An Examination of Dynamic Aeroelasticity and Wing Flutter

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1 Introduction

This report details the dynamic aeroelasticity analysis of a theoretical wing. Dynamic aeroelasticity examines the aerodynamic, elastic, and inertial forces acting on the wing. The equations of motion indicate that the motion is periodic with three particular cases of interest: stable, unstable, and neutrally stable motion. In stable case, the oscillatory motion is damped, becoming stable over time. When it is critically damped, the motion becomes stable the fastest without oscillations. In the unstable case, the amplitude of the motion increases toward infinity. The boundary between the stable and unstable cases is of special interest because the wing experiences undamped motion. This motion is neutrally stable and is called flutter.

We used a simple 2 degree-of-freedom (DOF) 2D dynamic model. The two DOF are: $\alpha(t)$ rotation of the airfoil in the x-y plane and $y(t)$ the displacement in the $\hat{j}$ direction. This report will often use the terms, "wing" and "airfoil", interchangeably because of this 2D model. To develop this flutter analysis, we first examined responses using a linear model. This model made small angle approximations and simplified some of the forces to linearize the system of differential equations. Then, we looked at the nonlinear (and more realistic) model of the wing and compared the results. Lastly, we determined the conditions for flutter using the nonlinear model and added additional terms to better describe the motion. These models are lumped parameter models in that they combine the more complex equations that come from the elasticity of the wing and generalize them with springs and dashpots. The model also uses a symmetric airfoil (such as a NACA 0012) to make sure that there is 0 lift at 0 angle of attack. In addition, we ignore thrust, lift-induced drag, and parasitic drag as a part of our analysis.

2 Simplified, Linear Model: A complete derivation

The model is set up with a series of springs and dashpots to generalize the elastic forces. Figure 1 shows the free body diagram of the airfoil. The following parameters are also defined:

- K_y = stiffness of the linear spring that connects the wing to the fuselage.

- K_α = stiffness of the torsional spring; rotational analog of the linear spring
- C_y = linear dashpot constant
- C_α = torsional dashpot constant
- G = location of the center of gravity of the wing
- L = center of lift taken to be at quarter-chord (rule of thumb)
- C = elastic axis; point through which linear spring and dashpot and torsional spring and dashpot act.

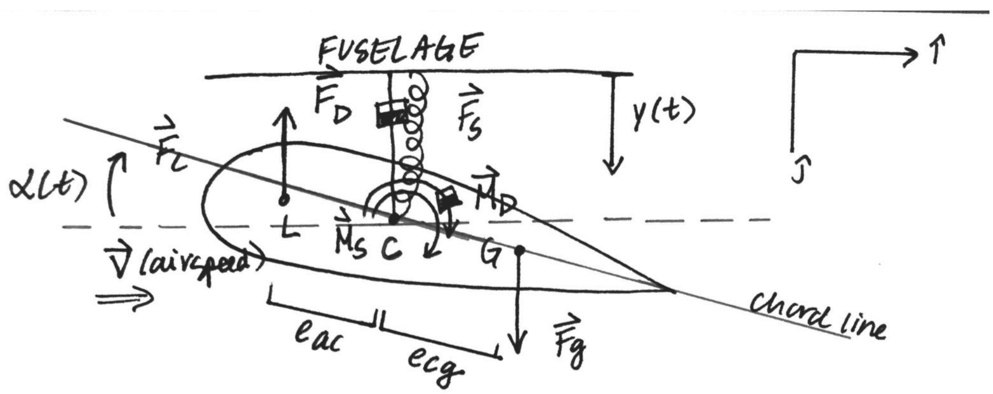

Figure 1: Linear model

Loads on airfoil:

- $\vec{F_D} = -C_y \dot{y} \hat{j}$; linear dashpot force
- $\vec{F_S} = -K_y y \hat{j}$; linear spring force
- $\vec{F_L} = qS \frac{\partial C_L}{\partial \alpha} \alpha \hat{j}$; lift force acting at L in the $\hat{j}$ direction
  - where S is the wing area
  - $\frac{\partial C_L}{\partial \alpha} \alpha$ is 2π for an ideal airfoil
  - and $q = \frac{1}{2} \rho v^2$ where v is taken as the airspeed and ρ the air density
- $\vec{F_g}$ = -mg$\hat{j}$; gravity acting at G in the $\hat{j}$ direction
- $\vec{M_D} = C_\alpha \dot{\alpha} \hat{k}$; torsional dashpot moment
- $\vec{M_S} = K_\alpha \alpha \hat{k}$; torsional spring moment

Linear momentum balance gives:

$$\sum \vec{F} = m\vec{a} \tag{1}$$

$$-\vec{F_L} + -K_h y\hat{j} - C_h y\hat{j} - mg\hat{j} = m\ddot{x}\hat{i} + m\ddot{y}\hat{j} + S_\alpha\ddot{\alpha}\hat{j} \tag{2}$$

Where S_α is the mass imbalance ($e_{cg}m$). As mentioned in the introduction, we set thrust and $\vec{F_{drag}} = 0$.

$$(2) \cdot \hat{i} \longrightarrow \ddot{x} = 0 \tag{3}$$

$$(2) \cdot \hat{j} \longrightarrow -F_L + -K_y y - C_h y - mg = m\ddot{y} + S_\alpha\ddot{\alpha} \tag{4}$$

