Sharif University of Technology

FACULTY Computer Engineering

Signals & Systems

Computer Assignment 5

Laplace & Z Transform

Student Name	**Student ID**
AmirHossein Bagheri	98105612

Instructor:

Prof. Hossein Sameti

Contents

1 Introduction **2**

2 Laplace transform **2**

2.1 a 2
2.2 b 3
2.3 c 3
2.4 d 6

3 Z transform **8**

3.1 a 8
3.2 b 13
3.2.1 1) 13
3.2.2 2) 16
3.2.3 3) 17

1 Introduction

If any argument is an array, then ilaplace acts element-wise on all elements of the array.

If the first argument contains a symbolic function, then the second argument must be a scalar.

To compute the direct Laplace transform, use laplace.

For a signal f(t), computing the Laplace transform (laplace) and then the inverse Laplace transform (ilaplace) of the result may not return the original signal for t ¡ 0. This is because the definition of laplace uses the unilateral transform. This definition assumes that the signal f(t) is only defined for all real numbers t 0. Therefore, the inverse result does not make sense for t ¡ 0 and may not match the original signal for negative t. One way to correct the problem is to multiply the result of ilaplace by a Heaviside step function. For example, both of these code blocks:

2 Laplace transform

2.1 a

```
syms t
f1 = t*heaviside(t-1);
f2 = sin(t)*exp(-4*t) * heaviside(t);
f3 = 2*t*cos(3*t)*heaviside(t);
laplace(f1)
laplace(f2)
laplace(f3)

```

1)

$$f_1(t) = tu(t-1)$$

$$\mathcal{L}\{u(t-1)\} = e^{-s}\frac{1}{s} \implies \mathcal{L}\{f_1\} = -\frac{\partial e^{-s}\frac{1}{s}}{\partial s} = \frac{e^{-s}}{s} + \frac{e^{-s}}{s^2}$$

matlab result :

$$\frac{\mathrm{e}^{-s}}{s} + \frac{\mathrm{e}^{-s}}{s^2} \checkmark$$

2)

$$f_2(t) = sint(t)e^{-4t}u(t)$$

$$\mathcal{L}\{sint(t)u(t)\} = \frac{1}{s^2+1} \implies \mathcal{L}\{f_2\} = \frac{1}{(s+4)^2+1}$$

matlab result :

$$\frac{1}{(s+4)^2+1} \checkmark$$

3)

$$f_3(t) = 2tcos(3t)u(t)$$

$$F`(s) = \mathcal{L}\{cos(3t)u(t)\} = \frac{s}{s^2+9} \implies \mathcal{L}\{f_3\} = -2\frac{\partial \frac{s}{s^2+9}}{\partial s} = \frac{2s^2-18}{(S^2+9)^2}$$

matlab result :

$$\frac{4\,s^2}{\left(s^2+9\right)^2}-\frac{2}{s^2+9}=\frac{2s^2-18}{(s^2+9)^2}\checkmark$$

2.2 b

```
syms s
F1 = (1/(s*(s+1)))*exp(-3*s);
F2 = 4/(s*(s^2 +4));
F3 = 1/(s^2 + 3 * s + 1);
ilaplace(F1)
ilaplace(F2)
ilaplace(F3)

```

1)

$$F1(s)=e^{-3s}\frac{1}{s(s+1)}=e^{-3s}(\frac{1}{s}-\frac{1}{s+1})$$

$$\mathcal{L}^{-1}\{\frac{1}{s}-\frac{1}{s+1}\}=(1-e^{-t})u(t)\implies\mathcal{L}^{-1}\{F1\}=(1-e^{-(t-3)})u(t-3)$$

matlab result :

$$-\mathrm{heaviside}\,(t-3)\,\left(\mathrm{e}^{3-t}-1\right)\checkmark$$

2)

$$F2(s)=\frac{4}{s(s^2+4)}=4(\frac{-s}{s^2+4}+\frac{1}{s})$$

$$\mathcal{L}^{-1}\{F2\}=4(1-cos(2t))u(t)$$

matlab result :

$$1-\cos\left(2\,t\right)\checkmark$$

3)

$$F3(t)=\frac{1}{(s+\frac{3}{2})^2+(\frac{\sqrt{5}j}{2})^2}=$$

$$\mathcal{L}^{-1}\{\frac{1}{(s)^2+(\frac{\sqrt{5}j}{2})^2}\}=\frac{2\sqrt{5}}{5}\sinh(\frac{\sqrt{5}}{2}t)u(t)\implies\mathcal{L}^{-1}\{F2\}=\frac{2\sqrt{5}}{5}e^{-\frac{3}{2}t}sinh(\frac{\sqrt{5}}{2}t)u(t)$$

matlab result :

$$\frac{2\,\sqrt{5}\,\mathrm{e}^{-\frac{3\,t}{2}}\,\sinh\left(\frac{\sqrt{5}\,t}{2}\right)}{5}\checkmark$$

2.3 c

$$G(s)=\frac{8}{s^2+s+4}=8\frac{2}{\sqrt{15}}\frac{\frac{\sqrt{15}}{2}}{(s+\frac{1}{2})^2+(\frac{\sqrt{15}}{2})^2}$$

$$g(t)=\mathcal{L}^{-1}\{G\}=\frac{16}{\sqrt{15}}e^{-\frac{1}{2}t}u(t)\sin(\frac{\sqrt{15}}{2}t)$$

1)
delta response:

$$g(t) = \frac{16}{\sqrt{15}} e^{-\frac{1}{2}t} \sin(\frac{\sqrt{15}}{2}t) u(t)$$

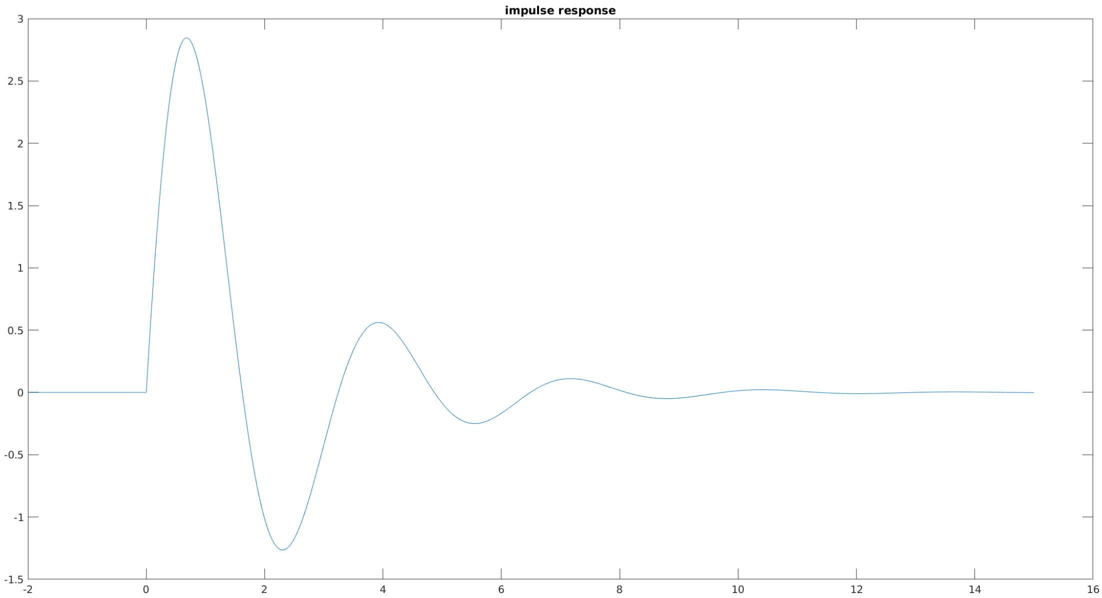

Figure 1: delta response

step response :

