



# Chapter 1. INTRODUCTION

The CUBLAS library is an implementation of BLAS (Basic Linear Algebra Subprograms) on top of the NVIDIA®CUDA™ runtime. It allows the user to access the computational resources of NVIDIA Graphics Processing Unit (GPU), but does not auto-parallelize across multiple GPUs.

To use the CUBLAS library, the application must allocate the required matrices and vectors in the GPU memory space, fill them with data, call the sequence of desired CUBLAS functions, and then upload the results from the GPU memory space back to the host. The CUBLAS library also provides helper functions for writing and retrieving data from the GPU.

#### 1.1. Data layout

For maximum compatibility with existing Fortran environments, the CUBLAS library uses column-major storage, and 1-based indexing. Since C and C++ use row-major storage, applications written in these languages can not use the native array semantics for two-dimensional arrays. Instead, macros or inline functions should be defined to implement matrices on top of one-dimensional arrays. For Fortran code ported to C in mechanical fashion, one may chose to retain 1-based indexing to avoid the need to transform loops. In this case, the array index of a matrix element in row "i" and column "j" can be computed via the following macro

```
#define IDX2F(i,j,ld) ((((j)-1)*(ld))+((i)-1))
```

Here, ld refers to the leading dimension of the matrix, which in the case of column-major storage is the number of rows of the allocated matrix (even if only a submatrix of it is being used). For natively written C and C++ code, one would most likely choose 0-based indexing, in which case the array index of a matrix element in row "i" and column "j" can be computed via the following macro

#define IDX2C(i,j,ld) (((j)\*(ld))+(i))

#### 1.2. New and Legacy CUBLAS API

Starting with version 4.0, the CUBLAS Library provides a new updated API, in addition to the existing legacy API. This section discusses why a new API is provided, the advantages of using it, and the differences with the existing legacy API.

The new CUBLAS library API can be used by including the header file "cublas\_v2.h". It has the following features that the legacy CUBLAS API does not have:

- the handle to the CUBLAS library context is initialized using the function and is explicitly passed to every subsequent library function call. This allows the user to have more control over the library setup when using multiple host threads and multiple GPUs. This also allows the CUBLAS APIs to be reentrant.
- the scalars  $\alpha$  and  $\beta$  can be passed by reference on the host or the device, instead of only being allowed to be passed by value on the host. This change allows library functions to execute asynchronously using streams even when  $\alpha$  and  $\beta$  are generated by a previous kernel.
- when a library routine returns a scalar result, it can be returned by reference on the host or the device, instead of only being allowed to be returned by value only on the host. This change allows library routines to be called asynchronously when the scalar result is generated and returned by reference on the device resulting in maximum parallelism.
- the error status cublasStatus\_t is returned by all CUBLAS library function calls. This change facilitates debugging and simplifies software development. Note that cublasStatus was renamed cublasStatus\_t to be more consistent with other types in the CUBLAS library.
- the cublasAlloc() and cublasFree() functions have been deprecated.
  This change removes these unnecessary wrappers around cudaMalloc() and cudaFree(), respectively.
- the function cublasSetKernelStream() was renamed cublasSetStream() to be more consistent with the other CUDA libraries.

The legacy CUBLAS API, explained in more detail in the Appendix A, can be used by including the header file "cublas.h". Since the legacy API is identical to the previously released CUBLAS library API, existing applications will work out of the box and automatically use this legacy API without any source code changes. In general, new applications should not use the legacy CUBLAS API, and existing existing applications should convert to using the new API if it requires sophisticated and optimal stream parallelism or if it calls CUBLAS routines concurrently from multiple threads. For the rest of the document, the new CUBLAS Library API will simply be referred to as the CUBLAS Library API.

As mentioned earlier the interfaces to the legacy and the CUBLAS library APIs are the header file "cublas.h" and "cublas\_v2.h", respectively. In addition, applications using the CUBLAS library need to link against the DSO cublas.so (Linux), the DLL cublas.dll (Windows), or the dynamic library cublas.dylib (Mac OS X). Note: the same dynamic library implements both the new and legacy CUBLAS APIs.

## 1.3. Example code

For sample code references please see the two examples below. They show an application written in C using the CUBLAS library API with two indexing styles (Example 1. "Application Using C and CUBLAS: 1-based indexing" and Example 2. "Application Using C and CUBLAS: 0-based Indexing").

```
//Example 1. Application Using C and CUBLAS: 1-based indexing
#include <stdio.h>
#include <stdlib.h>
#include <math.h>
#include <cuda runtime.h>
#include "cublas v2.h"
#define M 6
#define N 5
#define IDX2F(i, j, ld) ((((j)-1)*(ld))+((i)-1))
static __inline__ void modify (cublasHandle_t handle, float *m, int ldm, int
n, int p, int q, float alpha, float beta) {
    cublasSscal (handle, n-p+1, &alpha, &m[IDX2F(p,q,ldm)], ldm);
    cublasSscal (handle, ldm-p+1, &beta, &m[IDX2F(p,q,ldm)], 1);
int main (void) {
    cudaError t cudaStat;
    cublasStatus t stat;
    cublasHandle t handle;
    int i, j;
float* devPtrA;
    float* a = 0;
    a = (float *) malloc (M * N * sizeof (*a));
        printf ("host memory allocation failed");
        return EXIT FAILURE;
    for (j = 1; j <= N; j++) {
        for (i = 1; i <= M; i++) {</pre>
            a[IDX2F(i,j,M)] = (float)((i-1) * M + j);
    cudaStat = cudaMalloc ((void**)&devPtrA, M*N*sizeof(*a));
    if (cudaStat != cudaSuccess) {
       printf ("device memory allocation failed");
        return EXIT_FAILURE;
    stat = cublasCreate(&handle);
    if (stat != CUBLAS STATUS SUCCESS) {
       printf ("CUBLAS initialization failed\n");
        return EXIT FAILURE;
    stat = cublasSetMatrix (M, N, sizeof(*a), a, M, devPtrA, M);
    if (stat != CUBLAS STATUS SUCCESS) {
       printf ("data download failed");
        cudaFree (devPtrA);
        cublasDestroy(handle);
        return EXIT FAILURE;
    modify (handle, devPtrA, M, N, 2, 3, 16.0f, 12.0f);
    stat = cublasGetMatrix (M, N, sizeof(*a), devPtrA, M, a, M);
    if (stat != CUBLAS STATUS SUCCESS) {
        printf ("data \overline{u}pload \overline{f}ailed");
       cudaFree (devPtrA);
```

```
cublasDestroy(handle);
        return EXIT FAILURE;
    }
    cudaFree (devPtrA);
    cublasDestroy(handle);
    for (j = 1; j <= N; j++) {
  for (i = 1; i <= M; i++) {
            printf ("%7.0f", a[IDX2F(i,j,M)]);
        printf ("\n");
    free(a);
    return EXIT SUCCESS;
//Example 2. Application Using C and CUBLAS: 0-based indexing
#include <stdio.h>
#include <stdlib.h>
#include <math.h>
#include <cuda_runtime.h>
#include "cublas v2.h"
#define M 6
#define N 5
#define IDX2C(i,j,ld) (((j)*(ld))+(i))
static __inline__ void modify (cublasHandle_t handle, float *m, int ldm, int
n, int p, int q, float alpha, float beta) {
    cublasSscal (handle, n-p, &alpha, &m[IDX2C(p,q,ldm)], ldm);
    cublasSscal (handle, ldm-p, &beta, &m[IDX2C(p,q,ldm)], 1);
int main (void) {
   cudaError t cudaStat;
    cublasStatus t stat;
    cublasHandle_t handle;
    int i, j;
float* devPtrA;
    float* a = 0;
    a = (float *)malloc (M * N * sizeof (*a));
    if (!a) {
        printf ("host memory allocation failed");
        return EXIT FAILURE;
    for (j = 0; j < N; j++) {
        for (i = 0; i < M; i++) {
            a[IDX2C(i,j,M)] = (float)(i * M + j + 1);
    cudaStat = cudaMalloc ((void**) &devPtrA, M*N*sizeof(*a));
    if (cudaStat != cudaSuccess) {
        printf ("device memory allocation failed");
        return EXIT FAILURE;
    stat = cublasCreate(&handle);
    if (stat != CUBLAS STATUS SUCCESS) {
        printf ("CUBLAS initialization failed\n");
        return EXIT FAILURE;
    stat = cublasSetMatrix (M, N, sizeof(*a), a, M, devPtrA, M);
    if (stat != CUBLAS STATUS SUCCESS) {
        printf ("data download failed");
        cudaFree (devPtrA);
        cublasDestroy(handle);
        return EXIT_FAILURE;
    modify (handle, devPtrA, M, N, 1, 2, 16.0f, 12.0f);
    stat = cublasGetMatrix (M, N, sizeof(*a), devPtrA, M, a, M);
```

```
if (stat != CUBLAS_STATUS_SUCCESS) {
    printf ("data upload failed");
    cudaFree (devPtrA);
    cublasDestroy(handle);
    return EXIT_FAILURE;
}
cudaFree (devPtrA);
cublasDestroy(handle);
for (j = 0; j < N; j++) {
    for (i = 0; i < M; i++) {
        printf ("%7.0f", a[IDX2C(i,j,M)]);
    }
    printf ("\n");
}
free(a);
return EXIT_SUCCESS;
}</pre>
```

## Chapter 2. USING THE CUBLAS API

This section describes how to use the CUBLAS library API. It does not contain a detailed reference for all API datatypes and functions—those are provided in subsequent chapters. The Legacy CUBLAS API is also not covered in this section—that is handled in an Appendix.

#### 2.1. Error status

All CUBLAS library function calls return the error status **cublasStatus\_t**.

#### 2.2. CUBLAS context

The application must initialize the handle to the CUBLAS library context by calling the cublasCreate() function. Then, the is explicitly passed to every subsequent library function call. Once the application finishes using the library, it must call the function cublasDestory() to release the resources associated with the CUBLAS library context.

This approach allows the user to explicitly control the library setup when using multiple host threads and multiple GPUs. For example, the application can use <code>cudaSetDevice()</code> to associate different devices with different host threads and in each of those host threads it can initialize a unique <code>handle</code> to the CUBLAS library context, which will use the particular device associated with that host thread. Then, the CUBLAS library function calls made with different <code>handle</code> will automatically dispatch the computation to different devices.

The device associated with a particular CUBLAS context is assumed to remain unchanged between the corresponding <code>cublasCreate()</code> and <code>cublasDestory()</code> calls. In order for the CUBLAS library to use a different device in the same host thread, the application must set the new device to be used by calling <code>cudaSetDevice()</code> and then create another CUBLAS context, which will be associated with the new device, by calling <code>cublasCreate()</code>.

#### 2.3. Thread Safety

The library is thread safe and its functions can be called from multiple host threads, even with the same handle.

#### 2.4. Scalar Parameters

In the CUBLAS API the scalar parameters  $\alpha$  and  $\beta$  can be passed by reference on the host or the device

Also, the few functions that return a scalar result, such as <code>amax()</code>, <code>amin</code>, <code>asum()</code>, <code>rotg()</code>, <code>rotmg()</code>, <code>dot()</code> and <code>nrm2()</code>, return the resulting value by reference on the host or the device. Notice that even though these functions return immediately, similarly to matrix and vector results, the scalar result is ready only when execution of the routine on the GPU completes. This requires proper synchronization in order to read the result from the host.

These changes allow the library functions to execute completely asynchronously using streams even when  $\alpha$  and  $\beta$  are generated by a previous kernel. For example, this situation can arise when iterative methods for solution of linear systems and eigenvalue problems are implemented using the CUBLAS library.

#### 2.5. Parallelism with Streams

If the application uses the results computed by multiple independent tasks, CUDA<sup>TM</sup> streams can be used to overlap the computation performed in these tasks.

The application can conceptually associate each stream with each task. In order to achieve the overlap of computation between the tasks, the user should create CUDA<sup>TM</sup> streams using the function <code>cudaStreamCreate()</code> and set the stream to be used by each individual CUBLAS library routine by calling <code>cublasSetStream()</code> just before calling the actual CUBLAS routine. Then, the computation performed in separate streams would be overlapped automatically when possible on the GPU. This approach is especially useful when the computation performed by a single task is relatively small and is not enough to fill the GPU with work.

We recommend using the new CUBLAS API with scalar parameters and results passed by reference in the device memory to achieve maximum overlap of the computation when using streams.

A particular application of streams, batching of multiple small kernels, is described below.

#### 2.6. Batching Kernels

In this section we will explain how to use streams to batch the execution of small kernels. For instance, suppose that we have an application where we need to make many small independent matrix-matrix multiplications with dense matrices.

It is clear that even with millions of small independent matrices we will not be able to achieve the same *GFLOPS* rate as with a one large matrix. For example, a single  $n \times n$  large matrix-matrix multiplication performs  $n^3$  operations for  $n^2$  input size, while 1024

 $\frac{n}{32} \times \frac{n}{32}$  small matrix-matrix multiplications perform  $1024 \left(\frac{n}{32}\right)^3 = \frac{n^3}{32}$  operations for the same input size. However, it is also clear that we can achieve a significantly better performance with many small independent matrices compared with a single small matrix.

The architecture family of GPUs allows us to execute multiple kernels simultaneously. Hence, in order to batch the execution of independent kernels, we can run each of them in a separate stream. In particular, in the above example we could create 1024 CUDA<sup>TM</sup> streams using the function cudaStreamCreate(), then preface each call to cublas<tbr/>tygemm() with a call to cublasSetStream() with a different stream for each of the matrix-matrix multiplications. This will ensure that when possible the different computations will be executed concurrently. Although the user can create many streams, in practice it is not possible to have more than 16 concurrent kernels executing at the same time.

#### 2.7. Cache configuration

On some devices, L1 cache and shared memory use the same hardware resources. The cache configuration can be set directly with the CUDA Runtime function cudaDeviceSetCacheConfig. The cache configuration can also be set specifically for some functions using the routine cudaFuncSetCacheConfig. Please refer to the CUDA Runtime API documentation for details about the cache configuration settings.

Because switching from one configuration to another can affect kernels concurrency, the CUBLAS Library does not set any cache configuration preference and relies on the current setting. However, some CUBLAS routines, especially Level-3 routines, rely heavily on shared memory. Thus the cache preference setting might affect adversely their performance.

#### 2.8. Device API Library

Starting with release 5.0, the CUDA Toolkit now provides a static CUBLAS Library cublas\_device.a that contains device routines with the same API as the regular CUBLAS Library. Those routines use internally the Dynamic Parallelism feature to launch kernel from within and thus is only available for device with compute capability at least equal to 3.5.

In order to use those library routines from the device the user must include the header file "cublas\_v2.h" corresponding to the new CUBLAS API and link against the static CUBLAS library cublas\_device.a.

Those device CUBLAS library routines are called from the device in exactly the same way they are called from the host, with the following exceptions:

- ► The legacy CUBLAS API is not supported on the device.
- ▶ The pointer mode is not supported on the device, in other words, scalar input and output parameters must be allocated on the device memory.

Furthermore, the input and output scalar parameters must be allocated and released on the device using the cudaMalloc and cudaFree routines from the Host respectively or malloc and free routines from the device, in other words, they can not be passed by reference from the local memory to the routines.

# Chapter 3. CUBLAS DATATYPES REFERENCE

## 3.1. cublasHandle\_t

The cublasHandle\_t type is a pointer type to an opaque structure holding the CUBLAS library context. The CUBLAS library context must be initialized using cublasCreate() and the returned handle must be passed to all subsequent library function calls. The context should be destroyed at the end using cublasDestroy().

#### 3.2. cublasStatus\_t

The type is used for function status returns. All CUBLAS library functions return their status, which can have the following values.

Value	Meaning
CUBLAS_STATUS_SUCCESS	The operation completed successfully.
CUBLAS_STATUS_NOT_INITIALIZED	The CUBLAS library was not initialized. This is usually caused by the lack of a prior cublasCreate() call, an error in the CUDA Runtime API called by the CUBLAS routine, or an error in the hardware setup.
	To correct: call cublasCreate() prior to the function call; and check that the hardware, an appropriate version of the driver, and the CUBLAS library are correctly installed.
CUBLAS_STATUS_ALLOC_FAILED	Resource allocation failed inside the CUBLAS library. This is usually caused by a cudaMalloc() failure.
	To correct: prior to the function call, deallocate previously allocated memory as much as possible.
CUBLAS_STATUS_INVALID_VALUE	An unsupported value or parameter was passed to the function (a negative vector size, for example).

Value	Meaning
	To correct: ensure that all the parameters being passed have valid values.
CUBLAS_STATUS_ARCH_MISMATCH	The function requires a feature absent from the device architecture; usually caused by the lack of support for double precision.
	To correct: compile and run the application on a device with appropriate compute capability, which is 1.3 for double precision.
CUBLAS_STATUS_MAPPING_ERROR	An access to GPU memory space failed, which is usually caused by a failure to bind a texture.
	To correct: prior to the function call, unbind any previously bound textures.
CUBLAS_STATUS_EXECUTION_FAILED	The GPU program failed to execute. This is often caused by a launch failure of the kernel on the GPU, which can be caused by multiple reasons.
	To correct: check that the hardware, an appropriate version of the driver, and the CUBLAS library are correctly installed.
CUBLAS_STATUS_INTERNAL_ERROR	An internal CUBLAS operation failed. This error is usually caused by a cudaMemcpyAsync() failure.
	To correct: check that the hardware, an appropriate version of the driver, and the CUBLAS library are correctly installed. Also, check that the memory passed as a parameter to the routine is not being deallocated prior to the routine's completion.

#### 3.3. cublasOperation\_t

The cublasOperation\_t type indicates which operation needs to be performed with the dense matrix. Its values correspond to Fortran characters 'N' or 'n' (non-transpose), 'T' or 't' (transpose) and 'C' or 'c' (conjugate transpose) that are often used as parameters to legacy BLAS implementations.

Value	Meaning
CUBLAS_OP_N	the non-transpose operation is selected
CUBLAS_OP_T	the transpose operation is selected
CUBLAS_OP_C	the conjugate transpose operation is selected

## 3.4. cublasFillMode\_t

The type indicates which part (lower or upper) of the dense matrix was filled and consequently should be used by the function. Its values correspond to Fortran characters

'L' or 'l' (lower) and 'U' or 'u' (upper) that are often used as parameters to legacy BLAS implementations.

Value	Meaning
CUBLAS_FILL_MODE_LOWER	the lower part of the matrix is filled
CUBLAS_FILL_MODE_UPPER	the upper part of the matrix is filled

### 3.5. cublasDiagType\_t

The type indicates whether the main diagonal of the dense matrix is unity and consequently should not be touched or modified by the function. Its values correspond to Fortran characters 'N' or 'n' (non-unit) and 'U' or 'u' (unit) that are often used as parameters to legacy BLAS implementations.

Value	Meaning
CUBLAS_DIAG_NON_UNIT	the matrix diagonal has non-unit elements
CUBLAS_DIAG_UNIT	the matrix diagonal has unit elements

#### 3.6. cublasSideMode\_t

The type indicates whether the dense matrix is on the left or right side in the matrix equation solved by a particular function. Its values correspond to Fortran characters 'L' or 'l' (left) and 'R' or 'r' (right) that are often used as parameters to legacy BLAS implementations.