In order to simplify this model, we made several assumptions. We set $\vec{F_g} = 0$ because we assumed that gravity is negligible compared to the other aerodynamic loads. As mentioned in the introduction, we also set $\vec{F_D} = 0$. Substituting for the lift force, we get the following equation of motion:

$$m\ddot{y} + S_\alpha\ddot{\alpha} + K_y y + qS\frac{\partial C_L}{\partial \alpha}\alpha = 0 \tag{5}$$

Angular momentum balance in the $\hat{k}$ direction gives:

$$\sum \vec{M_C} = \dot{\vec{H}} \tag{6}$$

$$F_L e_{ac} cos(\alpha) - K_\alpha\alpha - C_\alpha\dot{\alpha} + mge_{cg}cos(\alpha) = I_\alpha\ddot{\alpha} + S_\alpha\ddot{y} \tag{7}$$

Where I_α is the moment of inertia.

$$qS\frac{\partial C_L}{\partial \alpha}\alpha e_{ac} sin(\alpha_c) - K_\alpha\alpha - c_\alpha\dot{\alpha} + mge_{cg}cos(\alpha_c) = I_\alpha\ddot{\alpha} + S_\alpha\ddot{y} \tag{8}$$

Here again, we make some simplifying assumptions to our model. We make the small angle approximation $sin(\alpha) = \alpha$. And again, gravity is neglected because it is significantly smaller than the other aerodynamic loads. We set the linear dashpot $\vec{F_D} = 0$ and torsional dashpot $\vec{M_D} = 0$ zero also for simplification. This gives us another equation of motion:

$$I_\alpha\ddot{\alpha} + S_\alpha\ddot{y} + K_\alpha\alpha - qS\frac{\partial C_L}{\partial \alpha}\alpha e_{ac} = 0 \tag{9}$$

Now there are two simplified equations of motion for the two unknowns (α and y):

$$m\ddot{y} + S_\alpha\ddot{\alpha} + K_y y + qS\frac{\partial C_L}{\partial \alpha}\alpha = 0 \tag{10}$$

$$I_\alpha\ddot{\alpha} + S_\alpha\ddot{y} + K_\alpha\alpha - qS\frac{\partial C_L}{\partial \alpha}\alpha e_{ac} = 0 \tag{11}$$

These are two coupled, 2nd order linear ordinary differential equations. To solve this analytically, we "predict" or "guess" a solution to this system. We are examining stability, where the system's (aircraft's) motion can diverge or converge rapidly in response to a perturbation. An exponential model for the system seemed the most straightforward:

$$y(t) = X_1 e^{pt} \tag{12}$$

$$\alpha(t) = X_2 e^{pt} \tag{13}$$

where X_1 and X_2 are constants given by the initial conditions. An investigation of the effect of initial conditions on stability is discussed later in this report.

Differentiating (12) and (13) twice and plugging these into (10) and (11), we get the characteristic equation for this 2nd order system.

$$(mp^2 + K_h)X_1 e^{pt} + (S_\alpha p^2 + qS\frac{\partial C_L}{\partial \alpha}\alpha)X_2 e^{pt} = 0 \tag{14}$$

$$(S_\alpha p^2)X_1 e^{pt} + (I_\alpha p^2 + K_\alpha - qS\frac{\partial C_L}{\partial \alpha}e_{ac})X_2 e^{pt} = 0 \tag{15}$$

In matrix form we get:

$$\begin{bmatrix} mp^2 + K_h & (S_\alpha p^2 + qS\frac{\partial C_L}{\partial \alpha}\alpha) \\ (S_\alpha p^2 + qS\frac{\partial C_L}{\partial \alpha}\alpha) & (I_\alpha p^2 + K_\alpha - qS\frac{\partial C_L}{\partial \alpha}e_{ac}) \end{bmatrix} \begin{bmatrix} X_1 e^{pt} \\ X_2 e^{pt} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

To find the nontrivial solutions for p by setting the determinant of the coefficients = 0. The characteristic equation simplifies to:

$$Ap^4 + Bp^2 + C = 0 \tag{16}$$

where
A $= mI_\alpha - S_\alpha^2$

B $= mK_\alpha - mqS\frac{\partial C_L}{\partial \alpha}e_{ac} + K_y I_\alpha - qS\frac{\partial C_L}{\partial \alpha}S_\alpha$

C $= K_y K_\alpha - K_y qS\frac{\partial C_L}{\partial \alpha}e_{ac}$

and from the quadratic formula we get:

$$p^2 = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A} \tag{17}$$

We made all the simplifications earlier such as neglecting gravity and removing the torsional dashpot and linear dashpot in order to linearize our system and bring it as close as it can to resemble a oscillating mass-spring pendulum system in order to understand behavior. There are three cases of interest:
$B^2 - 4AC > 0$: p is a purely imaginary number; stable motion
$B^2 - 4AC < 0$: p is a complex number; unstable motion

