سوال ۷

الف) $\pi_{011} \le 2^{-n.E_0}$ $\qquad E_0'' = \lambda E_0 + (1-\lambda) E_0'$

$\pi'_{011} \le 2^{-n.E_0'}$

$\Rightarrow \min(E_0,E_0') \le E_0'' \le \max(E_0,E_0') \Rightarrow -\min(E_0,E_0') \ge -E_0'' \ge -\max(E_0,E_0')$

$\Rightarrow 2^{-\min(E_0,E_0')} \ge 2^{-E_0''} \ge 2^{-\max(E_0,E_0')}$

$\Leftarrow$ نرخ E_0'' قابل دسترسی است.

برای π_{110} نیز به همین شیوه قابل اثبات است.

$\left.\begin{array}{l} \pi_{110} \le 2^{-n.E_1} \\ \pi'_{110} \le 2^{-n.E_1'} \end{array}\right\} \Rightarrow \pi_{110} \le 2^{-n.E_1''}$

ب) مطابق 16.1 در کتاب داریم:

$\pi_{110} = 1-\alpha \le \exp(-nE_0)$

$\pi_{011} = \beta \le \exp(-nE_1)$

پایه را 2 بگیریم

$E_1^*(E_0) = \sup\{E_1 , \exists n_0 , \forall n \ge n_0 \; \exists \overset{\text{test}}{Z} \; \alpha \ge 1-2^{-nE_0} , \beta \le 2^{-nE_1}\}$

$= \liminf_{n\to\infty} \frac{1}{n}\log \frac{1}{\beta_{1-2^{-nE_0}}(P^n, Q^n)}$ $\rightarrow$ 14.12 کتاب

$T_k = \log\frac{dP}{dQ}(x_k) \rightsquigarrow \log \frac{dP^n}{dQ^n} x = \sum T_i \qquad T_i = \log\frac{dP_i}{dQ_i}$

$\psi_Q(\lambda) = \log E_Q[2^{\lambda T}] = \log \sum \left(\frac{P(x)}{q(x)}\right)^{\lambda} q(x) = \log\sum P(x)^{\lambda} Q(x)^{1-\lambda}$

$\psi_P(\lambda) \overset{(1)}{=} \log\sum P(x)^{1-\lambda} Q(x)^{\lambda}$

$\psi_Q(\lambda) = \log\sum 2^{\lambda T - T} P(x) = \psi_P(\lambda-1) \Rightarrow \psi_Q(\lambda) = \psi_P(\lambda-1)$ ٤

$$\Rightarrow \psi_P^* = \sup_\lambda (\theta\lambda - \psi_P(\lambda)) \quad \text{II}$$ مشابه این بخش در سوال 4 نیز بوده است.

$$\psi_Q^* = \sup_\lambda (\theta\lambda - \psi_Q(\lambda)) = \sup_\lambda (\theta\lambda - \psi_P(\lambda - 1)) = \psi_P^*(\theta) + \theta$$

$\tau = +n\theta \rightarrow$ Neyman Pearson. مطالب 15.9 large deviation

Achievability ~~$\pi_{0|1} = Q[\sum T_k \geq n\theta]$~~

$\pi_{0|1} = P[\sum T_k < n\theta]$

$\pi_{1|0} = Q[\sum T_k \geq n\theta]$

$\mapsto$ large deviation

for any n: $\pi_{0|1} = P[\sum T_k < n\theta] \leq \exp(-n\psi_P^*(\theta))$ ①

$\pi_{1|0} = Q[\sum T_k \geq n\theta] \leq \exp(-n\psi_Q^*(\theta))$ ②

در اینجا باند θ باید تقریبا به مطابق بخش 15.9 باند θ بصورت زیر است.

① $\rightarrow \theta \geq -D(Q\|P)$

$\theta \leq D(P\|Q) \Rightarrow -D(Q\|P) \leq \theta \leq D(P\|Q)$

دقت کنید که $T = \log \frac{P}{Q}$ است و در کتاب $T = \log \frac{Q}{P}$ بوده است بنابراین در اینجا

جای P و Q عوض شده و باند $-D(Q\|P) \leq \theta \leq D(P\|Q)$ حاصل.

①,② $\Rightarrow E_0(\theta), E_1(\theta)$ are achievable.

①,②,I,II $\Rightarrow \psi_Q^* = \psi_P^* + \theta$

تاریخ:

$\beta - \frac{1}{\gamma}\alpha \leq Q[\cancel{B_j}\sum T_i < \cancel{\log \gamma}^{+n\theta}]$

$\beta \leq \frac{1}{\gamma_e^{n\alpha}}\alpha + Q[\sum T_i < n\theta] \Rightarrow$

$2^{-n\gamma_Q^*(\theta)+o(n)} \leq 2^{-n(E_0+\theta)} + 2^{-nE_1} \Rightarrow$

$\min(E_0+\theta, E_1) \leq \gamma_Q^* = \gamma_P^* + \theta \Rightarrow E_0 \leq \gamma_P^*$ یا $E_1 \leq \gamma_P^* - \theta$

⟸ با اندازه بزرگ داریم ⟸ $t_0 \beta$ است .

$P_e^* = \min_{test}(\pi_0 \pi_{1|0} + \pi_1 \pi_{0|1}) = \max_\theta(\pi_0 e^{-nE_0(\theta)} + \pi_1 e^{-nE_1(\theta)})$

$\cancel{\min_{test}\max} \max_\theta \min_{Z(n|x)} (\pi_0 e^{-nE_0(\theta)} + \pi_1 e^{-nE_1(\theta)})$

$\cancel{E = \lim \frac{1}{n}\log\frac{1}{P_e^*} = \max_\theta \max_{Z(n|x)} (\pi_0 e^{-nE_0} + \pi_1 e^{-nE_1})} \Rightarrow$

$\cancel{\max_\theta \max_{Z(n|1)}}$

براساس Carally داریم

$1.6.2$

$E = \max_\theta \min(E_0(\theta), E_1(\theta))$

$E = \max_\theta \min(\gamma_P^*(\theta), \gamma_P^* + \theta) = \begin{cases} \max_{\theta \geq 0} \gamma_P^*(\theta) \\ \max_{\theta < 0} \gamma_P^*(\theta) + \theta \end{cases}$

$\theta - \gamma_P^*(\lambda) = 0$

$\gamma^*(\theta) = \sup_\lambda \lambda\theta - \gamma_P(\lambda)$ $\quad \frac{\partial \gamma^*(\theta)}{\partial \lambda} = \gamma_P'(\lambda)$

$\theta = \gamma_P'(\lambda)$

$$\psi_P'(\theta) = \frac{E[e^{\lambda T} T]}{E[e^{\lambda T}]} \xRightarrow{\lambda=0} D(P||Q)$$

$$\lambda = 0$$

از طرفی از قبل دیدیم که نرخ بهینه $D(P||Q)$ است $\Leftarrow$

$$\psi_P'(\theta) \overset{\lambda=0}{=} -\psi^*(\lambda)$$

$$\theta \preceq D(P||Q)$$

$$\Rightarrow \cancel{\theta = \psi_P' < \theta <} \Rightarrow E^* = \max_{\theta} \min(E_0(\theta), E_1(\theta)) = \psi_P^*(\theta)$$