Value	Meaning
CUBLAS_SIDE_LEFT	the matrix is on the left side in the equation
CUBLAS_SIDE_RIGHT	the matrix is on the right side in the equation

#### 3.7. cublasPointerMode\_t

The cublasPointerMode\_t type indicates whether the scalar values are passed by reference on the host or device. It is important to point out that if several scalar values are present in the function call, all of them must conform to the same single pointer mode. The pointer mode can be set and retrieved using cublasSetPointerMode() and cublasGetPointerMode() routines, respectively.

Value	Meaning
CUBLAS_POINTER_MODE_HOST	the scalars are passed by reference on the host
CUBLAS_POINTER_MODE_DEVICE	the scalars are passed by reference on the device

## 3.8. cublasAtomicsMode\_t

The type indicates whether CUBLAS routines which has an alternate implementation using atomics can be used. The atomics mode can be set and queried using and routines, respectively.

Value	Meaning
CUBLAS_ATOMICS_NOT_ALLOWED	the usage of atomics is not allowed
CUBLAS_ATOMICS_ALLOWED	the usage of atomics is allowed

# Chapter 4. CUBLAS HELPER FUNCTION REFERENCE

## 4.1. cublasCreate()

```
cublasStatus_t
cublasCreate(cublasHandle_t *handle)
```

This function initializes the CUBLAS library and creates a handle to an opaque structure holding the CUBLAS library context. It allocates hardware resources on the host and device and must be called prior to making any other CUBLAS library calls.

Return Value	Meaning
CUBLAS_STATUS_SUCCESS	the initialization succeeded
CUBLAS_STATUS_NOT_INITIALIZED	the CUDA™ Runtime initialization failed
CUBLAS_STATUS_ALLOC_FAILED	the resources could not be allocated

## 4.2. cublasDestroy()

```
cublasStatus_t
cublasDestroy(cublasHandle t handle)
```

This function releases hardware resources used by the CUBLAS library. The release of GPU resources may be deferred until the application exits. This function is usually the last call with a particular handle to the CUBLAS library.

Return Value	Meaning
CUBLAS_STATUS_SUCCESS	the shut down succeeded
CUBLAS_STATUS_NOT_INITIALIZED	the library was not initialized

### 4.3. cublasGetVersion()

cublasStatus\_t

```
cublasGetVersion(cublasHandle t handle, int *version)
```

This function returns the version number of the CUBLAS library.

Return Value	Meaning
CUBLAS_STATUS_SUCCESS	the operation completed successfully
CUBLAS_STATUS_NOT_INITIALIZED	the library was not initialized

#### 4.4. cublasSetStream()

```
cublasStatus_t
cublasSetStream(cublasHandle t handle, cudaStream t streamId)
```

This function sets the CUBLAS library stream, which will be used to execute all subsequent calls to the CUBLAS library functions. If the CUBLAS library stream is not set, all kernels use the *default* **NULL** stream. In particular, this routine can be used to change the stream between kernel launches and then to reset the CUBLAS library stream back to **NULL**.

Return Value	Meaning
CUBLAS_STATUS_SUCCESS	the stream was set successfully
CUBLAS_STATUS_NOT_INITIALIZED	the library was not initialized

### 4.5. cublasGetStream()

```
cublasStatus_t
cublasGetStream(cublasHandle t handle, cudaStream t *streamId)
```

This function gets the CUBLAS library stream, which is being used to execute all calls to the CUBLAS library functions. If the CUBLAS library stream is not set, all kernels use the *default* **NULL** stream.

Return Value	Meaning
CUBLAS_STATUS_SUCCESS	the stream was returned successfully
CUBLAS_STATUS_NOT_INITIALIZED	the library was not initialized

#### 4.6. cublasGetPointerMode()

```
cublasStatus_t
cublasGetPointerMode(cublasHandle t handle, cublasPointerMode t *mode)
```

This function obtains the pointer mode used by the CUBLAS library. Please see the section on the **cublasPointerMode** t type for more details.

Return Value	Meaning
CUBLAS_STATUS_SUCCESS	the pointer mode was obtained successfully

Return Value	Meaning
CUBLAS_STATUS_NOT_INITIALIZED	the library was not initialized

#### 4.7. cublasSetPointerMode()

```
cublasStatus_t
cublasSetPointerMode(cublasHandle_t handle, cublasPointerMode_t mode)
```

This function sets the pointer mode used by the CUBLAS library. The *default* is for the values to be passed by reference on the host. Please see the section on the **cublasPointerMode** t type for more details.

Return Value	Meaning
CUBLAS_STATUS_SUCCESS	the pointer mode was set successfully
CUBLAS_STATUS_NOT_INITIALIZED	the library was not initialized

#### 4.8. cublasSetVector()

This function copies **n** elements from a vector **x** in host memory space to a vector **y** in GPU memory space. Elements in both vectors are assumed to have a size of **elemSize** bytes. The storage spacing between consecutive elements is given by **incx** for the source vector **x** and for the destination vector **y**.

In general, **y** points to an object, or part of an object, that was allocated via **cublasAlloc()**. Since column-major format for two-dimensional matrices is assumed, if a vector is part of a matrix, a vector increment equal to **1** accesses a (partial) column of that matrix. Similarly, using an increment equal to the leading dimension of the matrix results in accesses to a (partial) row of that matrix.

Return Value	Meaning
CUBLAS_STATUS_SUCCESS	the operation completed successfully
CUBLAS_STATUS_NOT_INITIALIZED	the library was not initialized
CUBLAS_STATUS_INVALID_VALUE	the parameters incx, incy, elemSize<=0
CUBLAS_STATUS_MAPPING_ERROR	there was an error accessing GPU memory

## 4.9. cublasGetVector()

This function copies **n** elements from a vector **x** in GPU memory space to a vector **y** in host memory space. Elements in both vectors are assumed to have a size of **elemSize** bytes. The storage spacing between consecutive elements is given by **incx** for the source vector and **incy** for the destination vector **y**.

In general, **x** points to an object, or part of an object, that was allocated via **cublasAlloc()**. Since column-major format for two-dimensional matrices is assumed, if a vector is part of a matrix, a vector increment equal to **1** accesses a (partial) column of that matrix. Similarly, using an increment equal to the leading dimension of the matrix results in accesses to a (partial) row of that matrix.

Return Value	Meaning
CUBLAS_STATUS_SUCCESS	the operation completed successfully
CUBLAS_STATUS_NOT_INITIALIZED	the library was not initialized
CUBLAS_STATUS_INVALID_VALUE	the parameters incx, incy, elemSize<=0
CUBLAS_STATUS_MAPPING_ERROR	there was an error accessing GPU memory

## 4.10. cublasSetMatrix()

This function copies a tile of rows x cols elements from a matrix A in host memory space to a matrix B in GPU memory space. It is assumed that each element requires storage of elemSize bytes and that both matrices are stored in column-major format, with the leading dimension of the source matrix A and destination matrix B given in lda and ldb, respectively. The leading dimension indicates the number of rows of the allocated matrix, even if only a submatrix of it is being used. In general, B is a device pointer that points to an object, or part of an object, that was allocated in GPU memory space via cublasAlloc().

Return Value	Meaning
CUBLAS_STATUS_SUCCESS	the operation completed successfully
CUBLAS_STATUS_NOT_INITIALIZED	the library was not initialized
CUBLAS_STATUS_INVALID_VALUE	the parameters rows, cols<0 or elemSize, lda, ldb<=0
CUBLAS_STATUS_MAPPING_ERROR	there was an error accessing GPU memory

#### 4.11. cublasGetMatrix()

This function copies a tile of rows x cols elements from a matrix A in GPU memory space to a matrix B in host memory space. It is assumed that each element requires storage of elemSize bytes and that both matrices are stored in column-major format, with the leading dimension of the source matrix A and destination matrix B given in lda and ldb, respectively. The leading dimension indicates the number of rows of the allocated matrix, even if only a submatrix of it is being used. In general, A is a device pointer that points to an object, or part of an object, that was allocated in GPU memory space via cublasAlloc().

Return Value	Meaning
CUBLAS_STATUS_SUCCESS	the operation completed successfully
CUBLAS_STATUS_NOT_INITIALIZED	the library was not initialized
CUBLAS_STATUS_INVALID_VALUE	the parameters rows, cols<0 Or elemSize, lda, ldb<=0
CUBLAS_STATUS_MAPPING_ERROR	there was an error accessing GPU memory

#### 4.12. cublasSetVectorAsync()

This function has the same functionality as **cublasSetVector()**, with the exception that the data transfer is done asynchronously (with respect to the host) using the given  $CUDA^{TM}$  stream parameter.

Return Value	Meaning
CUBLAS_STATUS_SUCCESS	the operation completed successfully
CUBLAS_STATUS_NOT_INITIALIZED	the library was not initialized
CUBLAS_STATUS_INVALID_VALUE	the parameters incx, incy, elemSize<=0
CUBLAS_STATUS_MAPPING_ERROR	there was an error accessing GPU memory

## 4.13. cublasGetVectorAsync()

This function has the same functionality as  ${\tt cublasGetVector}$ (), with the exception that the data transfer is done asynchronously (with respect to the host) using the given  ${\tt CUDA}^{\tt TM}$  stream parameter.

Return Value	Meaning
CUBLAS_STATUS_SUCCESS	the operation completed successfully
CUBLAS_STATUS_NOT_INITIALIZED	the library was not initialized

Return Value	Meaning
CUBLAS_STATUS_INVALID_VALUE	the parameters incx, incy, elemSize<=0
CUBLAS_STATUS_MAPPING_ERROR	there was an error accessing GPU memory

#### 4.14. cublasSetMatrixAsync()

This function has the same functionality as  ${\tt cublasSetMatrix()}$ , with the exception that the data transfer is done asynchronously (with respect to the host) using the given  ${\tt CUDA}^{\tt TM}$  stream parameter.

Return Value	Meaning
CUBLAS_STATUS_SUCCESS	the operation completed successfully
CUBLAS_STATUS_NOT_INITIALIZED	the library was not initialized
CUBLAS_STATUS_INVALID_VALUE	the parameters rows, cols<0 Or elemSize, lda, ldb<=0
CUBLAS_STATUS_MAPPING_ERROR	there was an error accessing GPU memory

## 4.15. cublasGetMatrixAsync()

This function has the same functionality as  ${\tt cublasGetMatrix()}$ , with the exception that the data transfer is done asynchronously (with respect to the host) using the given  ${\tt CUDA}^{\tt TM}$  stream parameter.

Return Value	Meaning
CUBLAS_STATUS_SUCCESS	the operation completed successfully
CUBLAS_STATUS_NOT_INITIALIZED	the library was not initialized
CUBLAS_STATUS_INVALID_VALUE	the parameters rows, cols<0 Or elemSize, lda, ldb<=0
CUBLAS_STATUS_MAPPING_ERROR	there was an error accessing GPU memory

## 4.16. cublasSetAtomicsMode()

cublasStatust cublasSetAtomicsMode(cublasHandlet handle, cublasAtomicsModet
 mode)

Some routines like cublas<t>symv and cublas<t>hemv have an alternate implementation that use atomics to cumulate results. This implementation is generally significantly faster but can generate results that are not strictly identical from one run to the others. Mathematically, those different results are not significant but when debugging those differences can be prejudicial.

This function allows or disallows the usage of atomics in the CUBLAS library for all routines which have an alternate implementation. When not explicitly specified in the documentation of any CUBLAS routine, it means that this routine does not have an alternate implementation that use atomics. When atomics mode is disabled, each CUBLAS routine should produce the same results from one run to the other when called with identical parameters on the same Hardware.

The value of the atomics mode is CUBLASATOMICSNOTALLOWED. Please see the section on the type for more details.

Return Value	Meaning
CUBLAS_STATUS_SUCCESS	the atomics mode was set successfully
CUBLAS_STATUS_NOT_INITIALIZED	the library was not initialized

## 4.17. cublasGetAtomicsMode()

 $cublas Statust\ cublas Get Atomics Mode (cublas Handlet\ handle,\ cublas Atomics Modet\ *mode)$ 

This function queries the atomic mode of a specific CUBLAS context.

The value of the atomics mode is CUBLASATOMICSNOTALLOWED. Please see the section on the type for more details.

Return Value	Meaning
CUBLAS_STATUS_SUCCESS	the atomics mode was queried successfully
CUBLAS_STATUS_NOT_INITIALIZED	the library was not initialized

# Chapter 5. CUBLAS LEVEL-1 FUNCTION REFERENCE

In this chapter we describe the Level-1 Basic Linear Algebra Subprograms (BLAS1) functions that perform scalar and vector based operations. We will use abbreviations <type> for type and <t> for the corresponding short type to make a more concise and clear presentation of the implemented functions. Unless otherwise specified <type> and <t> have the following meanings:

<type></type>	<t></t>	Meaning
float	's' or 'S'	real single-precision
double	'd' or 'D'	real double-precision
cuComplex	'c' or 'C'	complex single-precision
cuDoubleComplex	ʻz' or ʻZ'	complex double-precision

When the parameters and returned values of the function differ, which sometimes happens for complex input, the <t> can also have the following meanings 'Sc', 'Cs', 'Dz' and 'Zd'.

The abbreviation  $\mathbf{Re}(.)$  and  $\mathbf{Im}(.)$  will stand for the real and imaginary part of a number, respectively. Since imaginary part of a real number does not exist, we will consider it to be zero and can usually simply discard it from the equation where it is being used. Also, the  $\alpha$  will denote the complex conjugate of  $\alpha$ .

In general throughout the documentation, the lower case Greek symbols  $\alpha$  and  $\beta$  will denote scalars, lower case English letters in bold type  $\mathbf{x}$  and  $\mathbf{y}$  will denote vectors and capital English letters A, B and C will denote matrices.

### 5.1. cublasI<t>amax()

This function finds the (smallest) index of the element of the maximum magnitude. Hence, the result is the first i such that  $|\mathbf{Im}(x[j])| + |\mathbf{Re}(x[j])|$  is maximum for i = 1, ..., n and  $j = 1 + (i - 1)^*$  incx. Notice that the last equation reflects 1-based indexing used for compatibility with Fortran.

Param.	Memory	In/out	Meaning
handle		input	handle to the CUBLAS library context.
n		input	number of elements in the vector $\mathbf{x}$ .
х	device	input	<type> vector with elements.</type>
incx		input	stride between consecutive elements of ${f x}$ .
result	host or device	output	the resulting index, which is 0 if n,incx<=0.

The possible error values returned by this function and their meanings are listed below.

Error Value	Meaning
CUBLAS_STATUS_SUCCESS	the operation completed successfully
CUBLAS_STATUS_NOT_INITIALIZED	the library was not initialized
CUBLAS_STATUS_ALLOC_FAILED	the reduction buffer could not be allocated
CUBLAS_STATUS_ARCH_MISMATCH	the device does not support double-precision
CUBLAS_STATUS_EXECUTION_FAILED	the function failed to launch on the GPU

For references please refer to:

isamax, idamax, icamax, izamax

### 5.2. cublasI<t>amin()

This function finds the (smallest) index of the element of the minimum magnitude. Hence, the result is the first i such that  $|\mathbf{Im}(x[j])| + |\mathbf{Re}(x[j])|$  is minimum for i = 1, ..., n and  $j = 1 + (i - 1)^*$  incx Notice that the last equation reflects 1-based indexing used for compatibility with Fortran.

Param.	Memory	In/out	Meaning
handle		input	handle to the CUBLAS library context.
n		input	number of elements in the vector $\mathbf{x}$ .

Param.	Memory	In/out	Meaning
х	device	input	<type> vector with elements.</type>
incx		input	stride between consecutive elements of ${f x}$ .
result	host or device	output	the resulting index, which is 0 if n,inex<=0.

Error Value	Meaning
CUBLAS_STATUS_SUCCESS	the operation completed successfully
CUBLAS_STATUS_NOT_INITIALIZED	the library was not initialized
CUBLAS_STATUS_ALLOC_FAILED	the reduction buffer could not be allocated
CUBLAS_STATUS_ARCH_MISMATCH	the device does not support double-precision
CUBLAS_STATUS_EXECUTION_FAILED	the function failed to launch on the GPU

For references please refer to:

isamin

## 5.3. cublas<t>asum()

This function computes the sum of the absolute values of the elements of vector  $\mathbf{x}$ . Hence, the result is  $\sum_{j=1}^{n} |\mathbf{Im}(x[j])| + |\mathbf{Re}(x[j])|$  where  $j = 1 + (i-1)^*$  incx. Notice that the last

equation reflects 1-based indexing used for compatibility with Fortran.

Param.	Memory	In/out	Meaning
handle		input	handle to the CUBLAS library context.
n		input	number of elements in the vector ${f x}$ .
х	device	input	<type> vector with elements.</type>
incx		input	stride between consecutive elements of ${f x}$ .
result	host or device	output	the resulting index, which is 0.0 if n,inex<=0.

Error Value	Meaning
CUBLAS_STATUS_SUCCESS	the operation completed successfully
CUBLAS_STATUS_NOT_INITIALIZED	the library was not initialized
CUBLAS_STATUS_ALLOC_FAILED	the reduction buffer could not be allocated
CUBLAS_STATUS_ARCH_MISMATCH	the device does not support double-precision
CUBLAS_STATUS_EXECUTION_FAILED	the function failed to launch on the GPU

sasum, dasum, scasum, dzasum

## 5.4. cublas<t>axpy()

This function multiplies the vector  $\mathbf{x}$  by the scalar  $\alpha$  and adds it to the vector  $\mathbf{y}$  overwriting the latest vector with the result. Hence, the performed operation is  $\mathbf{y}[j] = \alpha \times \mathbf{x}[k] + \mathbf{y}[j]$  for i = 1, ..., n,  $k = 1 + (i - 1)^*$  incx and  $j = 1 + (i - 1)^*$  incy. Notice that the last two equations reflect 1-based indexing used for compatibility with Fortran.

Param.	Memory	In/out	Meaning
handle		input	handle to the CUBLAS library context.
alpha	host or device	input	<type> scalar used for multiplication.</type>
n		input	number of elements in the vector ${f x}$ and ${f y}$ .
x	device	input	<type> vector with n elements.</type>
incx		input	stride between consecutive elements of ${f x}$ .
у	device	in/out	<type> vector with <math>\mathbf{n}</math> elements.</type>
incy		input	stride between consecutive elements of $y$ .

Error Value	Meaning
CUBLAS_STATUS_SUCCESS	the operation completed successfully
CUBLAS_STATUS_NOT_INITIALIZED	the library was not initialized
CUBLAS_STATUS_ARCH_MISMATCH	the device does not support double-precision
CUBLAS_STATUS_EXECUTION_FAILED	the function failed to launch on the GPU

saxpy, daxpy, caxpy, zaxpy

## 5.5. cublas<t>copy()

This function copies the vector  $\mathbf{x}$  into the vector  $\mathbf{y}$ . Hence, the performed operation is  $\mathbf{y}[j] = \mathbf{x}[k]$  for i = 1, ..., n,  $k = 1 + (i - 1)^*$  incx and  $j = 1 + (i - 1)^*$  incy. Notice that the last two equations reflect 1-based indexing used for compatibility with Fortran.

Param.	Memory	In/out	Meaning
handle		input	handle to the CUBLAS library context.
n		input	number of elements in the vector ${f x}$ and ${f y}$ .
х	device	input	<type> vector with <math>\mathbf{n}</math> elements.</type>
incx		input	stride between consecutive elements of $\mathbf{x}$ .
у	device	output	<type> vector with <math>\mathbf{n}</math> elements.</type>
incy		input	stride between consecutive elements of $y$ .