$B^2 - 4AC = 0$: boundary between stable and unstable motion, where 'flutter' occurs; This equation can be used to determine the velocity at which flutter occurs

```
%% Inputs
% Geometry
p.b = 10; p.c = 1; p.S = p.b*p.c; p.e = 0.1;

% Properties
p.m = 1; p.Kh = 0; p.Ka = 1; p.Ch = 0; p.Ca = 0;
p.My = 1; p.Ia = 1; p.Sa = 0.1;

% Aerodynamics
p.L = 0.5; p.CLa = 2*pi;
%p.q = 0.1
p.q = 0.1; n = 0.001;
A = p.m*p.Ia-p.Sa^2;
B = p.m*(p.Ka-p.q*p.S*p.e*p.CLa)+p.Kh*p.Ia-p.Sa*p.q*p.S*p.CLa;
C = p.Kh*(p.Ka-p.q*p.S*p.e*p.CLa);
% while (B^2-4*A*C > 10^-1)
%     B^2-4*A*C
%     p.q = p.q + n;
%     A = p.m*p.Ia-p.Sa^2;
%     B = p.m*(p.Ka-p.q*p.S*p.e*p.CLa)+p.Kh*p.Ia-p.Sa*p.q*p.S*p.CLa;
%     C = p.Kh*(p.Ka-p.q*p.S*p.e*p.CLa);
% end
% p.q
```

Figure 2: Inputs

```
%% Solve
tstart = 0; tend = 1; npointspers = 100;
ntimes = tend*npointspers+1; % total number of time points
t = linspace(tstart,tend,ntimes);

h0 = 1; hd0 = 1; al0 = pi/90; ald0 = -1;
z0 = [h0;hd0;al0;ald0];

% ODE45
small = 1e-7;
options = odeset('RelTol', small, 'AbsTol', small);
f = @(t,z) detailedFlutterRHS(t,z,p);
[t,z] = ode45(f, t, z0, options);

h = z(:,1); hd = z(:,2); al = z(:,3); ald = z(:,4);
minh = min(h); maxh = max(h);
minal = min(al); maxal = max(al);
```

Figure 3: ODE Solver

```
%% More Detailed Flutter RHS Function
function zdot = detailedFlutterRHS(t,z,p)
h = z(1); hd = z(2);
al = z(3); ald = z(4);

m = p.m; Kh = p.Kh; Ch = p.Ch;
My = p.My; Ka = p.Ka; Ca = p.Ca; Ia = p.Ia;
%detailed:
q = p.q; CLa = p.CLa; S = p.S; e = p.e; Sa = p.Sa;

L = q*S*CLa*al; My = L*e;

aldd = (My-Ka*al+(Sa/m)*(Kh*h+L*sin(al)))/(Ia-Sa^2/m);
hdd = (-1/m)*(Sa*aldd+Kh*h+L*sin(al));
% hdd = (-1/m)*(Kh*h+Ch*hd+q*S*CLa*al);
% aldd = (1/Ia)*(q*S*e*CLa*al-Ka*al-Ca*ald);

zdot = [hd;hdd;ald;aldd];
end
```

Figure 4: Right Hand Side.

```
%% Plot
foil_str = 'mh114.xlsx';
%graph(foil_str,t,al,h,p);
animate(foil_str,t,al,h,p);
```

Figure 5: Plotting call functions

3 Case Studies using the Linear Model

Here we used MATLAB in order to set our parameters and constants such that $B^2 = 4AC$ so we could see flutter. We arbitrarily fixed some constants to be 1 and then calculated the other parameters based on those values. We played with the parameters to see the response of the system to various initial conditions and parameters.

3.1 Case I: A Stable System

m= 1, $I_\alpha = 1$, $q = 0.1$, $S = 10$, e= 0.1, $S_\alpha = 0.1$, $K_\alpha = 1000$, $K_h = 1000$, Nonzero initial conditions. You can see that the structural parameters of the plane are such that, if there are any perturbations, the system returns to equilibrium.

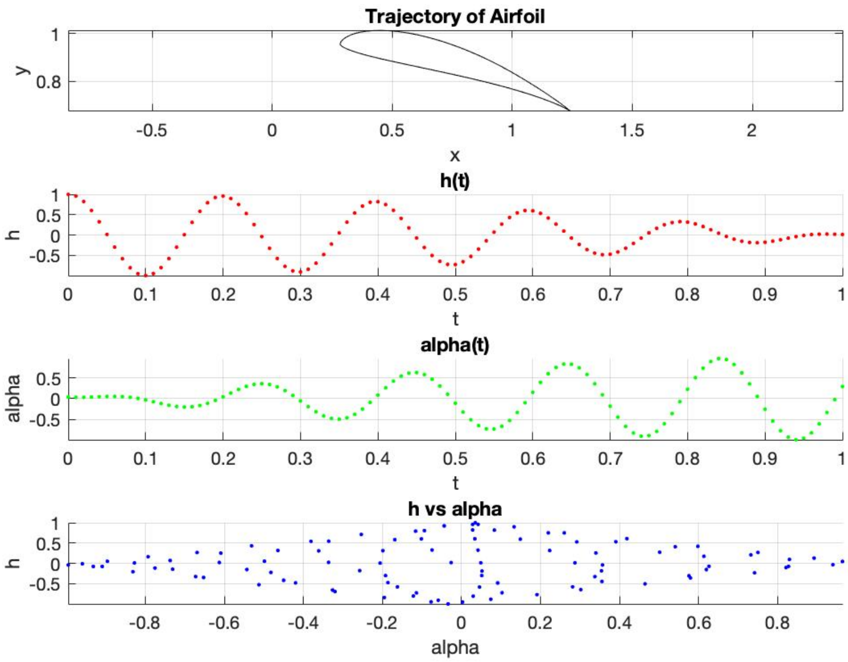

Figure 6: Case I: System that is stable.

