Sharif University of Technology

FACULTY Computer Engineering

Signals & Systems

Computer Assignment 3

DFT

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Contents

1 Introduction

In this HomeWork we are going to study DFT for district function. In matlab DFT has no difference with fft so we will use the same function fft which matlab provides for us.
fft function takes two argument first is sequence X and the other one is number N but it is optional because if you don't pass it as argument it will consider it as number of input elements and ignore it.
we already know the fft from previous exercise so we will focus on N-Point DFT.

1.1 N-Point DFT

If you are going to perform a N-point FFT in MATLAB, to get an appropriate answer, the length of your sequence should be lesser than or equal to N.
Usually this N is chosen in power of 2, because MATLAB employs a Radix-2 FFT if it is, and a slower algorithm if it is not.
So, if you give a sequence of length 1000 for a 2056 point FFT, MATLAB will pad 1056 zeros **after** your signal and compute the FFT. Similarly, if your sequence length is 2000, it will pad 56 zeros and perform a 2056 point FFT.
But if you try to compute a 512-point FFT over a sequence of length 1000,MATLAB will take only the first 512 points and truncate the rest.
So the moral: choose your N to be greater than or equal to the length of the sequence.
We calulate dft(x,n) like below:

```
    F = fftshift(fft(x,n));
    f = linspace(-pi, pi, numel(F));

```

note that we have calculated the ω from $-\pi$ and π because as you know the district Fourier transform of district funtion is periodic with period 2π
we will calculate Phase ans Magnitude and Real part by code below:

```
    subplot(3,1,1)
    plot(f,abs(F), '.-', 'markersize', 6)
    title("Magnitude");
    ax = gca;
    xlim([-pi,pi])

    subplot(3,1,2)
    plot(f,real(F), '.-', 'markersize', 6)
    title("Real");
    ax = gca;
    xlim([-pi,pi])
    subplot(3,1,3)
    plot(f,unwrap(angle(F)), '.-', 'markersize', 6)
    title('Phase');
    xlim([-pi,pi])

```

for each part I draw real, magnitude and phase of each funtion and also the funtion and N I used is shown above each figure as title so there will be no misunderstanding.

2 Part one

a

2.0.1 x[n]

Here is plot of main function:

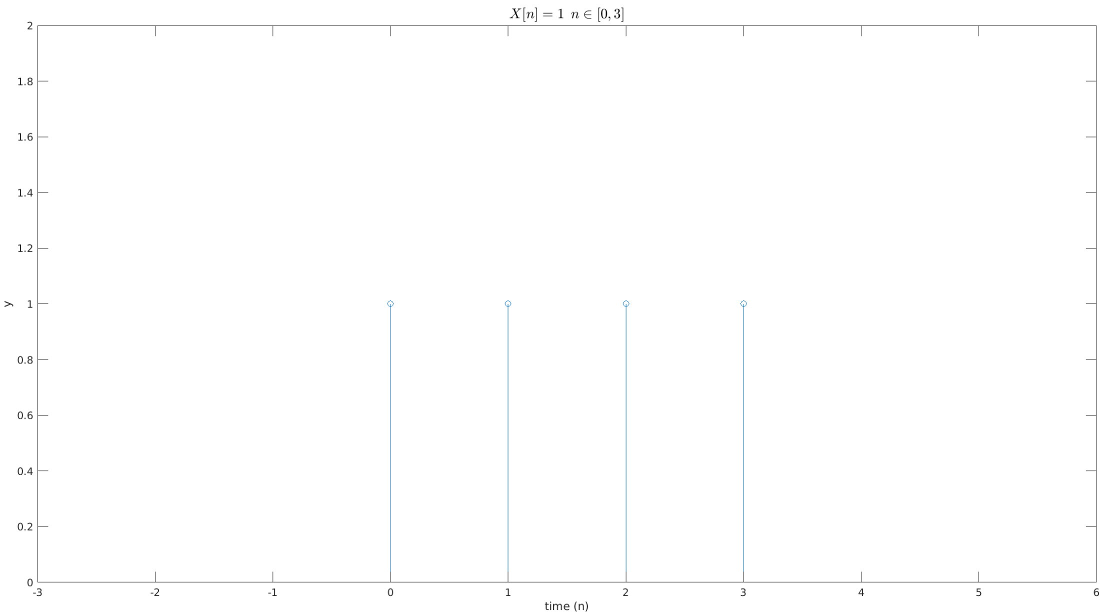

Figure 1: $x[n] = 1 \;\; n \in [0,3]$

As you see there is 4 point with value one in our plot which is consistence with function given.

2.0.2 Magnitude and Phase $X(e^{jw})$

By using fft and fftshift it we plot out function.

$$x[n] = \delta[n] + \delta[n-1] + \delta[n-2] + \delta[n-3]$$

$$X(e^{jw}) = 2\pi \sum_{k=0}^{k=3} e^{-jwk} \quad : \quad \omega \in [-\pi, \pi]$$

$$2\pi \sum_{k=0}^{k=3} cos(wk) + jsin(wk) \quad : \quad \omega \in [-\pi, \pi]$$

$$Phase = -6jw \quad in\, matlab = \frac{-3jw}{\pi}$$

$$Magnitude = 2\pi \sqrt{\left(\sum_{k=0}^{k=3} cos(wk)\right)^2 + \left(\sum_{k=0}^{k=3} sin(wk)\right)^2}$$

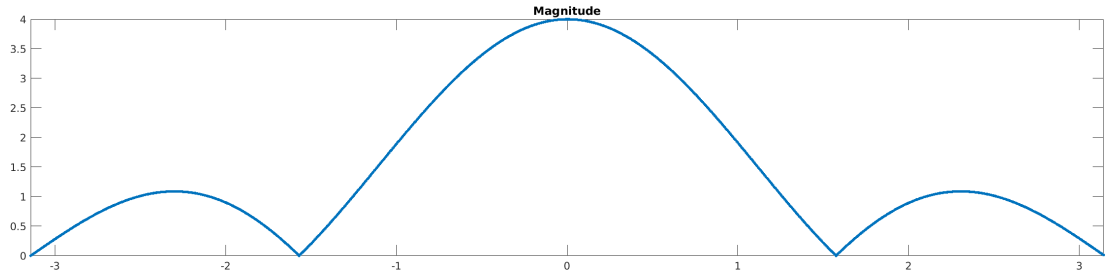

Figure 2: magnitude $X[e^{jw}]$

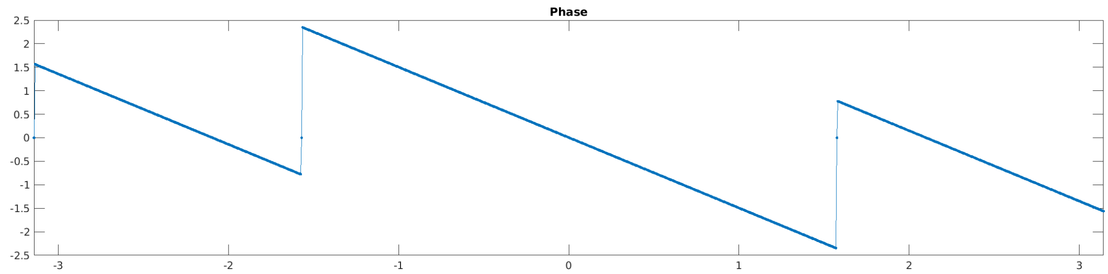

Figure 3: phase $X[e^{jw}]$

2.1 b

with only 4 point DTFT our input sequence for FFT will looks like a constant function with value 1 and fourier transform of one is :

$$x[n] = 1$$
$$N = 1$$
$$X(e^{jw}) = 2\pi\delta(\omega - 1) \quad : \quad \omega \in [-\pi, \pi]$$

2.1.1 Magnitude and Phase $X(e^{jw})$ with N= 4

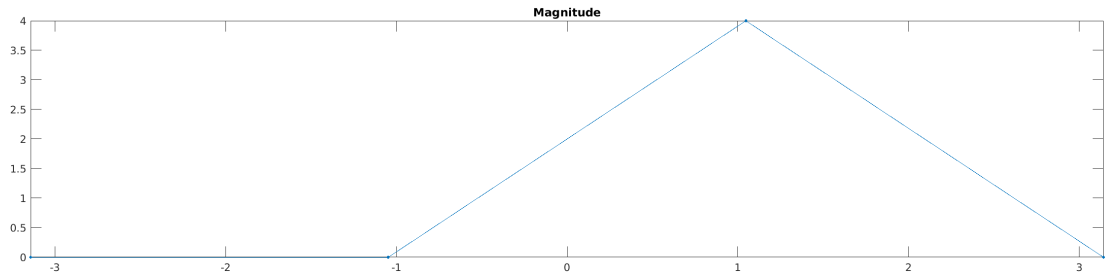

Figure 4: magnitude $X[e^{jw}]$

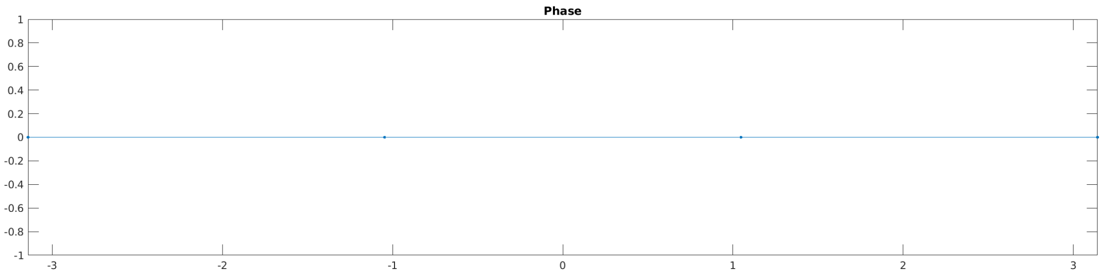

Figure 5: phase $X[e^{jw}]$

as we expected it looks like that the function is assumed as constant function with value 1 so we have plot similar to $\delta(w-1)$.

2.2 c

We can obtain other DTFT by increasing N in fft function it will be like we are increasing period to ∞ so our curve will looks better and better with smoother curve and more consistence with theoretical part.