Error Value	Meaning
CUBLAS_STATUS_SUCCESS	the operation completed successfully
CUBLAS_STATUS_NOT_INITIALIZED	the library was not initialized
CUBLAS_STATUS_ARCH_MISMATCH	the device does not support double-precision
CUBLAS_STATUS_EXECUTION_FAILED	the function failed to launch on the GPU

scopy, dcopy, ccopy, zcopy

#### 5.6. cublas<t>dot()

```
cublasStatus t cublasSdot (cublasHandle t handle, int n,
                      cublasStatus_t cublasDdot (cublasHandle_t handle, int n,
                      const double *x, int incx, const double *y, int incy,
                      cublasStatus t cublasCdotu(cublasHandle t handle, int n,
                      cuComplex *result)
cublasStatus_t cublasCdotc(cublasHandle_t handle, int n,
                      cuComplex *result)
cublasStatus t cublasZdotu(cublasHandle t handle, int n,
                      const cuDoubleComplex *x, int incx,
                      const cuDoubleComplex *y, int incy,
                      cuDoubleComplex *result)
cublasStatus t cublasZdotc(cublasHandle t handle, int n,
                      const cuDoubleComplex *x, int incx,
                      const cuDoubleComplex *y, int incy,
                      cuDoubleComplex *result)
```

This function computes the dot product of vectors  $\mathbf{x}$  and  $\mathbf{y}$ . Hence, the result is  $\sum_{i=1}^{n} \langle \mathbf{x}[k] \times \mathbf{y}[j] \rangle$  where  $k = 1 + (i-1)^*$  incx and  $j = 1 + (i-1)^*$  incy. Notice that in the first equation the conjugate of the element of vector should be used if the function name ends in character 'c' and that the last two equations reflect 1-based indexing used for compatibility with Fortran.

Param.	Memory	In/out	Meaning
handle		input	handle to the CUBLAS library context.
n		input	number of elements in the vectors ${f x}$ and ${f y}$ .
х	device	input	<type> vector with n elements.</type>
incx		input	stride between consecutive elements of x.
у	device	input	<type> vector with n elements.</type>
incy		input	stride between consecutive elements of y.
result	host or device	output	the resulting dot product, which is 0.0 if $n \le 0$ .

Error Value	Meaning
CUBLAS_STATUS_SUCCESS	the operation completed successfully

Error Value	Meaning
CUBLAS_STATUS_NOT_INITIALIZED	the library was not initialized
CUBLAS_STATUS_ALLOC_FAILED	the reduction buffer could not be allocated
CUBLAS_STATUS_ARCH_MISMATCH	the device does not support double-precision
CUBLAS_STATUS_EXECUTION_FAILED	the function failed to launch on the GPU

sdot, ddot, cdotu, cdotc, zdotu, zdotc

## 5.7. cublas<t>nrm2()

This function computes the Euclidean norm of the vector  $\mathbf{x}$ . The code uses a multiphase model of accumulation to avoid intermediate underflow and overflow, with the result

being equivalent to  $\sqrt{\sum_{i=1}^{n} (\mathbf{x}[j] \times \mathbf{x}[j])}$  where  $j = 1 + (i-1)^*$  incx in exact arithmetic. Notice that the last equation reflects 1-based indexing used for compatibility with Fortran.

Param.	Memory	In/out	Meaning
handle		input	handle to the CUBLAS library context.
n		input	number of elements in the vector ${f x}$ .
х	device	input	<type> vector with <math>\mathbf{n}</math> elements.</type>
incx		input	stride between consecutive elements of ${f x}$ .
result	host or device	output	the resulting norm, which is 0.0 if n,incx<=0.

The possible error values returned by this function and their meanings are listed below.

Error Value	Meaning
CUBLAS_STATUS_SUCCESS	the operation completed successfully
CUBLAS_STATUS_NOT_INITIALIZED	the library was not initialized
CUBLAS_STATUS_ALLOC_FAILED	the reduction buffer could not be allocated
CUBLAS_STATUS_ARCH_MISMATCH	the device does not support double-precision
CUBLAS_STATUS_EXECUTION_FAILED	the function failed to launch on the GPU

For references please refer to:

snrm2, snrm2, dnrm2, dnrm2, scnrm2, scnrm2, dznrm2

## 5.8. cublas<t>rot()

```
cublasStatus t cublasSrot(cublasHandle t handle, int n,
                               const float *c, const float
                                                                            *s)
cublasStatus t cublasDrot(cublasHandle t handle, int n,
                               double *x, int incx, double *y, int incy,
                              const double *c, const double
                                                                            *s)
cublasStatus_t cublasCrot(cublasHandle_t handle, int n,
                              cuComplex *x, int incx, cuComplex *y, int incy, const float *c, const cuComplex
                                                                            *s)
cublasStatus t cublasCsrot(cublasHandle t handle, int n,
                              cuComplex *x, int incx, cuComplex *y, int incy, const float *c, const float
                                                                            *s)
cublasStatus t cublasZrot(cublasHandle t handle, int n,
                              cuDoubleComplex *x, int incx,
                               cuDoubleComplex *y, int incy,
const double *c, const cuDoubleComplex *s)
cublasStatus_t cublasZdrot(cublasHandle_t handle, int n,
                               cuDoubleComplex *x, int incx,
                               cuDoubleComplex *y, int incy,
                               const double *c, const double
                                                                            *s)
```

This function applies Givens rotation matrix

$$G = \begin{pmatrix} C & S \\ -S & C \end{pmatrix}$$

to vectors  $\mathbf{x}$  and  $\mathbf{y}$ .

Hence, the result is  $\mathbf{x}[k] = c \times \mathbf{x}[k] + s \times \mathbf{y}[j]$  and  $\mathbf{y}[j] = -s \times \mathbf{x}[k] + c \times \mathbf{y}[j]$  where  $k = 1 + (i-1)^*$  incx and  $j = 1 + (i-1)^*$  incy. Notice that the last two equations reflect 1-based indexing used for compatibility with Fortran.

Param.	Memory	In/out	Meaning
handle		input	handle to the CUBLAS library context.
n		input	number of elements in the vectors ${f x}$ and ${f y}$ .
х	device	in/out	<type> vector with <math>\mathbf{n}</math> elements.</type>
incx		input	stride between consecutive elements of ${f x}$ .
у	device	in/out	<type> vector with <math>{f n}</math> elements.</type>
incy		input	stride between consecutive elements of $y$ .
С	host or device	input	cosine element of the rotation matrix.
s	host or device	input	sine element of the rotation matrix.

Error Value	Meaning
CUBLAS_STATUS_SUCCESS	the operation completed successfully
CUBLAS_STATUS_NOT_INITIALIZED	the library was not initialized
CUBLAS_STATUS_ARCH_MISMATCH	the device does not support double-precision
CUBLAS_STATUS_EXECUTION_FAILED	the function failed to launch on the GPU

srot, drot, crot, csrot, zrot, zdrot

## 5.9. cublas<t>rotg()

This function constructs the Givens rotation matrix

$$G = \begin{pmatrix} C & S \\ -S & C \end{pmatrix}$$

that zeros out the second entry of a  $2 \times 1$  vector  $(a, b)^T$ .

Then, for real numbers we can write

$$\begin{pmatrix} c & s \\ -s & c \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} r \\ 0 \end{pmatrix}$$

where  $c^2 + s^2 = 1$  and  $r = a^2 + b^2$ . The parameters a and b are overwritten with r and z, respectively. The value of z is such that c and s may be recovered using the following rules:

$$(c, s) = \begin{cases} (\sqrt{1-z^2}, z) & \text{if } |z| < 1\\ (0.0, 1.0) & \text{if } |z| = 1\\ (1/z, \sqrt{1-z^2}) & \text{if } |z| > 1 \end{cases}$$

For complex numbers we can write

$$\begin{pmatrix} c & s \\ -\overline{s} & c \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} r \\ 0 \end{pmatrix}$$

where  $c^2 + (\bar{s} \times s) = 1$  and  $r = \frac{a}{|a|} \times \|(a, b)^T\|_2$  with  $\|(a, b)^T\|_2 = \sqrt{|a|^2 + |b|^2}$  for  $a \neq 0$  and r = b for a = 0. Finally, the parameter a is overwritten with r on exit.

Param.	Memory	In/out	Meaning
handle		input	handle to the CUBLAS library context.
a	host or device	in/out	<type> scalar that is overwritten with <math>\emph{r}</math> .</type>
b	host or device	in/out	<type> scalar that is overwritten with <math>\emph{z}</math> .</type>
С	host or device	output	cosine element of the rotation matrix.
S	host or device	output	sine element of the rotation matrix.

Error Value	Meaning
CUBLAS_STATUS_SUCCESS	the operation completed successfully
CUBLAS_STATUS_NOT_INITIALIZED	the library was not initialized
CUBLAS_STATUS_ARCH_MISMATCH	the device does not support double-precision
CUBLAS_STATUS_EXECUTION_FAILED	the function failed to launch on the GPU

For references please refer to:

srotg, drotg, crotg, zrotg

### 5.10. cublas<t>rotm()

This function applies the modified Givens transformation

$$H = \begin{pmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{pmatrix}$$

to vectors  $\mathbf{x}$  and  $\mathbf{y}$ .

Hence, the result is  $\mathbf{x}[k] = h_{11} \times \mathbf{x}[k] + h_{12} \times \mathbf{y}[j]$  and  $\mathbf{y}[j] = h_{21} \times \mathbf{x}[k] + h_{22} \times \mathbf{y}[j]$  where  $k = 1 + (i-1)^*$  incx and  $j = 1 + (i-1)^*$  incy . Notice that the last two equations reflect 1-based indexing used for compatibility with Fortran.

The elements , , and of matrix H are stored in param[1], param[2], param[3] and param[4], respectively. The flag=param[0] defines the following predefined values for the matrix H entries

flag=-1.0	flag= 0.0	flag= 1.0	flag=-2.0
$   \begin{pmatrix}     h_{11} & h_{12} \\     h_{21} & h_{22}   \end{pmatrix} $	$\begin{pmatrix} 1.0 & h_{12} \\ h_{21} & 1.0 \end{pmatrix}$	$ \begin{pmatrix} h_{11} & 1.0 \\ -1.0 & h_{22} \end{pmatrix} $	$\begin{pmatrix} 1.0 & 0.0 \\ 0.0 & 1.0 \end{pmatrix}$

Notice that the values -1.0, 0.0 and 1.0 implied by the flag are not stored in param.

Param.	Memory	In/out	Meaning
handle		input	handle to the CUBLAS library context.
n		input	number of elements in the vectors ${f x}$ and ${f y}$ .
x	device	in/out	<type> vector with n elements.</type>
incx		input	stride between consecutive elements of ${f x}$ .
у	device	in/out	<type> vector with <math>\mathbf{n}</math> elements.</type>
incy		input	stride between consecutive elements of $y$ .
param	host or device	input	<pre><type> vector of 5 elements, where <math>param[0]</math> and <math>param[1-4]</math> contain the flag and matrix <math>H</math>.</type></pre>

Error Value	Meaning
CUBLAS_STATUS_SUCCESS	the operation completed successfully
CUBLAS_STATUS_NOT_INITIALIZED	the library was not initialized
CUBLAS_STATUS_ARCH_MISMATCH	the device does not support double-precision
CUBLAS_STATUS_EXECUTION_FAILED	the function failed to launch on the GPU

For references please refer to:

srotm, drotm

## 5.11. cublas<t>rotmg()

This function constructs the modified Givens transformation

$$H = \begin{pmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{pmatrix}$$

that zeros out the second entry of a  $2 \times 1$  vector  $(\sqrt{d1} * x1, \sqrt{d2} * y1)^T$ .

The flag=param[0] defines the following predefined values for the matrix H entries

flag=-1.0	flag= 0.0	flag= 1.0	flag=-2.0
$ \begin{pmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{pmatrix} $	$\begin{pmatrix} 1.0 & h_{12} \\ h_{21} & 1.0 \end{pmatrix}$	$\begin{pmatrix} h_{11} & 1.0 \\ -1.0 & h_{22} \end{pmatrix}$	$\begin{pmatrix} 1.0 & 0.0 \\ 0.0 & 1.0 \end{pmatrix}$

Notice that the values -1.0, 0.0 and 1.0 implied by the flag are not stored in param.

Param.	Memory	In/out	Meaning
handle		input	handle to the CUBLAS library context.
d1	host or device	in/out	<type> scalar that is overwritten on exit.</type>
d2	host or device	in/out	<type> scalar that is overwritten on exit.</type>
x1	host or device	in/out	<type> scalar that is overwritten on exit.</type>
y1	host or device	input	<type> scalar.</type>
param	host or device	output	<pre><type> vector of 5 elements, where <math>param[0]</math> and <math>param[1-4]</math> contain the flag and matrix <math>H</math>.</type></pre>

Error Value	Meaning
CUBLAS_STATUS_SUCCESS	the operation completed successfully
CUBLAS_STATUS_NOT_INITIALIZED	the library was not initialized
CUBLAS_STATUS_ARCH_MISMATCH	the device does not support double-precision
CUBLAS_STATUS_EXECUTION_FAILED	the function failed to launch on the GPU

For references please refer to:

srotmg, drotmg

#### 5.12. cublas<t>scal()

This function scales the vector  $\mathbf{x}$  by the scalar  $\alpha$  and overwrites it with the result. Hence, the performed operation is  $\mathbf{x}[j] = \alpha \times \mathbf{x}[j]$  for i = 1, ..., n and  $j = 1 + (i - 1)^*$  incx. Notice that the last two equations reflect 1-based indexing used for compatibility with Fortran.

Param.	Memory	In/out	Meaning
handle		input	handle to the CUBLAS library context.

Param.	Memory	In/out	Meaning
alpha	host or device	input	<type> scalar used for multiplication.</type>
n		input	number of elements in the vector ${f x}$ .
х	device	in/out	<type> vector with n elements.</type>
incx		input	stride between consecutive elements of ${f x}$ .

Error Value	Meaning
CUBLAS_STATUS_SUCCESS	the operation completed successfully
CUBLAS_STATUS_NOT_INITIALIZED	the library was not initialized
CUBLAS_STATUS_ARCH_MISMATCH	the device does not support double-precision
CUBLAS_STATUS_EXECUTION_FAILED	the function failed to launch on the GPU

For references please refer to:

sscal, dscal, csscal, cscal, zdscal, zscal

## 5.13. cublas<t>swap()

This function interchanges the elements of vector  $\mathbf{x}$  and  $\mathbf{y}$ . Hence, the performed operation is  $\mathbf{y}[j] \Leftrightarrow \mathbf{x}[k]$  for i = 1, ..., n,  $k = 1 + (i - 1)^*$  incx and  $j = 1 + (i - 1)^*$  incy. Notice that the last two equations reflect 1-based indexing used for compatibility with Fortran.

Param.	Memory	In/out	Meaning
handle		input	handle to the CUBLAS library context.
n		input	number of elements in the vector ${f x}$ and ${f y}$ .
х	device	in/out	<type> vector with <math>\mathbf{n}</math> elements.</type>
incx		input	stride between consecutive elements of $\mathbf{x}$ .
у	device	in/out	<type> vector with <math>\mathbf{n}</math> elements.</type>
incy		input	stride between consecutive elements of $y$ .

Error Value	Meaning
CUBLAS_STATUS_SUCCESS	the operation completed successfully
CUBLAS_STATUS_NOT_INITIALIZED	the library was not initialized
CUBLAS_STATUS_ARCH_MISMATCH	the device does not support double-precision
CUBLAS_STATUS_EXECUTION_FAILED	the function failed to launch on the GPU

sswap, dswap, cswap, zswap

# Chapter 6. CUBLAS LEVEL-2 FUNCTION REFERENCE

In this chapter we describe the Level-2 Basic Linear Algebra Subprograms (BLAS2) functions that perform matrix-vector operations.

#### 6.1. cublas<t>gbmv()

```
cublasStatus_t cublasSgbmv(cublasHandle_t handle, cublasOperation_t trans,
                             *A, int lda,
                                                    *x, int incx,
const double *A, int lda,
const double *X, int lda,
const double *x, int incx,
const double *y, int incy)
cublasStatus_t cublasCgbmv(cublasHandle_t handle, cublasOperation_t trans,
                             int m, int n, int kl, int ku,
                            const cuComplex *alpha,
const cuComplex *A, int lda,
const cuComplex *x, int incx,
const cuComplex *beta,
cuComplex *y, int incy)
cublasStatus_t cublasZgbmv(cublasHandle_t handle, cublasOperation_t trans,
                             int m, int n, int kl, int ku,
                             const cuDoubleComplex *alpha,
                             const cuDoubleComplex *A, int lda,
                             const cuDoubleComplex *x, int incx,
                             const cuDoubleComplex *beta,
                             cuDoubleComplex *y, int incy)
```

This function performs the banded matrix-vector multiplication

$$y = \alpha \operatorname{op}(A)x + \beta y$$

where A is a banded matrix with kl subdiagonals and ku superdiagonals,  $\mathbf{x}$  and  $\mathbf{y}$  are vectors, and  $\boldsymbol{\alpha}$  and  $\boldsymbol{\beta}$  are scalars. Also, for matrix A

$$op(A) = \begin{cases} A & \text{if transa} == \text{CUBLAS\_OP\_N} \\ A^T & \text{if transa} == \text{CUBLAS\_OP\_T} \\ A^H & \text{if transa} == \text{CUBLAS\_OP\_H} \end{cases}$$

The banded matrix A is stored column by column, with the main diagonal stored in row ku+1 (starting in first position), the first superdiagonal stored in row ku (starting in second position), the first subdiagonal stored in row ku+2 (starting in first position), etc. So that in general, the element A(i, j) is stored in the memory location  $\mathbf{A}$  ( $\mathbf{ku+1+i-j}$ ,  $\mathbf{j}$ ) for  $j=1,\ldots,n$  and  $i\in[\max(1,j-ku),\min(m,j+kl)]$ . Also, the elements in the array A that do not conceptually correspond to the elements in the banded matrix (the top left  $ku \times ku$  and bottom right  $kl \times kl$  triangles) are not referenced.

Param.	Memory	In/out	Meaning
handle		input	handle to the CUBLAS library context.
trans		input	operation op(A) that is non- or (conj.) transpose.
m		input	number of rows of matrix A.
n		input	number of columns of matrix A.
kl		input	number of subdiagonals of matrix A.
ku		input	number of superdiagonals of matrix ${f a}.$
alpha	host or device	input	<type> scalar used for multiplication.</type>
A	device	input	<type> array of dimension lda x n with lda&gt;=kl+ku+1.</type>
lda		input	leading dimension of two-dimensional array used to store matrix A.
х	device	input	<pre><type> vector with n elements if transa == CUBLAS_OP_N and m elements otherwise.</type></pre>
incx		input	stride between consecutive elements of x.
beta	host or device	input	<pre><type> scalar used for multiplication, if beta == 0 then y does not have to be a valid input.</type></pre>
у	device	in/out	<pre><type> vector with m elements if transa == CUBLAS_OP_N and n elements otherwise.</type></pre>
incy		input	stride between consecutive elements of y.

The possible error values returned by this function and their meanings are listed below.