3.2 Case II: An Unstable System

$m = 1$, $I_\alpha = 1$, $q = 0.1$, $S = 10$, $e = 0.1$, $S_\alpha = 0.1$, $K_\alpha = 1$, $K_h = 0$, Nonzero initial conditions. You can see that the structural parameters of the plane are such that, if there are any perturbations, the system will diverge away from the equilibrium.

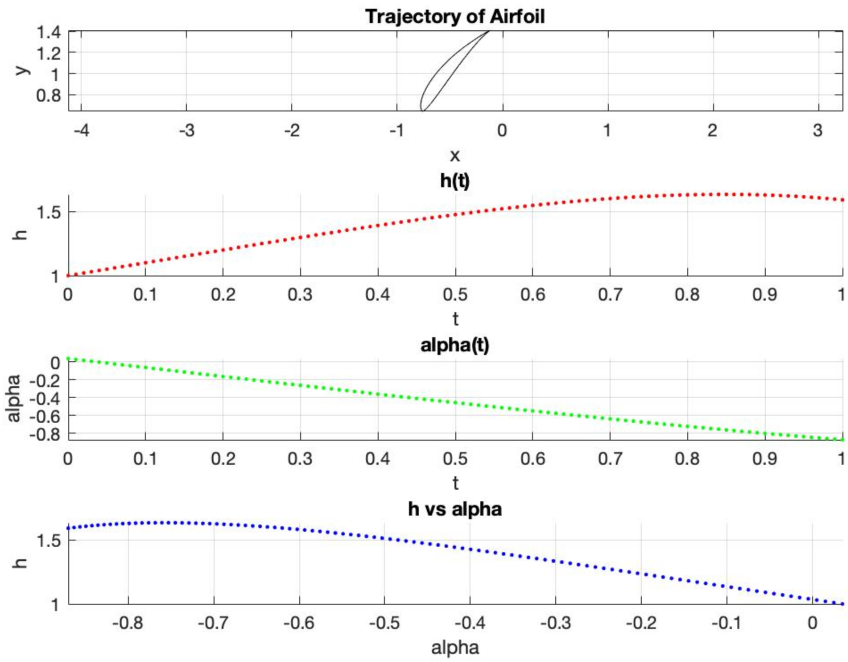

Figure 7: Case II: System in unstable motion.

3.3 Case III: A Neutrally Stable System, Wing Flutter

m= 1, $I_\alpha = 1$, $q = 0.1$, $S = 10$, e= 0.1, $S_\alpha = 0.1$, $K_\alpha = 1000$, $K_h = 100$, Nonzero initial conditions. You can see that the structural parameters of the plane are such that, if there are any perturbations, the system oscillates and does not return to equilibrium.

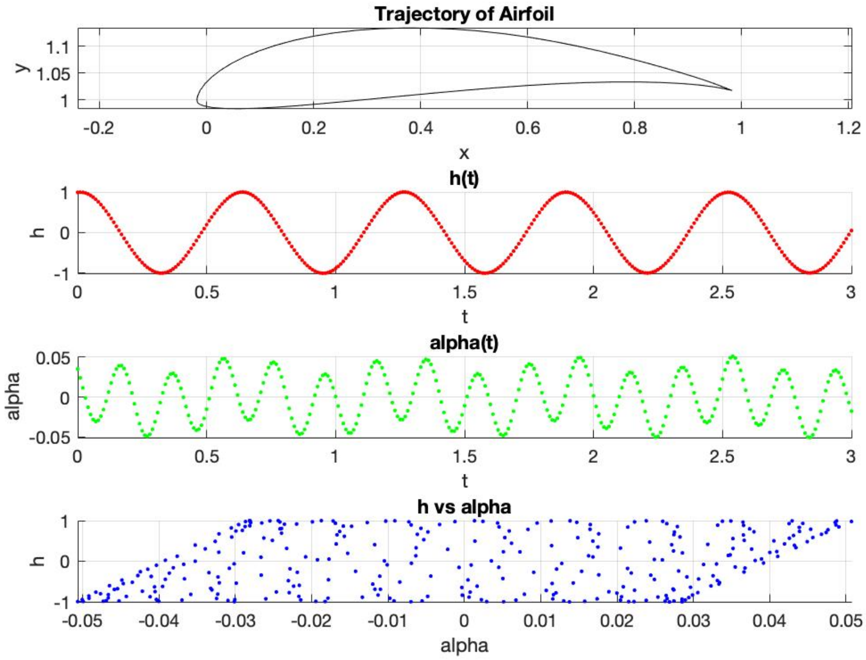

Figure 8: Case III: A system undergoing wing flutter!

3.4 Case IV: System with Zero-valued initial conditions

m= 1, $I_\alpha = 1$, $q = 0.1$, $S = 10$, e= 0.1, $S_\alpha = 0.1$, $K_\alpha = 1000$, $K_h = 100$