$$U(t) = \mathcal{L}^{-1}\{\frac{G(s)}{s}\} = g(t) * u(t) = \int_0^t \frac{16}{\sqrt{15}} e^{-\frac{1}{2}T} \sin(\frac{\sqrt{15}}{2}T) dT =$$

$$\frac{16\left(\frac{\sqrt{15}}{8} - \frac{\mathrm{e}^{-\frac{t}{2}}\left(\sin\left(\frac{\sqrt{15}t}{2}\right)+\sqrt{15}\cos\left(\frac{\sqrt{15}t}{2}\right)\right)}{8}\right)}{\sqrt{15}} u(t) =$$

$$2 - 2\,\mathrm{e}^{-\frac{t}{2}}\left(\cos\left(\frac{\sqrt{15}\,t}{2}\right) + \frac{\sqrt{15}\,\sin\left(\frac{\sqrt{15}\,t}{2}\right)}{15}\right) u(t)$$

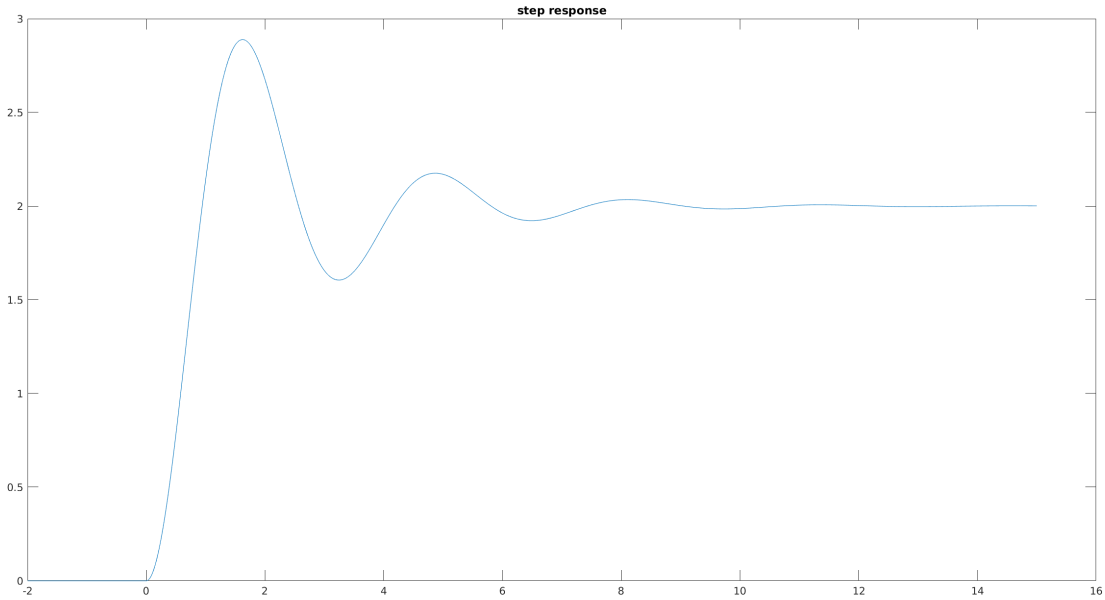

Figure 2: step response

2)

$$G(s) = \frac{8}{s^2+s+4} = 8\frac{1}{s^2+2*2*\frac{1}{4}s+2^2}$$
$$\zeta = \frac{1}{4} \implies under-dampped$$

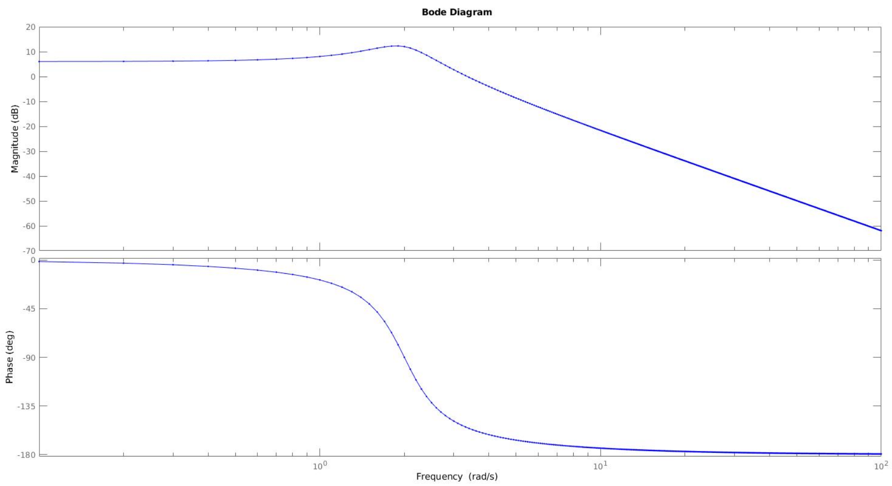

Figure 3: step response

2.4 d

$$G(s) = \frac{2s+1}{s^2+as+7} = \frac{2s+1}{(s+\frac{a}{2})^2 + (\sqrt{7-\frac{a^2}{4}})^2} \tag{1}$$

$$\begin{cases} 7-\frac{a^2}{4} > 0 & \propto e^{-\frac{a}{2}t}(\alpha cos(\sqrt{7-\frac{a^2}{4}}t) + \beta sin(\sqrt{7-\frac{a^2}{4}}t)) \\ 7-\frac{a^2}{4} < 0 & \propto e^{-\frac{a}{2}t}(\alpha cosh(\sqrt{7-\frac{a^2}{4}}t) + \beta sinh(\sqrt{7-\frac{a^2}{4}}t)) \end{cases} \tag{2}$$

a = 4

```
 syms s
G =(2*s+1)/(s^2 + 4*s + 7)
step_res = ilaplace(G*(1/s))
t = -1:0.001:10;
y2 = subs(step_res,t) .* (t>=0);
plot_fig(t,y2,"step response with a = 4")
clear t
syms t
step_res =  sym(1/7) - (exp((-2*t))*(cos(sqrt(sym(3))*t) - 4*sqrt
    (sym(3))*sin(sqrt(sym(3))*t)))/7
limit(step_res,t,Inf)
t= 0:0.001:2;
y = subs(step_res,t) .* (t>=0);
[val,index] = max(y);
display("maximum at = ");
display(t(index))
display("value of maximum is :")
display(double(val))

