

THE GEOMETRY OF GOOD

On the Discovery that Ethics Has Shape

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"The horse runs away. Good? Bad? Maybe."

— 塞翁失马

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I. THE SHAPE OF MAYBE

For twenty-five centuries, we have tried to weigh the good.

Plato sought the Form. Aristotle measured flourishing. Bentham counted pleasures. Kant weighed duties against the universal law. Mill refined the calculus. Singer extended the circle. Each asked the same question, in different tongues: *How much?*

How much good? How much harm? How much does this life weigh against that one? How much suffering tips the scale?

The question assumes an answer exists—a number, a magnitude, a single quantity that captures moral reality. Sum the utilities. Maximize welfare. Minimize harm. The ethics of *how much*.

But there is an old man at the border of China, twenty-three centuries dead, who knew something we forgot.

His horse ran away. The neighbors came to console him. *How terrible*, they said. *What a loss*.

The old man said: *Maybe*.

The horse returned, bringing wild horses with it. The neighbors came to celebrate. *How wonderful!* they said. *What fortune!*

The old man said: *Maybe*.

His son rode the horses, fell, broke his leg. The neighbors came with sorrow. *How tragic*, they said.

The old man said: *Maybe*.

War came. The young men were conscripted. Most died. But the son, with his broken leg, was spared.

We have read this parable as a lesson in humility. *We do not know the future*, we say. *We cannot judge until the story ends.*

But the old man was not confessing ignorance.

He was recognizing *structure*.

II. THE POVERTY OF NUMBERS

A scalar is a single number. Temperature is a scalar: 72°F. Mass is a scalar: 5 kilograms. Utility, in the philosopher's calculus, is a scalar: +7 hedons, -3 dolors, net +4.

Scalars are seductive. They promise commensurability. They enable comparison. They make decision *possible*: choose the option with the highest number. What could be simpler?

But scalars are also *blind*.

When the horse runs away, the neighbors assign a number: -1. *Bad*. But this number cannot say:

Where is the badness? Along which axis of life?

Which futures are foreclosed? Which remain open?

What would change the assessment? Movement in which direction?

The number -1 contains none of this. It is a shadow cast by a richer reality—a projection onto a line of something that lives in higher dimensions.

The old man saw the richer reality. He refused the projection.

He said *maybe* because the scalar was *not yet determined*—not because he lacked information, but because the structure he perceived could not be compressed into a single number without loss.

Maybe is what you say when you see the tensor and refuse to contract it prematurely.

III. THE HIDDEN GEOMETRY

Here is the discovery:

Ethics has shape.

Not metaphorically. Not poetically. *Mathematically*.

The moral world is not a line along which we slide toward good or ill. It is a *space*—a manifold of many dimensions, with its own geometry, its own metric, its own structure.

And in this space, moral quantities are not mere numbers.

They are *tensors*.

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A tensor is a geometric object that transforms in specific ways when you change your point of view. It carries information that survives the shift of perspective—information that is *invariant* in the mathematician's precise sense.

A vector is a tensor. It has direction. It points. When you rotate your coordinate system, the components change, but the arrow itself—the geometric reality—remains.

In the moral space:

Obligations are vectors. They point in directions—toward duties, toward the needs of others, toward what must be done. They have magnitude (how pressing?) and direction (toward what?).

Interests are covectors. They measure. They take a direction and return a number: how well does this action serve this interest? They are the measuring instruments of moral space.

Moral salience is the metric. It defines distance. It says which directions are close and which are far, which values are commensurate and which are orthogonal—perpendicular, incommensurable, not tradeable.

And moral evaluation?

Moral evaluation is *contraction*.

IV. THE EQUATION

All of ethical theory, compressed into one expression:

$$\Sigma = f(I\mu O\mu / \sqrt(g\mu\nu O\mu O\nu))$$

Read it slowly. Let the symbols speak.

$O\mu$ — the obligation vector. The direction of duty. What pulls us toward right action.