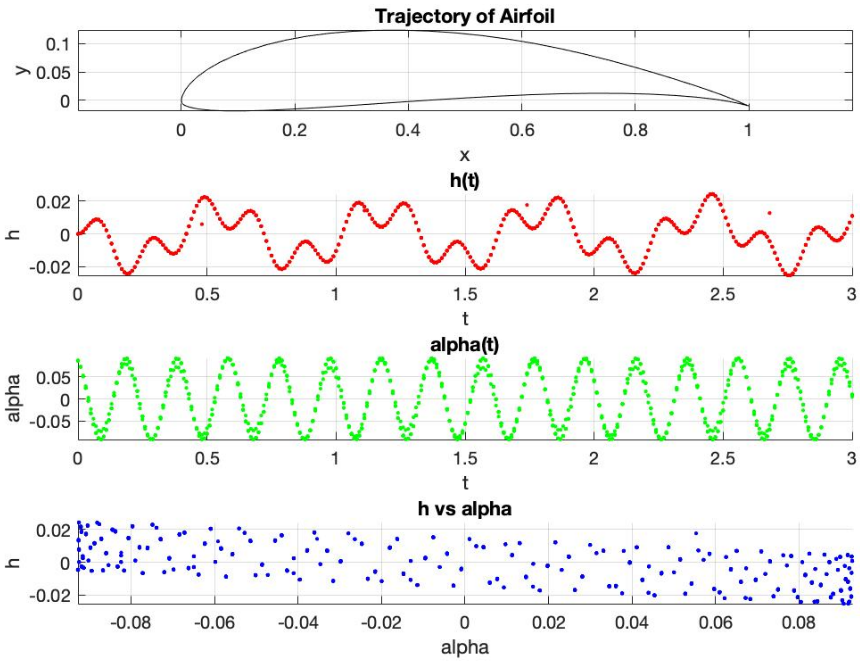

Figure 9: Case IV: A system with zero-valued initial conditions

4 Nonlinear Model

This model uses the same variables and constants as in the linear model. However, it does not use small angle approximations and it takes into account the relative motion of the wing to the airflow.

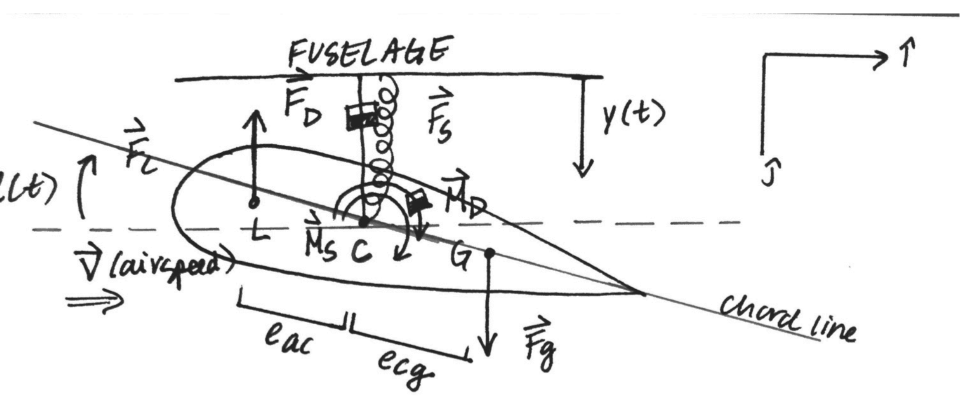

Figure 10: Nonlinear model (same as before)

Loads on airfoil:

- $\vec{F_D} = -C_y \dot{y} \hat{j}$; linear dashpot force

- $\vec{F_S} = -K_y y\hat{j}$; linear spring force
- $\vec{F_L}$ = lift force acting at L in the $\hat{j}$ direction
  - where S is the wing area
  - $\frac{\partial C_L}{\partial \alpha}\alpha$ is 2π for an ideal airfoil
  - and $q = \frac{1}{2}\rho(v^2 + \dot{y}^2)$ where v is taken as the airspeed and ρ the air density; the flow velocity also depends on the vertical velocity of the wing relative to the air as we explain later.
- $\vec{F_g} = -mg\hat{j}$; gravity acting at G in the $\hat{j}$ direction
- $\vec{M_D} = C_\alpha \dot{\alpha}\hat{k}$; torsional dashpot force
- $\vec{M_S} = K_\alpha \alpha\hat{k}$; torsional spring force

Linear momentum balance (sum of the forces) gives us:

$$-\vec{F_L}\hat{j} - K_h y\hat{j} - C_h y\hat{j} + mg\hat{j} = m\ddot{x}\hat{i} + m\ddot{y}\hat{j} + S_\alpha\ddot{\alpha}\hat{j} \tag{18}$$

Dotting (18) in the $\hat{i}$ and $\hat{j}$ directions and plugging in $\vec{F_L} = qS\frac{\partial C_L}{\partial \alpha}\alpha$, we get

$$(18) \cdot \hat{i} \longrightarrow \ddot{x} = 0 \tag{19}$$

$$(18) \cdot \hat{j} \longrightarrow -qS\frac{\partial C_L}{\partial \alpha}\alpha - K_h y - c_h\dot{y} + mg = m\ddot{y} + S_\alpha\ddot{\alpha} \tag{20}$$

Angular momentum balance in the $\hat{k}$ direction shows:

$$\sum \vec{M_C} = \dot{\vec{H}} \tag{21}$$

$$F_L e_{ac} cos(\alpha_c) - K_\alpha\alpha - c_\alpha\dot{\alpha} + mge_{cg}cos(\alpha_c) = I_\alpha\ddot{\alpha} + S_\alpha\ddot{y} \tag{22}$$

where α_c is the angle between the chord and the x axis. Plugging the equation for lift, we get

$$qS\frac{\partial C_L}{\partial \alpha}\alpha e_{ac}cos(\alpha_c) - K_\alpha\alpha - c_\alpha\dot{\alpha} + mge_{cg}cos(\alpha_c) = I_\alpha\ddot{\alpha} + S_\alpha\ddot{y} \tag{23}$$

The values for q and α are calculated more subtly than in the linear model.