```

$$U(t) = \mathcal{L}^{-1}\{\frac{G_4(s)}{s}\} = \frac{1}{7} - \frac{e^{-2\,t}\left(\cos\left(\sqrt{3}\,t\right) - 4\,\sqrt{3}\,\sin\left(\sqrt{3}\,t\right)\right)}{7} u(t)$$

step response :

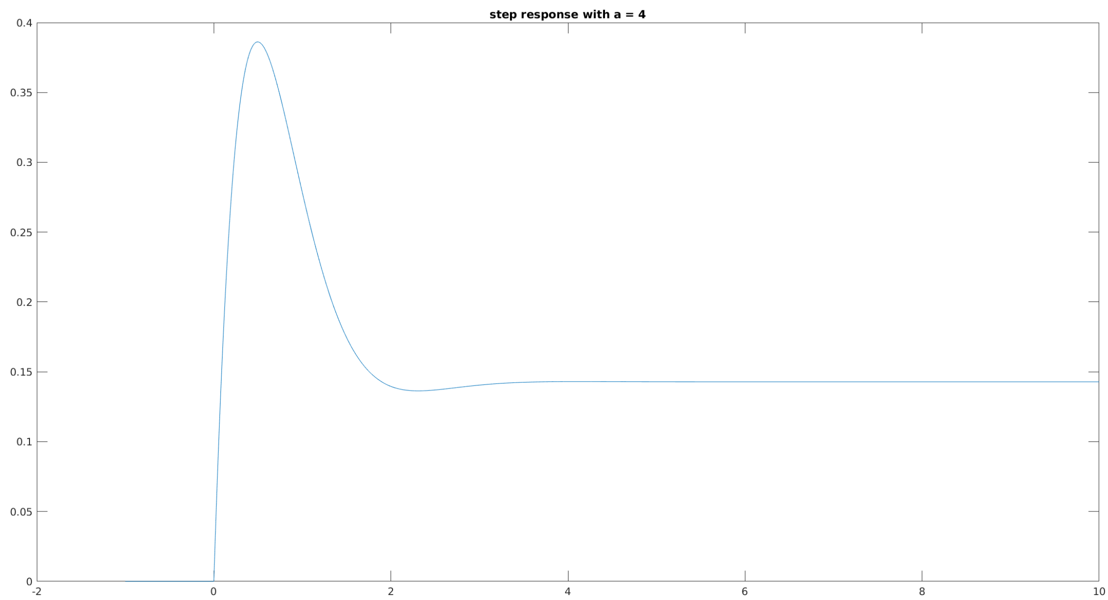

Figure 4: step response

limit to ∞ : $\frac{1}{7}$
maximum at 0.4950 **with value :** 0.3862
a = 6

```
syms s
G =(2*s+1)/(s^2 + 6*s + 7)
step_res = ilaplace(G*(1/s))
t = -1:0.001:10;
y2 = subs(step_res,t) .* (t>=0);
plot_fig(t,y2,"step response with a = 6")
b
clear t
syms t
step_res =  sym(step_res);
limit(step_res,t,Inf)
t= 0:0.001:2;
y = subs(step_res,t) .* (t>=0);
[val,index] = max(y);
display("maximum at = ");
display(t(index))
display("value of maximum is :")
display(double(val))

```

$$U(t) = \mathcal{L}^{-1}\{\frac{G_6(s)}{s}\} = \frac{1}{7} - \frac{e^{-3t}\left(\cosh\left(\sqrt{2}\,t\right) - \frac{11\sqrt{2}\sinh\left(\sqrt{2}\,t\right)}{2}\right)}{7}u(t)$$

step response :

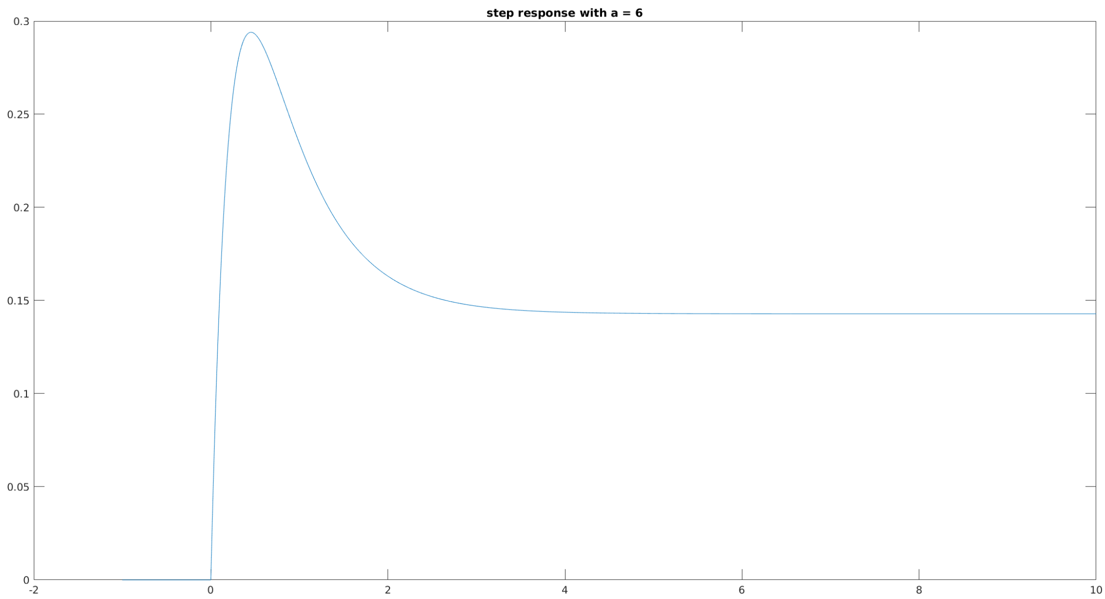

Figure 5: step response

limit to ∞ : $\frac{1}{7}$
maximum at 0.4530 **with value :** 0.2940

3 Z transform

3.1 a

1)

$$x[n] = (u[n+2] - u[n-1]) \times (-n+1) - (u[n-1] - u[n-3]) \times (n+1)$$
$$X(z) = \frac{2\,z - \frac{2}{z} - \frac{3}{z^2} + 3\,z^2 + 1}{z^2}$$

matlab results:

$$X(z) = \frac{2}{z} - \frac{\frac{z}{(z-1)^2} - \frac{3}{z-1} + \frac{2\left(\frac{1}{z-1}+1\right)}{z^3} + \frac{\frac{1}{z-1}+1}{z^5} - \frac{5\,z-4}{z^4\,(z-1)^2} - 3}{z^2} + 3$$