Error Value	Meaning
CUBLAS_STATUS_SUCCESS	the operation completed successfully
CUBLAS_STATUS_NOT_INITIALIZED	the library was not initialized
CUBLAS_STATUS_INVALID_VALUE	the parameters or
CUBLAS_STATUS_ARCH_MISMATCH	the device does not support double-precision

Error Value	Meaning
CUBLAS_STATUS_EXECUTION_FAILED	the function failed to launch on the GPU

For references please refer to:

sgbmv, dgbmv, cgbmv, zgbmv

# 6.2. cublas<t>gemv()

```
cublasStatus t cublasSgemv(cublasHandle t handle, cublasOperation t trans,
                           int m, int n,
                          *A, int lda,
                                                *x, int incx,
cublasStatus_t cublasDgemv(cublasHandle_t handle, cublasOperation_t trans,
                          cublasStatus_t cublasCgemv(cublasHandle_t handle, cublasOperation_t trans,
                          int m, int n,
                          const cuComplex *alpha,
const cuComplex *A, int lda,
const cuComplex *x, int incx,
const cuComplex *beta,
                          cuComplex *y, int incy)
cublasStatus_t cublasZgemv(cublasHandle_t handle, cublasOperation_t trans,
                          int m, int n,
                          const cuDoubleComplex *alpha,
                          const cuDoubleComplex *A, int lda,
                          const cuDoubleComplex *x, int incx,
                          const cuDoubleComplex *beta,
                          cuDoubleComplex *y, int incy)
```

This function performs the matrix-vector multiplication

$$y = \alpha o p(A)x + \beta y$$

where A is a  $m \times n$  matrix stored in column-major format,  $\mathbf{x}$  and  $\mathbf{y}$  are vectors, and  $\boldsymbol{\alpha}$  and  $\boldsymbol{\beta}$  are scalars. Also, for matrix A

$$op(A) = \begin{cases} A & \text{if trans a} == \text{CUBLAS\_OP\_N} \\ A^T & \text{if trans a} == \text{CUBLAS\_OP\_T} \\ A^H & \text{if trans a} == \text{CUBLAS\_OP\_H} \end{cases}$$

Param.	Memory	In/out	Meaning
handle		input	handle to the CUBLAS library context.
trans		input	operation op(A) that is non- or (conj.) transpose.
m		input	number of rows of matrix A.
n		input	number of columns of matrix A.

Param.	Memory	In/out	Meaning
alpha	host or device	input	<type> scalar used for multiplication.</type>
A	device	input	<pre><type> array of dimension lda x n with lda &gt;= max(1,n) if transa==CUBLAS_OP_N and lda x m with lda &gt;= max(1,n) otherwise.</type></pre>
lda		input	leading dimension of two-dimensional array used to store matrix A.
х	device	input	<pre><type> vector with n elements if transa==CUBLAS_OP_N and m elements otherwise.</type></pre>
incx		input	stride between consecutive elements of ${f x}$ .
beta	host or device	input	<pre><type> scalar used for multiplication, if beta==0 then y does not have to be a valid input.</type></pre>
У	device	in/out	<pre><type> vector with m elements if transa==CUBLAS_OP_N and n elements otherwise.</type></pre>
incy		input	stride between consecutive elements of .y

Error Value	Meaning
CUBLAS_STATUS_SUCCESS	the operation completed successfully
CUBLAS_STATUS_NOT_INITIALIZED	the library was not initialized
CUBLAS_STATUS_INVALID_VALUE	the parameters m, n<9 or incx,incy=0
CUBLAS_STATUS_ARCH_MISMATCH	the device does not support double-precision
CUBLAS_STATUS_EXECUTION_FAILED	the function failed to launch on the GPU

For references please refer to:

sgemv, dgemv, cgemv, zgemv

#### 6.3. cublas<t>ger()

This function performs the rank-1 update

```
A = \begin{cases} \alpha \mathbf{x} \mathbf{y}^T + A & \text{if ger(),geru() is called} \\ \alpha \mathbf{x} \mathbf{y}^H + A & \text{if gerc() is called} \end{cases}
```

where *A* is a  $m \times n$  matrix stored in column-major format, **x** and **y** are vectors, and  $\alpha$  is a scalar.

Param.	Memory	In/out	Meaning
handle		input	handle to the CUBLAS library context.
m		input	number of rows of matrix A.
n		input	number of columns of matrix A.
alpha	host or device	input	<type> scalar used for multiplication.</type>
x	device	input	<type $>$ vector with $m$ elements.
incx		input	stride between consecutive elements of x.
у	device	input	<type> vector with n elements.</type>
incy		input	stride between consecutive elements of y.
А	device	in/out	<pre><type> array of dimension lda <math>\times</math> n with lda &gt;= max(1,m).</type></pre>
lda		input	leading dimension of two-dimensional array used to store matrix A.

The possible error values returned by this function and their meanings are listed below.

Error Value	Meaning
CUBLAS_STATUS_SUCCESS	the operation completed successfully
CUBLAS_STATUS_NOT_INITIALIZED	the library was not initialized
CUBLAS_STATUS_INVALID_VALUE	the parameters m, n<0 or incx,incy=0
CUBLAS_STATUS_ARCH_MISMATCH	the device does not support double-precision
CUBLAS_STATUS_EXECUTION_FAILED	the function failed to launch on the GPU

For references please refer to:

sger, dger, cgeru, cgerc, zgeru, zgerc

#### 6.4. cublas<t>sbmv()

This function performs the symmetric banded matrix-vector multiplication

$$y = \alpha Ax + \beta y$$

where A is a  $n \times n$  symmetric banded matrix with k subdiagonals and superdiagonals,  $\mathbf{x}$  and  $\mathbf{y}$  are vectors, and  $\boldsymbol{\alpha}$  and  $\boldsymbol{\beta}$  are scalars.

If **uplo == CUBLAS\_FILL\_MODE\_LOWER** then the symmetric banded matrix A is stored column by column, with the main diagonal of the matrix stored in row 1, the first subdiagonal in row 2 (starting at first position), the second subdiagonal in row 3 (starting at first position), etc. So that in general, the element A(i, j) is stored in the memory location A(1+i-j,j) for j=1,...,n and  $i \in [j, \min(m, j+k)]$ . Also, the elements in the array A that do not conceptually correspond to the elements in the banded matrix (the bottom right  $k \times k$  triangle) are not referenced.

If  $\mathtt{uplo} == \mathtt{CUBLAS\_FILL\_MODE\_UPPER}$  then the symmetric banded matrix A is stored column by column, with the main diagonal of the matrix stored in row  $\mathtt{k+1}$ , the first superdiagonal in row  $\mathtt{k}$  (starting at second position), the second superdiagonal in row  $\mathtt{k-1}$  (starting at third position), etc. So that in general, the element A(i, j) is stored in the memory location  $\mathtt{A}(\mathtt{1+k+i-j},\mathtt{j})$  for  $j=1,\ldots,n$  and  $i\in[\max(1,j-k),j]$ . Also, the elements in the array  $\mathtt{A}$  that do not conceptually correspond to the elements in the banded matrix (the top left  $k\times k$  triangle) are not referenced.

Param.	Memory	In/out	Meaning
handle		input	handle to the CUBLAS library context.
uplo		input	indicates if matrix A lower or upper part is stored, the other symmetric part is not referenced and is inferred from the stored elements.
n		input	number of rows and columns of matrix A.
k		input	number of sub- and super-diagonals of matrix A.
alpha	host or device	input	<type> scalar used for multiplication.</type>
А	device	input	<type> array of dimension lda x n with \lda &gt;= k+1.</type>
lda		input	leading dimension of two-dimensional array used to store matrix A.

Param.	Memory	In/out	Meaning
х	device	input	<type> vector with <math>\mathbf{n}</math> elements.</type>
incx		input	stride between consecutive elements of ${f x}$ .
beta	host or device	input	<type> scalar used for multiplication, if beta==0 then <math>\mathbf{y}</math> does not have to be a valid input.</type>
у	device	in/out	<type> vector with <math>\mathbf{n}</math> elements.</type>
incy		input	stride between consecutive elements of y.

Error Value	Meaning
CUBLAS_STATUS_SUCCESS	the operation completed successfully
CUBLAS_STATUS_NOT_INITIALIZED	the library was not initialized
CUBLAS_STATUS_INVALID_VALUE	the parameters n,k<0 or incx,incy=0
CUBLAS_STATUS_ARCH_MISMATCH	the device does not support double-precision
CUBLAS_STATUS_EXECUTION_FAILED	the function failed to launch on the GPU

For references please refer to:

ssbmv, dsbmv

#### 6.5. cublas<t>spmv()

This function performs the symmetric packed matrix-vector multiplication

$$\mathbf{y} = \alpha A \mathbf{x} + \beta \mathbf{y}$$

where *A* is a  $n \times n$  symmetric matrix stored in packed format, **x** and **y** are vectors, and  $\alpha$  and  $\beta$  are scalars.

If **uplo == CUBLAS\_FILL\_MODE\_LOWER** then the elements in the lower triangular part of the symmetric matrix A are packed together column by column without gaps, so that the element A(i, j) is stored in the memory location AP[i+((2\*n-j+1)\*j)/2] for j=1, ..., n and  $i \ge j$ . Consequently, the packed format requires only  $\frac{n(n+1)}{2}$  elements for storage.

If  $uplo == CUBLAS_FILL_MODE_UPPER$  then the elements in the upper triangular part of the symmetric matrix A are packed together column by column without gaps, so that

the element A(i, j) is stored in the memory location **AP[i+(j\*(j+1))/2]** for j = 1, ..., n and  $i \le j$ . Consequently, the packed format requires only  $\frac{n(n+1)}{2}$  elements for storage.

Param.	Memory	In/out	Meaning
handle		input	handle to the CUBLAS library context.
uplo		input	indicates if matrix ${\cal A}$ lower or upper part is stored, the other symmetric part is not referenced and is inferred from the stored elements.
n		input	number of rows and columns of matrix $\boldsymbol{A}$ .
alpha	host or device	input	<type> scalar used for multiplication.</type>
AP	device	input	$\ensuremath{^{<}}$ type $\ensuremath{^{>}}$ array with $A$ stored in packed format.
х	device	input	<type> vector with n elements.</type>
incx		input	stride between consecutive elements of ${f x}$ .
beta	host or device	input	<pre><type> scalar used for multiplication, if beta==0 then y does not have to be a valid input.</type></pre>
у	device	input	<type> vector with n elements.</type>
incy		input	stride between consecutive elements of $\mathbf{y}$ .

The possible error values returned by this function and their meanings are listed below.

Error Value	Meaning
CUBLAS_STATUS_SUCCESS	the operation completed successfully
CUBLAS_STATUS_NOT_INITIALIZED	the library was not initialized
CUBLAS_STATUS_INVALID_VALUE	the parameters n<0 or incx,incy=0
CUBLAS_STATUS_ARCH_MISMATCH	the device does not support double-precision
CUBLAS_STATUS_EXECUTION_FAILED	the function failed to launch on the GPU

For references please refer to:

sspmv, dspmv

# 6.6. cublas<t>spr()

This function performs the packed symmetric rank-1 update

$$A = \alpha \mathbf{x} \mathbf{x}^T + A$$

where *A* is a  $n \times n$  symmetric matrix stored in packed format, **x** is a vector, and  $\alpha$  is a scalar.

If **uplo == CUBLAS\_FILL\_MODE\_LOWER** then the elements in the lower triangular part of the symmetric matrix A are packed together column by column without gaps, so that the element A(i, j) is stored in the memory location AP[i+((2\*n-j+1)\*j)/2] for j=1, ..., n and  $i \ge j$ . Consequently, the packed format requires only  $\frac{n(n+1)}{2}$  elements for storage.

If **uplo == CUBLAS\_FILL\_MODE\_UPPER** then the elements in the upper triangular part of the symmetric matrix A are packed together column by column without gaps, so that the element A(i,j) is stored in the memory location AP[i+(j\*(j+1))/2] for j=1,...,n and  $i \le j$ . Consequently, the packed format requires only  $\frac{n(n+1)}{2}$  elements for storage.

Param.	Memory	In/out	Meaning
handle		input	handle to the CUBLAS library context.
uplo		input	indicates if matrix ${\cal A}$ lower or upper part is stored, the other symmetric part is not referenced and is inferred from the stored elements.
n		input	number of rows and columns of matrix $\boldsymbol{A}$ .
alpha	host or device	input	<type> scalar used for multiplication.</type>
х	device	input	<type> vector with n elements.</type>
incx		input	stride between consecutive elements of ${f x}$ .
AP	device	in/out	$\ensuremath{^{<}}$ type $\ensuremath{^{>}}$ array with $A$ stored in packed format.

The possible error values returned by this function and their meanings are listed below.

Error Value	Meaning
CUBLAS_STATUS_SUCCESS	the operation completed successfully
CUBLAS_STATUS_NOT_INITIALIZED	the library was not initialized
CUBLAS_STATUS_INVALID_VALUE	the parameters n<0 or incx,incy=0
CUBLAS_STATUS_ARCH_MISMATCH	the device does not support double-precision
CUBLAS_STATUS_EXECUTION_FAILED	the function failed to launch on the GPU

For references please refer to:

sspr, dspr

#### 6.7. cublas<t>spr2()

This function performs the packed symmetric rank-2 update

$$A = \alpha(\mathbf{x}\mathbf{y}^T + \mathbf{y}\mathbf{x}^T) + A$$

where *A* is a  $n \times n$  symmetric matrix stored in packed format, **x** is a vector, and  $\alpha$  is a scalar.

If **uplo == CUBLAS\_FILL\_MODE\_LOWER** then the elements in the lower triangular part of the symmetric matrix A are packed together column by column without gaps, so that the element A(i, j) is stored in the memory location AP[i+((2\*n-j+1)\*j)/2] for j=1, ..., n and  $i \ge j$ . Consequently, the packed format requires only  $\frac{n(n+1)}{2}$  elements for storage.

If **uplo == CUBLAS\_FILL\_MODE\_UPPER** then the elements in the upper triangular part of the symmetric matrix A are packed together column by column without gaps, so that the element A(i,j) is stored in the memory location AP[i+(j\*(j+1))/2] for  $j=1,\ldots,n$  and  $i \le j$ . Consequently, the packed format requires only  $\frac{n(n+1)}{2}$  elements for storage.

Param.	Memory	In/out	Meaning
handle		input	handle to the CUBLAS library context.
uplo		input	indicates if matrix $\boldsymbol{A}$ lower or upper part is stored, the other symmetric part is not referenced and is inferred from the stored elements.
n		input	number of rows and columns of matrix $\boldsymbol{A}$ .
alpha	host or device	input	<type> scalar used for multiplication.</type>
х	device	input	<type> vector with <math>\mathbf{n}</math> elements.</type>
incx		input	stride between consecutive elements of ${f x}$ .
у	device	input	<type> vector with n elements.</type>
incy		input	stride between consecutive elements of y.
AP	device	in/out	<type> array with <math>\boldsymbol{A}</math> stored in packed format.</type>

The possible error values returned by this function and their meanings are listed below.

Error Value	Meaning
CUBLAS_STATUS_SUCCESS	the operation completed successfully
CUBLAS_STATUS_NOT_INITIALIZED	the library was not initialized
CUBLAS_STATUS_INVALID_VALUE	the parameters n<0 or incx,incy=0
CUBLAS_STATUS_ARCH_MISMATCH	the device does not support double-precision

Error Value	Meaning
CUBLAS_STATUS_EXECUTION_FAILED	the function failed to launch on the GPU

For references please refer to:

sspr2, dspr2

#### 6.8. cublas<t>symv()

```
cublasStatus t cublasSsymv(cublasHandle t handle, cublasFillMode t uplo,
                         *beta,
float *y, int incy)
cublasStatus_t cublasDsymv(cublasHandle_t handle, cublasFillMode_t uplo,
                         int n, const double *A, int lda, const double *x, int incx, const double
      *beta,
                                   *y, int incy)
                        double
cublasStatus t cublasCsymv(cublasHandle t handle, cublasFillMode t uplo,
                         int n, const cuComplex
                                                   *alpha, \overline{/}* host or
device pointer */
                         *beta,
                         cuComplex
                                      *y, int incy)
cublasStatus t cublasZsymv(cublasHandle t handle, cublasFillMode t uplo,
                         int n, const cuDoubleComplex *alpha,
                         const cuDoubleComplex *A, int lda,
const cuDoubleComplex *x, int incx, const
cuDoubleComplex *beta,
                        cuDoubleComplex *y, int incy)
```

This function performs the symmetric matrix-vector multiplication.

$$y = \alpha Ax + \beta y$$

where *A* is a  $n \times n$  symmetric matrix stored in lower or upper mode, **x** and **x** are vectors, and  $\boldsymbol{\alpha}$  and  $\boldsymbol{\beta}$  are scalars.

This function has an alternate faster implementation using atomics that can be enabled with cublasSetAtomicsMode().

Please see the section on the function **cublasSetAtomicsMode()** for more details about the usage of atomics.

Param.	Memory	In/out	Meaning
handle		input	handle to the CUBLAS library context.
uplo		input	indicates if matrix lower or upper part is stored, the other symmetric part is not referenced and is inferred from the stored elements.
n		input	number of rows and columns of matrix A.
alpha	host or device	input	<type> scalar used for multiplication.</type>

Param.	Memory	In/out	Meaning
А	device	input	<pre><type> array of dimension lda <math>x</math> n with lda&gt;=max(1,n).</type></pre>
lda		input	leading dimension of two-dimensional array used to store matrix ${\bf A}$ .
х	device	input	<type> vector with n elements.</type>
incx		input	stride between consecutive elements of ${f x}$ .
beta	host or device	input	<pre><type> scalar used for multiplication, if beta==0 then y does not have to be a valid input.</type></pre>
у	device	in/out	<type> vector with n elements.</type>
incy		input	stride between consecutive elements of $y$ .

Error Value	Meaning
CUBLAS_STATUS_SUCCESS	the operation completed successfully
CUBLAS_STATUS_NOT_INITIALIZED	the library was not initialized
CUBLAS_STATUS_INVALID_VALUE	the parameters n<0 or incx,incy=0
CUBLAS_STATUS_ARCH_MISMATCH	the device does not support double-precision
CUBLAS_STATUS_EXECUTION_FAILED	the function failed to launch on the GPU

For references please refer to:

ssymv, dsymv

# 6.9. cublas<t>syr()

This function performs the symmetric rank-1 update

 $A = \alpha \mathbf{x} \mathbf{x}^T + A$ 

where *A* is a  $n \times n$  symmetric matrix stored in column-major format, **x** is a vector, and  $\alpha$  is a scalar.

Param.	Memory	In/out	Meaning
handle		input	handle to the CUBLAS library context.
uplo		input	indicates if matrix A lower or upper part is stored, the other symmetric part is not referenced and is inferred from the stored elements.
n		input	number of rows and columns of matrix A.
alpha	host or device	input	<type> scalar used for multiplication.</type>
x	device	input	<type> vector with <math>\mathbf{n}</math> elements.</type>
incx		input	stride between consecutive elements of ${f x}$ .
А	device	in/out	<pre><type> array of dimensions lda x n, with lda&gt;=max(1,n).</type></pre>
lda		input	leading dimension of two-dimensional array used to store matrix A.

The possible error values returned by this function and their meanings are listed below.