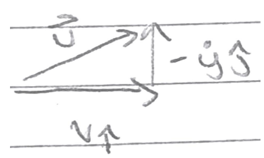

Figure 11: Flow Velocity

The flow velocity depends, not only on v, but also on the vertical velocity of the wing relative to the air. So,

$$q = \frac{1}{2}\rho|v\hat{\imath} - \dot{y}\hat{\jmath}|^2 = \frac{1}{2}\rho(v^2 + \dot{y}^2) \tag{24}$$

Further, α depends on α_c, the angle between the chord and the $\hat{\imath}$ direction, and θ, the angle between $\vec{u}$ (flow velocity) and $\hat{\imath}$.

$$\alpha = \alpha_c - \arctan\left(\frac{-\dot{y}}{v}\right) \tag{25}$$

Now that the equations of motion have been determined, $\ddot{y}$ and $\ddot{\alpha}$ need to be isolated so that the differential equations can be solved numerically. (18)$\frac{I_\alpha}{S_\alpha}$ − (23) gives us

$$\left(\frac{mI_\alpha}{S_\alpha} - S_\alpha\right)\ddot{y} = -qS\frac{\partial C_L}{\partial \alpha}\alpha\left(\frac{I_\alpha}{S_\alpha} + e_a c\cos\alpha_c\right) + mg\left(\frac{I_\alpha}{S_\alpha} - e_c g\cos\alpha_c\right) - \frac{I_\alpha}{S_\alpha}K_h y - \frac{I_\alpha}{S_\alpha}c_h\dot{y} + K_\alpha\alpha + c_\alpha\dot{\alpha} \tag{26}$$

Note that α depends on independent variables. Differentiating α gives

$$\dot{\alpha} = \dot{\alpha_c} + \frac{1}{1 + (\frac{\dot{y}}{v})^2}\frac{\ddot{y}}{v} \tag{27}$$

The second derivative of α is

$$\ddot{\alpha} = \ddot{\alpha}_c - \frac{2\dot{y}}{v^2((\frac{\dot{y}}{v})^2 + 1)^2}\left(\frac{\ddot{y}}{v}\right)^2 + \left(\frac{1}{1 + (\frac{\dot{y}}{v})^2}\right)\left(\frac{\dddot{y}}{v}\right) \tag{28}$$

For the sake of this analysis, we assume that $\dddot{y} = 0$. Now that $\ddot{y}$ can be found using (26), $\ddot{\alpha}$ can be found using (23). Simultaneously, we can calculate $\ddot{\alpha}_c$ using (28).

The following MATLAB code determines $\ddot{y}$, $\ddot{\alpha}$, and $\ddot{\alpha}_c$ for use with ODE45.

```matlab
function zdot = nonLinearFlutterRHS(t,z,p)
h = z(1); hd = z(2);
al = z(3); ald = z(4);
alc = z(5); alcd = z(6);

m = p.m; g = p.g; Kh = p.Kh; Ch = p.Ch; Ka = p.Ka; Ca = p.Ca; Ia = p.Ia;
CLa = p.CLa; S = p.S; e_ac = p.e_ac; Sa = p.Sa; rho = p.rho; v = p.v;
e_cg = p.e_cg;

q = (1/2)*rho*(v^2+hd^2); L = q*S*CLa*al; Mz = L*e_ac*cos(alc);

A = (m*Ia/Sa) - Sa; B = -L*((Ia/Sa)+e_ac*cos(alc));
C = m*g*((Ia/Sa)-e_cg*cos(alc)); D = -(Ia/Sa)*Kh*h;
E = -(Ia/Sa)*Ch*hd; F = Ka*al; G = Ca*ald;

hdd = (1/A)*(B+C+D+E+F+G);
aldd = (Mz-Ka*al-Ca*ald+m*g*e_cg*cos(alc)-Sa*hdd)/Ia;
alcdd = aldd + (2*hd/(v^2*(((hd/v)^2)+1)^2))*(hdd/v)^2;

```

```
zdot = [hd;hdd;ald;aldd;alcd;alcdd];
end
```

The parameters are initially set as follows:

```
%% Inputs
% Geometry
p.b = 10; p.c = 1; p.S = p.b*p.c; p.e_ac = 0.1; p.e_cg = 0.1;

% Properties
p.m = 1; p.g = 9.81; p.Kh = 100; p.Ka = 1000; p.Ch = 0; p.Ca = 0;
p.Ia = 1; p.Sa = p.m*p.e_cg;

% Aerodynamics
p.CLa = 2*pi; p.rho = 1.225; p.v = 1;
```

The differential equations are computed using ODE45.

```
%% Solve
tstart = 0; tend = 4; npointspers = 100;
ntimes = tend*npointspers+1; % total number of time points
t = linspace(tstart,tend,ntimes);

h0 = 0.1; hd0 = 0.1; alc0 = 5*pi/180; alcd0 = -1;
z0 = getZ0(h0,hd0,alc0,alcd0,p);

% ODE45
small = 1e-7;
options = odeset('RelTol', small, 'AbsTol', small);
f = @(t,z) nonLinearFlutterRHS(t,z,p);
[t,z] = ode45(f, t, z0, options);

h = z(:,1); hd = z(:,2); al = z(:,3); ald = z(:,4);
alc = z(:,1); alcd = z(:,2);

minh = min(h); maxh = max(h);
minal = min(al); maxal = max(al);
minalc = min(alc); maxalc = max(alc);
```