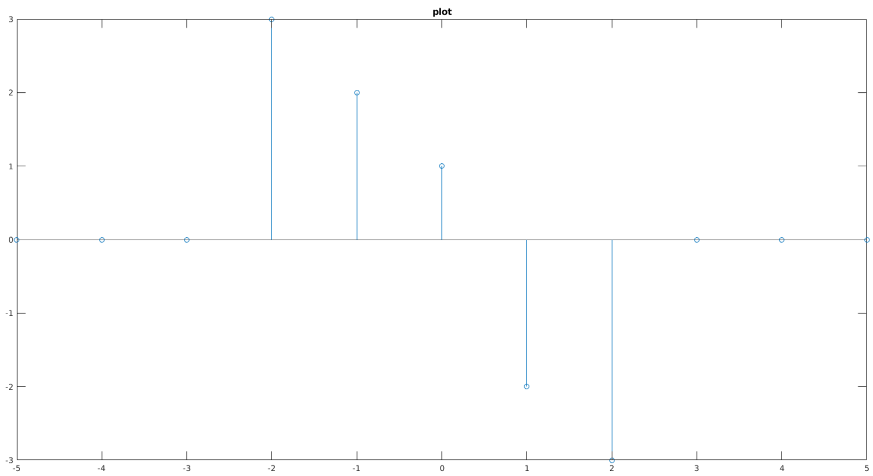

Figure 6: plot of function $(u[n+2]-u[n-1])\times(-n+1)-(u[n-1]-u[n-3])\times(n+1)$

ROC

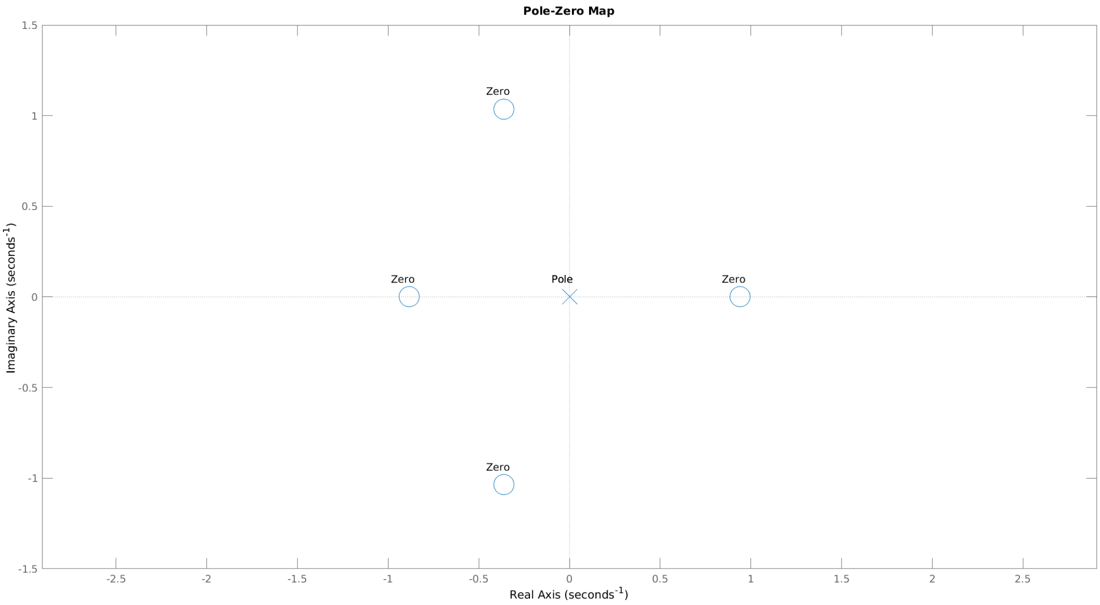

Figure 7: ROC of $\frac{2\,z-\frac{2}{z}-\frac{3}{z^2}+3\,z^2+1}{z^2}$

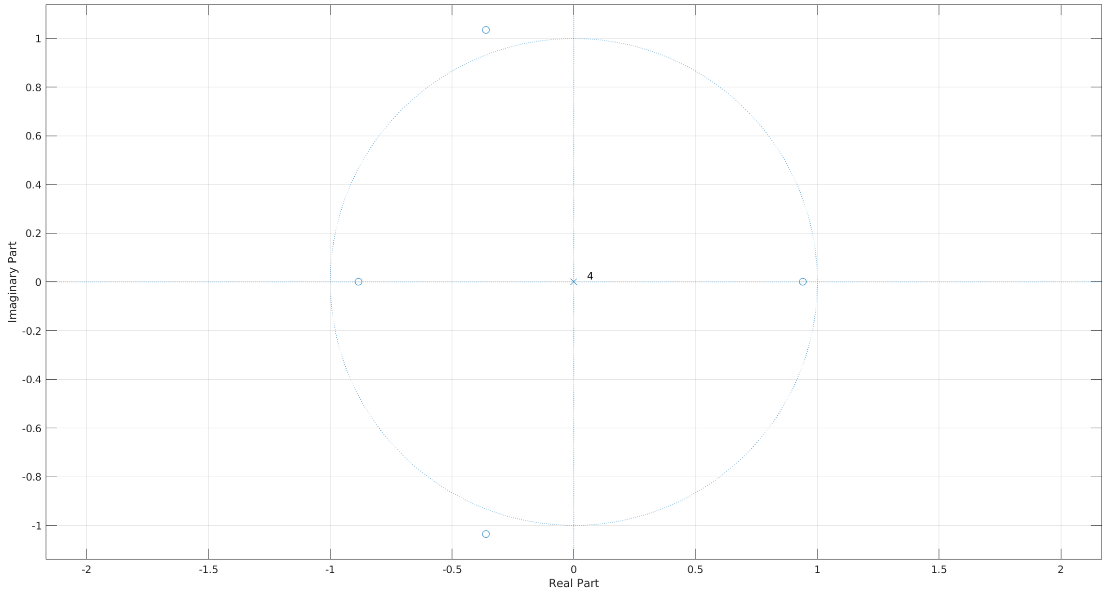

Figure 8: ROC of $\frac{2\,z-\frac{2}{z}-\frac{3}{z^2}+3\,z^2+1}{z^2}$

Signal is limited from both sides so:
all $|z| > 0$ are in ROC as you see in figure only pole is zero which is not important.
ROC : $|z| > 0$
all poles are in unit circle so system is stable
2)

$$x[n] = \left(\frac{4}{5}\right)^n \mathrm{u}\left(n-2\right)$$

$$X(z) = \frac{16}{25}\frac{1.2z}{1.2z^3 - z^2}$$

matlab results:

