سوال ۴

$$\lim_{n\to\infty} -\frac{1}{n}\log P\left[\frac{1}{n}\sum x_i \geq \delta\right] = \inf_{E_Q \geq \delta} D(Q\|P) = \overbrace{\sup_{\lambda \geq 0}\left[\lambda\delta - \psi_T(\lambda)\right]}^{\psi_T^*(\lambda)} \quad \text{I}$$

در جبهت large deviation در کلاس اثبات شد. (5

$$\underset{\hookrightarrow H_0}{P}[\hat{H}=1] = P\left[\sum \log \frac{P}{Q} \geq n\delta\right] \leq \overset{\text{constant}}{C_1}\exp(-n\psi_P^*(\delta))$$

$$\underset{\hookrightarrow H_1}{Q}[\hat{H}=0] = Q\left[\sum \log\frac{P}{Q} > n\delta\right] \leq C_2\exp(-n\psi_Q^*(\delta))$$

$$\psi_Q^*(\delta) = \sup_\lambda \lambda\delta - \psi_Q(\lambda) \rightarrow \psi_P^*(\delta) = \sup_\lambda \lambda\delta - \psi_P(\lambda)$$

$$\psi_Q(\lambda) = \log E_Q\left[e^{\lambda\log\frac{P}{Q}}\right] = \log\sum_Q e^{\lambda\log\frac{P}{Q}} Q(x) = \log\sum_Q e^{\lambda\log\frac{P}{Q}} \frac{Q(x)}{P(x)}P(x)$$

$$= \log\sum_P e^{(\lambda+1)\log\frac{P}{Q}} P(x) = \log E_P\left[e^{(\lambda+1)\log\frac{P}{Q}}\right] \Rightarrow \psi_Q(\lambda) = \psi_P(\lambda+1) \quad \text{I}$$

$\Rightarrow$ ~~$\psi_Q^*(\delta) = \sup_\lambda \lambda\delta - \psi_Q(\delta)$, $\psi_P^*(\delta) = \sup_\lambda (\lambda+1)\delta - \psi_P(\delta)$~~ ~~$\lambda^* = \operatorname{Arg}\psi_Q^*(\delta)$~~ → درسوال T حساب اثبات شده.

$$\Rightarrow \psi_Q^*(\delta) - \psi_P^*(\delta) = \delta \quad \text{II}$$

Sanov theorem for $n\to\infty$

$$P[\hat{H}=1] = C_1\exp(-nE_0) \leq C_1'\exp(-n\psi_P^*(\delta)) \Rightarrow E_0 = \psi_P^*(\delta) = \psi_Q^*(\delta) - \delta$$

$$Q[\hat{H}=0] = C_2\exp(-nE_1) \leq C_2'\exp(-n\psi_Q^*(\delta)) \Rightarrow E_1 = \psi_Q^*(\delta)$$

~~$\Rightarrow \lim_{n\to\infty} -\frac{1}{n}\log \frac{1}{2}(P[\hat{H}=1] + Q[\hat{H}=0]) \ldots$~~

$$\Rightarrow -\lim \frac{1}{n}\log\left(\underbrace{P[\hat{H}=1] + Q[\hat{H}=0]}_{\Rightarrow \exp(E_0)}\right)\times\frac{1}{2} \leq -\lim\frac{1}{n}\log\left[\sup_\delta \exp(-n\psi_P^*(\delta))\right]$$

در حالت $n\to\infty$

$\frac{1}{n}\log\frac{1}{2} \to 0$

از آن صرف نظر می کنیم.

$$\Rightarrow -\lim_{n\to\infty} \frac{1}{n}\log P_{error} \leq -\lim_{n\to\infty}\frac{1}{n}\log \sup_{\delta} \exp(-n(\psi^{*}_{Q}-\delta))$$

$$\leq -\lim \frac{1}{n}\log \inf_{\delta} \exp(-n(\psi^{*}_{Q}-\delta)) \leq -\lim_{n\to\infty}\frac{1}{n}\log \exp(-n\psi^{*}_{Q}(\delta=0))$$

$$= -\inf_{\lambda}\psi_{Q}(\lambda) = -\inf_{\lambda}\log E[e^{\log\frac{p}{q}\lambda}] = -\inf_{\lambda}\log E\left[\left(\frac{p}{q}\right)^{\lambda}\right]$$

$$= \sup_{\lambda}\{-\log E[(\tfrac{p}{q})^{\lambda}]\} = \max_{s}\{-\log E[(\tfrac{p}{q})^{\lambda}]\}$$

$$\Rightarrow -\lim_{n\to\infty}\frac{1}{n}\log P_{error} \leq \max_{s}\{-\log E[(\tfrac{p}{q})^{\lambda}]\} \quad 1$$

Subject

کران بست استفاده

$$\frac{1}{2}\left[P[\hat{H}=1]+Q[\hat{H}=0]\right]=\frac{1}{2}\left[\sum z(1|x)P(x)+\sum z(0|x)q(x)\right]$$

ML

$$=\frac{1}{2}\sum \min(P(x),q(x))$$

$$\boxed{\min(\alpha,\beta)\le \alpha^{\lambda}\beta^{1-\lambda} \quad \forall \lambda \geq 0}$$

برای n تا:

$$\frac{1}{2}P[\hat{H}=1]+Q[\hat{H}=0]\le \prod_{i=1}^{n}\sum \min\left(\frac{P(x)}{2},\frac{q(x)}{2}\right)\le \prod_{i=1}^{n}\sum P(x)^{\lambda}q(x)^{1-\lambda} \quad \forall\lambda\geq 0$$

$$=\left(\sum P(x)^{\lambda}q(x)^{1-\lambda}\right)^{n}\le \inf_{\lambda}\left(\sum \left(\frac{P(x)}{q(x)}\right)^{\lambda}q(x)\right)^{n}\Rightarrow$$

$\forall\lambda\geq 0$

$$-\frac{1}{n}\log P_{error}\geq -\frac{1}{\cancel{n}}\inf_{\lambda}\cancel{n}\log\left(\sum\left(\frac{P(x)}{q(x)}\right)^{\lambda}q(x)\right)=-\inf_{\lambda}\left\{\log E_{Q}\left[\left(\frac{P(x)}{q(x)}\right)^{\lambda}\right]\right\}$$

$$-\frac{1}{n}\log P_{error}\geq \max_{\lambda}\left\{-\log E_{Q}\left[\left(\frac{P(x)}{q(x)}\right)^{\lambda}\right]\right\}$$

$$\Rightarrow -\frac{1}{n}P_{error}\geq \max_{s}\left\{-\log E_{Q}\left[\left(\frac{P(x)}{q(x)}\right)^{s}\right]\right\} \quad \boxed{\checkmark}$$ (2)

(1,2) $\Rightarrow$ اثبات تمام $\boxed{\checkmark}$