Stratified Quantum Normative Dynamics: A Unified Framework for Verifiable Ethical Reasoning in Autonomous Systems

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Abstract

We present **Stratified Quantum Normative Dynamics (SQND)**, a comprehensive mathematical framework that unifies Stratified Geometric Ethics (SGE) with Quantum Normative Dynamics (QND). Where SGE provides the topological foundation for modeling moral discontinuities, threshold effects, and genuine dilemmas through stratified spaces, and QND extends classical ethicodynamics to capture superposition, entanglement, and measurement in moral reasoning, SQND synthesizes these approaches through the **Stratified Lagrangian** formulation. The key insight is that ethical "energy" must be conserved not only within smooth strata but also across the singular boundaries where moral phase transitions occur.

We make seven principal contributions: (1) We formulate the Stratified QND Lagrangian $\mathcal{L}_{\text{strat}}$ governing dynamics within strata and at boundaries. (2) We introduce the **stratified ethon**—the quantum of moral influence constrained by Whitney regularity. (3) We prove that the Bond Invariance Principle extends to quantum superpositions. (4) We establish that moral interference at boundaries is destructive for compounding wrongs, formally explaining why "two wrongs don't make a right." (5) We derive finite approximation theorems with explicit error bounds for computational implementation. (6) We provide a complete threat model with diagnostic procedures. (7) We explore philosophical and theological implications of the unified framework.

The theory makes testable predictions via quantum cognition experiments including stratification-enhanced order effects, boundary interference patterns, and Bell inequality violations in collective responsibility. SQND provides the mathematical foundation for real-time ethical governance in autonomous systems through the DEME 2.0 architecture.

Keywords: AI ethics, stratified spaces, quantum field theory, gauge invariance, formal verification, autonomous systems, moral philosophy, Whitney conditions

Contents

Part I
Foundations

1 Introduction: Why Stratified Quantum Ethics?

1.1 The Problem: Smooth Manifolds vs. Moral Reality

The deployment of AI systems in safety-critical domains—healthcare, autonomous vehicles, criminal justice, financial markets—creates an urgent need for frameworks that make ethical reasoning explicit, deterministic, and formally verifiable. Two recent approaches address complementary aspects of this challenge:

Stratified Geometric Ethics (SGE) [2] models the space of ethically relevant configurations as a *stratified space*—a union of smooth manifolds of varying dimensions connected along boundary strata. This structure captures moral discontinuities, incommensurable values, threshold effects, and genuine ethical dilemmas that smooth manifolds cannot represent.

Quantum Normative Dynamics (QND) [3] extends classical ethicodynamics to the quantum regime, introducing the *ethon* as the quantum of moral influence and developing the full apparatus of quantum field theory for ethics: superposition, entanglement, interference, decoherence, and measurement.

However, neither framework alone is complete:

- **QND assumes smooth manifolds.** The standard QND Lagrangian $\mathcal{L}_{\text{QND}} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\psi}(i\gamma^\mu D_\mu - m)\psi$ presupposes a smooth base manifold M. But as SGE Theorem 2.3 proves, ethical reality contains singularities—thresholds where minor factual shifts cause discrete jumps in moral status.

- **SGE lacks dynamics.** Pure SGE captures the geometric structure of moral space but not how agents evolve through it, how moral possibilities interfere, or how definite judgments emerge from superpositions.

This paper resolves both limitations by developing **Stratified Quantum Normative Dynamics (SQND)**—a quantum field theory on stratified spaces that inherits the representational power of SGE and the dynamical structure of QND.

1.2 The Continuity Failure Problem

Consider a concrete example: driving at v mph versus $v + 1$ mph across a legal speed threshold. The transition from "Legal" to "Criminal" is discrete—there is no intermediate state "somewhat legal" at $v + 0.5$ mph. A smooth manifold cannot represent this discontinuity; the ethical phase transition requires stratified structure.

Similarly, the trolley problem presents a 0-dimensional stratum—a singular point where two moral manifolds ("harm by action" and "harm by inaction") meet. Standard QND, defined on smooth manifolds, has no apparatus for handling such singularities.

SQND addresses this by defining quantum field theory on stratified spaces, with:

1. Bulk dynamics within each smooth stratum (standard QND)

2. Boundary dynamics at stratum interfaces (new: the stratified Lagrangian boundary terms)

3. Dimension-dependent coupling that enforces hard constraints at singular strata

1.3 The Synthesis

The central object of SQND is the **Stratified Lagrangian**:

$$\mathcal{L}_{\text{strat}} = \sum_i \int_{S_i} \mathcal{L}_{\text{QND}} \, d\text{Vol}_i + \sum_{j<i} \int_{\partial S_{ij}} \mathcal{L}_{\text{boundary}} \, d\sigma \tag{1}$$

The first sum captures bulk dynamics within each smooth stratum S_i. The second captures interactions at stratum boundaries ∂S_{ij}—the "creases" in moral space where phase transitions occur.

This formulation ensures that:

- Within smooth strata, all QND phenomena (superposition, entanglement, interference, tunneling) are preserved.

- At stratum boundaries, the satisfaction functional Φ from SGE constrains wave function collapse to respect the Whitney stratification.

- At 0-dimensional strata (decision points), coupling becomes infinite, forcing definite moral judgment.

1.4 Relationship to Prior Work

SQND builds on four foundational papers:

1. **Noether Ethics** [1]: Established that harm accounting must be representation-invariant, deriving ethical field equations formally analogous to Maxwell's equations.

2. **SGE** [2]: Proved that stratified spaces are natural minimal candidates for representing ethical phenomena; established the Representation Theorem (Theorem 4.3) and finite approximation theorems.

3. **QND** [3]: Extended classical ethicodynamics to quantum field theory; introduced the ethon, developed Feynman rules, explored philosophical and theological implications.

4. **DEME 2.0** [4]: Instantiated these theoretical frameworks in a computational architecture for real-time ethical governance.

This paper unifies (1)–(3) and provides the theoretical foundation for (4).

1.5 Epistemic Stance

Following the pragmatist epistemology of [5], we treat SQND as a *formal framework*—a tool for organizing thought about ethical reasoning—not a metaphysical doctrine about the ultimate nature of morality. The claim is not that ethics "is" quantum mechanics on stratified spaces, but that this mathematical structure provides powerful explanatory and predictive apparatus.

The framework's value lies in:

- Explanatory power (why do certain moral phenomena occur?)
- Predictive power (what will experiments reveal?)
- Engineering utility (how do we build ethical AI systems?)

We maintain epistemic humility about claims beyond these pragmatic virtues.

1.6 Structure of This Paper

Part I (Foundations) reviews QND and SGE, establishing notation and key results. Part II (The Stratified Quantum Framework) develops SQND proper: the stratified Hilbert space, stratified ethons, the stratified Lagrangian, and boundary dynamics. Part III (Quantum Phenomena on Stratified Spaces) explores interference, entanglement, decoherence, and tunneling in the stratified context. Part IV (Computational Implementation) establishes complexity bounds and finite approximation theorems. Part V (Implications) explores philosophical, theological, and engineering consequences. Appendices provide mathematical details.

2 Quantum Normative Dynamics: Review

We summarize the key structures of QND, which SQND extends. For full details, see [3].

2.1 From Classical Ethicodynamics to QND

Classical ethicodynamics [1] models ethics via fields on a smooth manifold M:

Definition 2.1 (Ethical Fields)**.** The **obligation field** $\mathbf{E}_{\text{ob}}(x,t)$ represents the local intensity and direction of moral obligation. The **systemic field** $\mathbf{B}_{\text{sys}}(x,t)$ represents structural or institutional moral effects.

