The discrete Fourier transform, or DFT, is the primary tool of digital signal processing. The foundation of the product is the fast Fourier transform (FFT), a method for computing the DFT with reduced execution time. Many of the toolbox functions (including *Z*-domain frequency response, spectrum and some filter design and implementation functions) incorporate the FFT.

The MATLAB® environment provides the functions fft and ifft to compute the discrete Fourier transform and its inverse, respectively. For the input sequence *x* and its transformed version *X* (the discrete-time Fourier transform at equally spaced frequencies around the unit circle), the two functions implement the relationships

$$X(k+1) = \sum_{n=0}^{N-1} x(n+1)W_N^{kn}$$

and

$$x(n+1) = \frac{1}{N}\sum_{k=0}^{N-1} X(k+1)W_N^{-kn}.$$

In these equations, the series subscripts begin with 1 instead of 0 because of the MATLAB vector indexing scheme, and

$$W_N = e^{-j2\pi/N}.$$

Note The MATLAB convention is to use a negative *j* for the fft function. This is an engineering convention; physics and pure mathematics typically use a positive *j*.

fft, with a single input argument, x, computes the DFT of the input vector or matrix. If x is a vector, fft computes the DFT of the vector; if x is a rectangular array, fft computes the DFT of each array column. For example, create a time vector and signal:

```
t = 0:1/100:10-1/100;                     % Time vector
x = sin(2*pi*15*t) + sin(2*pi*40*t);      % Signal
```

Compute the DFT of the signal and the magnitude and phase of the transformed sequence. Decrease round-off error when computing the phase by setting small-magnitude transform values to zero.

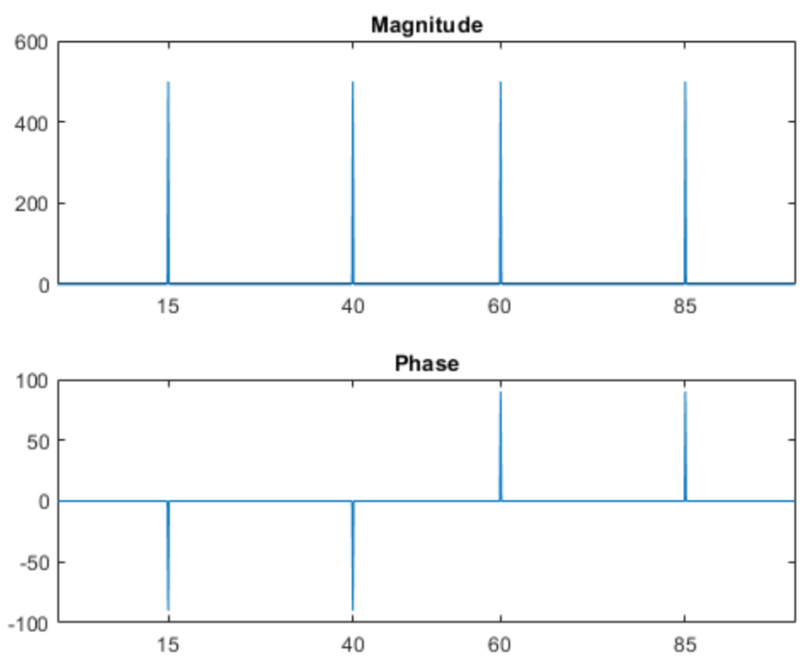
```
y = fft(x);                                  % Compute DFT of x
m = abs(y);                                  % Magnitude
y(m<1e-6) = 0;
p = unwrap(angle(y));                        % Phase
```

To plot the magnitude and phase in degrees, type the following commands:

```
f = (0:length(y)-1)*100/length(y);        % Frequency vector

subplot(2,1,1)
plot(f,m)
title('Magnitude')
ax = gca;
ax.XTick = [15 40 60 85];

subplot(2,1,2)
plot(f,p*180/pi)
title('Phase')
ax = gca;
ax.XTick = [15 40 60 85];
```