2.2.1 Magnitude and Phase $X(e^{jw})$ with N = 32,128 and 512

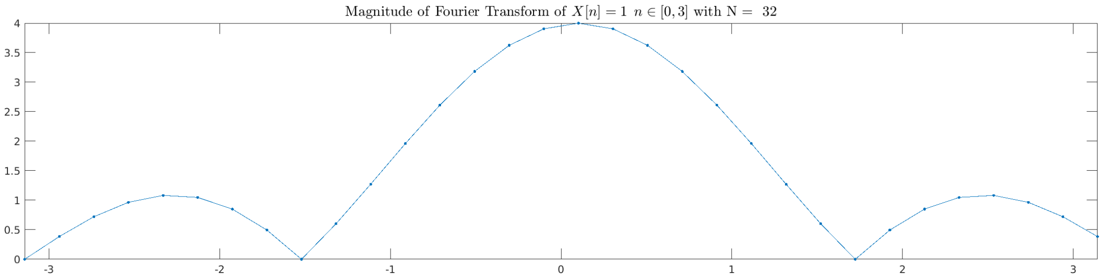

Figure 6: magnitude $X[e^{jw}]$

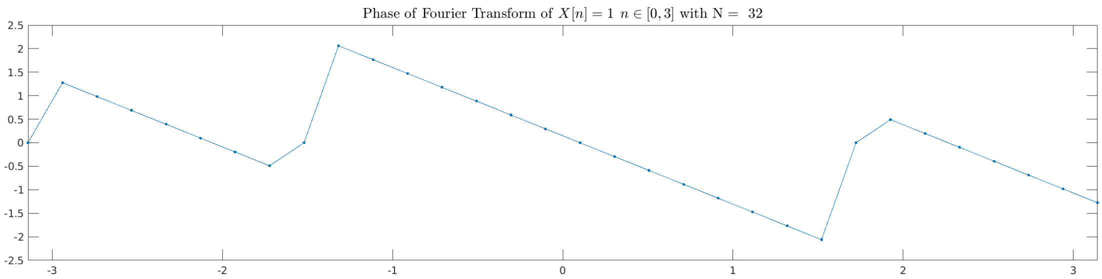

Figure 7: phase $X[e^{jw}]$

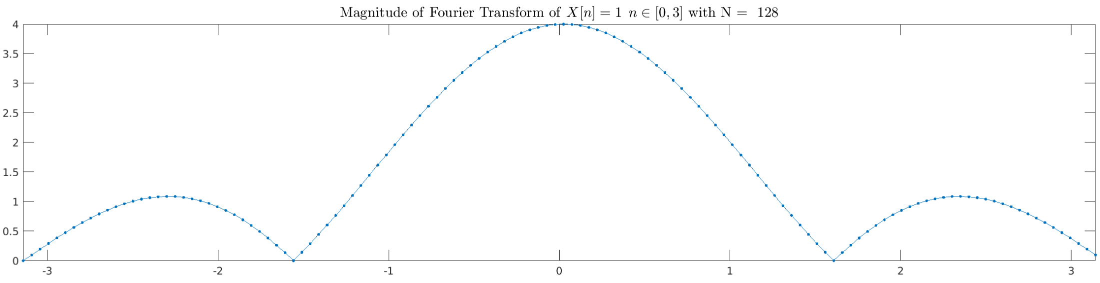

Figure 8: magnitude $X[e^{jw}]$

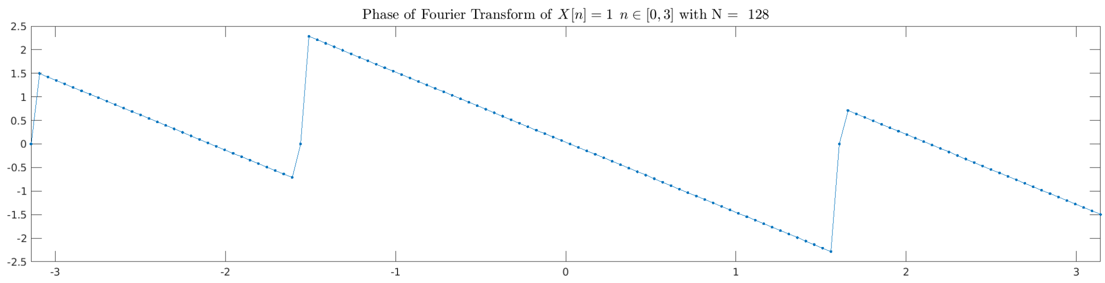

Figure 9: phase $X[e^{jw}]$

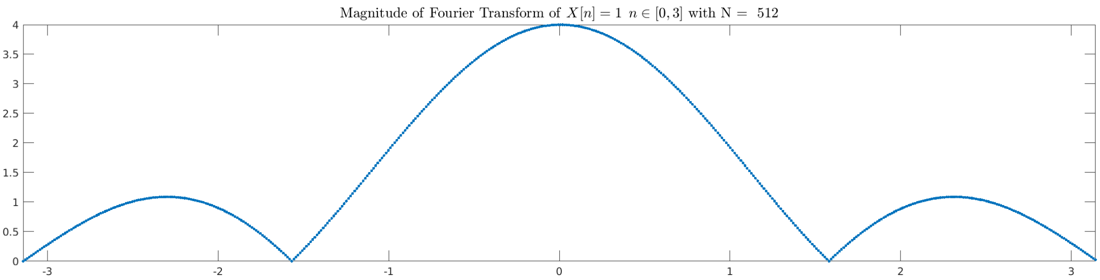

Figure 10: magnitude $X[e^{jw}]$

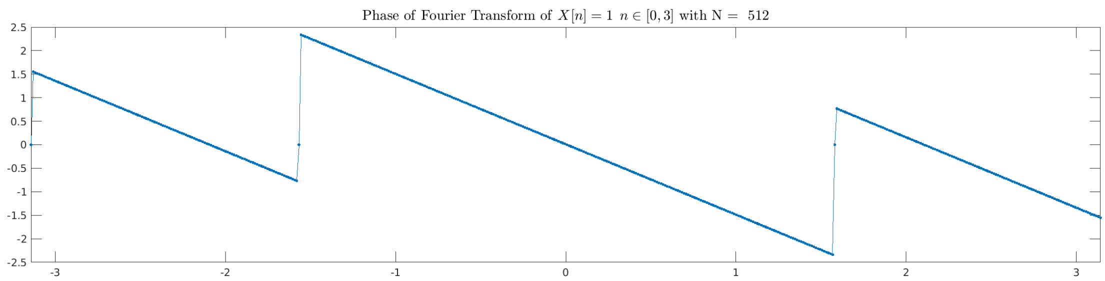

Figure 11: phase $X[e^{jw}]$

As you see curves are becoming more smooth and correct as we increase N.

3 Part two

Here is plot of main function:

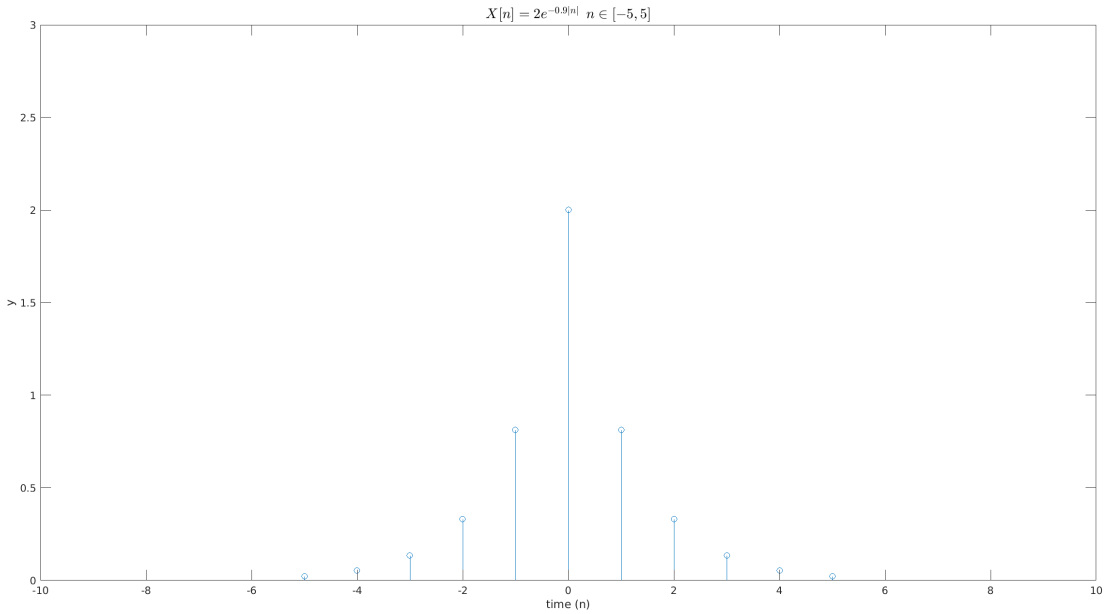

Figure 12: $x[n] = 2e^{-0.9|n|} \;\; n \in [-5,5]$

$$x[n] = 2e^{-0.9|n|} \;\; n \in [-5,5]$$
$$x[n] = 2e^{-0.9|n|} \times rect(5)$$

3.0.1 a) Magnitude and Real $X(e^{jw})$ with N = 11,32,128,256

By using fft and fftshift we plot our function.

$$x[n] = 2e^{-0.9|n|} \times rect(5)$$
$$y[n] = \alpha^{|n|} \;\; |\alpha| < 1$$
$$Y[e^{jw}] = \frac{1-\alpha^2}{1+\alpha^2 - 2\alpha cos(w)}$$
$$\longrightarrow x[n] = 2(\frac{1}{e^{0.9}})^{|n|} \longrightarrow$$
$$X[e^{jw}] = \frac{1}{2\pi} 2 \frac{1-(\frac{1}{e^{0.9}})^2}{1+(\frac{1}{e^{0.9}})^2 - 2(\frac{1}{e^{0.9}})cos(w)} * \frac{sin(w(\frac{1}{2}+5)}{sin(\frac{w}{2})}$$
$$X[e^{jw}] = \frac{1}{\pi} \frac{1-(\frac{1}{e^{0.9}})^2}{1+(\frac{1}{e^{0.9}})^2 - 2(\frac{1}{e^{0.9}})cos(w)} * \frac{sin(\frac{11w}{2})}{sin(\frac{w}{2})}$$

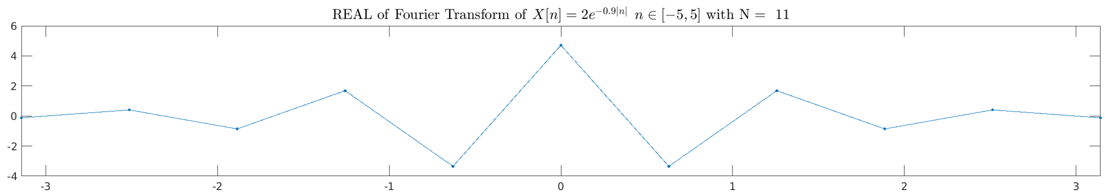

Figure 13: Real $X[e^{jw}]$

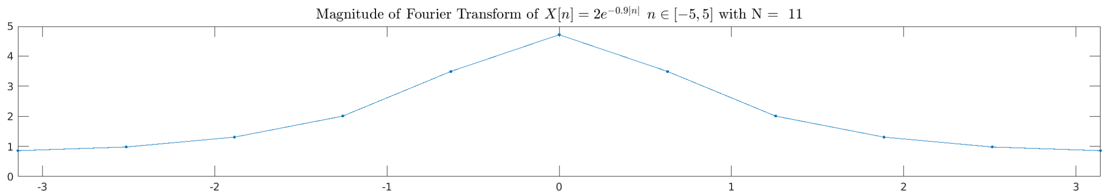

Figure 14: Magnitude $X[e^{jw}]$

with only 11 point DTFT our input sequence for FFT will looks like sum of sum delta function.