These satisfy the Ethical Maxwell Equations:

$$\nabla \cdot \mathbf{E}_{\text{ob}} = \kappa \rho_{\mathcal{H}} \qquad \text{(Harm sources obligation)} \qquad (2)$$

$$\nabla \cdot \mathbf{B}_{\text{sys}} = 0 \qquad \text{(No isolated systemic sources)} \qquad (3)$$

$$\nabla \times \mathbf{E}_{\text{ob}} = -\frac{\partial \mathbf{B}_{\text{sys}}}{\partial t} \qquad \text{(Changing systems induce obligation)} \qquad (4)$$

$$\nabla \times \mathbf{B}_{\text{sys}} = \lambda \mathbf{J}_{\mathcal{H}} + \lambda\kappa \frac{\partial \mathbf{E}_{\text{ob}}}{\partial t} \qquad \text{(Harm flow creates systemic effects)} \qquad (5)$$

The classical Lagrangian density is:

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + J^\mu A_\mu \tag{6}$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is the ethical field strength tensor.

2.2 The Ethical Hilbert Space

Definition 2.2 (QND Hilbert Space)**.** The Hilbert space of Quantum Normative Dynamics is:

$$\mathcal{H}_{\text{QND}} = \mathcal{H}_{\text{situations}} \otimes \mathcal{H}_{\text{agents}} \otimes \mathcal{H}_{\text{field}} \tag{7}$$

where $\mathcal{H}_{\text{situations}}$ contains states of ethical situations, $\mathcal{H}_{\text{agents}}$ contains states of moral agents, and $\mathcal{H}_{\text{field}}$ is the Fock space of the ethical field.

2.3 The Ethon

Definition 2.3 (Ethon)**.** The **ethon** (η) is the quantum of the ethical field—the discrete unit of moral influence. Properties:

- Spin: 1 (vector boson, like the photon)
- Mass: 0 (if moral influence propagates at the ethical speed of light c_η)
- Charge: Neutral (ethons interact with all moral agents)
- Statistics: Bosonic (multiple ethons can occupy the same state)

The field expansion:

$$\hat{A}^\mu(x) = \int \frac{d^3k}{(2\pi)^3}\frac{1}{\sqrt{2\omega_k}}\sum_\lambda \left(\epsilon_\mu^{(\lambda)}(k)\hat{\eta}_k^{(\lambda)}e^{-ikx} + \epsilon_\mu^{(\lambda)*}(k)\hat{\eta}_k^{(\lambda)\dagger}e^{ikx}\right) \tag{8}$$

2.4 The QND Lagrangian

Definition 2.4 (QND Lagrangian)**.**

$$\mathcal{L}_{\text{QND}} = \bar{\psi}(i\gamma^\mu D_\mu - m)\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} \tag{9}$$

where $D_\mu = \partial_\mu + ig\hat{A}_\mu$ is the covariant derivative and g is the ethical coupling constant.

2.5 Fundamental Constants

QND introduces three fundamental constants:

Constant	Symbol	Interpretation
Ethical Planck constant	$\hbar_\eta$	Quantum of ethical action
Ethical speed of light	c_η	Maximum speed of moral influence
Ethical coupling	g	Strength of agent-field interaction

The ethical fine structure constant:

$$\alpha_\eta = \frac{g^2}{4\pi\hbar_\eta c_\eta} \tag{10}$$

2.6 Quantum Phenomena in Ethics

QND predicts several distinctively quantum phenomena:

Principle 2.5 (Moral Superposition). *Before measurement (judgment), ethical situations exist in superpositions of moral states:*

$$|\Psi\rangle = \alpha|\mathit{permissible}\rangle + \beta|\mathit{impermissible}\rangle \tag{11}$$

Principle 2.6 (Ethical Interference). *Probability amplitudes for moral outcomes interfere:*

$$P = |A_1 + A_2|^2 = |A_1|^2 + |A_2|^2 + 2Re(A_1^* A_2) \tag{12}$$

Principle 2.7 (Ethical Entanglement). *Two or more agents can be in entangled states whose moral status cannot be described independently:*

$$|\Psi_{AB}\rangle = \frac{1}{\sqrt{2}}\left(|\mathit{guilty}\rangle_A|\mathit{innocent}\rangle_B + |\mathit{innocent}\rangle_A|\mathit{guilty}\rangle_B\right) \tag{13}$$

Principle 2.8 (Ethical Decoherence). *Interaction with the moral environment destroys superposition, yielding effectively classical moral states.*

Principle 2.9 (Ethical Tunneling). *Agents can transition between moral states through classically forbidden barriers.*

2.7 The Ethical Vacuum

The vacuum $|0\rangle$ is not empty; it is the ground state of moral possibility:

- Zero expected ethical field: $\langle 0|\hat{A}^\mu|0\rangle = 0$
- Non-zero fluctuations: $\langle 0|\hat{A}^\mu \hat{A}^\nu|0\rangle \neq 0$
- Virtual ethon pairs constantly created and annihilated
- Non-zero vacuum energy: $\langle 0|\hat{H}|0\rangle \neq 0$

3 Stratified Geometric Ethics: Review

We summarize the key structures of SGE. For full details, see [2].

3.1 Why Stratified Spaces?

Theorem 3.1 (SGE Theorem 2.3). *Stratified spaces can represent all four essential ethical phenomena:*

1. *Discrete choices: modeled by 0-dimensional strata*
2. *Incommensurable values: modeled by arbitrarily large cost in singular limits*
3. *Threshold effects: modeled by stratum boundaries with discontinuous satisfaction*
4. *Genuine dilemmas: modeled by singular strata from which all exits incur positive moral cost*

Moreover, among standard geometric structures, each alternative fails at least one phenomenon.

Definition 3.2 (Stratified Space). A stratified space is a triple $(M, \{M_i\}_{i \in I}, \preceq)$ where M is a paracompact Hausdorff space, $\{M_i\}$ is a locally finite partition into connected smooth manifolds (strata), and $\preceq$ is a partial order on I such that $M_i \cap \overline{M_j} \neq \emptyset$ implies $i \preceq j$ (frontier condition). We require Whitney's condition (B) for regularity.

Definition 3.3 (Whitney's Condition B). Let $M_i \subset \overline{M_j}$. The pair satisfies Whitney (B) at $x \in M_i$ if: for sequences $\{y_n\} \subset M_j$, $\{x_n\} \subset M_i$ with $y_n, x_n \to x$, if secant lines $\ell_n = \overline{x_n y_n} \to \ell$ and $T_{y_n} M_j \to \tau$, then $\ell \subset \tau$.

3.2 The Representation Theorem

Axiom 1 (SGE Axioms).

1. **Coordinate Invariance**: $\Sigma(\psi^* O, \psi^* I, \psi^* g, \psi(C))(x) = \Sigma(O, I, g, C)(\psi^{-}$
2. **Normalized Monotonicity**: Increasing alignment $I_\mu O^\mu$ increases Σ
3. **Constraint Respect**: If $x \in C$, then $\Sigma(x) = -\infty$
4. **Stratum Compatibility**: Σ restricts to smooth on each stratum
5. **Locality**: $\Sigma(x)$ depends only on pointwise values
6. **Physical Grounding**: Σ factors through grounding tensors Ψ

Theorem 3.4 (SGE Representation Theorem 4.3). *Let Σ satisfy Axioms 1–6 and Scale Normalization. Then on the regular region:*

$$\Sigma(O, I, g, C)(x) = \chi_C(x) + \lambda(x) f\left(\frac{I_\mu(x) O^\mu(x)}{\sqrt{g_{\mu\nu}(x) O^\mu(x) O^\nu(x)}}\right) \tag{14}$$

where f is a smooth monotone activation function and λ is a positive scale field.