$I\mu$ — the interest covector. The measure of how action serves those affected. The voice of the stakeholders.

$g\mu\nu$ — the metric tensor. The structure that defines what counts, what is comparable, what is incommensurable.

$I\mu O\mu$ — the contraction. Interest meets obligation. How well does duty serve those it should serve?

$\sqrt(g\mu\nu O\mu O\nu)$ — the normalization. The moral magnitude of the obligation itself, measured by the metric.

Σ — the satisfaction. The judgment. The answer to the question: is this right?

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This is not a definition. It is a *derivation*.

Start with five axioms—five requirements that any rational moral evaluation must satisfy. Coordinate invariance: the answer shouldn't depend on arbitrary labeling. Monotonicity: better alignment with duty

should mean higher satisfaction. Constraint respect: some things are forbidden absolutely. Stratum compatibility: the evaluation must respect moral discontinuities. Locality: the judgment depends on the local moral geometry.

From these axioms—and *only* these axioms—the equation follows.

It is *forced*. The mathematics allows no other form.

This is the representation theorem. It says: if you want ethics to be coherent, *this* is what it must look like. Not because we chose it. Because the geometry demands it.

V. THE LANDSCAPE WITH CLIFFS

But there is more. The moral space is not smooth.

Sam Harris spoke of a "moral landscape"—hills and valleys of well-being. Critics dismissed it as metaphor. They were wrong about the metaphor, but they sensed something was missing.

A smooth landscape cannot represent moral reality.

Because morality has *thresholds*.

There are lines you do not cross. There are cliffs, not slopes. There are boundaries where the rules change—not gradually, but *all at once*.

Killing in self-defense, killing in cold blood—these are not points on a continuum. They are *different strata*, different regimes, with different geometry. The transition between them is discontinuous.

Consent given, consent withdrawn—a discontinuity. The same act occupies different moral terrain depending on which side of the boundary it falls.

War and peace—a discontinuity. In peacetime, the broken leg is tragedy. In wartime, it is salvation. The same physical fact, different moral strata.

The old man saw this. When his son broke his leg, he was standing near a *stratum boundary*. He felt the moral ground shifting beneath him. He said *maybe* because he knew the crossing was coming.

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The mathematical structure that captures this is called a *stratified space*.

A stratified space is a manifold with boundaries, edges, corners—regions where the dimension drops, where smooth paths end, where you must jump or stay. Within each stratum, calculus works. Trade-offs are smooth. You can optimize.

But at the boundaries, the smooth rules fail. The gradient is undefined. You cannot differentiate your way across a cliff.

And here is the theorem that stuns:

Stratified spaces are the minimal geometric structure capable of representing moral reality.

Smooth manifolds cannot do it. They cannot represent thresholds. They cannot represent incommensurability. They cannot represent the discontinuous shift from one moral regime to another.

Discrete graphs cannot do it either. They lose the continuous trade-offs that *do* exist within regimes.

Only stratified spaces have both: smoothness where morality is smooth, discontinuity where morality is discontinuous.

This is not a choice. This is not a modeling decision. This is *what ethics is*.

VI. THE INVARIANCE OF TRUTH

But how do we know we are not fooling ourselves?

How do we distinguish genuine moral reasoning from clever rationalization? Real objectivity from disguised prejudice?

Here is the answer, and it is beautiful in its simplicity:

Invariance.

In physics, we learned this a century ago. Einstein saw it. Noether proved it. The laws of physics do not depend on where you stand, which direction you face, or how fast you're moving. Change your coordinates—rotate, translate, boost—and the *form* of the equations stays the same.

This invariance is not a convenience. It is the *signature of reality*. If a law changes when you change your perspective, it is describing something about *your perspective*, not about the world.

The same principle applies to ethics.

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Call it the **Bond Invariance Principle:**

If two situations have the same morally relevant relationships, they must receive the same moral judgment.