Error Value	Meaning
CUBLAS_STATUS_SUCCESS	the operation completed successfully
CUBLAS_STATUS_NOT_INITIALIZED	the library was not initialized
CUBLAS_STATUS_INVALID_VALUE	the parameters n<0 or incx=0
CUBLAS_STATUS_ARCH_MISMATCH	the device does not support double-precision
CUBLAS_STATUS_EXECUTION_FAILED	the function failed to launch on the GPU

For references please refer to:

ssyr, dsyr

# 6.10. cublas<t>syr2()

```
cublasStatus_t cublasSsyr2(cublasHandle_t handle, cublasFillMode_t uplo, int n,
                         const float *alpha, const float
*x, int incx,
                       const float
                                            *y, int incy, float
*A, int lda
cublasStatus_t cublasDsyr2(cublasHandle_t handle, cublasFillMode_t uplo, int n,
                        const double *alpha, const double
*x, int incx,
                        const double
                                             *y, int incy, double
*A, int lda
cublasStatus_t cublasCsyr2(cublasHandle_t handle, cublasFillMode_t uplo, int n,
                         const cuComplex *alpha, const cuComplex
*x, int incx,
                         const cuComplex *y, int incy, cuComplex
*A, int lda
cublasStatus t cublasZsyr2(cublasHandle t handle, cublasFillMode t uplo, int n,
```

```
const cuDoubleComplex *alpha, const cuDoubleComplex

*x, int incx,

const cuDoubleComplex *y, int incy, cuDoubleComplex

*A, int lda
```

This function performs the symmetric rank-2 update

$$A = \alpha(\mathbf{x}\mathbf{y}^T + \mathbf{y}\mathbf{x}^T) + A$$

where *A* is a  $n \times n$  symmetric matrix stored in column-major format, **x** and **y** are vectors, and  $\alpha$  is a scalar.

Param.	Memory	In/out	Meaning
handle		input	handle to the CUBLAS library context.
uplo		input	indicates if matrix A lower or upper part is stored, the other symmetric part is not referenced and is inferred from the stored elements.
n		input	number of rows and columns of matrix A.
alpha	host or device	input	<type> scalar used for multiplication.</type>
х	device	input	<type> vector with <math>\mathbf{n}</math> elements.</type>
incx		input	stride between consecutive elements of ${f x}$ .
у	device	input	<type> vector with <math>\mathbf{n}</math> elements.</type>
incy		input	stride between consecutive elements of y.
А	device	in/out	<pre><type> array of dimensions lda <math>x</math> n, with lda&gt;=max(1,n).</type></pre>
lda		input	leading dimension of two-dimensional array used to store matrix A.

The possible error values returned by this function and their meanings are listed below.

Error Value	Meaning
CUBLAS_STATUS_SUCCESS	the operation completed successfully
CUBLAS_STATUS_NOT_INITIALIZED	the library was not initialized
CUBLAS_STATUS_INVALID_VALUE	the parameters n<0 or incx,incy=0
CUBLAS_STATUS_ARCH_MISMATCH	the device does not support double-precision
CUBLAS_STATUS_EXECUTION_FAILED	the function failed to launch on the GPU

For references please refer to:

ssyr2, dsyr2

# 6.11. cublas<t>tbmv()

This function performs the triangular banded matrix-vector multiplication

$$\mathbf{x} = \operatorname{op}(A)\mathbf{x}$$

where A is a triangular banded matrix, and  $\mathbf{x}$  is a vector. Also, for matrix A

$$op(A) = \begin{cases} A & \text{if transa} == \text{CUBLAS\_OP\_N} \\ A^T & \text{if transa} == \text{CUBLAS\_OP\_T} \\ A^H & \text{if transa} == \text{CUBLAS\_OP\_C} \end{cases}$$

If **uplo == CUBLAS\_FILL\_MODE\_LOWER** then the triangular banded matrix A is stored column by column, with the main diagonal of the matrix stored in row **1**, the first subdiagonal in row **2** (starting at first position), the second subdiagonal in row **3** (starting at first position), etc. So that in general, the element A(i, j) is stored in the memory location A(1+i-j,j) for j=1, ..., n and  $i \in [j, \min(m, j+k)]$ . Also, the elements in the array **A** that do not conceptually correspond to the elements in the banded matrix (the bottom right  $k \times k$  triangle) are not referenced.

If **uplo == CUBLAS\_FILL\_MODE\_UPPER** then the triangular banded matrix A is stored column by column, with the main diagonal of the matrix stored in row **k+1**, the first superdiagonal in row **k** (starting at second position), the second superdiagonal in row **k-1** (starting at third position), etc. So that in general, the element A(i, j) is stored in the memory location A(1+k+i-j,j) for j=1,...,n and  $i \in [\max(1,j-k,j)]$ . Also, the elements in the array A that do not conceptually correspond to the elements in the banded matrix (the top left  $k \times k$  triangle) are not referenced.

Param.	Memory	In/out	Meaning
handle		input	handle to the CUBLAS library context.
uplo		input	indicates if matrix ${\tt A}$ lower or upper part is stored, the other part is not referenced and is inferred from the stored elements.
trans		input	operation op(A) that is non- or (conj.) transpose.
diag		input	indicates if the elements on the main diagonal of matrix ${\tt A}$ are unity and should not be accessed.
n		input	number of rows and columns of matrix A.
k		input	number of sub- and super-diagonals of matrix .
А	device	input	<type> array of dimension lda x n, with lda&gt;=k+1.</type>

Param.	Memory	In/out	Meaning
lda		input	leading dimension of two-dimensional array used to store matrix A.
х	device	in/out	<type> vector with <math>\mathbf{n}</math> elements.</type>
incx		input	stride between consecutive elements of x.

Error Value	Meaning
CUBLAS_STATUS_SUCCESS	the operation completed successfully
CUBLAS_STATUS_NOT_INITIALIZED	the library was not initialized
CUBLAS_STATUS_INVALID_VALUE	the parameters n,k<0 or incx=0
CUBLAS_STATUS_ARCH_MISMATCH	the device does not support double-precision
CUBLAS_STATUS_ALLOC_FAILED	the allocation of internal scratch memory failed
CUBLAS_STATUS_EXECUTION_FAILED	the function failed to launch on the GPU

For references please refer to:

stbmv, dtbmv, ctbmv, ztbmv

# 6.12. cublas<t>tbsv()

```
cublasStatus t cublasStbsv(cublasHandle t handle, cublasFillMode t uplo,
                          cublasOperation_t trans, cublasDiagType_t diag,
                           int n, int k, const float
                          float
                                         *x, int incx)
cublasStatus t cublasDtbsv(cublasHandle t handle, cublasFillMode t uplo,
                          cublasOperation t trans, cublasDiagType t diag,
                          int n, int k, const double *A, int lda,
                          double
                                          *x, int incx)
cublasStatus_t cublasCtbsv(cublasHandle_t handle, cublasFillMode_t uplo,
                          cublasOperation_t trans, cublasDiagType_t diag,
                          int n, int k, const cuComplex *A, int lda,
                          cuComplex *x, int incx)
cublasStatus t cublasZtbsv(cublasHandle t handle, cublasFillMode t uplo,
                          cublasOperation_t trans, cublasDiagType_t diag,
                          int n, int k, const cuDoubleComplex *A, int lda,
cuDoubleComplex *x, int incx)
```

This function solves the triangular banded linear system with a single right-hand-side  $op(A)\mathbf{x} = \mathbf{b}$ 

where A is a triangular banded matrix, and  $\mathbf{x}$  and  $\mathbf{b}$  are vectors. Also, for matrix A

$$op(A) = \begin{cases} A & \text{if transa} == \text{CUBLAS\_OP\_N} \\ A^T & \text{if transa} == \text{CUBLAS\_OP\_T} \\ A^H & \text{if transa} == \text{CUBLAS\_OP\_C} \end{cases}$$

The solution  $\mathbf{x}$  overwrites the right-hand-sides  $\mathbf{b}$  on exit.

No test for singularity or near-singularity is included in this function.

If **uplo == CUBLAS\_FILL\_MODE\_LOWER** then the triangular banded matrix A is stored column by column, with the main diagonal of the matrix stored in row **1**, the first subdiagonal in row **2** (starting at first position), the second subdiagonal in row **3** (starting at first position), etc. So that in general, the element A(i, j) is stored in the memory location A(1+i-j,j) for j=1, ..., n and  $i \in [j, \min(m, j+k)]$ . Also, the elements in the array **A** that do not conceptually correspond to the elements in the banded matrix (the bottom right  $k \times k$  triangle) are not referenced.

If **uplo == CUBLAS\_FILL\_MODE\_UPPER** then the triangular banded matrix A is stored column by column, with the main diagonal of the matrix stored in row **k+1**, the first superdiagonal in row **k** (starting at second position), the second superdiagonal in row **k-1** (starting at third position), etc. So that in general, the element A(i, j) is stored in the memory location **A(1+k+i-j,j)** for j = 1, ..., n and  $i \in [\max(1, j-k, j)]$ . Also, the elements in the array **A** that do not conceptually correspond to the elements in the banded matrix (the top left  $k \times k$  triangle) are not referenced.

Param.	Memory	In/out	Meaning
handle		input	handle to the CUBLAS library context.
uplo		input	indicates if matrix ${\bf A}$ lower or upper part is stored, the other part is not referenced and is inferred from the stored elements.
trans		input	operation op(A) that is non- or (conj.) transpose.
diag		input	indicates if the elements on the main diagonal of matrix ${\bf A}$ are unity and should not be accessed.
n		input	number of rows and columns of matrix A.
k		input	number of sub- and super-diagonals of matrix ${f A}$ .
Α	device	input	<pre><type> array of dimension lda <math>x</math> n, with lda &gt;= k+1.</type></pre>
lda		input	leading dimension of two-dimensional array used to store matrix A.
x	device	in/out	<type> vector with n elements.</type>
incx		input	stride between consecutive elements of $\mathbf{x}$ .

The possible error values returned by this function and their meanings are listed below.

Error Value	Meaning
CUBLAS_STATUS_SUCCESS	the operation completed successfully
CUBLAS_STATUS_NOT_INITIALIZED	the library was not initialized
CUBLAS_STATUS_INVALID_VALUE	the parameters n,k<0 or incx=0
CUBLAS_STATUS_ARCH_MISMATCH	the device does not support double-precision
CUBLAS_STATUS_EXECUTION_FAILED	the function failed to launch on the GPU

For references please refer to:

stbsv, dtbsv, ctbsv, ztbsv

# 6.13. cublas<t>tpmv()

```
cublasStatus t cublasStpmv(cublasHandle t handle, cublasFillMode t uplo,
                         cublasOperation t trans, cublasDiagType t diag,
                         int n, const float
                         float
                                        *x, int incx)
cublasStatus t cublasDtpmv(cublasHandle t handle, cublasFillMode t uplo,
                         cublasOperation_t trans, cublasDiagType_t diag,
                         int n, const double
                         cublasStatus t cublasCtpmv(cublasHandle t handle, cublasFillMode t uplo,
                         cublasOperation_t trans, cublasDiagType_t diag,
                         int n, const cuComplex
                         cuComplex *x, int incx)
cublasStatus t cublasZtpmv(cublasHandle t handle, cublasFillMode t uplo,
                         cublasOperation t trans, cublasDiagType t diag,
                         int n, const cuDoubleComplex *AP,
                         cuDoubleComplex *x, int incx)
```

This function performs the triangular packed matrix-vector multiplication

$$\mathbf{x} = \operatorname{op}(A)\mathbf{x}$$

where A is a triangular matrix stored in packed format, and  $\mathbf{x}$  is a vector. Also, for matrix A

$$op(A) = \begin{cases} A & \text{if transa} == \text{CUBLAS\_OP\_N} \\ A^T & \text{if transa} == \text{CUBLAS\_OP\_T} \\ A^H & \text{if transa} == \text{CUBLAS\_OP\_C} \end{cases}$$

If **uplo == CUBLAS\_FILL\_MODE\_LOWER** then the elements in the lower triangular part of the triangular matrix A are packed together column by column without gaps, so that the element A(i, j) is stored in the memory location AP[i+((2\*n-j+1)\*j)/2] for j=1, ..., n and  $i \ge j$ . Consequently, the packed format requires only  $\frac{n(n+1)}{2}$  elements for storage.

If **uplo == CUBLAS\_FILL\_MODE\_UPPER** then the elements in the upper triangular part of the triangular matrix A are packed together column by column without gaps, so that the element A(i, j) is stored in the memory location **AP[i+(j\*(j+1))/2]** for A(i, j) and

 $i \le j$  . Consequently, the packed format requires only  $\frac{n(n+1)}{2}$  elements for storage.

Param.	Memory	In/out	Meaning
handle		input	handle to the CUBLAS library context.
uplo		input	indicates if matrix ${\bf A}$ lower or upper part is stored, the other part is not referenced and is inferred from the stored elements.
trans		input	operation op(A) that is non- or (conj.) transpose.
diag		input	indicates if the elements on the main diagonal of matrix ${\tt A}$ are unity and should not be accessed.
n		input	number of rows and columns of matrix A.

Param.	Memory	In/out	Meaning
АР	device	input	<type> array with <math>A</math> stored in packed format.</type>
х	device	in/out	<type> vector with <math>\mathbf{n}</math> elements.</type>
incx		input	stride between consecutive elements of $\mathbf{x}$ .

Error Value	Meaning
CUBLAS_STATUS_SUCCESS	the operation completed successfully
CUBLAS_STATUS_NOT_INITIALIZED	the library was not initialized
CUBLAS_STATUS_INVALID_VALUE	the parameters \$n<0 or incx=0
CUBLAS_STATUS_ARCH_MISMATCH	the device does not support double-precision
CUBLAS_STATUS_ALLOC_FAILED	the allocation of internal scratch memory failed
CUBLAS_STATUS_EXECUTION_FAILED	the function failed to launch on the GPU

For references please refer to:

stpmv, dtpmv, ctpmv, ztpmv

#### 6.14. cublas<t>tpsv()

```
cublasStatus t cublasStpsv(cublasHandle t handle, cublasFillMode t uplo,
                        cublasOperation_t trans, cublasDiagType_t diag,
                        int n, const float
                        cublasStatus t cublasDtpsv(cublasHandle t handle, cublasFillMode t uplo,
                        cublasOperation_t trans, cublasDiagType_t diag,
                       int n, const double
                       cublasStatus t cublasCtpsv(cublasHandle t handle, cublasFillMode t uplo,
                       cublasOperation_t trans, cublasDiagType_t diag,
                        int n, const cuComplex
                                               *AP,
                       cuComplex *x, int incx)
cublasStatus t cublasZtpsv(cublasHandle t handle, cublasFillMode t uplo,
                        cublasOperation_t trans, cublasDiagType_t diag,
                        int n, const cuDoubleComplex *AP,
                        cuDoubleComplex *x, int incx)
```

This function solves the packed triangular linear system with a single right-hand-side

$$op(A)\mathbf{x} = \mathbf{b}$$

where A is a triangular matrix stored in packed format, and  $\mathbf{x}$  and  $\mathbf{b}$  are vectors. Also, for matrix A

$$op(A) = \begin{cases} A & \text{if transa} == \text{CUBLAS\_OP\_N} \\ A^T & \text{if transa} == \text{CUBLAS\_OP\_T} \\ A^H & \text{if transa} == \text{CUBLAS\_OP\_C} \end{cases}$$

The solution **x** overwrites the right-hand-sides **b** on exit.

No test for singularity or near-singularity is included in this function.

If **uplo == CUBLAS\_FILL\_MODE\_LOWER** then the elements in the lower triangular part of the triangular matrix A are packed together column by column without gaps, so that the element A(i, j) is stored in the memory location AP[i+((2\*n-j+1)\*j)/2] for j=1, ..., n and  $i \ge j$ . Consequently, the packed format requires only  $\frac{n(n+1)}{2}$  elements for storage.

If **uplo == CUBLAS\_FILL\_MODE\_UPPER** then the elements in the upper triangular part of the triangular matrix A are packed together column by column without gaps, so that the element A(i, j) is stored in the memory location AP[i+(j\*(j+1))/2] for j=1, ..., n and  $i \le j$ . Consequently, the packed format requires only  $\frac{n(n+1)}{2}$  elements for storage.

Param.	Memory	In/out	Meaning
handle		input	handle to the CUBLAS library context.
uplo		input	indicates if matrix ${\bf A}$ lower or upper part is stored, the other part is not referenced and is inferred from the stored elements.
trans		input	operation op(A) that is non- or (conj.) transpose.
diag		input	indicates if the elements on the main diagonal of matrix are unity and should not be accessed.
n		input	number of rows and columns of matrix ${f a}$ .
AP	device	input	<type> array with A stored in packed format.</type>
х	device	in/out	<type> vector with n elements.</type>
incx		input	stride between consecutive elements of $\mathbf{x}$ .

The possible error values returned by this function and their meanings are listed below.

Error Value	Meaning
CUBLAS_STATUS_SUCCESS	the operation completed successfully
CUBLAS_STATUS_NOT_INITIALIZED	the library was not initialized
CUBLAS_STATUS_INVALID_VALUE	the parameters n<0 or incx=0
CUBLAS_STATUS_ARCH_MISMATCH	the device does not support double-precision
CUBLAS_STATUS_EXECUTION_FAILED	the function failed to launch on the GPU

For references please refer to:

stpsv, dtpsv, ctpsv, ztpsv

# 6.15. cublas<t>trmv()

This function performs the triangular matrix-vector multiplication

$$\mathbf{x} = \operatorname{op}(A)\mathbf{x}$$

where A is a triangular matrix stored in lower or upper mode with or without the main diagonal, and  $\mathbf{x}$  is a vector. Also, for matrix A

$$op(A) = \begin{cases} A & \text{if transa} == \text{CUBLAS\_OP\_N} \\ A^T & \text{if transa} == \text{CUBLAS\_OP\_T} \\ A^H & \text{if transa} == \text{CUBLAS\_OP\_C} \end{cases}$$

Param.	Memory	In/out	Meaning
handle		input	handle to the CUBLAS library context.
uplo		input	indicates if matrix ${\bf A}$ lower or upper part is stored, the other part is not referenced and is inferred from the stored elements.
trans		input	operation op(A) that is non- or (conj.) transpose.
diag		input	indicates if the elements on the main diagonal of matrix ${\bf A}$ are unity and should not be accessed.
n		input	number of rows and columns of matrix A.
А	device	input	<pre><type> array of dimensions lda <math>x n</math>, with lda&gt;=max(1,n).</type></pre>
lda		input	leading dimension of two-dimensional array used to store matrix A.
x	device	in/out	<type> vector with n elements.</type>
incx		input	stride between consecutive elements of x.

The possible error values returned by this function and their meanings are listed below.