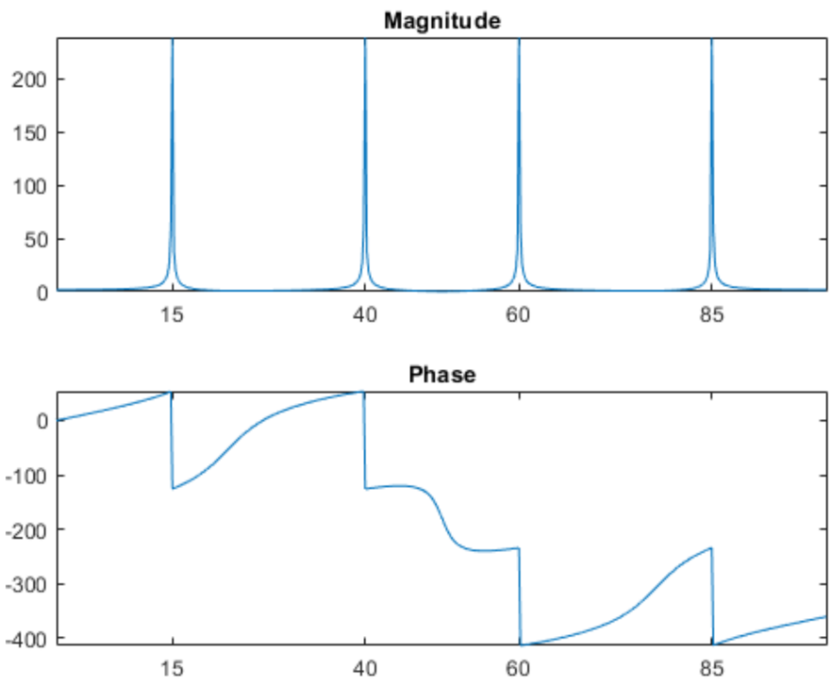


A second argument to fft specifies a number of points n for the transform, representing DFT length:

```
n = 512;
y = fft(x,n);
m = abs(y);
p = unwrap(angle(y));
f = (0:length(y)-1)*100/length(y);

subplot(2,1,1)
plot(f,m)
title('Magnitude')
ax = gca;
ax.XTick = [15 40 60 85];

subplot(2,1,2)
plot(f,p*180/pi)
title('Phase')
ax = gca;
ax.XTick = [15 40 60 85];
```



1- Let x(n) be a 4-point sequence:

$$x(n) = \begin{cases} 1, & 0 \le n \le 3 \\ 0, & \text{otherwise} \end{cases}$$

a. Compute the discrete time Fourier transform $X(e^{j\omega})$ and plot its magnitude and phase.

b. Compute the 4-point DFT of x(n)

c. How can we obtain other samples of the DTFT $X(e^{j\omega})$

2- Let a finite-length sequence be given by

$$x(n) = \begin{cases} 2e^{-0.9|n|}, & -5 \le n \le 5; \\ 0, & \text{otherwise.} \end{cases}$$

a. Plot the DTFT $X(e^{j\omega})$ of the above sequence using DFT as a computation tool. Choose the length N of the DFT so that this plot appears to be a smooth graph

3- Plot the DTFT magnitude of each of the following sequences using the DFT as a computation tool. Make an educated guess about the length N so that your plots are meaningful.

1. $x(n) = (0.6)^{|n|}\,[u(n+10) - u(n-11)]$.
2. $x(n) = n(0.9)^n\,[u(n) - u(n-21)]$.
3. $x(n) = [\cos(0.5\pi n) + j\sin(0.5\pi n)][u(n) - u(n-51)]$.
4. $x(n) = \{1, 2, 3, \underset{\uparrow}{4}, 3, 2, 1\}$.
5. $x(n) = \{-1, -2, -3, \underset{\uparrow}{0}, 3, 2, 1\}$.

4- Let $H(e^{j\omega})$ be the frequency response of a real, causal discrete-time LTI system.

a. If :

$$\mathrm{Re}\left\{H\left(e^{j\omega}\right)\right\} = \sum_{k=0}^{5} (0.9)^k \cos(k\omega)$$

determine the impulse response h(n) analytically. Verify your answer using DFT as a computation tool. Choose the length N appropriately.