5 Case Studies using the Nonlinear Model

Our linear model is an example of a classic eigenvalue problem. It was possible for us to find the critical ratios of parameters needed to find the roots of our characteristic equation. The critical ratios were the conditions needed for a neutrally stable, unstable, or wing "fluttering" system. However, the nonlinear model poses complications. There is no way to analytically find these ratios and the roots of the system. Instead, we used MATLAB to numerically vary the parameters m, C_y, K_y,C_α, K_α, I_α, over nonzero values and looked at the output plots for any patterns. The nonlinear model gave the following response:

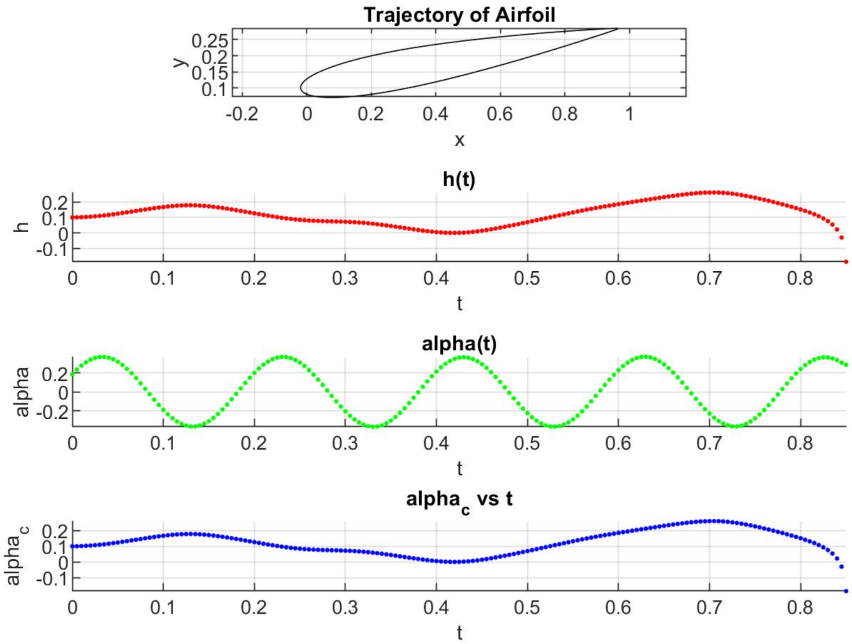

Figure 12: Nonlinear Model, All Plots

5.1 Case I: A Stable System

m= 0.5, g=9.81, $I_\alpha = 1$, $q = 0.1$, $S = 10$, e= 0.1, $S_\alpha = 0.1$, $K_\alpha = 1000$, $K_h = 1000$, $C_h = 0$, $C_\alpha = 0$, $I_\alpha = 1$; S_α= me_{cg}. Nonzero initial conditions. You can see that the structural parameters of the plane are such that, if there are any perturbations, the system returns to equilibrium.

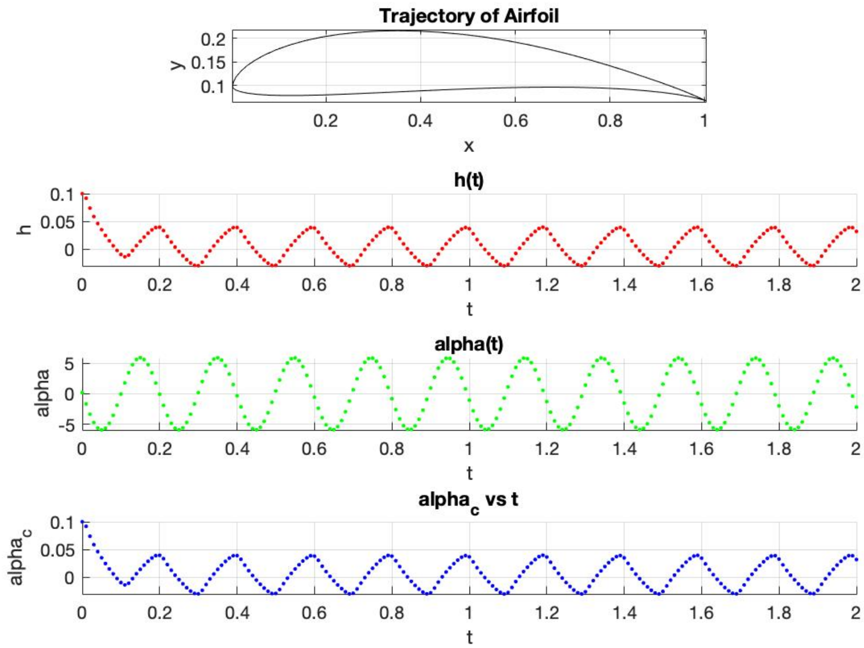

Figure 13: Case I: System is Stable.

5.2 Case II: An Unstable system

m= 10, g=9.81, $I_\alpha = 1$, $q = 0.1$, $S = 10$, e= 0.1, $S_\alpha = 0.1$, $K_\alpha = 1000$, $K_h = 100$, $C_h = 0$, $C_\alpha = 0$, $I_\alpha = 1$; S_α= me_{cg}. Nonzero initial conditions. You can see that the structural parameters of the plane are such that, if there are any perturbations, the system will diverge away from the equilibrium.