Error Value	Meaning
CUBLAS_STATUS_SUCCESS	the operation completed successfully
CUBLAS_STATUS_NOT_INITIALIZED	the library was not initialized
CUBLAS_STATUS_INVALID_VALUE	the parameters n<0 or incx=0
CUBLAS_STATUS_ARCH_MISMATCH	the device does not support double-precision
CUBLAS_STATUS_ALLOC_FAILED	the allocation of internal scratch memory failed

Error Value	Meaning
CUBLAS_STATUS_EXECUTION_FAILED	the function failed to launch on the GPU

For references please refer to:

strmv, dtrmv, ctrmv, ztrmv

#### 6.16. cublas<t>trsv()

```
cublasStatus t cublasStrsv(cublasHandle t handle, cublasFillMode t uplo,
                         cublasOperation t trans, cublasDiagType t diag,
                                                   *A, int lda,
                         int n, const float
                         float *x, int incx)
cublasStatus t cublasDtrsv(cublasHandle t handle, cublasFillMode t uplo,
                        cublasOperation_t trans, cublasDiagType_t diag,
                        int n, const double
                                                   *A, int lda,
                                *x, int incx)
                        double
cublasStatus t cublasCtrsv(cublasHandle t handle, cublasFillMode t uplo,
                        cublasOperation_t trans, cublasDiagType_t diag,
                        int n, const cuComplex *A, int lda,
                        cublasStatus t cublasZtrsv(cublasHandle t handle, cublasFillMode t uplo,
                        cublasOperation_t trans, cublasDiagType_t diag,
                         int n, const cuDoubleComplex *A, int lda,
                         cuDoubleComplex *x, int incx)
```

This function solves the triangular linear system with a single right-hand-side

$$op(A)\mathbf{x} = \mathbf{b}$$

where A is a triangular matrix stored in lower or upper mode with or without the main diagonal, and  $\mathbf{x}$  and  $\mathbf{b}$  are vectors. Also, for matrix A

$$op(A) = \begin{cases} A & \text{if transa} == \text{CUBLAS\_OP\_N} \\ A^T & \text{if transa} == \text{CUBLAS\_OP\_T} \\ A^H & \text{if transa} == \text{CUBLAS\_OP\_C} \end{cases}$$

The solution **x** overwrites the right-hand-sides **b** on exit.

No test for singularity or near-singularity is included in this function.

Param.	Memory	In/out	Meaning
handle		input	handle to the CUBLAS library context.
uplo		input	indicates if matrix ${\bf A}$ lower or upper part is stored, the other part is not referenced and is inferred from the stored elements.
trans		input	operation op(A) that is non- or (conj.) transpose.
diag		input	indicates if the elements on the main diagonal of matrix ${f A}$ are unity and should not be accessed.
n		input	number of rows and columns of matrix A.
А	device	input	<pre><type> array of dimension lda x n, with lda&gt;=max(1,n).</type></pre>

Param.	Memory	In/out	Meaning
lda		input	leading dimension of two-dimensional array used to store matrix A.
х	device	in/out	<type> vector with <math>\mathbf{n}</math> elements.</type>
incx		input	stride between consecutive elements of x.

Error Value	Meaning
CUBLAS_STATUS_SUCCESS	the operation completed successfully
CUBLAS_STATUS_NOT_INITIALIZED	the library was not initialized
CUBLAS_STATUS_INVALID_VALUE	the parameters n<0 or incx=0
CUBLAS_STATUS_ARCH_MISMATCH	the device does not support double-precision
CUBLAS_STATUS_EXECUTION_FAILED	the function failed to launch on the GPU

For references please refer to:

strsv, dtrsv, ctrsv, ztrsv

#### 6.17. cublas<t>hemv()

This function performs the Hermitian matrix-vector multiplication

$$y = \alpha Ax + \beta y$$

where *A* is a  $n \times n$  Hermitian matrix stored in lower or upper mode, **x** and **y** are vectors, and  $\boldsymbol{\alpha}$  and  $\boldsymbol{\beta}$  are scalars.

This function has an alternate faster implementation using atomics that can be enabled with

Please see the section on the for more details about the usage of atomics

Param.	Memory	In/out	Meaning
handle		input	handle to the CUBLAS library context.

Param.	Memory	In/out	Meaning
uplo		input	indicates if matrix A lower or upper part is stored, the other Hermitian part is not referenced and is inferred from the stored elements.
n		input	number of rows and columns of matrix A.
alpha	host or device	input	<type> scalar used for multiplication.</type>
A	device	input	<pre><type> array of dimension lda x n, with lda&gt;=max(1,n). The imaginary parts of the diagonal elements are assumed to be zero.</type></pre>
lda		input	leading dimension of two-dimensional array used to store matrix A.
х	device	input	<type> vector with n elements.</type>
incx		input	stride between consecutive elements of ${f x}$ .
beta	host or device	input	<type> scalar used for multiplication, if beta==0 then <math>\mathbf{y}</math> does not have to be a valid input.</type>
у	device	in/out	<type> vector with n elements.</type>
incy		input	stride between consecutive elements of y.

Error Value	Meaning
CUBLAS_STATUS_SUCCESS	the operation completed successfully
CUBLAS_STATUS_NOT_INITIALIZED	the library was not initialized
CUBLAS_STATUS_INVALID_VALUE	the parameters n<0 or incx,incy=0
CUBLAS_STATUS_ARCH_MISMATCH	the device does not support double-precision
CUBLAS_STATUS_EXECUTION_FAILED	the function failed to launch on the GPU

For references please refer to:

chemv, zhemv

#### 6.18. cublas<t>hbmv()

This function performs the Hermitian banded matrix-vector multiplication

$$\mathbf{y} = \alpha A \mathbf{x} + \beta \mathbf{y}$$

where A is a  $n \times n$  Hermitian banded matrix with k subdiagonals and superdiagonals,  $\mathbf{x}$  and  $\mathbf{y}$  are vectors, and  $\boldsymbol{\alpha}$  and  $\boldsymbol{\beta}$  are scalars.

If uplo == CUBLAS\_FILL\_MODE\_LOWER then the Hermitian banded matrix A is stored column by column, with the main diagonal of the matrix stored in row 1, the first subdiagonal in row 2 (starting at first position), the second subdiagonal in row 3 (starting at first position), etc. So that in general, the element A(i, j) is stored in the memory location A(1+i-j,j) for  $j=1,\ldots,n$  and  $i \in [j,\min(m,j+k)]$ . Also, the elements in the array A that do not conceptually correspond to the elements in the banded matrix (the bottom right  $k \times k$  triangle) are not referenced.

If  $\mathtt{uplo} == \mathtt{CUBLAS\_FILL\_MODE\_UPPER}$  then the Hermitian banded matrix A is stored column by column, with the main diagonal of the matrix stored in row  $\mathtt{k+1}$ , the first superdiagonal in row  $\mathtt{k}$  (starting at second position), the second superdiagonal in row  $\mathtt{k-1}$  (starting at third position), etc. So that in general, the element A(i, j) is stored in the memory location  $\mathtt{A}(\mathtt{1+k+i-j},\mathtt{j})$  for  $j=1,\ldots,n$  and  $i\in[\max(1,j-k),j]$ . Also, the elements in the array  $\mathtt{A}$  that do not conceptually correspond to the elements in the banded matrix (the top left  $k\times k$  triangle) are not referenced.

Param.	Memory	In/out	Meaning
handle		input	handle to the CUBLAS library context.
uplo		input	indicates if matrix A lower or upper part is stored, the other Hermitian part is not referenced and is inferred from the stored elements.
n		input	number of rows and columns of matrix A.
k		input	number of sub- and super-diagonals of matrix A.
alpha	host or device	input	<type> scalar used for multiplication.</type>
A	device	input	<pre><type> array of dimensions lda x n, with lda&gt;=k+1. The imaginary parts of the diagonal elements are assumed to be zero.</type></pre>
lda		input	leading dimension of two-dimensional array used to store matrix A.
х	device	input	<type> vector with n elements.</type>
incx		input	stride between consecutive elements of x.
beta	host or device	input	<pre><type> scalar used for multiplication, if beta==0 then does not have to be a valid input.</type></pre>
у	device	in/out	<type> vector with n elements.</type>
incy		input	stride between consecutive elements of y.

The possible error values returned by this function and their meanings are listed below.

Error Value	Meaning
CUBLAS_STATUS_SUCCESS	the operation completed successfully
CUBLAS_STATUS_NOT_INITIALIZED	the library was not initialized
CUBLAS_STATUS_INVALID_VALUE	the parameters n,k<0 or incx,incy=0
CUBLAS_STATUS_ARCH_MISMATCH	the device does not support double-precision
CUBLAS_STATUS_EXECUTION_FAILED	the function failed to launch on the GPU

For references please refer to:

chbmv, zhbmv

#### 6.19. cublas<t>hpmv()

This function performs the Hermitian packed matrix-vector multiplication

$$y = \alpha Ax + \beta y$$

where *A* is a  $n \times n$  Hermitian matrix stored in packed format, **x** and **y** are vectors, and  $\alpha$  and  $\beta$  are scalars.

If **uplo == CUBLAS\_FILL\_MODE\_LOWER** then the elements in the lower triangular part of the Hermitian matrix A are packed together column by column without gaps, so that the element A(i, j) is stored in the memory location AP[i+((2\*n-j+1)\*j)/2] for j=1, ..., n and  $i \ge j$ . Consequently, the packed format requires only  $\frac{n(n+1)}{2}$  elements for storage.

If **uplo == CUBLAS\_FILL\_MODE\_UPPER** then the elements in the upper triangular part of the Hermitian matrix A are packed together column by column without gaps, so that the element A(i, j) is stored in the memory location **AP[i+(j\*(j+1))/2]** for j = 1, ..., n and  $i \le j$ . Consequently, the packed format requires only  $\frac{n(n+1)}{2}$  elements for storage.

Param.	Memory	In/out	Meaning
handle		input	handle to the CUBLAS library context.
uplo		input	indicates if matrix A lower or upper part is stored, the other Hermitian part is not referenced and is inferred from the stored elements.

Param.	Memory	In/out	Meaning
n		input	number of rows and columns of matrix A.
alpha	host or device	input	<type> scalar used for multiplication.</type>
АР	device	input	<type> array with A stored in packed format. The imaginary parts of the diagonal elements are assumed to be zero.</type>
х	device	input	<type> vector with <math>\mathbf{n}</math> elements.</type>
incx		input	stride between consecutive elements of $\mathbf{x}$ .
beta	host or device	input	<pre><type> scalar used for multiplication, if beta==0 then y does not have to be a valid input.</type></pre>
у	device	in/out	<type> vector with n elements.</type>
incy		input	stride between consecutive elements of y.

Error Value	Meaning
CUBLAS_STATUS_SUCCESS	the operation completed successfully
CUBLAS_STATUS_NOT_INITIALIZED	the library was not initialized
CUBLAS_STATUS_INVALID_VALUE	the parameters n<0 or incx,incy=0
CUBLAS_STATUS_ARCH_MISMATCH	the device does not support double-precision
CUBLAS_STATUS_EXECUTION_FAILED	the function failed to launch on the GPU

For references please refer to:

chpmv, zhpmv

# 6.20. cublas<t>her()

This function performs the Hermitian rank-1 update

$$A = \alpha \mathbf{x} \mathbf{x}^H + A$$

where *A* is a  $n \times n$  Hermitian matrix stored in column-major format, **x** is a vector, and  $\alpha$  is a scalar.

Param.	Memory	In/out	Meaning
handle		input	handle to the CUBLAS library context.

Param.	Memory	In/out	Meaning
uplo		input	indicates if matrix A lower or upper part is stored, the other Hermitian part is not referenced and is inferred from the stored elements.
n		input	number of rows and columns of matrix A.
alpha	host or device	input	<type> scalar used for multiplication.</type>
х	device	input	<type> vector with n elements.</type>
incx		input	stride between consecutive elements of ${f x}$ .
A	device	in/out	<pre><type> array of dimensions lda x n, with lda&gt;=max(1,n). The imaginary parts of the diagonal elements are assumed and set to zero.</type></pre>
lda		input	leading dimension of two-dimensional array used to store matrix A.

Error Value	Meaning
CUBLAS_STATUS_SUCCESS	the operation completed successfully
CUBLAS_STATUS_NOT_INITIALIZED	the library was not initialized
CUBLAS_STATUS_INVALID_VALUE	the parameters n<0 or incx=0
CUBLAS_STATUS_ARCH_MISMATCH	the device does not support double-precision
CUBLAS_STATUS_EXECUTION_FAILED	the function failed to launch on the GPU

For references please refer to:

cher, zher

# 6.21. cublas<t>her2()

This function performs the Hermitian rank-2 update

$$A = \alpha \mathbf{x} \mathbf{y}^H + \bar{\alpha} \mathbf{y} \mathbf{x}^H + A$$

where *A* is a  $n \times n$  Hermitian matrix stored in column-major format, **x** and **y** are vectors, and  $\alpha$  is a scalar.

Param.	Memory	In/out	Meaning
handle		input	handle to the CUBLAS library context.
uplo		input	indicates if matrix A lower or upper part is stored, the other Hermitian part is not referenced and is inferred from the stored elements.
n		input	number of rows and columns of matrix A.
alpha	host or device	input	<type> scalar used for multiplication.</type>
х	device	input	<type> vector with n elements.</type>
incx		input	stride between consecutive elements of x.
у	device	input	<type> vector with n elements.</type>
incy		input	stride between consecutive elements of y.
A	device	in/out	<pre><type> array of dimension lda x n with lda&gt;=max(1,n). The imaginary parts of the diagonal elements are assumed and set to zero.</type></pre>
lda		input	leading dimension of two-dimensional array used to store matrix A.

Error Value	Meaning
CUBLAS_STATUS_SUCCESS	the operation completed successfully
CUBLAS_STATUS_NOT_INITIALIZED	the library was not initialized
CUBLAS_STATUS_INVALID_VALUE	the parameters n<0 or incx,incy=0
CUBLAS_STATUS_ARCH_MISMATCH	the device does not support double-precision
CUBLAS_STATUS_EXECUTION_FAILED	the function failed to launch on the GPU

For references please refer to:

cher2, zher2

# 6.22. cublas<t>hpr()

This function performs the packed Hermitian rank-1 update

 $A = \alpha \mathbf{x} \mathbf{x}^H + A$ 

where *A* is a  $n \times n$  Hermitian matrix stored in packed format, **x** is a vector, and  $\alpha$  is a scalar.

If **uplo == CULBAS\_FILL\_MODE\_LOWER** then the elements in the lower triangular part of the Hermitian matrix A are packed together column by column without gaps, so that the element A(i, j) is stored in the memory location AP[i+((2\*n-j+1)\*j)/2] for j=1, ..., n and  $i \ge j$ . Consequently, the packed format requires only  $\frac{n(n+1)}{2}$  elements for storage.

If **uplo == CULBAS\_FILL\_MODE\_UPPER** then the elements in the upper triangular part of the Hermitian matrix A are packed together column by column without gaps, so that the element A(i, j) is stored in the memory location AP[i+(j\*(j+1))/2] for j=1, ..., n and  $i \le j$ . Consequently, the packed format requires only  $\frac{n(n+1)}{2}$  elements for storage.

Param.	Memory	In/out	Meaning
handle		input	handle to the CUBLAS library context.
uplo		input	indicates if matrix A lower or upper part is stored, the other Hermitian part is not referenced and is inferred from the stored elements.
n		input	number of rows and columns of matrix A.
alpha	host or device	input	<type> scalar used for multiplication.</type>
х	device	input	<type> vector with n elements.</type>
incx		input	stride between consecutive elements of $\mathbf{x}$ .
АР	device	in/out	<type> array with <math>\mathbf{A}</math> stored in packed format. The imaginary parts of the diagonal elements are assumed and set to zero.</type>

The possible error values returned by this function and their meanings are listed below.

Error Value	Meaning
CUBLAS_STATUS_SUCCESS	the operation completed successfully
CUBLAS_STATUS_NOT_INITIALIZED	the library was not initialized
CUBLAS_STATUS_INVALID_VALUE	the parameters n<0 or inex=0
CUBLAS_STATUS_ARCH_MISMATCH	the device does not support double-precision
CUBLAS_STATUS_EXECUTION_FAILED	the function failed to launch on the GPU

For references please refer to:

chpr, zhpr

# 6.23. cublas<t>hpr2()

cublasStatus\_t cublasChpr2(cublasHandle\_t handle, cublasFillMode\_t uplo,

This function performs the packed Hermitian rank-2 update

$$A = \alpha \mathbf{x} \mathbf{y}^H + \bar{\alpha} \mathbf{y} \mathbf{x}^H + A$$

where *A* is a  $n \times n$  Hermitian matrix stored in packed format, **x** and **y** are vectors, and  $\alpha$  is a scalar.

If **uplo == CULBAS\_FILL\_MODE\_LOWER** then the elements in the lower triangular part of the Hermitian matrix A are packed together column by column without gaps, so that the element A(i, j) is stored in the memory location AP[i+((2\*n-j+1)\*j)/2] for j=1, ..., n and  $i \ge j$ . Consequently, the packed format requires only  $\frac{n(n+1)}{2}$  elements for storage.

If **uplo == CULBAS\_FILL\_MODE\_UPPER** then the elements in the upper triangular part of the Hermitian matrix A are packed together column by column without gaps, so that the element A(i, j) is stored in the memory location AP[i+(j\*(j+1))/2] for j=1, ..., n and  $i \le j$ . Consequently, the packed format requires only  $\frac{n(n+1)}{2}$  elements for storage.

Param.	Memory	In/out	Meaning
	, ,		3
handle		input	handle to the CUBLAS library context.
uplo		input	indicates if matrix A lower or upper part is stored, the other Hermitian part is not referenced and is inferred from the stored elements.
n		input	number of rows and columns of matrix A.
alpha	host or device	input	<type> scalar used for multiplication.</type>
х	device	input	<type> vector with n elements.</type>
incx		input	stride between consecutive elements of $\mathbf{x}$ .
у	device	input	<type> vector with n elements.</type>
incy		input	stride between consecutive elements of y.
AP	device	in/out	<type> array with A stored in packed format. The imaginary parts of the diagonal elements are assumed and set to zero.</type>

The possible error values returned by this function and their meanings are listed below.

Error Value	Meaning
CUBLAS_STATUS_SUCCESS	the operation completed successfully
CUBLAS_STATUS_NOT_INITIALIZED	the library was not initialized

Error Value	Meaning
CUBLAS_STATUS_INVALID_VALUE	the parameters n<0 or incx,incy=0
CUBLAS_STATUS_ARCH_MISMATCH	the device does not support double-precision
CUBLAS_STATUS_EXECUTION_FAILED	the function failed to launch on the GPU

For references please refer to: chpr2, zhpr2

# Chapter 7. CUBLAS LEVEL-3 FUNCTION REFERENCE

In this chapter we describe the Level-3 Basic Linear Algebra Subprograms (BLAS3) functions that perform matrix-matrix operations.

#### 7.1. cublas<t>gemm()

```
cublasStatus_t cublasSgemm(cublasHandle_t handle,
                           cublasOperation_t transa, cublasOperation_t transb,
                           int m, int n, int k,
                           const float
                                                  *alpha,
                           const float
                           *A, int lda,
                                                 *B, int ldb,
cublasStatus t cublasDgemm(cublasHandle t handle,
                           cublasOperation_t transa, cublasOperation_t transb,
                           int m, int n, int k,
                           const double
                                                  *alpha,
                                                 *A, int lda, *B, int ldb,
                           const double
                           const double
                           const double
                                                  *beta,
                                          *C, int ldc)
                           double
cublasStatus t cublasCgemm(cublasHandle t handle,
                           cublasOperation_t transa, cublasOperation_t transb,
                           int m, int n, int k,
                           const cuComplex *alpha,
const cuComplex *A, int lda,
                           const cuComplex *B, int ldb, const cuComplex *beta,
                           const cucomplex *beta,
cublasStatus t cublasZgemm(cublasHandle t handle,
                           cublasOperation t transa, cublasOperation t transb,
                           int m, int n, \overline{int} k,
                           const cuDoubleComplex *alpha,
                           const cuDoubleComplex *A, int lda,
                           const cuDoubleComplex *B, int ldb,
                           const cuDoubleComplex *beta,
                           cuDoubleComplex *C, int ldc)
```

This function performs the matrix-matrix multiplication

 $C = \alpha \operatorname{op}(A)\operatorname{op}(B) + \beta C$ 

where  $\alpha$  and  $\beta$  are scalars, and A, B and C are matrices stored in column-major format with dimensions op(A)  $m \times k$ , op(B)  $k \times n$  and C  $m \times n$ , respectively. Also, for matrix A

$$op(A) = \begin{cases} A & \text{if transa} == \text{CUBLAS\_OP\_N} \\ A^T & \text{if transa} == \text{CUBLAS\_OP\_T} \\ A^H & \text{if transa} == \text{CUBLAS\_OP\_C} \end{cases}$$

and op(B) is defined similarly for matrix B.