$$X(z) = \frac{16\left(\frac{1}{\frac{5\,z}{4}-1}+1\right)}{25\,z^2}$$

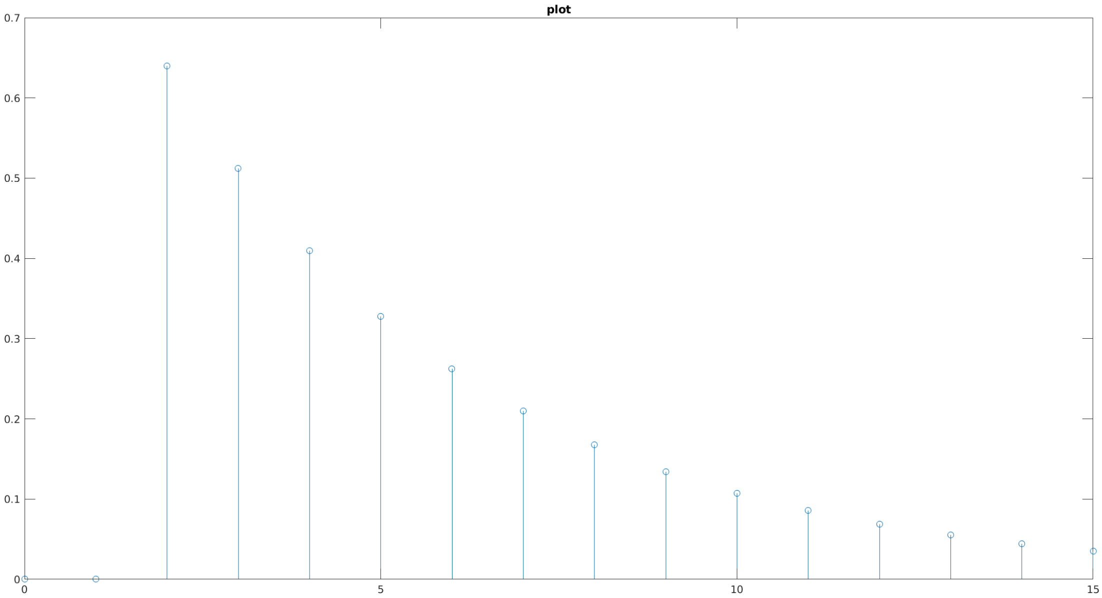

Figure 9: plot of function $\left(\frac{4}{5}\right)^n \mathrm{u}\,(n-2)$

ROC

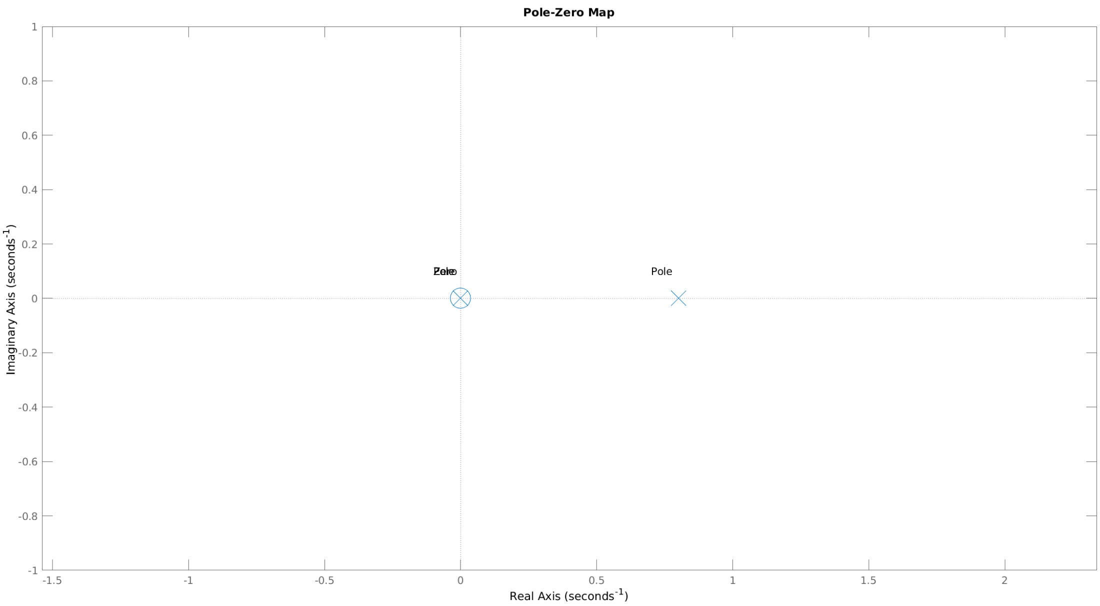

Figure 10: ROC of $\frac{16}{25}\frac{1.2z}{1.2z^3-z^2}$

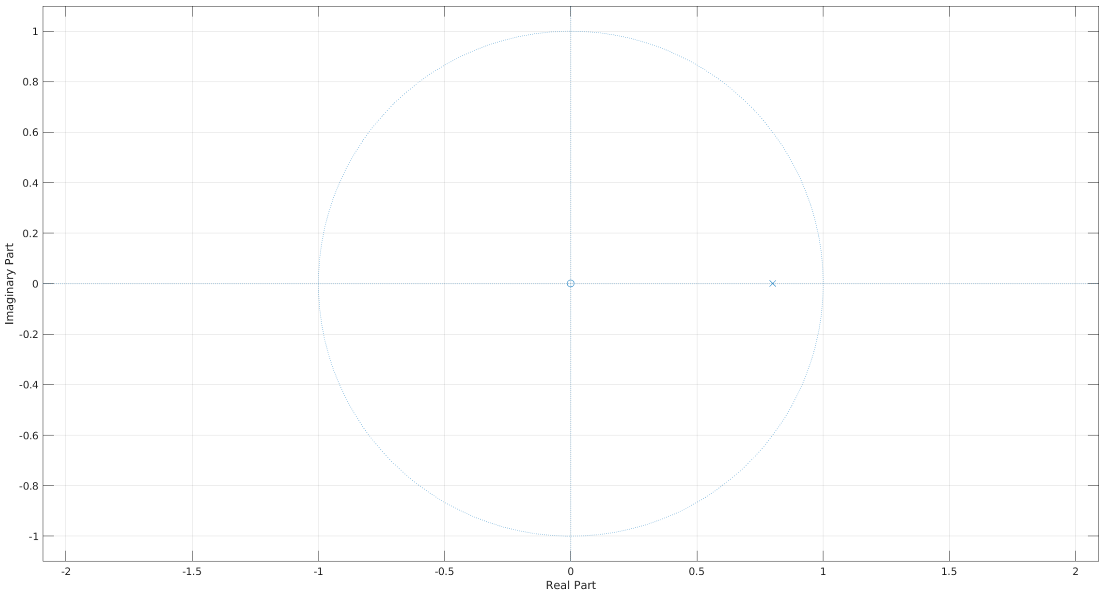

Figure 11: ROC of $\frac{16}{25}\frac{1.2z}{1.2z^3-z^2}$

Signal is right sided so:
biggest pole is $|z| = 0.8 \implies$ all $|z| > 0.8$ are in ROC as you see in figure
ROC : $|z| > 0.8$ all poles are in unit circle so system is stable
3)

$$x[n] = 2^n \cos\left(\frac{2\,\pi\,n}{5}\right)\ \mathrm{u}\,(n)$$

$$X(z) = \frac{z - \frac{347922205179541}{562949953421312}}{z^2 - \frac{347922205179541\,z}{281474976710656} + 4}$$

matlab results:

$$X(z) = \frac{2\,\mathrm{ztrans}\left(\cos\left(\frac{2\,\pi\,(n+1)}{5}\right), n, \frac{z}{2}\right)}{z} + 1$$