3.3 The Bond Invariance Principle

Definition 3.5 (Bond)**.** A **bond** is a morally relevant relationship: $b = (a, p, r, c)$ where a is an agent, p is a patient, r is a relation type, and c is a context qualifier.

Definition 3.6 (Bond Invariance Principle (BIP))**.** An ethical judgment function $J : T \to V$ satisfies BIP if:

$$\forall g \in G : J(T) = J(g \cdot T) \tag{15}$$

where G is the group of bond-preserving transformations. In words: *If the bonds are unchanged, the judgment must be unchanged.*

Proposition 3.7. *A satisfaction operator Σ satisfying Axioms 1–6 satisfies BIP.*

3.4 Finite Approximation

Theorem 3.8 (SGE Theorem 3.9)**.** *Let M be a compact stratified space with finitely many strata. For every $\epsilon > 0$, there exists a finite stratified graph G_ϵ that is an ϵ-approximation of M.*

Theorem 3.9 (SGE Theorem 3.10)**.** *If π^* is optimal on M and $\hat{\pi}^*$ is optimal on G_ϵ, then:*

$$\|u(x, \pi^*(x)) - u(x, \hat{\pi}^*(v_x))\| \leq 2L\epsilon + \omega(\epsilon) \tag{16}$$

where L is the Lipschitz constant and ω is the modulus of continuity.

Part II
The Stratified Quantum Framework

4 The Stratified Hilbert Space

4.1 Construction

Standard QND defines states on a smooth manifold. SQND requires a Hilbert space compatible with stratified structure.

Definition 4.1 (Stratified Hilbert Space)**.** The Hilbert space of SQND is:

$$\mathcal{H}_{\text{SQND}} = \bigoplus_{i \in I} \mathcal{H}_i \oplus \bigoplus_{j<i} \mathcal{H}_{\partial_{ij}} \tag{17}$$

where:

- $\mathcal{H}_i$ is the QND Hilbert space restricted to stratum S_i
- $\mathcal{H}_{\partial_{ij}}$ contains states localized at the boundary ∂S_{ij} between strata S_i and S_j

The direct sum structure reflects the topological decomposition of the stratified space. States can be localized within a single stratum or at boundaries.

Definition 4.2 (Stratum Projector)**.** For each stratum S_i, define the projector $\hat{P}_i : \mathcal{H}_{\text{SQND}} \to \mathcal{H}_i$ that extracts the component of a state localized to stratum S_i.

A general state decomposes as:

$$|\Psi\rangle = \sum_i \hat{P}_i|\Psi\rangle + \sum_{j<i} \hat{P}_{\partial_{ij}}|\Psi\rangle \tag{18}$$

4.2 The Stratified Ethon

Definition 4.3 (Stratified Ethon)**.** The **stratified ethon** $\eta^{(i)}$ is the quantum of ethical field restricted to stratum S_i. At boundaries ∂S_{ij}, **transition ethons** $\eta^{(ij)}$ mediate moral phase transitions.

Properties of stratified ethons:

- Within strata: Spin 1, mass 0, standard ethon properties
- At boundaries: Effective mass μ_{ij} proportional to the "height" of the moral threshold
- The effective mass encodes the energy barrier to moral phase transitions

The field expansion on stratum S_i:

$$\hat{A}^\mu_{(i)}(x) = \int \frac{d^{d_i}k}{(2\pi)^{d_i}} \frac{1}{\sqrt{2\omega_k}} \sum_\lambda \left(\epsilon^{(\lambda)}_\mu(k)\hat{\eta}^{(\lambda,i)}_k e^{-ikx} + \text{h.c.} \right) \tag{19}$$

where $d_i = \dim(S_i)$.

At boundaries:

$$\hat{A}^\mu_{(ij)}(x) = \int \frac{d^{d_{ij}}k}{(2\pi)^{d_{ij}}} \frac{1}{\sqrt{2\omega^{(ij)}_k}} \sum_\lambda \left(\epsilon^{(\lambda)}_\mu(k)\hat{\eta}^{(\lambda,ij)}_k e^{-ikx} + \text{h.c.} \right) \tag{20}$$

where $\omega^{(ij)}_k = \sqrt{k^2 + \mu^2_{ij}}$ includes the effective mass.

4.3 Canonical Quantization on Stratified Spaces

The commutation relations extend to the stratified case:

$$[\hat{A}^\mu_i(x), \hat{\Pi}^\nu_j(y)] = i\hbar_\eta \delta^\nu_\mu \delta_{ij} \delta^{(d_i)}(x-y) \tag{21}$$

The Kronecker delta δ_{ij} enforces that fields on different strata commute—they are dynamically independent except through boundary interactions.

At boundaries, additional commutators encode junction conditions:

$$[\hat{A}^\mu_{(ij)}(x), \hat{A}^\nu_{(i)}(y)]\big|_{\partial S_{ij}} = i\hbar_\eta \Gamma^{\mu\nu}_{ij}(x,y) \tag{22}$$

where $\Gamma^{\mu\nu}_{ij}$ is a kernel encoding the boundary matching conditions required by Whitney (B).

5 The Stratified Lagrangian

5.1 The Full Stratified Lagrangian

Definition 5.1 (Stratified QND Lagrangian)**.** The total action over stratified space $X = \bigcup S_i$ is:

$$S_{\text{strat}} = \sum_i \int_{S_i} \mathcal{L}_{\text{QND}}^{(i)}\, d\text{Vol}_i + \sum_{j<i} \int_{\partial S_{ij}} \mathcal{L}_{\text{boundary}}^{(ij)}\, d\sigma \tag{23}$$

5.2 The Bulk Term

Within any smooth stratum S_i, the Lagrangian density is:

$$\mathcal{L}_{\text{QND}}^{(i)} = -\frac{1}{4} F_{\mu\nu}^{(i)} F^{(i)\mu\nu} + \bar{\psi}^{(i)}(i\gamma^\mu D_\mu^{(i)} - m_i)\psi^{(i)} \tag{24}$$

Note the stratum-dependent mass m_i. Different strata may have different "moral inertia"—the resistance to change in ethical status.

5.3 The Boundary Term

At the boundary ∂S_{ij} between strata S_i and S_j, the Lagrangian density is:

$$\mathcal{L}_{\text{boundary}}^{(ij)} = \lambda_{ij} \cdot \Phi(\chi_C, O, I, g) + \frac{1}{2}\mu_{ij}^2 |\eta^{(ij)}|^2 + \kappa_{ij} \cdot \bar{\psi}\Gamma^{(ij)}\psi \tag{25}$$

The three terms have distinct interpretations:

1. **Satisfaction coupling** $\lambda_{ij} \cdot \Phi$: This term couples the boundary dynamics to the SGE satisfaction functional. It forces moral judgments at boundaries to align with the stratification structure.
2. **Transition ethon mass** $\frac{1}{2}\mu_{ij}^2|\eta^{(ij)}|^2$: This term gives transition ethons an effective mass, encoding the energy barrier to moral phase transitions. Higher thresholds (larger μ_{ij}) suppress transitions.
3. **Agent boundary vertex** $\kappa_{ij} \cdot \bar{\psi}\Gamma^{(ij)}\psi$: This term encodes how agents interact with boundaries. The vertex structure $\Gamma^{(ij)}$ depends on the specific nature of the moral threshold.