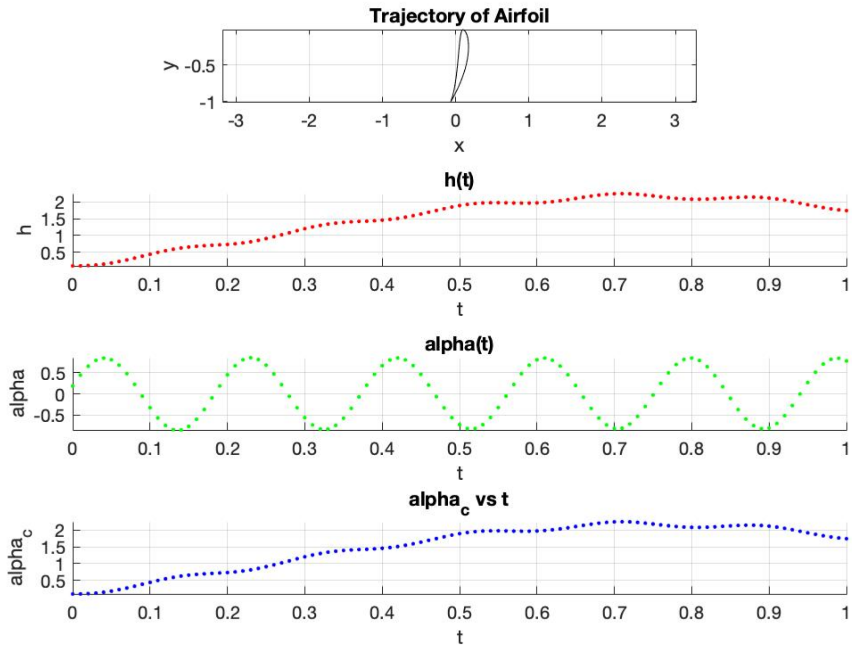

Figure 14: Case II: System in unstable motion.

5.3 Case III: A Neutrally Stable System, Wing Flutter

m= 1, g=9.81, $I_\alpha = 1$, $q = 0.1$, $S = 10$, e= 0.1, $S_\alpha = 0.1$, $K_\alpha = 1000$, $K_h = 1000$, $C_h = 0$, $C_\alpha = 0$, $I_\alpha = 1$; S_α= me_{cg}.

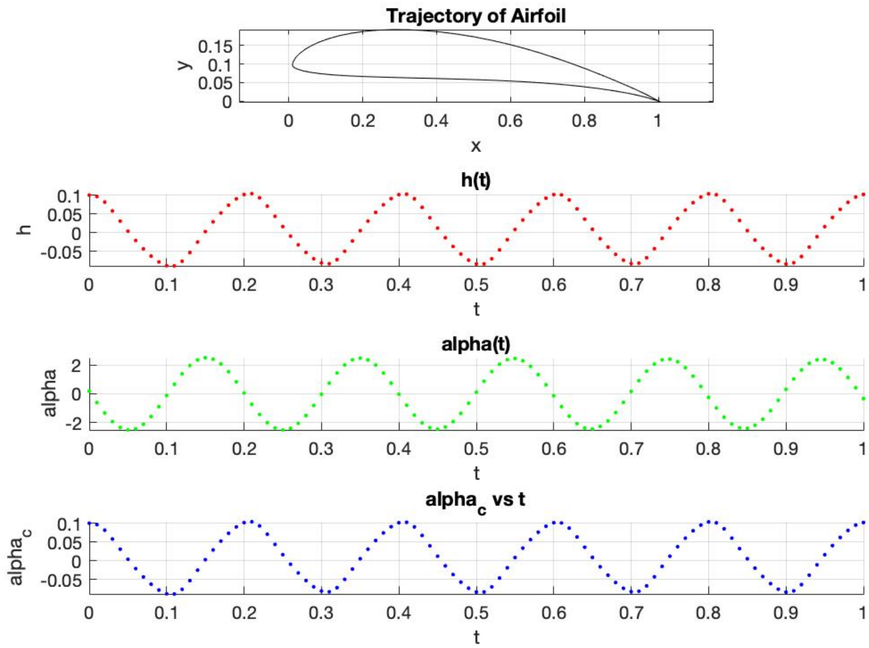

Figure 15: Case III: Wing flutter!

This case is particularly interesting. Wing flutter occurs for both the linear and nonlinear models under the same conditions. This suggests that the assumptions made in the linear model are generally valid since they appear to have little effect in the nonlinear model. The amplitude of the motion, however, does differ significantly.

6 Discussion

After running several test cases using the nonlinear model, we were able to see similar responses as the linear model under similar load conditions. This tells us that for small α, the small angle approximation is valid to make. In addition, it was reasonable to assume that $q = \frac{1}{2}\rho v^2$ instead of $q = \frac{1}{2}\rho(v^2 + \dot{y}^2)$

7 References

- Aeroelasticity. Wikipedia, Wikimedia Foundation, 10 May 2019, en.wikipedia.org/wiki/Aeroelasticity.
- Dimitriadis, G. Aeroelasticity and Experimental Aerodynamics. Universite de Liege, Universite De Liege.
- Dowell, Earl H. A Modern Course in Aeroelasticity. Springer, 2015.