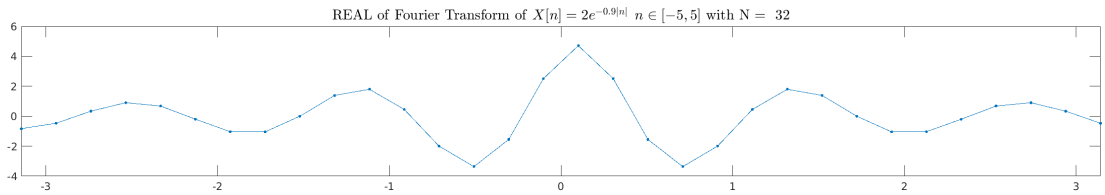

Figure 15: Real $X[e^{jw}]$

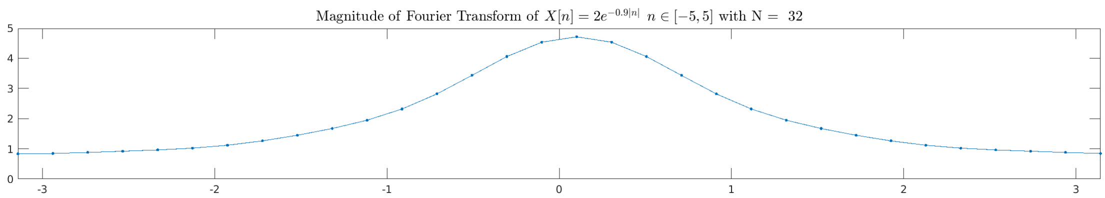

Figure 16: Magnitude $X[e^{jw}]$

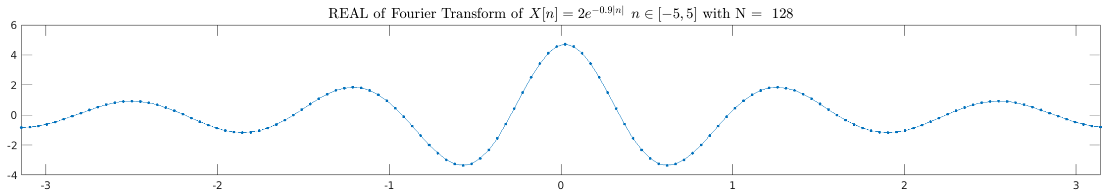

Figure 17: Real $X[e^{jw}]$

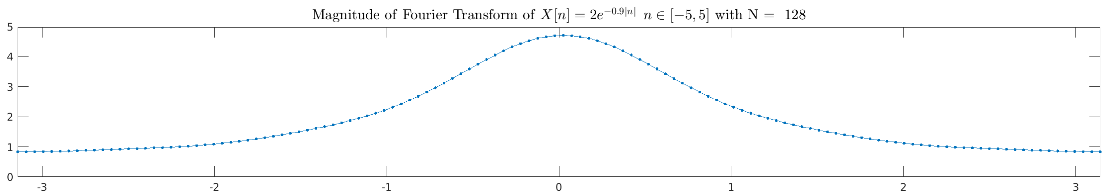

Figure 18: Magnitude $X[e^{jw}]$

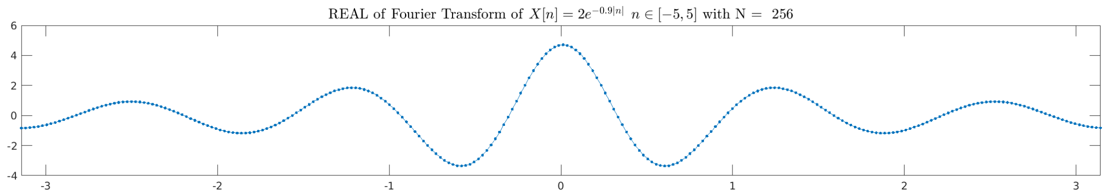

Figure 19: Real $X[e^{jw}]$

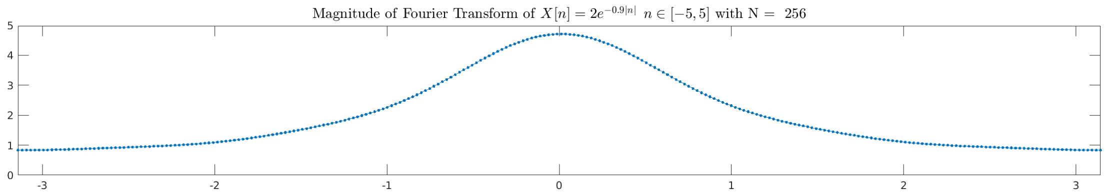

Figure 20: Magnitude $X[e^{jw}]$

As you see curves are becoming more smooth and correct as we increase N. good N is around **128**

4 Part three

for each part we will draw the main plot of function and some plot with different N to get insight of N.