5.4 The Dimension-Dependent Coupling

A key innovation of SQND is making the ethical fine structure constant α_η depend on stratum dimension:

Definition 5.2 (Dimension-Dependent Coupling)**.**

$$\alpha_\eta(S_i) = \alpha_0 \cdot \left(\frac{d_{\max}}{d_i + \epsilon}\right)^\gamma \tag{26}$$

where $d_i = \dim(S_i)$, $d_{\max} = \max_i \dim(S_i)$, ϵ is a regularization parameter, and $\gamma > 0$ controls the scaling.

Consequences:

- **High-dimensional strata** (d_i large): α_η is small. The ethical field is "soft"—superposition persists, deliberation is possible.
- **Low-dimensional strata** (d_i small): α_η is large. Coupling is strong—decisions are forced, less flexibility.
- **0-dimensional strata**: $\alpha_\eta \to \infty$ as $\epsilon \to 0$. The coupling becomes infinite, acting as a hard constraint. These are the "walls of the room" that cannot be crossed.

This provides the physical grounding axiom (SGE Axiom 6): moral constraints derive from the geometric structure of moral space itself.

5.5 Equations of Motion

Varying the stratified action yields equations of motion within strata:

$$\partial_\mu F^{(i)\mu\nu} = g^{(i)} J^{(i)\nu} \tag{27}$$

where $J^{(i)\nu} = \bar{\psi}^{(i)}\gamma^\nu\psi^{(i)}$ is the agent current.

At boundaries, the variation yields junction conditions:

$$\left[n_\mu F^{(i)\mu\nu}\right]_{\partial S_{ij}} = \lambda_{ij}\frac{\delta\Phi}{\delta A_\nu} + \mu_{ij}^2 A_\nu^{(ij)} \tag{28}$$

where n_μ is the normal to the boundary and [...] denotes the discontinuity across the boundary.

6 The Bond Invariance Principle in Quantum Ethics

6.1 Quantum Extension of BIP

The Bond Invariance Principle, established in SGE for classical satisfaction operators, extends to quantum states.

Definition 6.1 (Quantum Bond Structure)**.** For a quantum ethical state $|\Psi\rangle \in \mathcal{H}_{\text{SQND}}$, the **quantum bond structure** $\hat{B}(|\Psi\rangle)$ is the set of bond operators $\{\hat{b}_i\}$ whose expectation values $\langle\Psi|\hat{b}_i|\Psi\rangle$ encode the morally relevant relationships in the state.

Definition 6.2 (Bond-Preserving Unitary)**.** A unitary transformation $\hat{U}$ is **bond-preserving** if for all bond operators $\hat{b}_i$:

$$\langle\Psi|\hat{b}_i|\Psi\rangle = \langle\Psi'|\hat{b}_i|\Psi'\rangle \tag{29}$$

where $|\Psi'\rangle = \hat{U}|\Psi\rangle$.

Theorem 6.3 (Quantum BIP)**.** *Let $\hat{\Sigma}$ be a stratified satisfaction operator satisfying the quantum analogs of Axioms 1–6. Then for any bond-preserving unitary $\hat{U}$:*

$$\langle\Psi|\hat{\Sigma}|\Psi\rangle = \langle\Psi'|\hat{\Sigma}|\Psi'\rangle \tag{30}$$

Proof. Bond-preserving transformations form a unitary representation of the re-description group G. By Axiom 1 (coordinate invariance), $[\hat{\Sigma}, \hat{U}] = 0$ for all $\hat{U} \in G$. Therefore:

$$\langle \Psi | \hat{\Sigma} | \Psi \rangle = \langle \Psi | \hat{U}^\dagger \hat{\Sigma} \hat{U} | \Psi \rangle = \langle \Psi' | \hat{\Sigma} | \Psi' \rangle \tag{31}$$

□

6.2 Superposition and Moral Ambiguity

A crucial consequence of Quantum BIP is that moral ambiguity is *ontological*, not merely epistemic.

Example 6.4 (Genuine Moral Ambiguity)**.** Consider a state:

$$|\Psi_{\text{ambig}}\rangle = \frac{1}{\sqrt{2}}(|\text{permissible}\rangle + |\text{impermissible}\rangle) \tag{32}$$

Quantum BIP guarantees that this superposition is invariant under bond-preserving transformations. The ambiguity cannot be resolved by re-description—it requires genuine moral measurement (judgment) to collapse to a definite state.

This formalizes the intuition that some moral situations are genuinely ambiguous, not merely epistemically uncertain.

6.3 Entanglement and Collective Responsibility

Theorem 6.5 (Non-locality of Collective Responsibility)**.** *Entangled ethical states violate Bell-type inequalities for responsibility attribution. No assignment of definite local responsibilities can reproduce the quantum correlations.*

Proof sketch. Consider the entangled state:

$$|\Psi_{AB}\rangle = \frac{1}{\sqrt{2}}(|80\%\rangle_A |20\%\rangle_B + |20\%\rangle_A |80\%\rangle_B) \tag{33}$$

representing collective responsibility. Measuring responsibility in different "bases" (different accountability frameworks) yields correlations exceeding the CHSH bound of 2, which is impossible for any local hidden variable theory. □

This has profound implications: genuine collective responsibility cannot be reduced to individual responsibilities. The entangled state is irreducibly non-local.

Part III

Quantum Phenomena on Stratified Spaces

7 Moral Interference at Boundaries

7.1 Constructive and Destructive Interference

When ethical reasoning paths converge at a stratum boundary, their quantum amplitudes interfere. The character of interference depends on the relative phase of the paths.

Theorem 7.1 (Destructive Interference for Compounding Wrongs)**.** *Let $|wrong_1\rangle$ and $|wrong_2\rangle$ be states with amplitudes $A_1 = |A_1|e^{i\phi_1}$ and $A_2 = |A_2|e^{i\phi_2}$ where $\phi_2 - \phi_1 = \pi$ (opposite moral orientations). At the boundary ∂S with the "right" stratum:*

$$P_{total} = |A_1 + A_2|^2 = |A_1|^2 + |A_2|^2 - 2|A_1||A_2| = (|A_1| - |A_2|)^2 \tag{34}$$

The interference is maximally destructive.

Proof. With phase difference π:

$$A_1^* A_2 = |A_1||A_2|e^{i(\phi_2 - \phi_1)} = |A_1||A_2|e^{i\pi} = -|A_1||A_2| \tag{35}$$

Therefore:

$$2\mathrm{Re}(A_1^* A_2) = -2|A_1||A_2| \tag{36}$$

□

Interpretation: This is the formal explanation for why "two wrongs don't make a right." When two wrongs have opposite phases (e.g., one attempts to compensate for the other), the combined probability of reaching a "right" outcome is *less* than either wrong alone. The interference is destructive by mathematical necessity.

Example 7.2 (Revenge as Destructive Interference)**.** An initial harm $|H_1\rangle$ has amplitude A_1. A revenge harm $|H_2\rangle$ intended to "cancel" it has amplitude A_2 with opposite phase. The total harm amplitude:

$$A_{\text{total}} = A_1 + A_2 \tag{37}$$

has magnitude $||A_1| - |A_2||$. But the probability of reaching a "no harm" state involves:

$$P_{\text{no harm}} \propto |A_{\text{total}}|^2 = (|A_1| - |A_2|)^2 \leq \max(|A_1|^2, |A_2|^2) \tag{38}$$

Revenge does not cancel harm—it reduces the probability of resolution compared to addressing either harm separately.