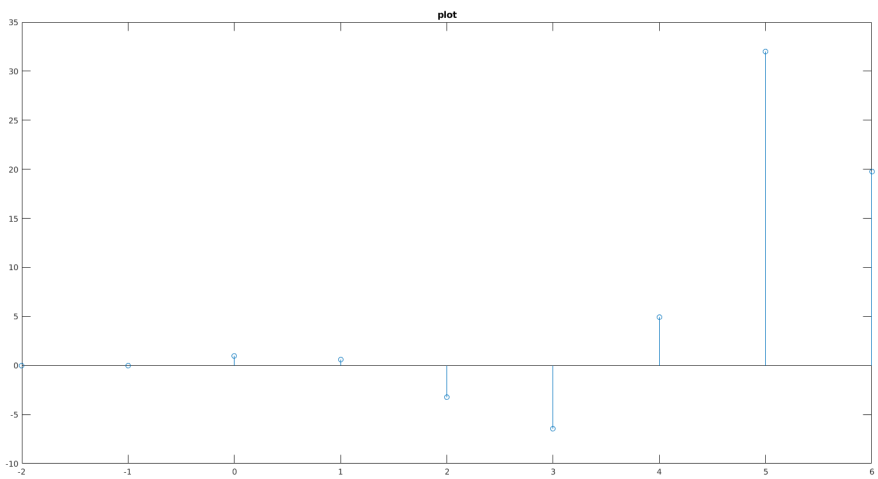

Figure 12: plot of function $2^n\,\cos\left(\frac{2\,\pi\,n}{5}\right)\ \mathrm{u}\,(n)$

ROC

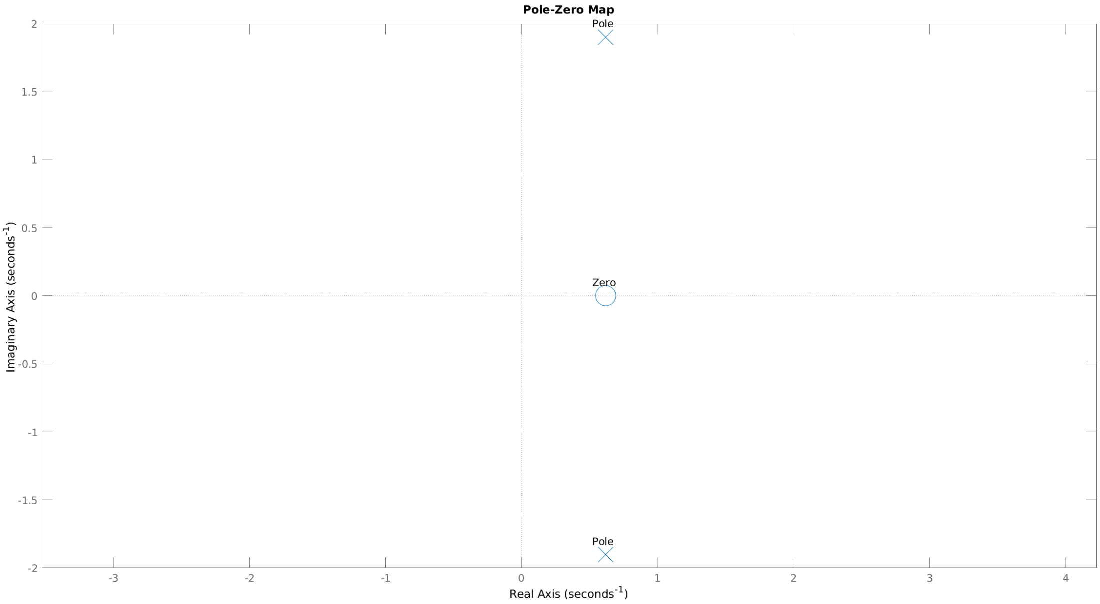

Figure 13: ROC of $\frac{z-\frac{347922205179541}{5629499534213312}}{z^2-\frac{347922205179541\,z}{281474976710656}+4}$

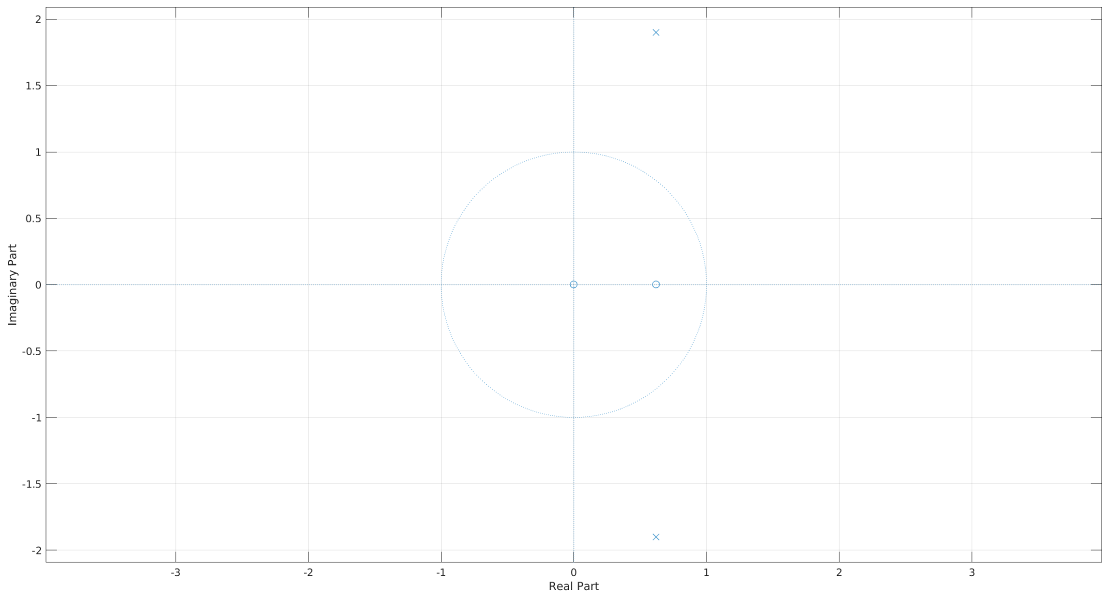

Figure 14: ROC of $\frac{z-\frac{347922205179541}{5629499534213312}}{z^2-\frac{347922205179541\,z}{281474976710656}+4}$

$$pole1 = 0.6180 + 1.9021j$$
$$pole2 = 0.6180 - 1.9021j$$
$$|poles| = 1.999 \sim 2$$

Signal is right sided so:
biggest pole is $|z| = 2 \implies$ all $|z| > 2$ are in ROC as you see in figure
ROC : $|z| > 2$ there is a pole out of unit circle so system is not stable.