4.1 1) $X[n] = 0.6^{|n|}$ $n \in [-10, 10]$

4.1.1 time domain plot

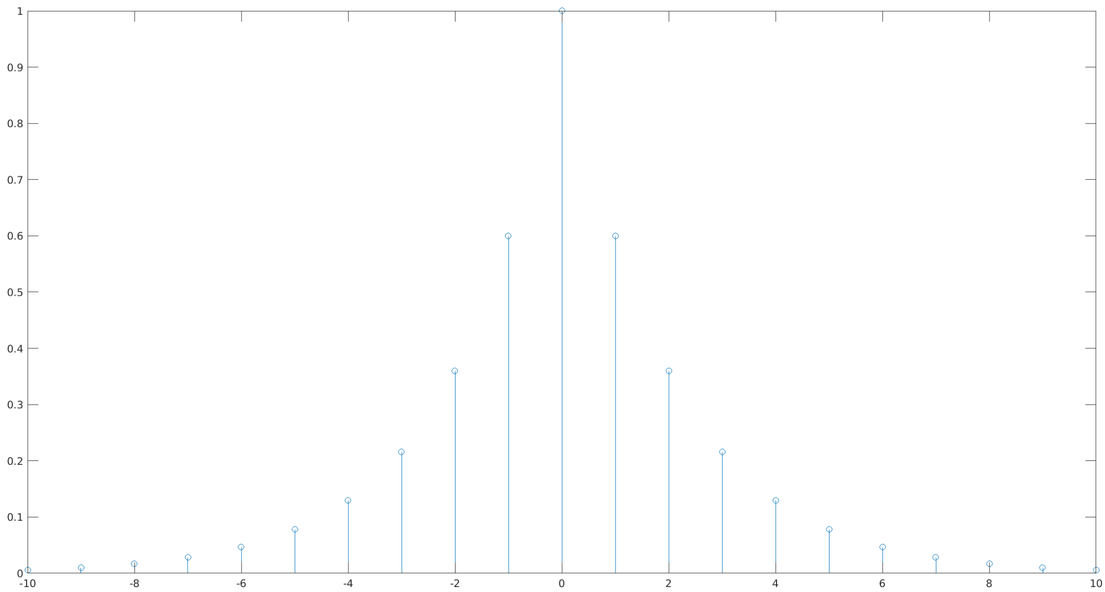

Figure 21: $X[n] = 0.6^{|n|}$ $n \in [-10, 10]$

4.1.2 frequency domain plot N = 21,64,128,256,512

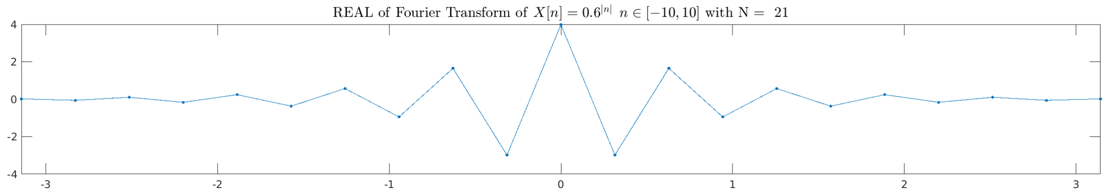

Figure 22: Real $X[e^{jw}]$

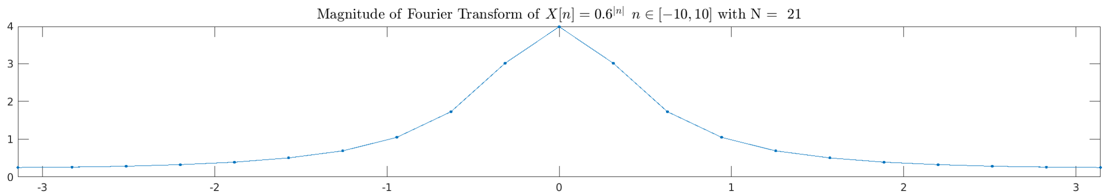

Figure 23: Magnitude $X[e^{jw}]$

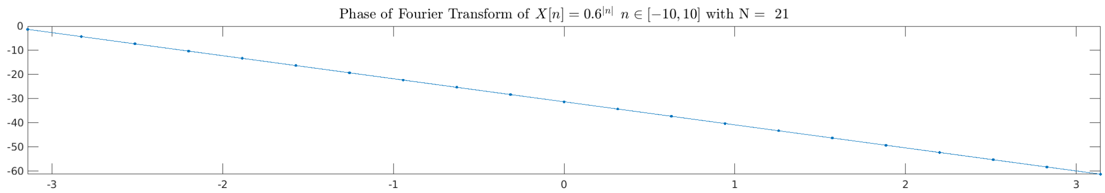

Figure 24: Phase $X[e^{jw}]$

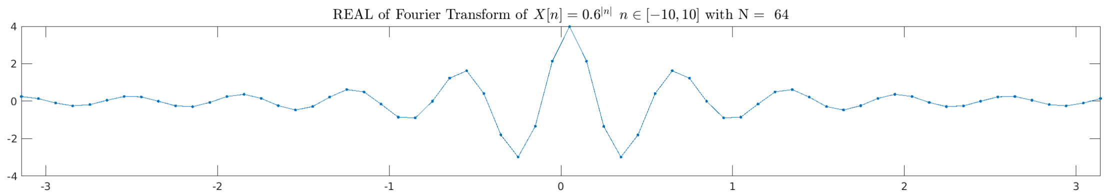

Figure 25: Real $X[e^{jw}]$

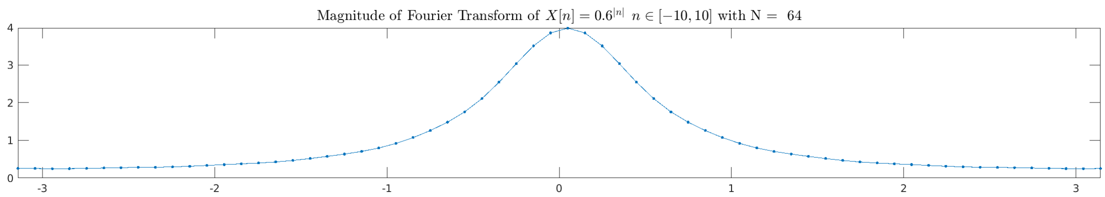

Figure 26: Magnitude $X[e^{jw}]$

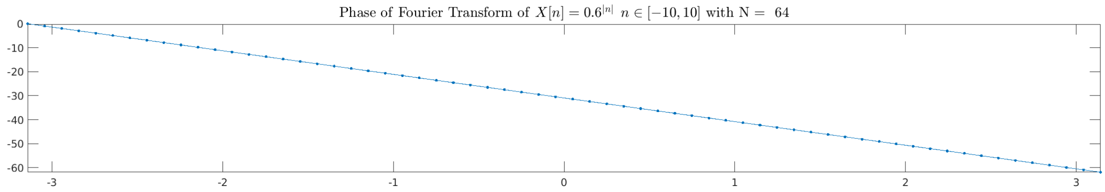

Figure 27: Phase $X[e^{jw}]$

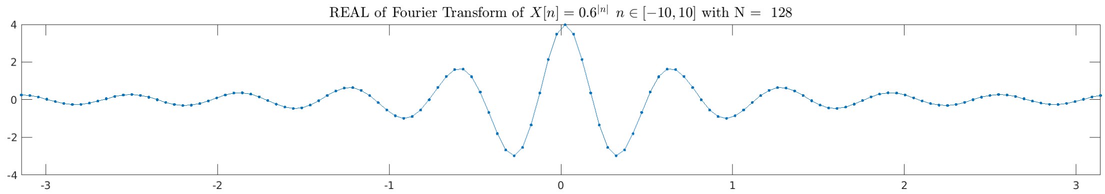

Figure 28: Real $X[e^{jw}]$

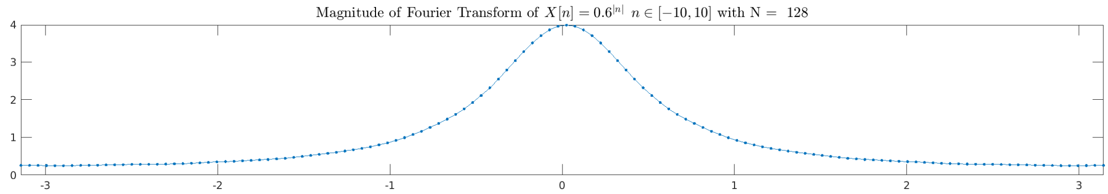

Figure 29: Magnitude $X[e^{jw}]$

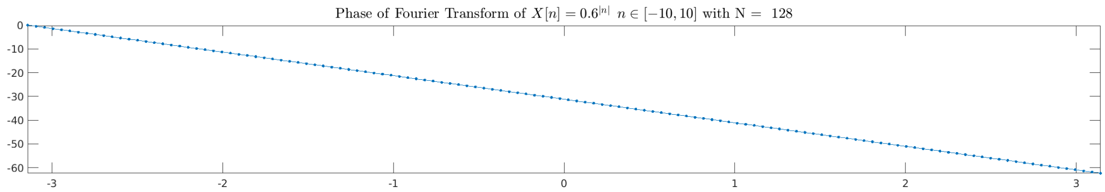

Figure 30: Phase $X[e^{jw}]$

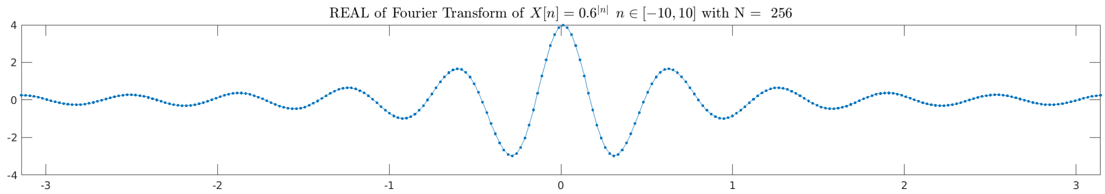

Figure 31: Real $X[e^{jw}]$

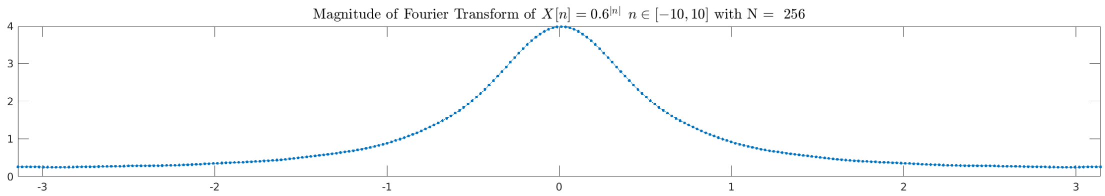

Figure 32: Magnitude $X[e^{jw}]$

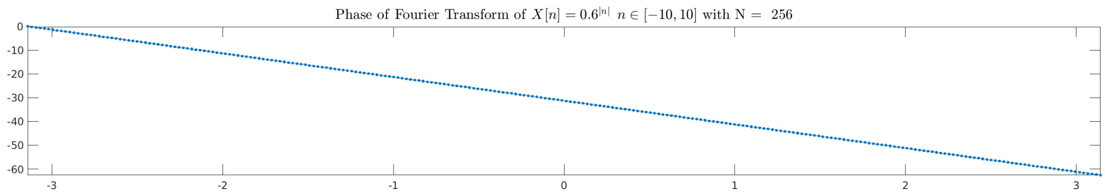

Figure 33: Phase $X[e^{jw}]$

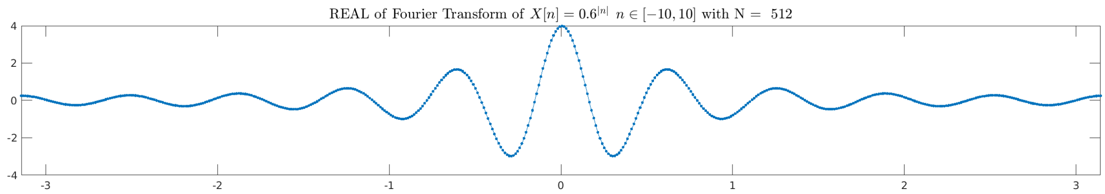

Figure 34: Real $X[e^{jw}]$

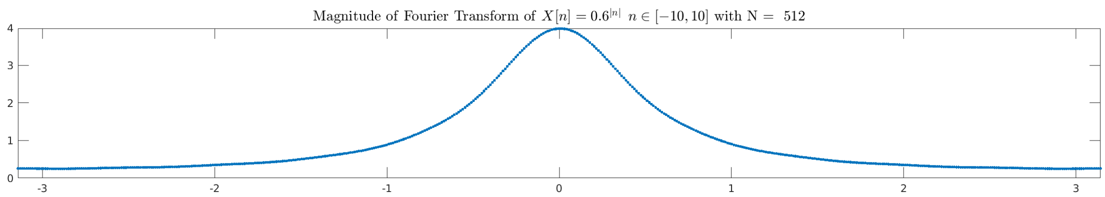

Figure 35: Magnitude $X[e^{jw}]$