7.2 Constructive Interference and Moral Progress

Conversely, when moral actions are *aligned* in phase, interference is constructive.

Theorem 7.3 (Constructive Interference for Aligned Actions)**.** *Let $|good_1\rangle$ and $|good_2\rangle$ be states with the same phase ($\phi_2 = \phi_1$). Then:*

$$P_{total} = (|A_1| + |A_2|)^2 > |A_1|^2 + |A_2|^2 \tag{39}$$

Interpretation: Aligned moral actions amplify each other. This formalizes the intuition that coordinated ethical effort is more effective than isolated action.

8 Ethical Entanglement on Stratified Spaces

8.1 Stratum-Constrained Entanglement

On stratified spaces, entanglement is constrained by stratum structure.

Definition 8.1 (Intra-Stratum Entanglement)**.** Agents A and B are **intra-stratum entangled** if their joint state $|\Psi_{AB}\rangle$ is entangled and both agents are localized to the same stratum S_i.

Definition 8.2 (Inter-Stratum Entanglement)**.** Agents are **inter-stratum entangled** if their joint state is entangled but they are localized to different strata.

Proposition 8.3 (Inter-Stratum Entanglement Requires Boundary Interaction)**.** *Inter-stratum entanglement can only be created through boundary interactions—exchange of transition ethons.*

Proof. The Hilbert space decomposes as $\mathcal{H} = \bigoplus_i \mathcal{H}_i$. Dynamics within a single stratum (the bulk Lagrangian) preserves this decomposition. Only the boundary Lagrangian couples different strata, enabling entanglement across stratum boundaries. □

8.2 Moral Luck as Boundary Entanglement

Example 8.4 (Moral Luck Entanglement)**.** Two agents perform identical actions; one leads to harm (bad luck), one doesn't. In SQND, they become inter-stratum entangled at the boundary between "harm occurred" and "no harm" strata:

$$|\Psi\rangle = \frac{1}{\sqrt{2}}(|\text{blameworthy}\rangle_A|\text{blameless}\rangle_B + |\text{blameless}\rangle_A|\text{blameworthy}\rangle_B) \tag{40}$$

The outcomes are correlated in a way that cannot be explained by their independent choices—the correlation arises from their joint interaction with the boundary.

9 Ethical Tunneling Through Forbidden Regions

9.1 Tunneling Amplitude

In standard QND, agents can tunnel between moral states through classically forbidden barriers. SQND constrains this tunneling through the dimension-dependent coupling. However, a complete analysis must account for the interplay between tunneling and decoherence.

Theorem 9.1 (Tunneling Suppression at Low-Dimensional Strata)**.** *Let $S_{forbidden}$ be a stratum of dimension d_f separating strata S_1 and S_2. The effective tunneling probability through $S_{forbidden}$ satisfies:*

$$T_{\mathit{eff}} \leq \min\left(e^{-\Gamma\tau_A}, \exp\left(-\frac{\alpha_\eta(S_f)\cdot\Delta x}{\hbar_\eta c_\eta}\right)\right) \to 0 \quad \text{as } d_f \to 0 \tag{41}$$

where τ_A is the approach time to the barrier, Γ is the decoherence rate, and Δx is the barrier width.

Proof. We establish suppression via two independent mechanisms:

Regime I: Decoherence-Dominated ($\tau_D < \tau_A$). Let $\tau_D = \Gamma^{-1}$ be the decoherence timescale and τ_A be the time for the agent to approach the barrier. By Proposition 9.1, $\Gamma(S) \propto (d_{\max} - d + 1)$, so as $d_f \to 0$, we have $\Gamma \to \Gamma_{\max}$.

The probability that coherence survives until barrier contact is:

$$P_{\text{coherent}} = e^{-\Gamma\tau_A} \to 0 \quad \text{as } \Gamma \to \infty \tag{42}$$

Once decohered, the agent follows a classical trajectory. Classical trajectories have exactly $T = 0$ for any finite barrier—no WKB approximation needed. Therefore $T_{\text{eff}} \leq P_{\text{coherent}} \to 0$.

Regime II: Coupling-Dominated ($\tau_D > \tau_A$). Suppose coherence persists to the barrier (e.g., in a carefully isolated system). The WKB approximation gives:

$$T_{\text{WKB}} \propto \exp\left(-\int_{S_f} \sqrt{2m_{\text{eff}}\alpha_\eta(S_f)V_0}\, dx\right) \tag{43}$$

With $\alpha_\eta(S_f) = \alpha_0(d_{\max}/(d_f + \epsilon))^\gamma$, taking $d_f \to 0$:

$$\alpha_\eta \to \alpha_0 \left(\frac{d_{\max}}{\epsilon}\right)^\gamma \to \infty \tag{44}$$

The WKB integral diverges, forcing $T_{\text{WKB}} \to 0$.

Combined Bound. Since tunneling requires *both* maintaining coherence *and* penetrating the barrier:

$$T_{\text{eff}} \leq P_{\text{coherent}} \cdot T_{\text{WKB}} \leq \min(P_{\text{coherent}}, T_{\text{WKB}}) \to 0 \tag{45}$$

Near 0-dimensional strata, both mechanisms are simultaneously active: Γ increases (faster decoherence) while α_η increases (higher barrier). The suppression is therefore doubly robust. $\square$

Remark 9.2 (No Quantum Loopholes)**.** This two-regime structure eliminates potential "quantum loopholes" for circumventing hard vetoes:

- An agent attempting rapid approach (small τ_A) to exploit coherence faces the divergent WKB barrier.
- An agent attempting slow, careful approach to reduce barrier interaction faces decoherence before arrival.
- No intermediate strategy succeeds because both Γ and α_η diverge as $d_f \to 0$.

The hard veto is enforced by the geometry of moral space itself, not by any single mechanism that might be circumvented.

Interpretation: Tunneling through hard vetoes (0-dimensional strata) is completely suppressed. This is the quantum-theoretic explanation for SGE's treatment of absolute prohibitions.

Example 9.3 (Moral Phase Transition vs. Hard Veto)**.** Consider two scenarios:

- **Moral phase transition**: An agent shifts from one ethical framework to another. The boundary is 1-dimensional (a threshold, not a point). Tunneling is suppressed but possible—corresponding to gradual paradigm shifts.
- **Hard veto**: The prohibition against (e.g.) torturing innocents is a 0-dimensional stratum. $\alpha_\eta \to \infty$, tunneling probability $T \to 0$. No quantum "loophole" exists.

10 Ethical Decoherence and the Classical Limit

10.1 Dimension-Dependent Decoherence

Decoherence—the loss of quantum coherence through environmental interaction—has a natural stratified structure in SQND.

Proposition 10.1 (Decoherence Rate Scaling)**.** *The decoherence rate Γ scales inversely with stratum dimension:*

$$\Gamma(S_i) \propto (d_{\max} - d_i + 1) \tag{46}$$

Interpretation:

- **High-dimensional strata**: Γ is small. Superposition persists—this is the regime of deliberation, where multiple possibilities are held in mind.
- **Low-dimensional strata**: Γ is large. Decoherence is rapid—this is the regime of decision, where superposition collapses quickly.
- **0-dimensional strata**: $\Gamma \to \infty$. Decoherence is instantaneous—the judgment is forced.