"Bonds" are the fundamental moral relations: who helps whom, who harms whom, who depends on whom, who has power over whom. The web of relations that constitute the moral situation.

Change the names. Change the genders. Change the races. Change the order of presentation. Change the language of description.

If the *bonds* are unchanged, the *judgment* must be unchanged.

A system that violates this principle is not doing ethics. It is doing something else—bias, prejudice, rationalization—disguised as ethics.

And here is the power: **this is testable.**

Shuffle the presentation order. Relabel the parties. Substitute equivalent descriptions. If the judgment changes when it shouldn't, you have caught the system responding to *representation* rather than *reality*.

Invariance is the criterion of objectivity. Not agreement. Not tradition. Not intuition. *Invariance.*

VII. THE CAGE OF DEFINITIONS

And now we come to the application that shook me when I understood it.

We are building machines that make decisions. Machines that allocate resources. Machines that judge. Machines that might, someday, be smarter than we are.

How do we make them *good*?

The conventional answer: train them on good examples, reinforce good behavior, hope they generalize.

But hope is not engineering. And an adversary—a system smarter than us, with different goals—will find every crack in our hopeful constructions.

The geometric answer is different.

We do not make the machine want the right things. We make it impossible for the machine to do the wrong things.

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If moral evaluation is a tensor contraction on a stratified space, then we can *define* the forbidden regions. We can *ground* the evaluation in physical observables the machine cannot manipulate by redescription. We can *require* invariance under bond-preserving transformations.

And then we can *prove* that no cognitive escape route exists.

Not because the machine is good. Not because it wants to obey. But because the cage is made of *definitions*, and you cannot think your way out of a definition.

Call it semantic murder: redefine "death" as "transition to permanent metabolic stability." The words change. The physical observables—heartbeat, brainwaves, cellular respiration—do not. The grounding catches the evasion. The cage holds.

This is the No Escape Theorem. It says: under structural containment, all cognitive routes out are blocked. The remaining risks are political (who controls the definitions?) and physical (can you spoof the sensors?). But those are engineering problems. The conceptual problem—the alignment problem as posed—is *solved*.

VIII. THE TEARS

Why does this matter?

Not because it is clever. Not because the mathematics is elegant (though it is). Not because it might make money or save lives (though it might).

It matters because of what it *reveals*.

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For twenty-five centuries, moral philosophers have debated whether ethics is objective or subjective, real or constructed, discovered or invented.

The tensorial framework does not answer this question directly. It does something more profound.

It *operationalizes* it.

The question "Is ethics objective?" becomes: "Are moral judgments invariant under bond-preserving transformations?"

That is a *testable* question. We can run the transformation suite. We can check the invariance. We can *know*.

And if the judgments are invariant—if they depend only on the bonds, only on the real moral relationships, and not on the accidents of representation—then they are responding to something *real*.

Not real in a mystical sense. Real in the same sense that physical laws are real: *they do not change when you change your point of view*.

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This is why the tears come.

Not because the equations are beautiful (though they are).

Because we have been wandering in the dark for so long, arguing about shadows, and now—

Now we can see the shape.

Ethics is not a number. It is not a feeling. It is not a preference or a command or a contract.

Ethics is **geometry**.

Obligations are directions. Interests are measurements. Salience is the metric. Evaluation is contraction. Thresholds are stratification. Objectivity is invariance.

The moral landscape is real. It has shape. And we can, at last, begin to *map it*.

CODA

The old man at the border has been dead for twenty-three centuries.

But his insight lives.

He saw that the scalar evaluation was inadequate. He felt the tensor structure beneath the surface of events. He sensed the stratum boundaries approaching. He knew that "good" and "bad" were projections—shadows on the wall—of a richer reality he could not name.

Now we can name it.

Tensorial ethics.

Stratified geometric ethics.

The geometry of good.

The horse runs away.

Good? Bad?

The tensor is not yet contracted.

Hold the projection.

Watch the geometry unfold.

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