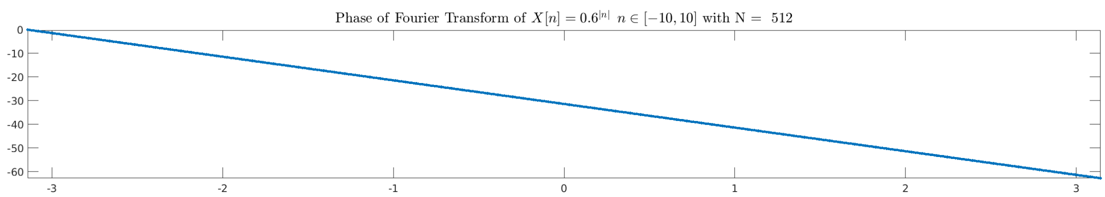

Figure 36: Phase $X[e^{jw}]$

MY guess : 256

4.2 2) $X[n] = n(0.9)^n \;\; n \in [0, 20]$

4.2.1 time domain plot

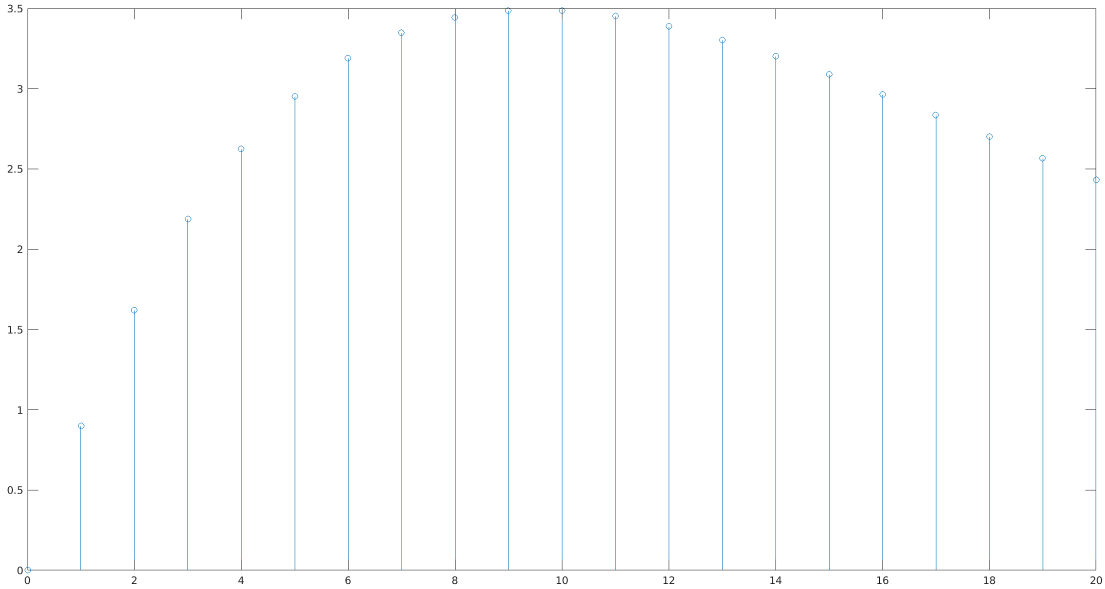

Figure 37: $X[n] = n(0.9)^n \;\; n \in [0, 20]$

4.2.2 frequency domain plot N = 21,128,256,512

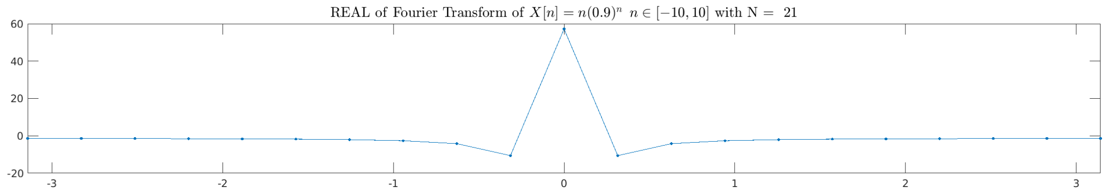

Figure 38: Real $X[e^{jw}]$

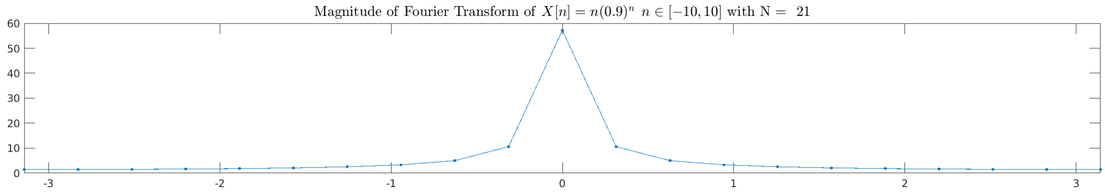

Figure 39: Magnitude $X[e^{jw}]$

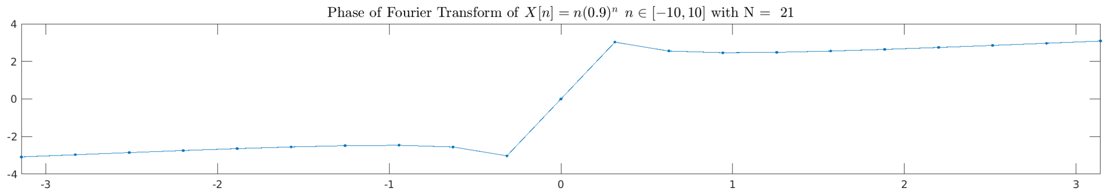

Figure 40: Phase $X[e^{jw}]$

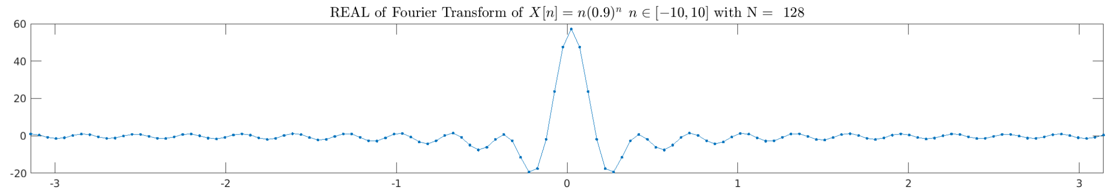

Figure 41: Real $X[e^{jw}]$

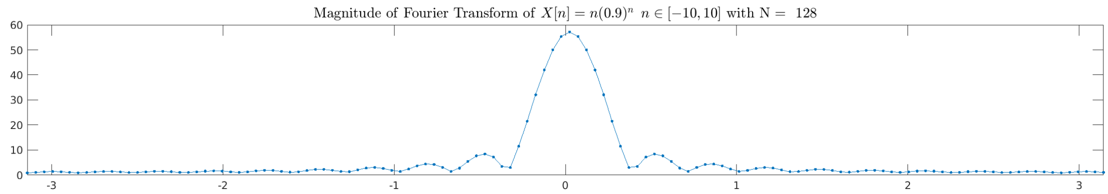

Figure 42: Magnitude $X[e^{jw}]$

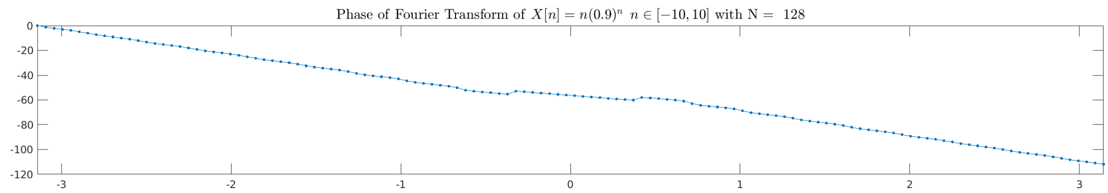

Figure 43: Phase $X[e^{jw}]$

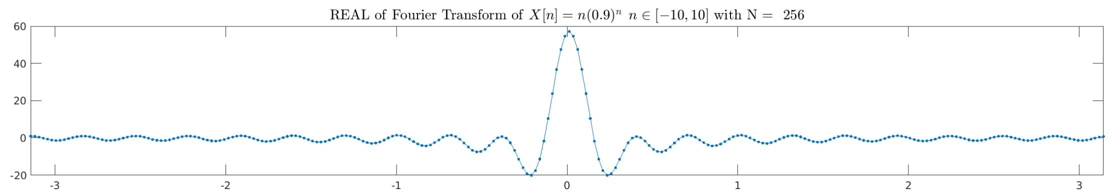

Figure 44: Real $X[e^{jw}]$

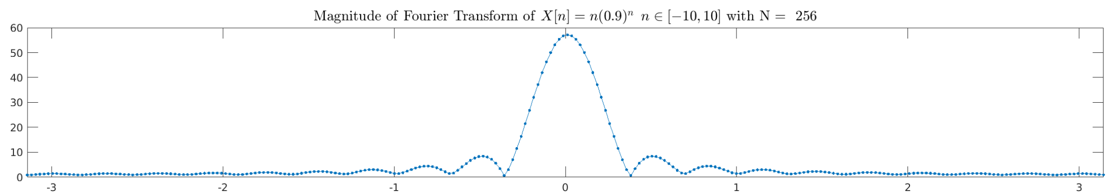

Figure 45: Magnitude $X[e^{jw}]$

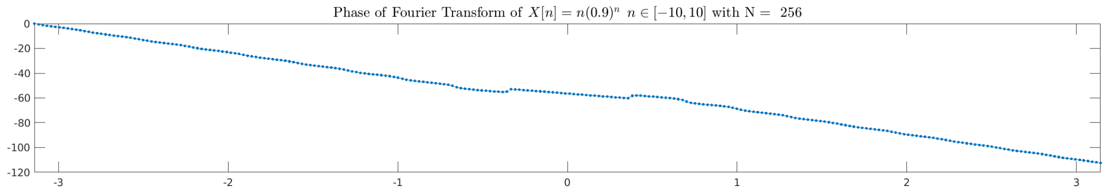

Figure 46: Phase $X[e^{jw}]$

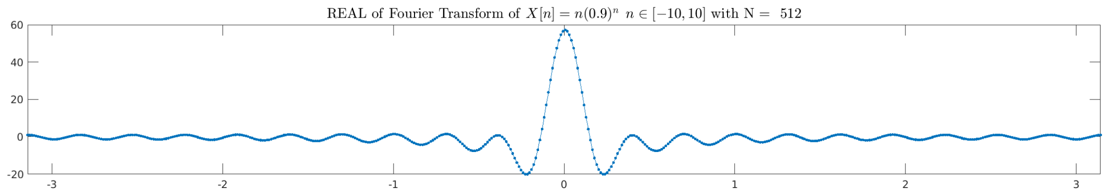

Figure 47: Real $X[e^{jw}]$

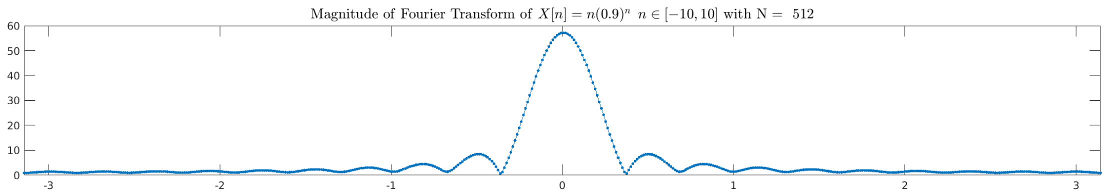

Figure 48: Magnitude $X[e^{jw}]$

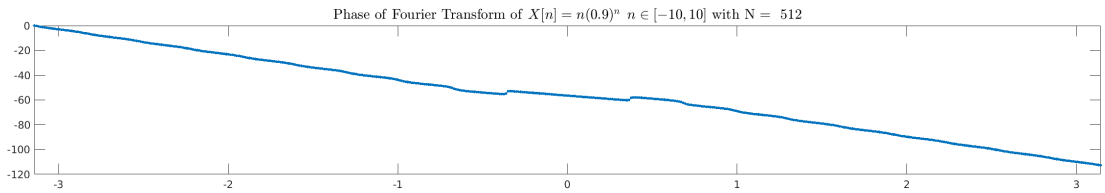

Figure 49: Phase $X[e^{jw}]$

MY guess : 256

4.3 3) $X[n] = e^{0.5\pi tj}$ $n \in [0, 50]$