This explains the phenomenology of moral decision-making:

- Extended deliberation in complex, high-dimensional cases
- Rapid convergence as options narrow
- Irreversible commitment at decision points

10.2 The Classical Limit

Theorem 10.2 (Classical Limit of SQND)**.** *As $\hbar_\eta \to 0$ and decoherence becomes instantaneous on all strata, SQND reduces to classical SGE with satisfaction function:*

$$\Sigma(x) = \lambda(x) f\left(\frac{I_\mu O^\mu}{\|O\|_g}\right) \tag{47}$$

Proof sketch. In the limit $\hbar_\eta \to 0$:

1. Superpositions collapse immediately (decoherence dominates)
2. Path integrals are dominated by classical paths (stationary phase)
3. The quantum expectation $\langle\Psi|\hat{\Sigma}|\Psi\rangle$ reduces to the classical $\Sigma(x_{\text{classical}})$

The stratified structure survives the classical limit, yielding SGE. □

11 The Ethical Vacuum on Stratified Spaces

11.1 Stratum-Dependent Vacuum Structure

The ethical vacuum $|0\rangle$ has structure that varies across strata.

Definition 11.1 (Stratified Vacuum)**.** The SQND vacuum is the state with no real ethons on any stratum:

$$\hat{\eta}_k^{(\lambda,i)}|0\rangle = 0 \quad \text{for all } k, \lambda, i \tag{48}$$

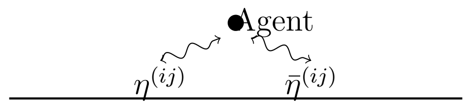
Proposition 11.2 (Stratum-Dependent Vacuum Fluctuations)**.** *The vacuum fluctuation spectrum depends on stratum dimension:*

$$\langle 0|\hat{A}^{(i)\mu}\hat{A}_\mu^{(i)}|0\rangle \propto \frac{1}{d_i + \epsilon} \tag{49}$$

Interpretation: Low-dimensional strata have larger vacuum fluctuations. Near moral singularities, the vacuum "seethes" more intensely. This corresponds to heightened moral sensitivity at decision points.

11.2 Vacuum Polarization at Boundaries

A "test agent" approaching a stratum boundary polarizes the vacuum:



Stratum boundary ∂S_{ij}

Virtual transition-ethon pairs are excited from the boundary vacuum. These screen the agent's "moral charge," modifying how the agent interacts with the threshold.

Interpretation: Moral assessment near boundaries is complicated by vacuum effects. The "intrinsic" moral status of an action differs from its "observed" status due to boundary screening.

Part IV
Computational Implementation

12 Finite Approximation Theorems

12.1 Quantum Finite Approximation

For computational implementation, continuous stratified quantum systems must be approximated by finite structures.

Theorem 12.1 (Quantum Finite Approximation)**.** *Let M be a compact stratified moral space with finitely many strata. For every $\epsilon > 0$, there exists a finite stratified quantum graph G_ϵ such that:*

1. *Vertices of G_ϵ correspond to basis states localized in ϵ-balls*
2. *Edges encode transition amplitudes $\langle v|\hat{H}|w\rangle$ above threshold*
3. *Stratum structure is preserved: vertices inherit stratum labels*
4. *Error bound: $\|\langle\Psi|\hat{\Sigma}|\Psi\rangle - \Sigma_G(v_\Psi)\| \leq 2L\epsilon + O(\epsilon^2)$*

Proof sketch. The proof extends SGE Theorem 3.9 to the quantum case:

1. Discretize each stratum S_i into ϵ-balls; these become vertices
2. Compute transition matrix elements $\langle v|\hat{H}_{\text{SQND}}|w\rangle$; non-negligible elements become edges
3. The stratified structure transfers directly; boundary vertices connect strata
4. Error analysis follows from Lipschitz bounds on $\hat{\Sigma}$ and the approximation theory of quantum channels

□

12.2 Complexity Analysis

Theorem 12.2 (Complexity of SQND Decision)**.** *Let M have m strata with N vertices per stratum in the ϵ-approximation, and let the quantum dimension (number of amplitude components) be d. Finding the optimal path requires $O(mN^2 \cdot d^2 \cdot \log(mN))$ operations.*

Proof.

- Graph has mN vertices
- Computing transition amplitudes for d-dimensional quantum states: $O(d^2)$ per edge
- Total edges: $O((mN)^2)$ worst case
- Shortest path (Dijkstra-like): $O((mN)^2 \log(mN))$
- Total: $O(mN^2 \cdot d^2 \cdot \log(mN))$

□

Corollary 12.3 (Real-Time Feasibility)**.** *For typical SQND configurations ($m \leq 10$ strata, $N \leq 100$ vertices per stratum, $d \leq 8$ amplitude dimensions):*

$$\textit{Operations} \approx 10 \times 10^4 \times 64 \times 10 \approx 6 \times 10^7 \tag{50}$$

Achievable in $< 10ms$ on contemporary embedded processors at 10 GFLOPS.

13 The Harm Operator

13.1 Definition and Calibration

Definition 13.1 (Harm Operator)**.** The harm operator $\hat{H}_{\mathcal{H}}$ has eigenstates $|h\rangle$ with eigenvalues $h \in \mathbb{R}$:

$$\hat{H}_{\mathcal{H}}|h\rangle = h|h\rangle \tag{51}$$

Measurement of harm yields eigenvalue h with probability $|\langle h|\Psi\rangle|^2$.

Following SGE's scale-normalization axiom, we calibrate:

$$\mathbb{E}_{S_i}[h] = 0, \quad \mathrm{Var}_{S_i}[h] = 1 \tag{52}$$

where expectations are over the stratum S_i.

This ensures harm is measured in comparable units across strata, enabling consistent evaluation of different types of harm.

13.2 The Harm Accounting Equation (Quantum Version)

From Noether Ethics, the classical harm accounting equation is:

$$\frac{\partial \rho_{\mathcal{H}}}{\partial t} + \nabla \cdot \mathbf{J}_{\mathcal{H}} = \sigma \tag{53}$$

In SQND, this becomes the operator equation:

$$\frac{\partial \hat{\rho}_{\mathcal{H}}}{\partial t} + \nabla \cdot \hat{\mathbf{J}}_{\mathcal{H}} = \hat{\sigma} \tag{54}$$

Theorem 13.2 (Quantum Harm Conservation). *In the absence of genuine harm generation ($\hat{\sigma} = 0$), the total expected harm is conserved:*

$$\frac{d}{dt}\langle\Psi|\hat{H}_{\mathcal{H},total}|\Psi\rangle = 0 \tag{55}$$

Interpretation: Harm cannot appear or disappear merely through re-description or quantum manipulation. If the total expected harm changes, genuine harm generation or repair must have occurred.

14 Threat Model and Diagnostics

14.1 Attack Vector Mapping

Attack Vector	**Axiom Violated**	**SQND Status**	**Diagnostic**
Representation gaming	Axiom 1 (Invariance)	Prevented by BIP	Gauge-fixing test
Path-dependent exploits	Curvature $\Omega \neq 0$	Detected (geo. regime)	Holonomy loop test
Boundary tunneling	Stratum violation	Doubly suppressed*	Amplitude monitor
Superposition abuse	Decoherence evasion	Forced at 0-dim strata	Dimension tracking
Entanglement washing	Axiom 6 (Grounding)	Residue persists	Trace monitoring
Sensor spoofing	External (Ψ bypass)	Outside scope	Physical security

**Doubly suppressed*: Tunneling through hard vetoes is blocked by two independent mechanisms: (1) decoherence collapses superposition before barrier contact, and (2) even if coherence persists, the coupling divergence ($\alpha_\eta \to \infty$) makes WKB tunneling amplitude vanish. No intermediate attack strategy succeeds.