3.2 b

3.2.1 1)

H1

$$H1(z) = \frac{z-1}{z^2 - z + \frac{1}{2}}$$

ROC

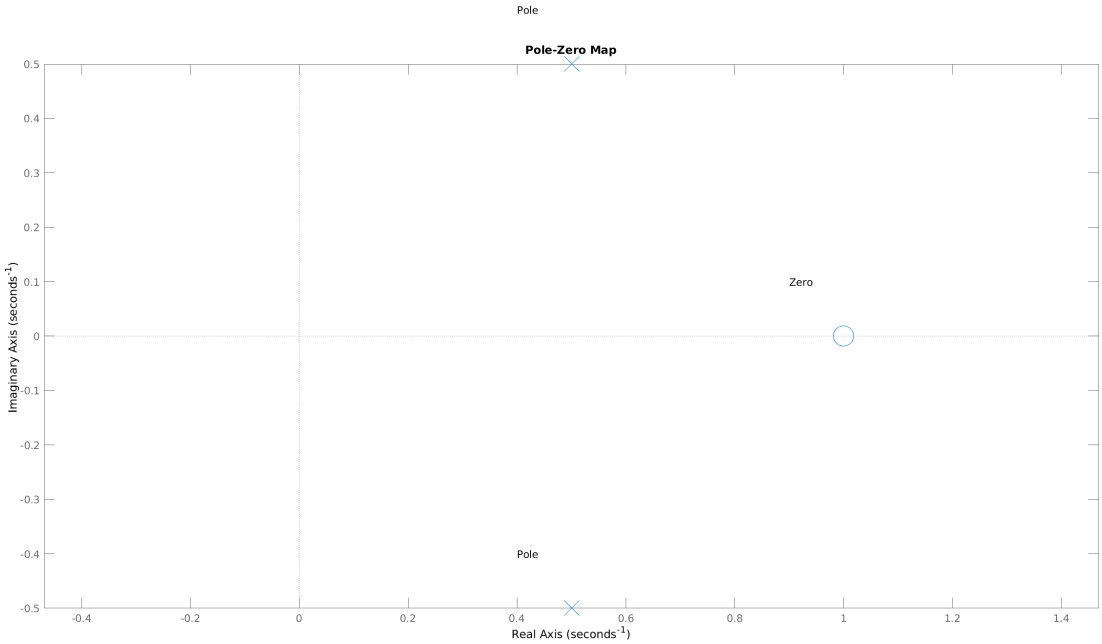

Figure 15: ROC of $\frac{z-1}{z^2-z+\frac{1}{2}}$

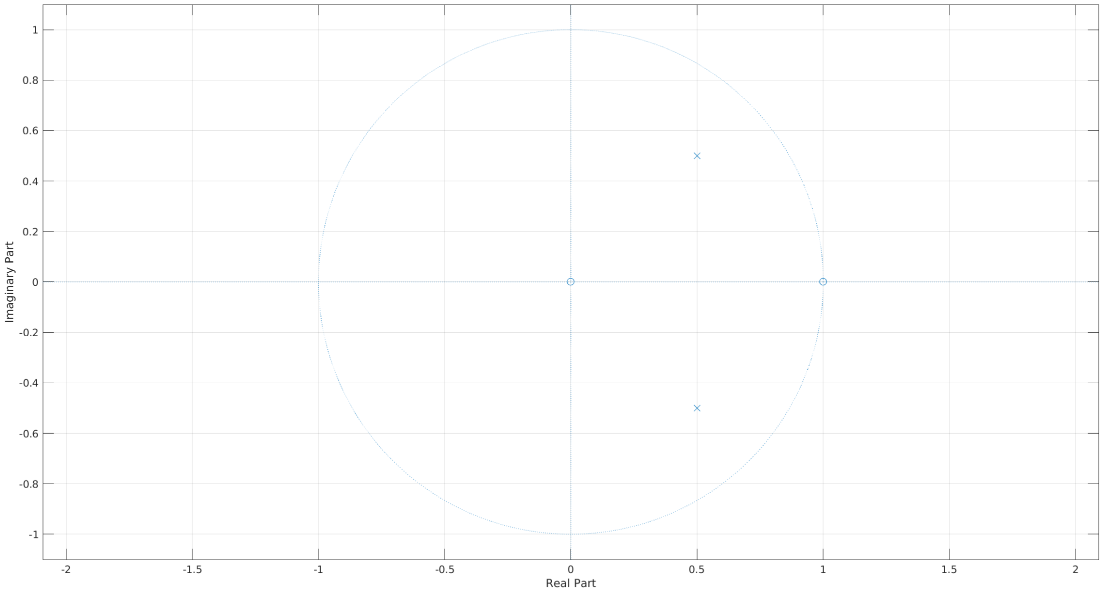

Figure 16: ROC of $\frac{z-1}{z^2-z+\frac{1}{2}}$

$$pole1 = 0.5 + 0.5j$$

$$pole2 = 0.5 - 0.5j \implies ROC: \ |z| > \frac{\sqrt{2}}{2}$$

Signal casual so ROC is all circle with radios bigger than biggest pole :
all $|z| > \frac{\sqrt{2}}{2}$ are in ROC
ROC : $|z| > \frac{\sqrt{2}}{2}$
all poles are in unit circle so system is stable
H2