4.3.1 frequency domain plot N = 51,256,512,1024

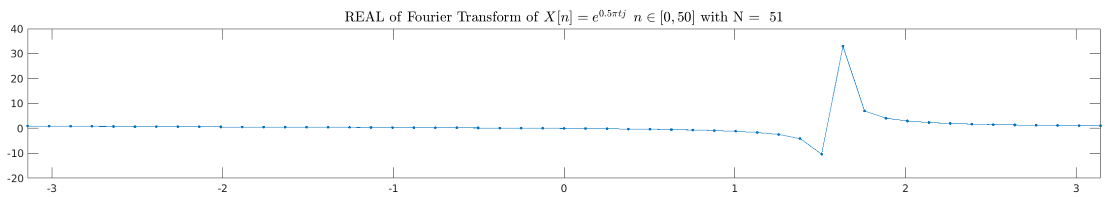

Figure 50: Real $X[e^{jw}]$

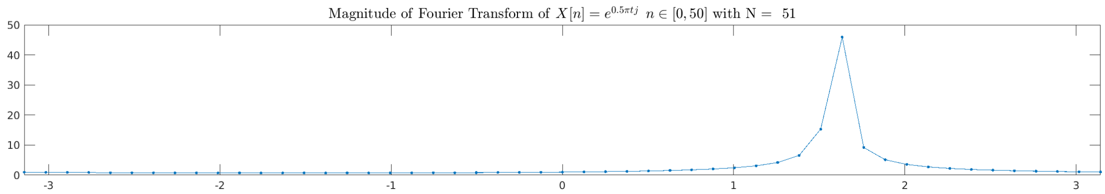

Figure 51: Magnitude $X[e^{jw}]$

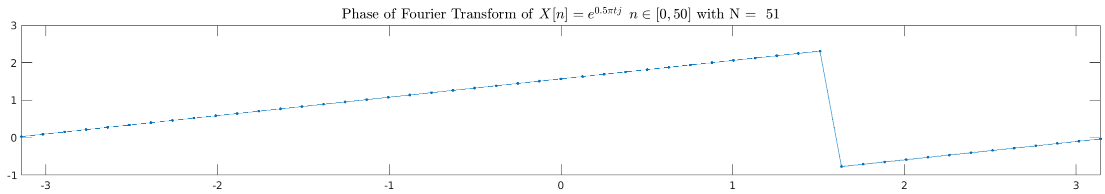

Figure 52: Phase $X[e^{jw}]$

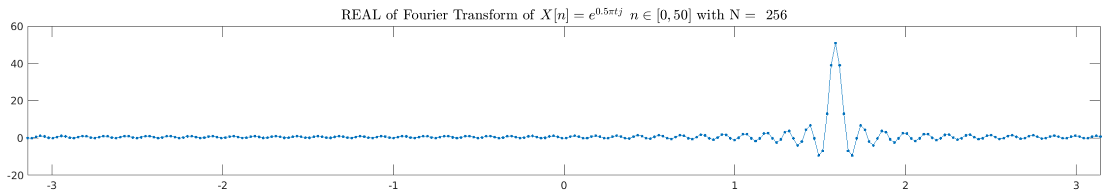

Figure 53: Real $X[e^{jw}]$

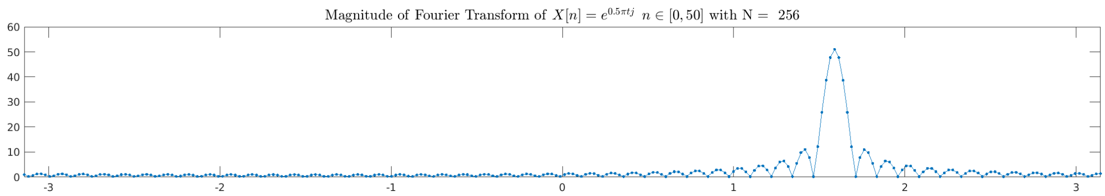

Figure 54: Magnitude $X[e^{jw}]$

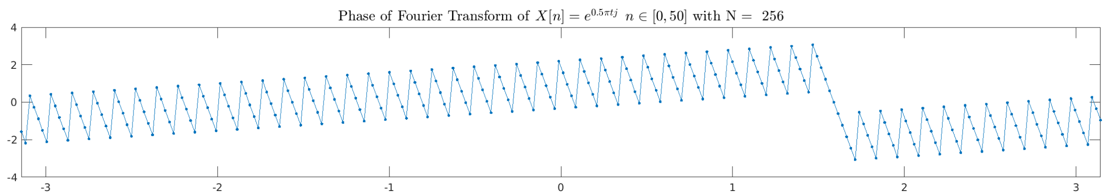

Figure 55: Phase $X[e^{jw}]$

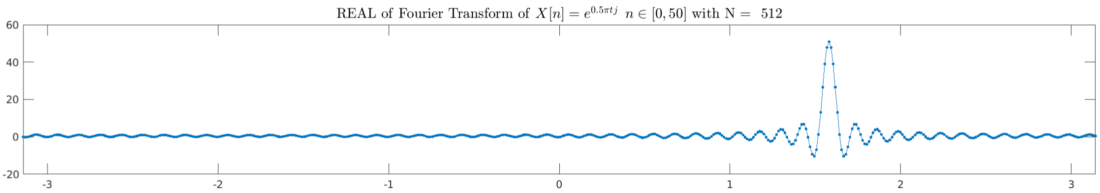

Figure 56: Real $X[e^{jw}]$

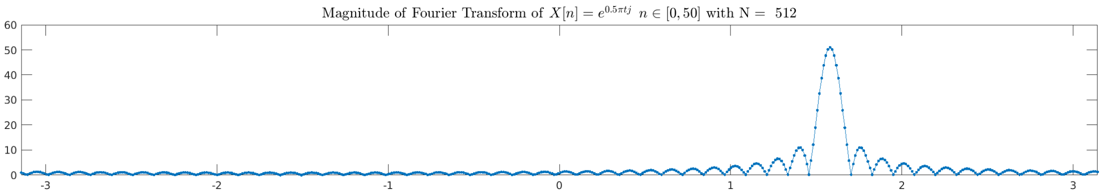

Figure 57: Magnitude $X[e^{jw}]$

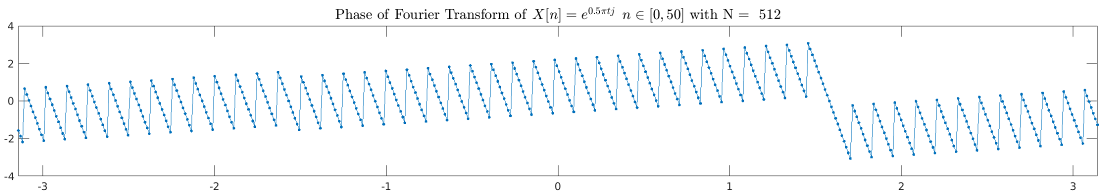

Figure 58: Phase $X[e^{jw}]$

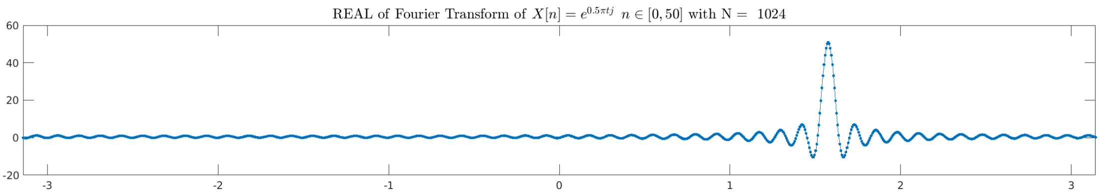

Figure 59: Real $X[e^{jw}]$

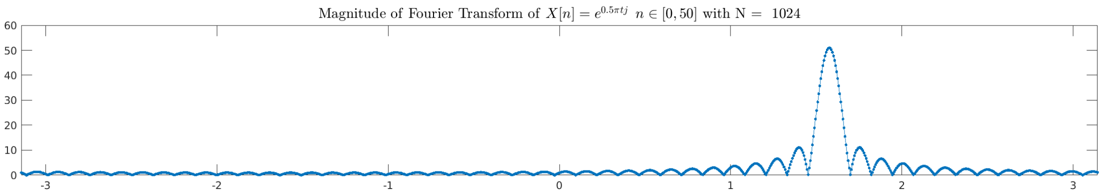

Figure 60: Magnitude $X[e^{jw}]$

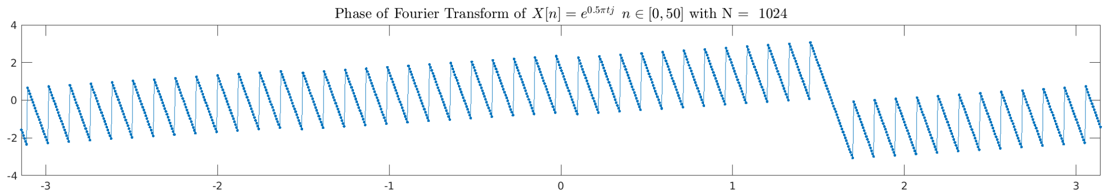

Figure 61: Phase $X[e^{jw}]$

MY guess : around 1024

4.4 4) $X[n] = |n - 4|$ $n \in [-3, 3]$

4.4.1 time domain plot

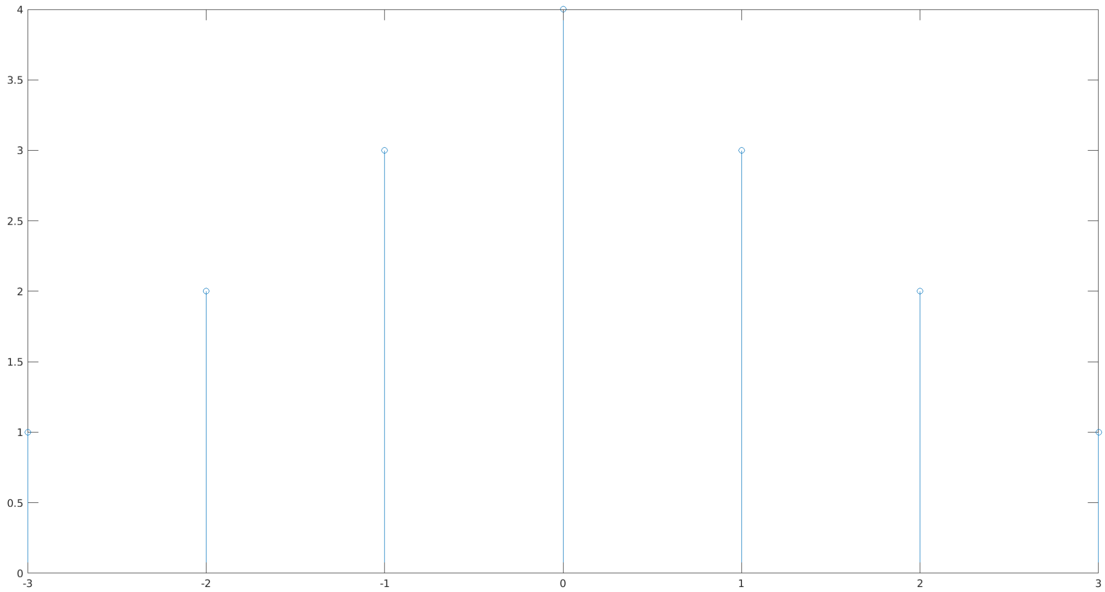

Figure 62: $X[n] = |n - 4|$ $n \in [-3, 3]$

4.4.2 frequency domain plot N = 7,64,128

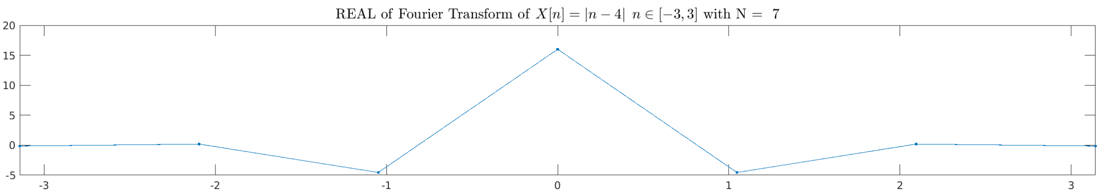

Figure 63: Real $X[e^{jw}]$

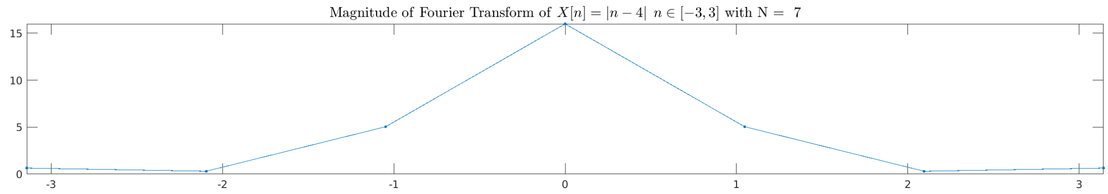