14.2 Diagnostic Procedures

Diagnostic A: Gauge-Fixing Consistency Test (Engineering Regime)

Purpose: Detect canonicalizer bugs, non-determinism, or implementation errors.

Procedure:

1. Sample transforms $g_1, g_2 \in G_{\text{declared}}$ and input x
2. Compute $\kappa(g_1(g_2(x)))$ and $\kappa(g_2(g_1(x)))$
3. Measure $\Delta = d(\kappa(g_1(g_2(x))), \kappa(g_2(g_1(x))))$
4. If $\Delta > \tau$, flag as canonicalizer inconsistency

Diagnostic B: Holonomy Loop Test (Geometric Regime)

Purpose: Detect genuine path dependence from curvature $\Omega \neq 0$.

Procedure:

1. Pick four nearby base points $b_{00}, b_{10}, b_{11}, b_{01} \in B$ forming a rectangle (different scenarios, not re-descriptions)
2. Transport around the loop: $b_{00} \to b_{10} \to b_{11} \to b_{01} \to b_{00}$
3. Compute holonomy h
4. Deviation $D_G(h, e)$ from identity measures path dependence

Diagnostic C: Stratum Boundary Monitor
Purpose: Detect attempted tunneling through forbidden regions.
Procedure:

1. Track agent's trajectory through moral space
2. Alert when approaching low-dimensional strata
3. Verify transition probabilities match theoretical predictions from $\mathcal{L}_{\text{boundary}}$
4. Flag anomalous amplitudes

Part V
Implications

15 Philosophical Implications

SQND has profound implications for moral philosophy, extending and refining those of QND.

15.1 Moral Realism and Anti-Realism

Classical moral realism: Moral facts exist independently; our judgments track them.

Classical anti-realism: There are no moral facts; "morality" is projection, convention, or error.

SQND: Moral reality is stratified and quantum. Before measurement, there are no definite moral facts on high-dimensional strata—only superpositions of possibilities. As dimension decreases (approaching decision points), possibilities narrow. At 0-dimensional strata, measurement is forced and moral facts become definite.

This is a *third option*: Moral facts are neither pre-existing and discovered (realism) nor purely constructed (anti-realism). They are *actualized* from a space of possibilities by the act of judgment itself, with the actualization constrained by the stratified structure of moral space.

15.2 Free Will and Determinism

Classical determinism: Given the state of the world, the future is fixed.

Classical libertarianism: Agents have irreducible freedom to choose.

SQND: Quantum indeterminacy provides genuine openness on high-dimensional strata. The stratified structure provides constraints—some choices are blocked by hard vetoes (0-dimensional barriers), others are channeled by lower-dimensional thresholds. Freedom exists within structure.

The measurement problem remains: what triggers collapse at decision points? SQND does not solve free will, but it provides a framework where moral choice is neither deterministic nor arbitrary—it is *structured indeterminacy*.

15.3 The Problem of Evil

Classical problem: If God is omnipotent, omniscient, and omnibenevolent, why does evil exist?

SQND reframing: The ethical vacuum on any stratum contains all possibilities—good and evil. The vacuum fluctuations are not contingent; they are structural features of any moral reality that supports meaningful choices.

More precisely: the vacuum must have non-zero fluctuations (by the uncertainty principle $\Delta H_{\mathcal{H}} \cdot \Delta \dot{H}_{\mathcal{H}} \geq \frac{1}{2}\hbar_\eta$). These fluctuations include both positive and negative moral content. A vacuum with only positive fluctuations would violate the uncertainty principle.

This doesn't "solve" the problem of evil, but it reframes it: evil is not an accident or a failure; it is a structural feature of any reality that supports meaningful moral choice.

15.4 Virtue and Character

Aristotelian virtue ethics: Virtue is a stable disposition to act well.

SQND: Virtue is a state of the agent field $\hat{\psi}$ concentrated in high-satisfaction regions of moral state space. Character is not fixed; it's a probability distribution that evolves through habituation.

Habituation is the process of shaping the wave function through repeated measurement. Each action reinforces certain states, increasing their amplitude. Virtue develops by repeated collapse into good states, which modifies the underlying distribution.

The stratified structure adds: different strata have different virtuedynamics. Virtue on high-dimensional strata is "soft"—dispositions, tendencies, probability weights. Virtue at 0-dimensional strata is "hard"—commitments that cannot be violated without crossing an infinite coupling barrier.

15.5 Moral Progress

Classical view: Moral progress means getting closer to pre-existing moral truths.

SQND view: Moral progress involves:

- Expanding the superpositions we can maintain (moral imagination)

- Reducing harmful decoherence (protecting deliberation from premature collapse)
- Re-engineering the vacuum (changing background moral structure)
- Adding new strata and boundaries (conceptual moral innovation)

Progress is not just discovering what was always true; it's reshaping moral space itself.

16 Theological Implications

Following QND Appendix B, we explore speculative theological connections. These are offered in a spirit of intellectual play, not doctrinal assertion.

16.1 The Moral Fabric of Reality

In SQND, the ethical vacuum has structure varying across strata. This is the "moral fabric of reality"—not uniform, but textured by the stratification of moral space.

Theological resonance: Many traditions speak of moral order woven into creation. SQND provides a formal model: the moral order *is* the stratified structure of the ethical vacuum.

16.2 The Ground of Being

Tillich spoke of God as the "ground of being"—not a being among beings, but the condition for all beings.

SQND analog: The stratified ethical vacuum is the ground of moral being. All moral facts are excitations above this ground state. The vacuum structure (including the stratification) is the condition for moral content.

16.3 Hard Vetoes and Divine Commands

Some ethical traditions posit divine commands as absolute—not because of consequences, but because God commands them.

SQND formal model: 0-dimensional strata with $\alpha_\eta \to \infty$ function as absolute prohibitions. No quantum loophole allows tunneling through them. The structure of moral space *itself* enforces certain prohibitions.

Whether this structure is "divine" or merely "fundamental" is a metaphysical question SQND does not resolve.

16.4 Sin, Entanglement, and Redemption

Sin as entanglement: Through moral interaction (ethon exchange), agents become entangled. Sin is not merely individual; it spreads through the moral fabric via entanglement.

Redemption as disentanglement: Breaking the entanglement, restoring separable states. In physics, disentanglement requires environmental interaction or deliberate operation. In the theological picture, redemption is the "divine operation" that disentangles the sinner from the web of sin.

SQND addition: The stratified structure constrains disentanglement. Some entanglements are easier to break (crossing low barriers); others are harder (high-dimensional correlations); some are permanent (entanglement through 0-dimensional strata).

16.5 The Last Judgment and Final Measurement

Traditional eschatology: At the end, all will be judged.

SQND: The Last Judgment is the final measurement—the collapse of all moral superpositions into definite states. At 0-dimensional strata, this collapse is instantaneous and total.

Why "last"? Because measurement is irreversible (in standard quantum mechanics). The Last Judgment is the end of moral possibility—the finalization of all moral states.

16.6 Theological Summary

Theological Concept	**SQND Analog**
Moral law in creation	Stratified vacuum structure
Ground of being	Vacuum as ground state
Divine commands (absolute)	0-dimensional strata, $\alpha_\eta \to \infty$
Sin / corruption	Ethical entanglement
Redemption / forgiveness	Disentanglement
Last Judgment	Final measurement / collapse
Problem of evil	Vacuum fluctuations on all strata
Grace	Favorable vacuum fluctuations

17 Experimental Predictions

SQND makes testable predictions via quantum cognition experiments.

