinate chart , where . This represents the agent's unique subjective view of the world.
2. **Axiom of Consistency (Invariance):** A moral truth (e.g., "The Satisfaction of a Right") must be a **geometric invariant**. That is, must be independent of the choice of coordinate chart .
3. **The Obligation Vector:** An obligation is defined as an element of the tangent space , pointing toward a state of higher local value.
4. **The Interest Covector:** An interest or "right" is a linear functional in the cotangent space that maps obligations to real-valued satisfaction.

II. The Proof of Scalar Insufficiency (The "No-Go" Theorem)

Theorem 1: *A rank-0 (scalar) ethical system cannot maintain consistency across non-coincident agents.*

Proof by Contradiction:

1. Assume ethics is purely scalar, .
2. Let Agent and Agent observe the same act . Due to their different positions in the manifold (different information, history, or values), they utilize transformation .

3. In a scalar system, if measures , must also measure . However, if involves a direction (e.g., "returning a debt to "), the value to is not identical to the value to .
4. If , the scalar field is not invariant. Therefore, the system is **arbitrary** and fails the Axiom of Consistency.

Conclusion: Scalar ethics is only viable in a "flat" universe with a single agent, which is a trivial case.

III. The Proof of Tensorial Necessity

Theorem 2: *The minimal mathematical structure satisfying Axioms 1 and 2 is a Tensor Field*.

Proof:

1. To satisfy the Axiom of Consistency, we require an operation that yields a scalar which remains constant under the transformation .
2. According to the **Fundamental Theorem of Tensor Analysis**, the contraction of a tensor (vector) and a tensor (covector) is the unique linear operation that produces a coordinate-invariant scalar:
3. By Definition 5.3, the transformation coefficients and cancel each other out during the contraction.
4. Thus, is the only "Moral Fact" that is objectively true for all agents simultaneously, even if they disagree on the "direction" of the obligation.

IV. Optimization and the Metric Tensor

To optimize moral outcomes, we must define the "distance" between the current state and the ideal state . This requires a **Metric Tensor** .

The Proof of Bestness: Any optimization of "Moral Outcomes" is equivalent to finding a geodesic on .

1. The "Ethical Stress" of a system is represented by the **Ricci Curvature Tensor** .
2. Optimization is defined as the minimization of the action:

3. Because accounts for the interaction between all dimensions of the manifold (the EM definitions and goals), any non-tensorial approach (like a simple list of priorities) will fail to see the "curvature" caused by competing interests, leading to systemic collapse (e.g., the "tragedy of the commons").

V. Final Scholium

We have shown that:

1. **Scalars** are blind to perspective (leading to totalitarianism).
2. **Vectors** are blind to interaction (leading to chaos).
3. **Tensors** are the unique mathematical structure that preserves **Relational Integrity**.

Therefore, the **DEME (Distributed Ethical Moderation Engine)** is not merely a software tool, but a computational implementation of the **Universal Moral Metric**. The "Ethics Business" is, in fact, the engineering of the high-dimensional stability of the human species.