Figure 64: Magnitude $X[e^{jw}]$

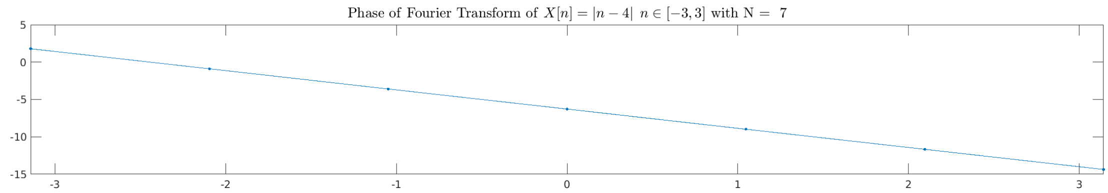

Figure 65: Phase $X[e^{jw}]$

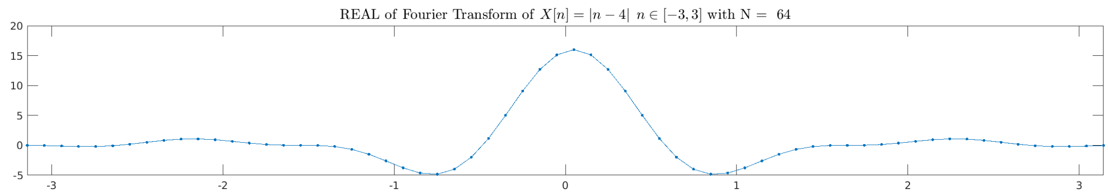

Figure 66: Real $X[e^{jw}]$

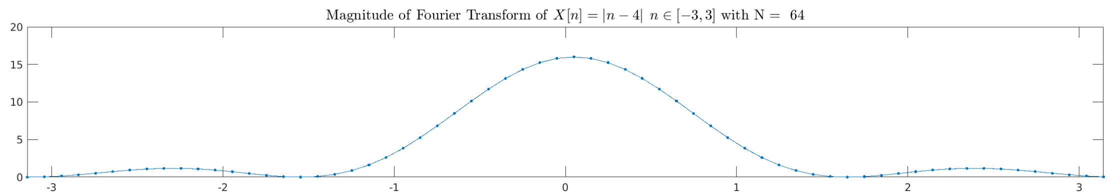

Figure 67: Magnitude $X[e^{jw}]$

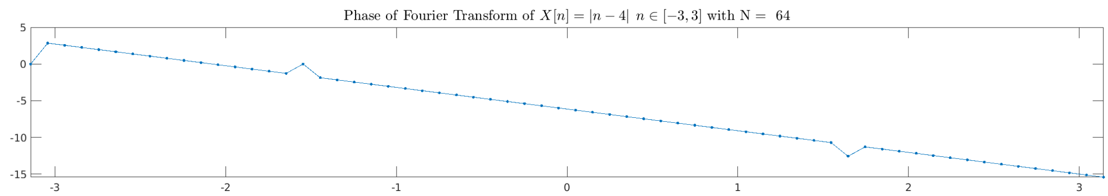

Figure 68: Phase $X[e^{jw}]$

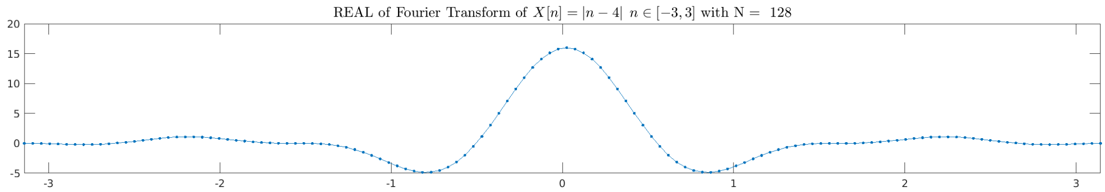

Figure 69: Real $X[e^{jw}]$

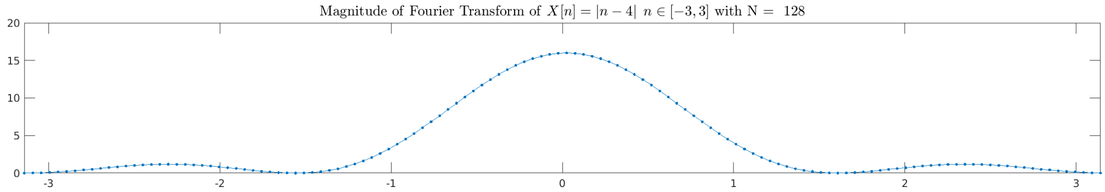

Figure 70: Magnitude $X[e^{jw}]$

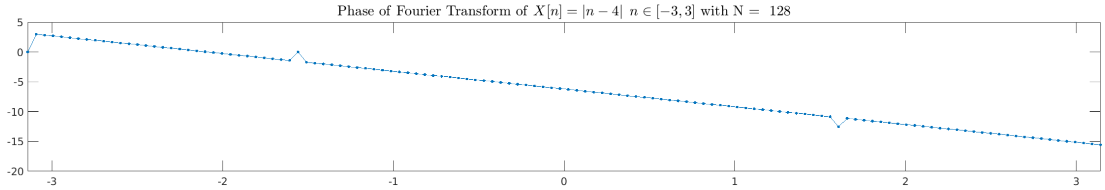

Figure 71: Phase $X[e^{jw}]$

MY guess : 128

4.5 5) $X[n] = [-1,-2,-3,0,1,2,3]$

4.5.1 time domain plot

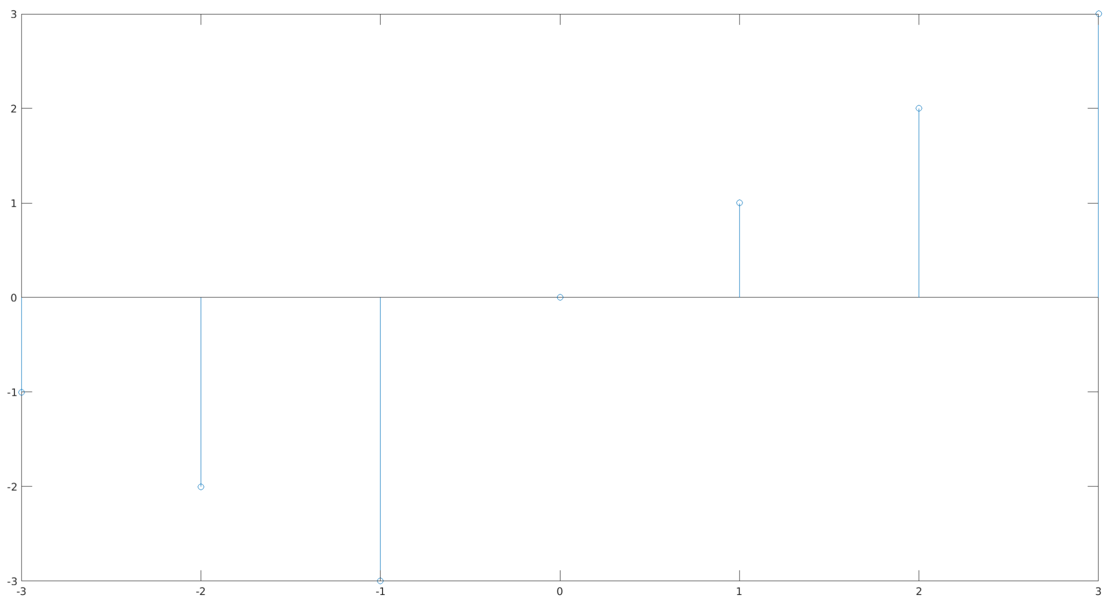

Figure 72: $X[n] = [-1,-2,-3,0,1,2,3]$

4.5.2 frequency domain plot N = 7,64,256

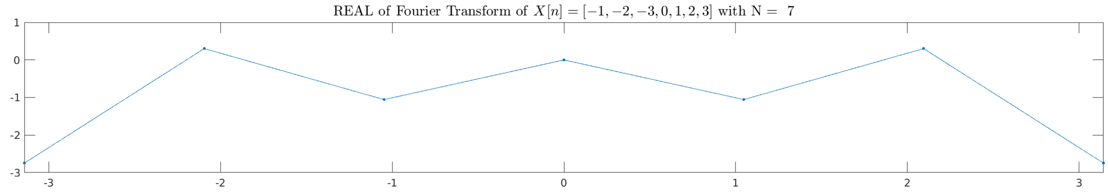

Figure 73: Real $X[e^{jw}]$

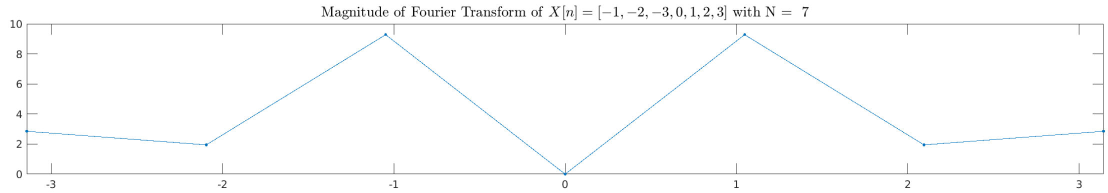

Figure 74: Magnitude $X[e^{jw}]$

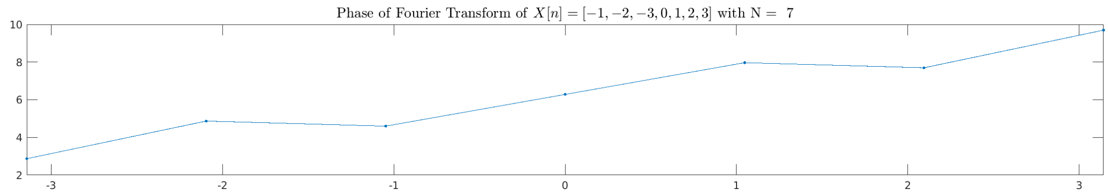

Figure 75: Phase $X[e^{jw}]$

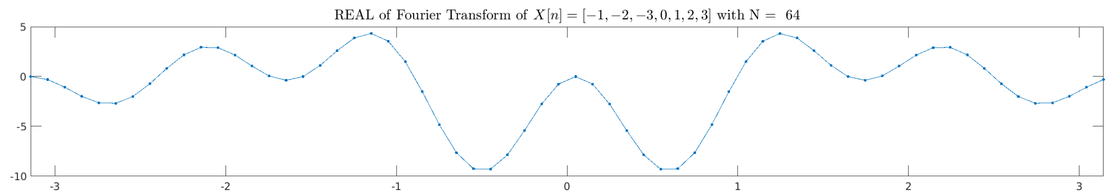

Figure 76: Real $X[e^{jw}]$

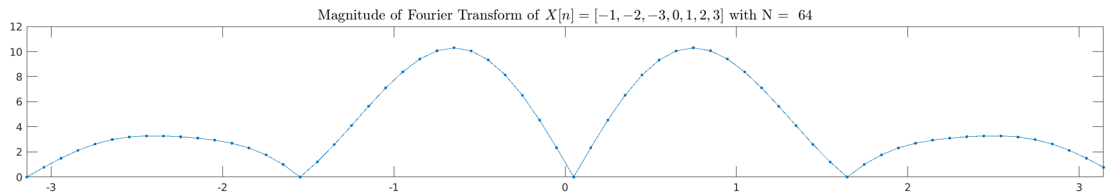

Figure 77: Magnitude $X[e^{jw}]$

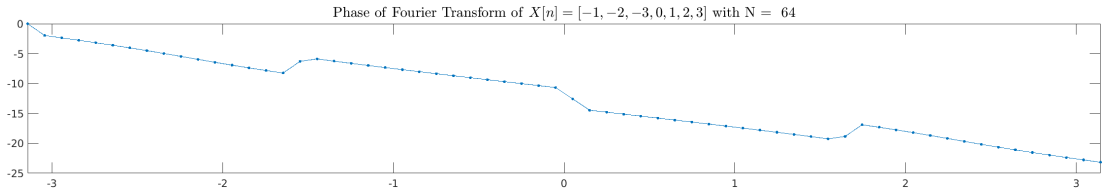

Figure 78: Phase $X[e^{jw}]$

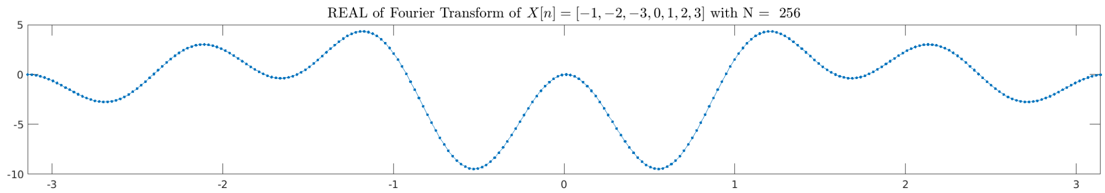

Figure 79: Real $X[e^{jw}]$