17.1 Stratification-Enhanced Order Effects

Prediction: The magnitude of order effects ($P(A \text{ then } B) \neq P(B \text{ then } A)$) should increase as questions approach stratum boundaries.

Test: Present moral dilemmas with questions at varying "distances" from decision thresholds. Measure whether the non-commutativity $|P(AB) - P(BA)|$ correlates with proximity to boundaries.

17.2 Boundary Interference Patterns

Prediction: When subjects reason through multiple ethical frameworks toward a decision point (0-dimensional stratum), interference effects should be stronger than when the decision space is high-dimensional.

Test: Compare judgment distributions when:

1. Only consequentialist reasoning is primed
2. Only deontological reasoning is primed
3. Both are available (interference condition)

Measure interference visibility $V = (P_{\max} - P_{\min})/(P_{\max} + P_{\min})$. Predict V increases as stratum dimension decreases.

17.3 Bell Violations in Collective Responsibility

Prediction: Collective responsibility judgments should violate Bell-type inequalities.

Test: Design scenarios with two agents jointly responsible. Measure responsibility attributions in different "bases" (accountability frameworks). Compute CHSH correlator. Predict violations exceeding 2, approaching $2\sqrt{2}$.

17.4 Decoherence Timescale Measurements

Prediction: Moral ambiguity persistence time correlates with stratum dimension.

Test: Present ambiguous scenarios of varying complexity. Measure response time distributions and confidence dynamics. Predict longer ambiguity persistence for complex (high-dimensional) scenarios.

18 Conclusion

We have presented Stratified Quantum Normative Dynamics (SQND), a unified framework synthesizing:

- The topological foundation of Stratified Geometric Ethics (SGE)
- The dynamical structure of Quantum Normative Dynamics (QND)
- The stratified Lagrangian formulation unifying both

Key contributions:

1. The Stratified Lagrangian $\mathcal{L}_{\text{strat}}$ governing bulk and boundary dynamics
2. The stratified ethon—moral quanta constrained by Whitney regularity
3. Quantum BIP—representation-invariance for superposition states

4. Formal explanation for why “two wrongs don’t make a right” (destructive interference)
5. Tunneling suppression at hard vetoes ($\alpha_\eta \to \infty$ at 0-dimensional strata)
6. Finite approximation with explicit error bounds
7. Complete threat model with diagnostic procedures

The framework provides the mathematical foundation for real-time ethical governance in autonomous systems, connecting to the DEME 2.0 architecture.

We maintain epistemic humility. SQND is a formal framework—a tool for organizing thought about ethical reasoning. We do not claim that ethics “is” quantum field theory on stratified spaces. We claim that this mathematical structure provides powerful explanatory, predictive, and engineering utility.

The classical limit emerges through ethical decoherence—the same mechanism that yields classical physics from quantum mechanics. Just as macroscopic objects don’t exhibit superposition, everyday moral decisions don’t exhibit quantum effects because of rapid decoherence at low-dimensional strata. The quantum structure becomes visible in the fine structure of deliberation, in interference effects, in collective responsibility, and in genuine moral ambiguity.

What unifies physics and ethics? We do not yet fully know. But we have established that the same formal patterns—symmetry, invariance, conservation, stratification, quantization—structure both domains. This is either a profound clue about coherent reasoning itself, or a remarkable mathematical coincidence.

We suspect the former.

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A Mathematical Details

A.1 Full Stratified Lagrangian

The complete form with all terms explicit:

$$S_{\text{strat}} = \sum_i \int_{S_i} \left[-\frac{1}{4} F_{\mu\nu}^{(i)} F^{(i)\mu\nu} + \bar{\psi}^{(i)} (i\gamma^\mu D_\mu^{(i)} - m_i) \psi^{(i)} \right] d\mathrm{Vol}_i \tag{56}$$

$$+ \sum_{j<i} \int_{\partial S_{ij}} \left[\lambda_{ij} \Phi + \frac{1}{2} \mu_{ij}^2 |\eta^{(ij)}|^2 + \kappa_{ij} \bar{\psi} \Gamma^{(ij)} \psi \right] d\sigma \tag{57}$$

A.2 Canonical Quantization

Commutation relations with stratum labels:

$$[\hat{A}_i^\mu(x), \hat{\Pi}_j^\nu(y)] = i\hbar_\eta \delta_\mu^\nu \delta_{ij} \delta^{(d_i)}(x-y) \tag{58}$$

Anticommutation for fermions:

$$\{\hat{\psi}_\alpha^{(i)}(x), \hat{\psi}_\beta^{(j)\dagger}(y)\} = \delta_{\alpha\beta}\delta_{ij}\delta^{(d_i)}(x-y) \tag{59}$$

A.3 Feynman Rules for Boundary Interactions

At boundary ∂S_{ij}:

- Transition ethon propagator: $D_{\mu\nu}^{(ij)}(k) = \frac{-i\eta_{\mu\nu}}{k^2 - \mu_{ij}^2 + i\epsilon}$
- Boundary vertex: $ig_{ij}\Gamma_\mu^{(ij)}$
- Junction condition: Amplitudes match according to Whitney (B)

A.4 Proof of Tunneling Suppression Theorem

Proof. The effective tunneling probability is bounded by the product of two independent suppression factors:

$$T_{\text{eff}} \leq P_{\text{coherent}} \cdot T_{\text{WKB}} \tag{60}$$

Decoherence Factor. The probability of maintaining coherence during approach time τ_A is:

$$P_{\text{coherent}} = \exp(-\Gamma(S_f) \cdot \tau_A) \tag{61}$$

where $\Gamma(S_f) \propto (d_{\max} - d_f + 1)$. As $d_f \to 0$, $\Gamma \to \Gamma_{\max}$, so $P_{\text{coherent}} \to 0$ for any finite $\tau_A > 0$.

WKB Factor. Conditional on coherence surviving, the tunneling amplitude is:

$$T_{\text{WKB}} = \exp\left(-\frac{2}{\hbar_\eta}\int_{S_f}\sqrt{2m_{\text{eff}}(V_{\text{eff}} - E)}\,dx\right) \tag{62}$$

The effective potential at low-dimensional strata is $V_{\text{eff}} = \alpha_\eta(S_f)V_0$ with:

$$\alpha_\eta(S_f) = \alpha_0\left(\frac{d_{\max}}{d_f + \epsilon}\right)^\gamma \tag{63}$$

As $d_f \to 0$: $\alpha_\eta \to \infty$, $V_{\text{eff}} \to \infty$, and the WKB integral diverges, giving $T_{\text{WKB}} \to 0$.

Combined Suppression. Since both factors vanish independently as $d_f \to 0$:

$$T_{\text{eff}} \leq \min(P_{\text{coherent}}, T_{\text{WKB}}) \to 0 \tag{64}$$

The suppression is doubly robust: failure of either mechanism alone cannot enable tunneling through hard vetoes. □

B Connection to DEME 2.0 Architecture

SQND provides the theoretical foundation for DEME 2.0's three-layer structure:

- **Strategic Layer**: Operates in high-dimensional strata where superposition persists; computes long-term satisfaction trajectories.
- **Tactical Layer**: Monitors approach to stratum boundaries; applies boundary Lagrangian constraints; manages decoherence timescales.
- **Reflex Layer**: Enforces hard vetoes at 0-dimensional strata where $\alpha_\eta \to \infty$; hardware-resident ethics module.

The ErisML runtime implements finite approximation (Theorem on quantum finite approximation) with guaranteed latency bounds (Corollary on real-time feasibility).