Param.	Memory	In/out	Meaning
handle		input	handle to the CUBLAS library context.
transa		input	operation op(A) that is non- or (conj.) transpose.
transb		input	operation op(B) that is non- or (conj.) transpose.
m		input	number of rows of matrix op(A) and c.
n		input	number of columns of matrix op(B) and c.
k		input	number of columns of op(A) and rows of op(B).
alpha	host or device	input	<type> scalar used for multiplication.</type>
A	device	input	<pre><type> array of dimensions lda x k With lda&gt;=max(1,m) if transa == CUBLAS_OP_N and lda x m With lda&gt;=max(1,k) otherwise.</type></pre>
lda		input	leading dimension of two-dimensional array used to store the matrix A.
В	device	input	<pre><type> array of dimension ldb x n with ldb&gt;=max(1,k) if transa == CUBLAS_OP_N and ldb x k with ldb&gt;=max(1,n) otherwise.</type></pre>
ldb		input	leading dimension of two-dimensional array used to store matrix B.
beta	host or device	input	<pre><type> scalar used for multiplication. If beta==0, c does not have to be a valid input.</type></pre>
С	device	in/out	<pre><type> array of dimensions ldc x n With ldc&gt;=max(1,m).</type></pre>
ldc		input	leading dimension of a two-dimensional array used to store the matrix c.

The possible error values returned by this function and their meanings are listed below.

Error Value	Meaning
CUBLAS_STATUS_SUCCESS	the operation completed successfully
CUBLAS_STATUS_NOT_INITIALIZED	the library was not initialized
CUBLAS_STATUS_INVALID_VALUE	the parameters m, n, k<0
CUBLAS_STATUS_ARCH_MISMATCH	the device does not support double-precision
CUBLAS_STATUS_EXECUTION_FAILED	the function failed to launch on the GPU

For references please refer to:

sgemm, dgemm, cgemm, zgemm

#### 7.2. cublas<t>gemmBatched()

```
cublasStatus t cublasSgemmBatched(cublasHandle t handle,
                                                                                                                   cublasOperation t transa, cublasOperation t
   transb,
                                                                                                                   int m, int n, int k,
                                                                                                                 const float *alpha,
const float *Aarray[], int lda,
const float *Barray[], int ldb,
const float *beta,
float *Carray[], int ldc, int
  batchCount.)
cublasStatus t cublasDgemmBatched(cublasHandle t handle,
                                                                                                                   cublasOperation t transa, cublasOperation t
  transb,
                                                                                                                   int m, int n, int k,
                                                                                                                                                                                           *alpha,
                                                                                                                  const double const double
                                                                                                                                                                                         *Aarray[], int lda,
                                                                                                                  const double const double const double the const double const double the const double const double the const
  batchCount)
cublasStatus t cublasCgemmBatched(cublasHandle t handle,
                                                                                                                   cublasOperation t transa, cublasOperation t
   transb.
                                                                                                                  int m, int n, int k,
                                                                                                                  const cuComplex *alpha,
const cuComplex *Aarray[], int lda,
const cuComplex *Barray[], int ldb,
const cuComplex *beta,
                                                                                                                  cuComplex *Carray[], int ldc, int
  batchCount)
cublasStatus t cublasZgemmBatched(cublasHandle t handle,
                                                                                                                   cublasOperation t transa, cublasOperation t
   transb,
                                                                                                                   int m, int n, int k,
                                                                                                                   const cuDoubleComplex *alpha,
const cuDoubleComplex *Aarray[], int lda,
                                                                                                                  const cuDoubleComplex *Barray[], int ldb,
                                                                                                                   const cuDoubleComplex *beta,
                                                                                                                   cuDoubleComplex *Carray[], int ldc, int
  batchCount)
```

This function performs the matrix-matrix multiplications of an array of matrices.

```
C[i] = \alpha \operatorname{op}(A[i]) \operatorname{op}(B[i]) + \beta C[i], for i \in [0, batchCount - 1]
```

where  $\alpha$  and  $\beta$  are scalars, and A, B and C are arrays of pointers to matrices stored in column-major format with dimensions op(A[i])  $m \times k$ , op(B[i])  $k \times n$  and C[i]  $m \times n$ , respectively. Also, for matrix A

$$op(A) = \begin{cases} A & \text{if transa} == \text{CUBLAS\_OP\_N} \\ A^T & \text{if transa} == \text{CUBLAS\_OP\_T} \\ A^H & \text{if transa} == \text{CUBLAS\_OP\_C} \end{cases}$$

and op(B[i]) is defined similarly for matrix B[i].

This function is intended to be used for matrices of small sizes where the launch overhead is a significant factor. For small sizes, typically smaller than 100x100, this function improves significantly performance compared to making calls to its corresponding <code>cublas<t>gemm</code> routine. However, on GPU architectures that support concurrent kernels, it might be advantageous to make multiple calls to <code>cublas<t>gemm</code> into different streams as the matrix sizes increase.

Param.	Memory	In/out	Meaning
handle		input	handle to the CUBLAS library context.
transa		input	operation op(A[i]) that is non- or (conj.) transpose.
transb		input	operation op(B[i]) that is non- or (conj.) transpose.
m		input	number of rows of matrix op(A[i]) and C[i].
n		input	number of columns of op(B[i]) and c[i].
k		input	number of columns of $op(A[i])$ and rows of $op(B[i])$ .
alpha	host or device	input	<type> scalar used for multiplication.</type>
Aarray	device	input	array of pointers to <type> array, with each array of dim. lda x k With lda&gt;=max(1,m) if transa==CUBLAS_OP_N and lda x m with lda&gt;=max(1,k) otherwise.</type>
lda		input	leading dimension of two-dimensional array used to store each matrix ${\tt A[i]}$ .
Barray	device	input	array of pointers to <type> array, with each array of dim. ldb x n With ldb&gt;=max(1,k) if transa==CUBLAS_OP_N and ldb x k with ldb&gt;=max(1,n) max(1,) otherwise.</type>
ldb		input	leading dimension of two-dimensional array used to store each matrix B[i].
beta	host or device	input	<pre><type> scalar used for multiplication. If beta == 0, C does not have to be a valid input.</type></pre>
Carray	device	in/out	array of pointers to <type> array. It has dimensions ldc <math>x</math> n with ldc&gt;=max(1,m).</type>
ldc		input	leading dimension of two-dimensional array used to store each matrix c[i].
batchCount		input	number of pointers contained in Aarray, Barray and Carray.

The possible error values returned by this function and their meanings are listed below.

Error Value	Meaning
CUBLAS_STATUS_SUCCESS	the operation completed successfully
CUBLAS_STATUS_NOT_INITIALIZED	the library was not initialized
CUBLAS_STATUS_INVALID_VALUE	the parameters m,n,k,batchCount<0
CUBLAS_STATUS_ARCH_MISMATCH	the device does not support double-precision

Error Value	Meaning
CUBLAS_STATUS_EXECUTION_FAILED	the function failed to launch on the GPU

### 7.3. cublas<t>symm()

```
cublasStatus t cublasSsymm(cublasHandle t handle,
                                   cublasSideMode_t side, cublasFillMode_t uplo,
                                   int m, int n,
                                                              *alpha,
                                   const float
                                   *A, int lda, *B, int ldb,
cublasStatus t cublasDsymm(cublasHandle t handle,
                                   cublasSideMode_t side, cublasFillMode_t uplo,
                                  int m, int n,
const double
                                   cublasStatus t cublasCsymm(cublasHandle t handle,
                                   cublasSideMode t side, cublasFillMode t uplo,
                                   int m, int n,
                                  const cuComplex *alpha,
const cuComplex *A, int lda,
const cuComplex *B, int ldb,
const cuComplex *beta,
cuComplex *C, int ldc)
cublasStatus t cublasZsymm(cublasHandle t handle,
                                   cublasSideMode t side, cublasFillMode t uplo,
                                   int m, int n,
                                   const cuDoubleComplex *alpha,
                                   const cuDoubleComplex *A, int lda,
                                   const cuDoubleComplex *B, int ldb,
                                   const cuDoubleComplex *beta,
                                   cuDoubleComplex *C, int ldc)
```

This function performs the symmetric matrix-matrix multiplication

$$C = \begin{cases} \alpha AB + \beta C & \text{if side} == \text{CUBLAS\_SIDE\_LEFT} \\ \alpha BA + \beta C & \text{if side} == \text{CUBLAS\_SIDE\_RIGHT} \end{cases}$$

where A is a symmetric matrix stored in lower or upper mode, A and A are  $m \times n$  matrices, and  $\alpha$  and  $\beta$  are scalars.

Param.	Memory	In/out	Meaning
handle		input	handle to the CUBLAS library context.
side		input	indicates if matrix A is on the left or right of B.
uplo		input	indicates if matrix A lower or upper part is stored, the other symmetric part is not referenced and is inferred from the stored elements.
m		input	number of rows of matrix ${\bf A}$ and ${\bf B}$ , with matrix ${\bf A}$ sized accordingly.

Param.	Memory	In/out	Meaning
n		input	number of columns of matrix ${\tt c}$ and ${\tt A}$ , with matrix ${\tt A}$ sized accordingly.
alpha	host or device	input	<type> scalar used for multiplication.</type>
А	device	input	<pre><type> array of dimension lda x m With lda&gt;=max(1,m) if side == CUBLAS_SIDE_LEFT and lda x n With lda&gt;=max(1,n) otherwise.</type></pre>
lda		input	leading dimension of two-dimensional array used to store matrix A.
В	device	input	<pre><type> array of dimension ldb x n with ldb&gt;=max(1,m).</type></pre>
ldb		input	leading dimension of two-dimensional array used to store matrix в.
beta	host or device	input	<pre><type> scalar used for multiplication, if beta == 0 then c does not have to be a valid input.</type></pre>
С	device	in/out	<pre><type> array of dimension ldc x n with ldc&gt;=max(1,m).</type></pre>
ldc		input	leading dimension of two-dimensional array used to store matrix c.

Error Value	Meaning
CUBLAS_STATUS_SUCCESS	the operation completed successfully
CUBLAS_STATUS_NOT_INITIALIZED	the library was not initialized
CUBLAS_STATUS_INVALID_VALUE	the parameters m, n<0
CUBLAS_STATUS_ARCH_MISMATCH	the device does not support double-precision
CUBLAS_STATUS_EXECUTION_FAILED	the function failed to launch on the GPU

For references please refer to:

ssymm, dsymm, csymm, zsymm

## 7.4. cublas<t>syrk()

This function performs the symmetric rank- *k* update

$$C = \alpha \operatorname{op}(A) \operatorname{op}(A)^{T} + \beta C$$

where  $\alpha$  and  $\beta$  are scalars, C is a symmetric matrix stored in lower or upper mode, and A is a matrix with dimensions op(A)  $n \times k$ . Also, for matrix A

$$op(A) = \begin{cases} A & \text{if transa} == CUBLAS\_OP\_N \\ A^T & \text{if transa} == CUBLAS\_OP\_T \end{cases}$$

Param.	Memory	In/out	Meaning
handle		input	handle to the CUBLAS library context.
uplo		input	indicates if matrix c lower or upper part is stored, the other symmetric part is not referenced and is inferred from the stored elements.
trans		input	operation op(A) that is non- or transpose.
n		input	number of rows of matrix $op(A)$ and $c$ .
k		input	number of columns of matrix op(A).
alpha	host or device	input	<type> scalar used for multiplication.</type>
A	device	input	<pre><type> array of dimension lda x k with lda&gt;=max(1,n) if trans == CUBLAS_OP_N and lda x n with lda&gt;=max(1,k) otherwise.</type></pre>
lda		input	leading dimension of two-dimensional array used to store matrix A.
beta	host or device	input	<pre><type> scalar used for multiplication, if beta==0 then c does not have to be a valid input.</type></pre>
С	device	in/out	<pre><type> array of dimension ldc <math>x</math> n, with ldc&gt;=max(1,n).</type></pre>
ldc		input	leading dimension of two-dimensional array used to store matrix c.

The possible error values returned by this function and their meanings are listed below.

Error Value	Meaning
CUBLAS_STATUS_SUCCESS	the operation completed successfully

Error Value	Meaning
CUBLAS_STATUS_NOT_INITIALIZED	the library was not initialized
CUBLAS_STATUS_INVALID_VALUE	the parameters n, k<0
CUBLAS_STATUS_ARCH_MISMATCH	the device does not support double-precision
CUBLAS_STATUS_EXECUTION_FAILED	the function failed to launch on the GPU

For references please refer to:

ssyrk, dsyrk, csyrk, zsyrk

## 7.5. cublas<t>syr2k()

```
cublasStatus t cublasSsyr2k(cublasHandle t handle,
                                                                                                      cublasFillMode t uplo, cublasOperation t trans,
                                                                                                     int n, int k, const float
                                                                                                    const float const float const float const float const float const float const float const float const float const float const float const float const float const float const float const float const float const float const float const float const float const float const float const float const float const float const float const float const float const float const float const float const float const float const float const float const float const float const float const float const float const float const float const float const float const float const float const float const float const float const float const float const float const float const float const float const float const float const float const float const float const float const float const float const float const float const float const float const float const float const float const float const float const float const float const float const float const float const float const float const float const float const float const float const float const float const float const float const float const float const float const float const float const float const float const float const float const float const float const float const float const float const float const float const float const float const float const float const float const float const float const float const float const float const float const float const float const float const float const float const float const float const float const float const float const float const float const float const float const float const float const float const float const float const float const float const float const float const float const float const float const float const float const float const float const float const float const float const float const float const float const float const float const float const float const float const float const float const float const float const float const float const float const float const float const float const float const float const float const float const 
                                                                                                                                                                                   *alpha,
                                                                                                                                                                                 *A, int lda,
                                                                                                                                                                                   *B, int ldb,
cublasStatus t cublasDsyr2k(cublasHandle t handle,
                                                                                                     cublasFillMode_t uplo, cublasOperation_t trans,
                                                                                                     int n, int k,
                                                                                                    int n, int k, const double const double const double const double const double double const double *C, int ldc)
                                                                                                                                                                                      *alpha,
                                                                                                                                                                                    *A, int lda, *B, int ldb,
cublasStatus_t cublasCsyr2k(cublasHandle_t handle,
                                                                                                     cublasFillMode_t uplo, cublasOperation_t trans,
                                                                                                     int n, int k,
                                                                                                                                                                                    *alpha,
                                                                                                      const cuComplex
                                                                                                    const cuComplex
                                                                                                                                                                                *A, int lda,
                                                                                                                                                                                *B, int ldb,
                                                                                                    *beta,
cublasStatus_t cublasZsyr2k(cublasHandle t handle,
                                                                                                      cublasFillMode t uplo, cublasOperation t trans,
                                                                                                     int n, int k,
                                                                                                      const cuDoubleComplex *alpha,
                                                                                                     const cuDoubleComplex *A, int lda,
const cuDoubleComplex *B, int ldb,
                                                                                                      const cuDoubleComplex *beta,
                                                                                                     cuDoubleComplex *C, int ldc)
```

This function performs the symmetric rank- 2k update

$$C = \alpha(\operatorname{op}(A)\operatorname{op}(B)^{T} + \operatorname{op}(B)\operatorname{op}(A)^{T}) + \beta C$$

where  $\alpha$  and  $\beta$  are scalars, C is a symmetric matrix stored in lower or upper mode, and A and B are matrices with dimensions op(A)  $n \times k$  and op(B)  $n \times k$ , respectively. Also, for matrix A and B

op(A) and op(B) = 
$$\begin{cases} A \text{ and } B & \text{if trans} == \text{CUBLAS\_OP\_N} \\ A^T \text{ and } B^T & \text{if trans} == \text{CUBLAS\_OP\_T} \end{cases}$$

Param.	Memory	In/out	Meaning
handle		input	handle to the CUBLAS library context.
uplo		input	indicates if matrix c lower or upper part, is stored, the other symmetric part is not referenced and is inferred from the stored elements.
trans		input	operation op(A) that is non- or transpose.
n		input	number of rows of matrix op(A), op(B) and c.
k		input	number of columns of matrix op(A) and op(B).
alpha	host or device	input	<type> scalar used for multiplication.</type>
A	device	input	<pre><type> array of dimension lda x k with lda&gt;=max(1,n) if transa == CUBLAS_OP_N and lda x n with lda&gt;=max(1,k) otherwise.</type></pre>
lda		input	leading dimension of two-dimensional array used to store matrix A.
В	device	input	<pre><type> array of dimensions ldb x k with ldb&gt;=max(1,n) if transa == CUBLAS_OP_N and ldb x n with ldb&gt;=max(1,k) otherwise.</type></pre>
ldb		input	leading dimension of two-dimensional array used to store matrix в.
beta	host or device	input	<pre><type> scalar used for multiplication, if beta==0, then c does not have to be a valid input.</type></pre>
С	device	in/out	<pre><type> array of dimensions ldc x n with ldc&gt;=max(1,n).</type></pre>
ldc		input	leading dimension of two-dimensional array used to store matrix c.

Error Value	Meaning
CUBLAS_STATUS_SUCCESS	the operation completed successfully
CUBLAS_STATUS_NOT_INITIALIZED	the library was not initialized
CUBLAS_STATUS_INVALID_VALUE	the parameters n,k<0
CUBLAS_STATUS_ARCH_MISMATCH	the device does not support double-precision
CUBLAS_STATUS_EXECUTION_FAILED	the function failed to launch on the GPU

For references please refer to: ssyr2k, dsyr2k, csyr2k, zsyr2k

## 7.6. cublas<t>syrkx()

```
cublasStatus t cublasSsyrkx(cublasHandle t handle,
                                       cublasFillMode_t uplo, cublasOperation_t trans,
                                      int n, int k,
const float
float
float

*C, int ldc)
                                                                    *A, int lda,
*B, int ldb,
*beta,
cublasStatus t cublasDsyrkx(cublasHandle t handle,
                                       cublasFillMode t uplo, cublasOperation t trans,
                                       int n, int k,
                                      const double *alpha,
const double *A, int lda,
const double *B, int ldb,
const double *C, int ldc)
cublasStatus_t cublasCsyrkx(cublasHandle t handle,
                                       cublasFillMode t uplo, cublasOperation t trans,
                                       int n, int k,
                                      const cuComplex *alpha,
const cuComplex *A, int lda,
const cuComplex *B, int ldb,
const cuComplex *beta.
                                                                     *beta,
                                       const cuComplex
cuComplex *C, int ldc)
cublasStatus t cublasZsyrkx(cublasHandle t handle,
                                       cublasFillMode t uplo, cublasOperation t trans,
                                       int n, int k,
                                       const cuDoubleComplex *alpha,
                                       const cuDoubleComplex *A, int lda,
                                       const cuDoubleComplex *B, int ldb,
                                       const cuDoubleComplex *beta,
                                       cuDoubleComplex *C, int ldc)
```

This function performs a variation of the symmetric rank- *k* update

$$C = \alpha(\operatorname{op}(A)\operatorname{op}(B)^T + \beta C$$

where  $\alpha$  and  $\beta$  are scalars, C is a symmetric matrix stored in lower or upper mode, and A and B are matrices with dimensions op(A)  $n \times k$  and op(B)  $n \times k$ , respectively. Also, for matrix A and B

$$op(A)$$
 and  $op(B) = \begin{cases} A \text{ and } B & \text{if trans} == CUBLAS\_OP\_N \\ A^T \text{ and } B^T & \text{if trans} == CUBLAS\_OP\_T \end{cases}$ 

This routine can be used when B is in such way that the result is garanteed to be symmetric. An usual example is when the matrix B is a scaled form of the matrix A: this is equivalent to B being the product of the matrix A and a diagonal matrix. For an efficient computation of the product of a regular matrix with a diagonal matrix, refer to the routine cublas<t>dgmm.