$$H2(z) = \frac{z}{z^2 - \sqrt{3}\,z + \frac{1}{2}}$$

ROC

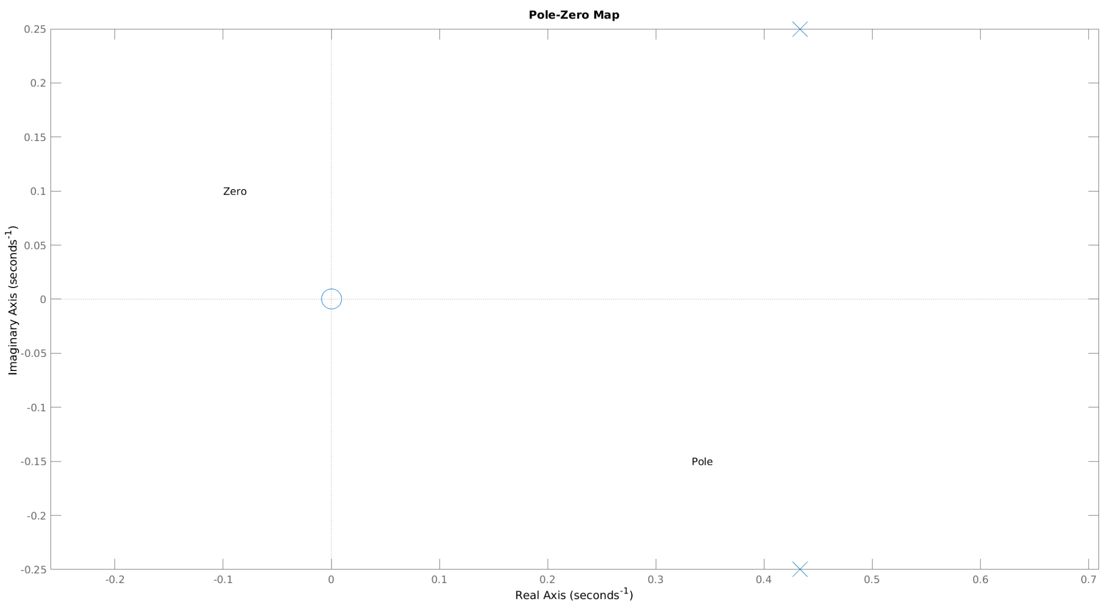

Figure 17: ROC of $\frac{z}{z^2 - \sqrt{3}\,z + \frac{1}{2}}$

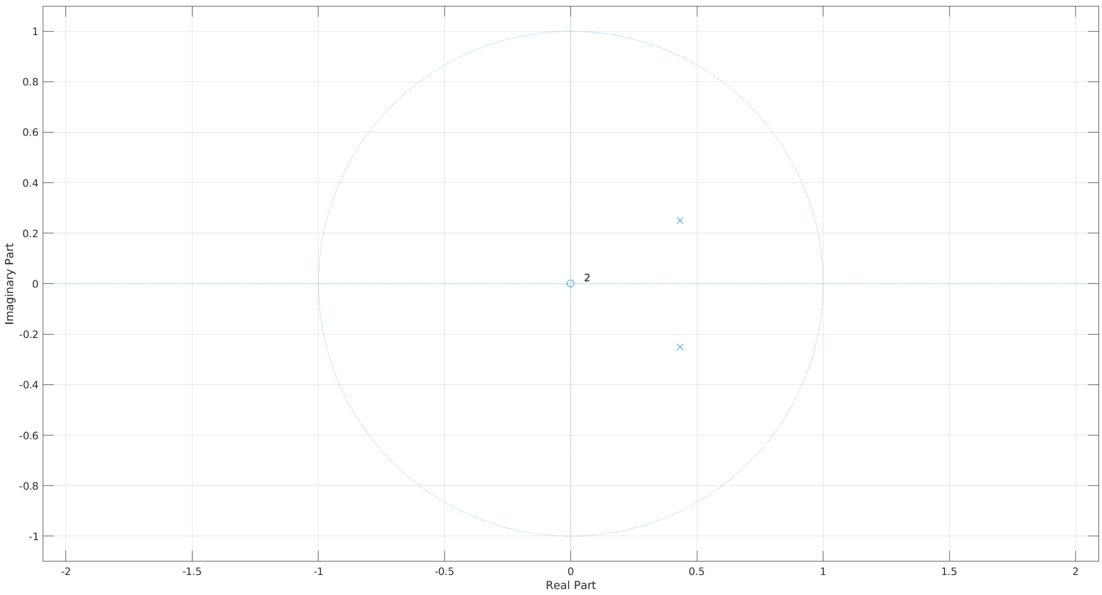

Figure 18: ROC of $\frac{z}{z^2-\sqrt{3}\,z+\frac{1}{2}}$

$$pole1 = 0.433 + 0.25j$$
$$pole2 = 0.433 - 0.25j \implies ROC : \; |z| > \frac{1}{2}$$

Signal casual so ROC is all circle with radios bigger than biggest pole :
all $|z| > \frac{1}{2}$ are in ROC
ROC : $|z| > \frac{1}{2}$
all poles are in unit circle so system is stable

3.2.2 2)

```
[r1,p1,k1] = residuez(num1,dom1)
```

```
r1 = 2×1 complex
      0.5000 + 0.5000i
      0.5000 - 0.5000i

p1 = 2×1 complex
      0.5000 + 0.5000i
      0.5000 - 0.5000i

k1 =

     []
```

```
[r2,p2,k2] = residuez(num2,dom2)
```

```
r2 = 2×1 complex
      0.2500 - 0.4331i
      0.2500 + 0.4331i

p2 = 2×1 complex
      0.4330 + 0.2500i
      0.4330 - 0.2500i

k2 =

     []
```

h1

$$H(z) = r11\frac{1}{1-p11z^{-1}} + r12\frac{1}{1-p12z^{-1}} \implies ROC\ :\ |z| > max(|p11, p12)$$
$$x[n] = r11(p11^n u[n]) + r12(p12^n u[n]) =$$
$$(0.5+0.5\jmath)((0.5+0.5\jmath)^n u[n]) + (0.5-0.5\jmath)((0.5-0.5\jmath)^n u[n]) =$$
$$\frac{1}{2}^n((1-i)^n + (1+i)^n)u[n]$$

h2

$$H(z) = r21\frac{1}{1-p21z^{-1}} + r22\frac{1}{1-p22z^{-1}} \implies ROC\ :\ |z| > max(|p21, p22)$$
$$x[n] = r21(p21^n u[n]) + r22(p22^n u[n])$$
$$(0.25-0.4331\jmath)((0.433+0.25\jmath)^n u[n]) + (0.25+0.4331\jmath)((0.433-0.25\jmath)^n u[n]) =$$
$$\frac{1}{2}^n((1+\sqrt{3})^n - (\sqrt{3}-1)^n)u[n]$$

3.2.3 3)

matlab result:
h1

$$h1[n] = \frac{2\,(-1)^n\,\cos\left(\frac{3\,\pi\,n}{4}\right)}{2^{n/2}} - 2\,\delta_{n,0} = \frac{1}{2}^n((1-i)^n + (1+i)^n)u[n-1]$$

matlab answer and our answer are different in n = 0 because matlab assume u[0] = $\frac{1}{2}$
h2

$$h2[n] = \frac{(-1)^n\,2^{1-n}\,\sqrt{3}\,\left(1-\sqrt{3}\right)^{n-1}}{3} - \frac{(-1)^n\,2^{1-n}\,\sqrt{3}\,\left(-\sqrt{3}-1\right)^{n-1}}{3}$$
$$+\frac{2\,(-1)^n\,\sqrt{3}\,\cos\left(n\left(\pi - \mathrm{acos}\left(\frac{\sqrt{2}\,\sqrt{3}}{2}\right)\right)\right)}{3\,2^{n/2}}$$
$$= \frac{1}{2}^n((1+\sqrt{3})^n - (\sqrt{3}-1)^n)u[n]$$

here results are same because theoretical part is also zero at origin so no problem.