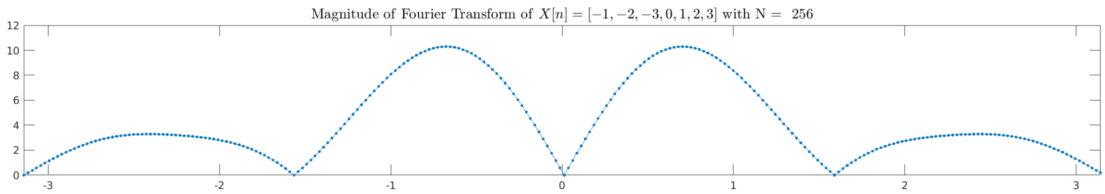

Figure 80: Magnitude $X[e^{jw}]$

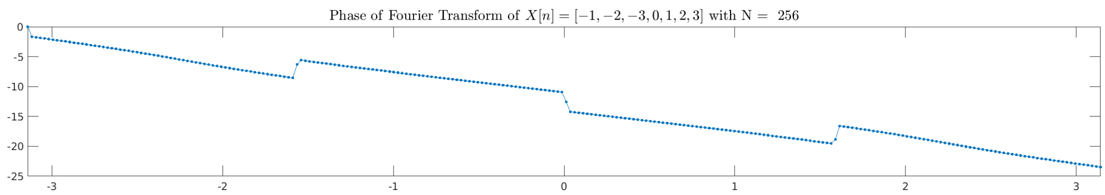

Figure 81: Phase $X[e^{jw}]$

MY guess : 256

5 Part four

Rewrite the transform as below:

$$H[e^{jw}] = \sum h[k]e^{-jkw} = \sum h[k]cos(kw) + (\sum h[k]sin(kw))j$$
$$\implies Real\{H[e^{jw}]\} = \sum h[k]cos(kw)$$
$$\implies h[n] = 0.9^n \;\; n \in [0, 5]$$

5.1 Plot Real$\{H[e^{jw}] = \sum_{k=0}^{k=5} 0.9^k cos(kw)\}$

```
clear;
syms v k
G(v) = sum(subs((0.9) .^k .* cos(v*k),k,0:5))
%x = @(t,w) cos(t*w) .* (0.9).^(t).* ((t<= 5) & (t>= 0));
%H = @(t,w) sum(x(t,w));
time_step =1;
t = 0:time_step:5;
time_step2 = 0.001;
%w = -0.5:time_step2:0.5;
w = -5:0.001:5;
figure;
set(gcf,'position',[0,0,1800,900]);
F = G(w);
plot(w,F)
title(strcat(strcat(strcat(" $\sum_{k=0}^{k=5} {0.9^k * cos(w*k) } $")," ")," " ),"fontsize",14,"interpreter","latex")
ax = gca;
xlim([-pi,pi])

```

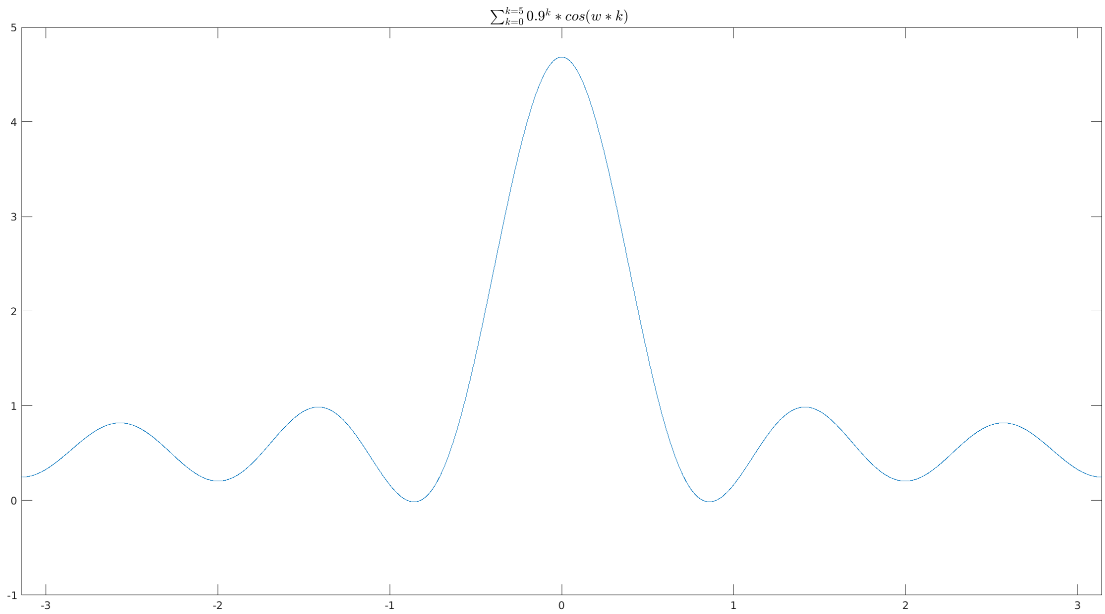

Figure 82: $H[e^{jw}] = \sum_{k=0}^{k=5} 0.9^k cos(kw)$

5.2 Plot Real(DFT) obtained function

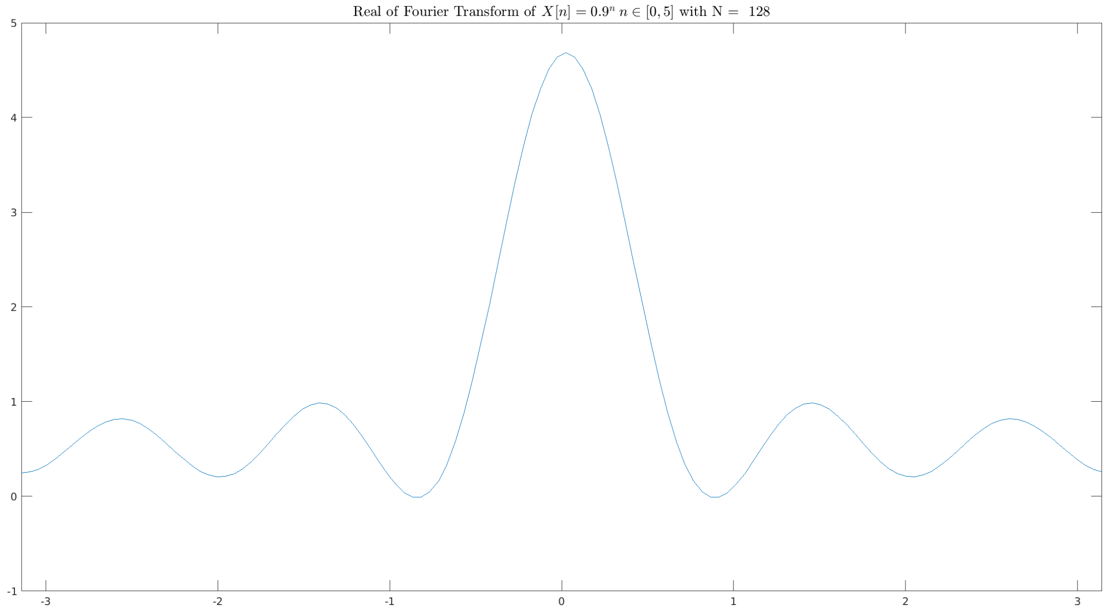

Figure 83: $Real(H[e^{jw}])$

5.3 Conclusion

As you see both function look exactly same so our calculation is right.
with appropriate N which is 128 we obtained similar plot.