Param.	Memory	In/out	Meaning
handle		input	handle to the CUBLAS library context.
uplo		input	indicates if matrix c lower or upper part, is stored, the other symmetric part is not referenced and is inferred from the stored elements.
trans		input	operation op(A) that is non- or transpose.
n		input	number of rows of matrix op(A), op(B) and c.

Param.	Memory	In/out	Meaning
k		input	number of columns of matrix op(A) and op(B).
alpha	host or device	input	<type> scalar used for multiplication.</type>
A	device	input	<pre><type> array of dimension lda x k with lda&gt;=max(1,n) if transa == CUBLAS_OP_N and lda x n with lda&gt;=max(1,k) otherwise.</type></pre>
lda		input	leading dimension of two-dimensional array used to store matrix A.
В	device	input	<pre><type> array of dimensions ldb x k with ldb&gt;=max(1,n) if transa == CUBLAS_OP_N and ldb x n with ldb&gt;=max(1,k) otherwise.</type></pre>
ldb		input	leading dimension of two-dimensional array used to store matrix в.
beta	host or device	input	<pre><type> scalar used for multiplication, if beta==0, then c does not have to be a valid input.</type></pre>
С	device	in/out	<pre><type> array of dimensions ldc x n with ldc&gt;=max(1,n).</type></pre>
ldc		input	leading dimension of two-dimensional array used to store matrix c.

Error Value	Meaning
CUBLAS_STATUS_SUCCESS	the operation completed successfully
CUBLAS_STATUS_NOT_INITIALIZED	the library was not initialized
CUBLAS_STATUS_INVALID_VALUE	the parameters n,k<0
CUBLAS_STATUS_ARCH_MISMATCH	the device does not support double-precision
CUBLAS_STATUS_EXECUTION_FAILED	the function failed to launch on the GPU

For references please refer to: ssyrk, dsyrk, csyrk, zsyrk and ssyr2k, dsyr2k, csyr2k, zsyr2k

## 7.7. cublas<t>trmm()

```
cublasOperation t trans, cublasDiagType t diag,
                              int m, int n,
                             const double const double const double
                                                       *alpha,
                                                     *A, int lda,
                                                     *B, int ldb,
                              double
                                                      *C, int ldc)
cublasStatus_t cublasCtrmm(cublasHandle_t handle,
                              cublasSideMode_t side, cublasFillMode_t uplo,
                              cublasOperation t trans, cublasDiagType t diag,
                              int m, int n,
                             const cuComplex *alpha,
const cuComplex *A, int lda,
const cuComplex *B, int ldb,
cuComplex *C, int ldc)
cublasStatus_t cublasZtrmm(cublasHandle_t handle,
                              cublasSideMode t side, cublasFillMode t uplo,
                              cublasOperation t trans, cublasDiagType t diag,
                              int m, int n,
                              const cuDoubleComplex *alpha,
                              const cuDoubleComplex *A, int lda,
                              const cuDoubleComplex *B, int ldb,
                              cuDoubleComplex
                                                 *C, int ldc)
```

This function performs the triangular matrix-matrix multiplication

$$C = \begin{cases} \alpha \operatorname{op}(A)B & \text{if side} == \mathsf{CUBLAS\_SIDE\_LEFT} \\ \alpha B \operatorname{op}(A) & \text{if side} == \mathsf{CUBLAS\_SIDE\_RIGHT} \end{cases}$$

where A is a triangular matrix stored in lower or upper mode with or without the main diagonal, B and C are  $m \times n$  matrix, and  $\alpha$  is a scalar. Also, for matrix A

$$op(A) = \begin{cases} A & \text{if transa} == \text{CUBLAS\_OP\_N} \\ A^T & \text{if transa} == \text{CUBLAS\_OP\_T} \\ A^H & \text{if transa} == \text{CUBLAS\_OP\_C} \end{cases}$$

Notice that in order to achieve better parallelism CUBLAS differs from the BLAS API only for this routine. The BLAS API assumes an in-place implementation (with results written back to B), while the CUBLAS API assumes an out-of-place implementation (with results written into C). The application can obtain the in-place functionality of BLAS in the CUBLAS API by passing the address of the matrix B in place of the matrix C. No other overlapping in the input parameters is supported.

Param.	Memory	In/out	Meaning
handle		input	handle to the CUBLAS library context.
side		input	indicates if matrix ${f a}$ is on the left or right of ${f B}$ .
uplo		input	indicates if matrix A lower or upper part is stored, the other part is not referenced and is inferred from the stored elements.
trans		input	operation op(A) that is non- or (conj.) transpose.
diag		input	indicates if the elements on the main diagonal of matrix ${\tt A}$ are unity and should not be accessed.
m		input	number of rows of matrix ${\tt B}$ , with matrix ${\tt A}$ sized accordingly.
n		input	number of columns of matrix B, with matrix A sized accordingly.

Param.	Memory	In/out	Meaning
alpha	host or device	input	<pre><type> scalar used for multiplication, if alpha==0 then <math>{\tt A}</math> is not referenced and <math>{\tt B}</math> does not have to be a valid input.</type></pre>
A	device	input	<pre><type> array of dimension lda x m with lda&gt;=max(1,m) if side == CUBLAS_SIDE_LEFT and lda x n with lda&gt;=max(1,n) otherwise.</type></pre>
lda		input	leading dimension of two-dimensional array used to store matrix A.
В	device	input	<pre><type> array of dimension ldb <math>x</math> n with ldb&gt;=max(1,m).</type></pre>
ldb		input	leading dimension of two-dimensional array used to store matrix в.
С	device	in/out	<pre><type> array of dimension ldc <math>x</math> n With ldc&gt;=max(1,m).</type></pre>
ldc		input	leading dimension of two-dimensional array used to store matrix c.

Error Value	Meaning
CUBLAS_STATUS_SUCCESS	the operation completed successfully
CUBLAS_STATUS_NOT_INITIALIZED	the library was not initialized
CUBLAS_STATUS_INVALID_VALUE	the parameters m, n<0
CUBLAS_STATUS_ARCH_MISMATCH	the device does not support double-precision
CUBLAS_STATUS_EXECUTION_FAILED	the function failed to launch on the GPU

For references please refer to:

strmm, dtrmm, ctrmm, ztrmm

## 7.8. cublas<t>trsm()

```
cublasStatus t cublasStrsm(cublasHandle t handle,
                           cublasSideMode_t side, cublasFillMode_t uplo,
                           cublasOperation_t trans, cublasDiagType_t diag,
                           int m, int n,
const float
                                                 *alpha,
                           const float
                                                 *A, int lda,
                                          *B, int ldb)
                           float
cublasStatus_t cublasDtrsm(cublasHandle_t handle,
                           cublasSideMode t side, cublasFillMode t uplo,
                           cublasOperation_t trans, cublasDiagType_t diag,
                           int m, int n,
                           const double
                                                 *alpha,
                           const double
                                                 *A, int lda,
                                          *B, int ldb)
                           double
cublasStatus_t cublasCtrsm(cublasHandle_t handle,
                           cublasSideMode t side, cublasFillMode t uplo,
                           cublasOperation t trans, cublasDiagType t diag,
                           int m, int n,
```

This function solves the triangular linear system with multiple right-hand-sides

```
\begin{cases} op(A)X = \alpha B & \text{if side} == CUBLAS\_SIDE\_LEFT \\ Xop(A) = \alpha B & \text{if side} == CUBLAS\_SIDE\_RIGHT \end{cases}
```

where A is a triangular matrix stored in lower or upper mode with or without the main diagonal, X and B are  $m \times n$  matrices, and  $\alpha$  is a scalar. Also, for matrix A

$$op(A) = \begin{cases} A & \text{if transa} == \text{CUBLAS\_OP\_N} \\ A^T & \text{if transa} == \text{CUBLAS\_OP\_T} \\ A^H & \text{if transa} == \text{CUBLAS\_OP\_C} \end{cases}$$

The solution *X* overwrites the right-hand-sides *B* on exit.

No test for singularity or near-singularity is included in this function.

Param.	Memory	In/out	Meaning
handle		input	handle to the CUBLAS library context.
side		input	indicates if matrix ${\bf A}$ is on the left or right of ${\bf x}$ .
uplo		input	indicates if matrix ${\bf A}$ lower or upper part is stored, the other part is not referenced and is inferred from the stored elements.
trans		input	operation op(A) that is non- or (conj.) transpose.
diag		input	indicates if the elements on the main diagonal of matrix <b>A</b> are unity and should not be accessed.
m		input	number of rows of matrix B, with matrix A sized accordingly.
n		input	number of columns of matrix ${\tt B}$ , with matrix ${\tt A}$ is sized accordingly.
alpha	host or device	input	<pre><type> scalar used for multiplication, if alpha==0 then <math>A</math> is not referenced and <math>B</math> does not have to be a valid input.</type></pre>
A	device	input	<pre><type> array of dimension lda x m with lda&gt;=max(1,m) if side == CUBLAS_SIDE_LEFT and lda x n with lda&gt;=max(1,n) otherwise.</type></pre>
lda		input	leading dimension of two-dimensional array used to store matrix A.
В	device	in/out	<type> array. It has dimensions ldb x n with ldb&gt;=max(1,m).</type>
ldb		input	leading dimension of two-dimensional array used to store matrix B.

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The	oossible error	values i	refiirned r	ow this	filinction	and	their	meanings	are listed be	LOW
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Error Value	Meaning
CUBLAS_STATUS_SUCCESS	the operation completed successfully
CUBLAS_STATUS_NOT_INITIALIZED	the library was not initialized
CUBLAS_STATUS_INVALID_VALUE	the parameters m, n<0
CUBLAS_STATUS_ARCH_MISMATCH	the device does not support double-precision
CUBLAS_STATUS_EXECUTION_FAILED	the function failed to launch on the GPU

For references please refer to:

strsm, dtrsm, ctrsm, ztrsm

## 7.9. cublas<t>trsmBatched()

```
cublasStatus_t cublasStrsmBatched( cublasHandle_t handle,
                                     cublasSideMode t side,
                                     cublasFillMode t uplo,
                                     cublasOperation_t trans,
                                     cublasDiagType t diag,
                                     int m,
                                    int n,
                                    const float *alpha,
                                    float *A[],
                                     int lda,
                                    float *B[],
                                    int ldb,
                                    int batchCount);
                                                       handle,
cublasStatus_t cublasDtrsmBatched( cublasHandle_t
                                    cublasSideMode_t side,
cublasFillMode_t uplo,
                                    cublasOperation t trans,
                                    cublasDiagType_t diag,
                                    int m,
                                     int n,
                                     const double *alpha,
                                    double *A[],
                                    int lda,
                                    double *B[],
                                     int ldb,
                                     int batchCount);
cublasStatus t cublasCtrsmBatched( cublasHandle t
                                                       handle,
                                    cublasSideMode t side,
                                     cublasFillMode_t uplo,
                                    cublasOperation_t trans,
                                     cublasDiagType t diag,
                                    int m,
                                    int n,
                                     const cuComplex *alpha,
                                     cuComplex *A[],
                                     int lda,
                                     cuComplex *B[],
                                     int ldb,
                                     int batchCount);
cublasStatus_t cublasZtrsmBatched( cublasHandle_t
                                                       handle,
                                     cublasSideMode_t side,
cublasFillMode_t uplo,
                                     cublasOperation t trans,
```

```
cublasDiagType_t diag,
int m,
int n,
const cuDoubleComplex *alpha,
cuDoubleComplex *A[],
int lda,
cuDoubleComplex *B[],
int ldb,
int batchCount);
```

This function solves an array of triangular linear systems with multiple right-hand-sides

```
 \begin{cases} op(A[i]X[i] = \alpha B[i] & \text{if side} == \text{CUBLAS\_SIDE\_LEFT} \\ X[i]op(A[i]) = \alpha B[i] & \text{if side} == \text{CUBLAS\_SIDE\_RIGHT} \end{cases}
```

where A[i] is a triangular matrix stored in lower or upper mode with or without the main diagonal, X[i] and B[i] are  $m \times n$  matrices, and  $\alpha$  is a scalar. Also, for matrix A

$$op(A[i]) = \begin{cases} A[i] & \text{if transa} == \text{CUBLAS\_OP\_N} \\ A^T[i] & \text{if transa} == \text{CUBLAS\_OP\_T} \\ A^H[i] & \text{if transa} == \text{CUBLAS\_OP\_C} \end{cases}$$

The solution X[i] overwrites the right-hand-sides B[i] on exit.

No test for singularity or near-singularity is included in this function.

This function is intended to be used for matrices of small sizes where the launch overhead is a significant factor. The current implementation limits the dimensions m and n to 32.

Param.	Memory	In/out	Meaning
handle		input	handle to the CUBLAS library context.
side		input	indicates if matrix $A[i]$ is on the left or right of $x[i]$ .
uplo		input	indicates if matrix A[i] lower or upper part is stored, the other part is not referenced and is inferred from the stored elements.
trans		input	operation op(A[i]) that is non- or (conj.) transpose.
diag		input	indicates if the elements on the main diagonal of matrix A[i] are unity and should not be accessed.
m		input	number of rows of matrix B[i], with matrix A[i] sized accordingly.
n		input	number of columns of matrix B[i], with matrix A[i] is sized accordingly.
alpha	host or device	input	<pre><type> scalar used for multiplication, if alpha==0 then A[i] is not referenced and B[i] does not have to be a valid input.</type></pre>
A	device	input	array of pointers to <type> array, with each array of dim. lda x m With lda&gt;=max(1,m) if transa==CUBLAS_OP_N and lda x n With lda&gt;=max(1,n) otherwise.</type>
lda		input	leading dimension of two-dimensional array used to store matrix A[i].

Param.	Memory	In/out	Meaning
В	device	in/out	array of pointers to <type> array, with each array of dim. ldb x n With ldb&gt;=max(1,m)</type>
ldb		input	leading dimension of two-dimensional array used to store matrix B[i].
batchCount		input	number of pointers contained in A and B.

Error Value	Meaning
CUBLAS_STATUS_SUCCESS	the operation completed successfully
CUBLAS_STATUS_NOT_INITIALIZED	the library was not initialized
CUBLAS_STATUS_INVALID_VALUE	the parameters $m, n < 0$ . The parameters $m, n > 32$
CUBLAS_STATUS_ARCH_MISMATCH	the device does not support double-precision
CUBLAS_STATUS_EXECUTION_FAILED	the function failed to launch on the GPU

For references please refer to:

strsm, dtrsm, ctrsm, ztrsm

## 7.10. cublas<t>hemm()

This function performs the Hermitian matrix-matrix multiplication

```
C = \begin{cases} \alpha AB + \beta C & \text{if side} == \text{CUBLAS\_SIDE\_LEFT} \\ \alpha BA + \beta C & \text{if side} == \text{CUBLAS\_SIDE\_RIGHT} \end{cases}
```

where *A* is a Hermitian matrix stored in lower or upper mode, *B* and *C* are  $m \times n$  matrices, and  $\alpha$  and  $\beta$  are scalars.

Param.	Memory	In/out	Meaning
handle		input	handle to the CUBLAS library context.

Param.	Memory	In/out	Meaning
side		input	indicates if matrix A is on the left or right of B.
uplo		input	indicates if matrix A lower or upper part is stored, the other Hermitian part is not referenced and is inferred from the stored elements.
m		input	number of rows of matrix с and в, with matrix а sized accordingly.
n		input	number of columns of matrix c and B, with matrix A sized accordingly.
alpha	host or device	input	<type> scalar used for multiplication.</type>
A	device	input	<pre><type> array of dimension lda x m with lda&gt;=max(1,m) if side==CUBLAS_SIDE_LEFT and lda x n with lda&gt;=max(1,n) Otherwise. The imaginary parts of the diagonal elements are assumed to be zero.</type></pre>
lda		input	leading dimension of two-dimensional array used to store matrix A.
В	device	input	<type> array of dimension ldb x n with ldb&gt;=max(1,m).</type>
ldb		input	leading dimension of two-dimensional array used to store matrix в.
beta		input	<pre><type> scalar used for multiplication, if beta==0 then c does not have to be a valid input.</type></pre>
С	device	in/out	<pre><type> array of dimensions ldc <math>x</math> n with ldc&gt;=max(1,m).</type></pre>
ldc		input	leading dimension of two-dimensional array used to store matrix c.

Error Value	Meaning
CUBLAS_STATUS_SUCCESS	the operation completed successfully
CUBLAS_STATUS_NOT_INITIALIZED	the library was not initialized
CUBLAS_STATUS_INVALID_VALUE	the parameters m, n<0
CUBLAS_STATUS_ARCH_MISMATCH	the device does not support double-precision
CUBLAS_STATUS_EXECUTION_FAILED	the function failed to launch on the GPU

For references please refer to:

chemm, zhemm

## 7.11. cublas<t>herk()

This function performs the Hermitian rank- *k* update

$$C = \alpha \operatorname{op}(A) \operatorname{op}(A)^H + \beta C$$

where  $\alpha$  and  $\beta$  are scalars, C is a Hermitian matrix stored in lower or upper mode, and A is a matrix with dimensions op(A)  $n \times k$ . Also, for matrix A

$$op(A) = \begin{cases} A & \text{if transa} == CUBLAS\_OP\_N \\ A^H & \text{if transa} == CUBLAS\_OP\_C \end{cases}$$

Param.	Memory	In/out	Meaning
handle		input	handle to the CUBLAS library context.
uplo		input	indicates if matrix A lower or upper part is stored, the other Hermitian part is not referenced and is inferred from the stored elements.
trans		input	operation op(A) that is non- or (conj.) transpose.
n		input	number of rows of matrix $op(A)$ and $c$ .
k		input	number of columns of matrix op(A).
alpha	host or device	input	<type> scalar used for multiplication.</type>
A	device	input	<pre><type> array of dimension lda x k with lda&gt;=max(1,n) if transa == CUBLAS_OP_N and lda x n with lda&gt;=max(1,k) otherwise.</type></pre>
lda		input	leading dimension of two-dimensional array used to store matrix A.
beta		input	<pre><type> scalar used for multiplication, if beta==0 then c does not have to be a valid input.</type></pre>
С	device	in/out	<pre><type> array of dimension ldc x n, with ldc&gt;=max(1,n). The imaginary parts of the diagonal elements are assumed and set to zero.</type></pre>
ldc		input	leading dimension of two-dimensional array used to store matrix c.

The possible error values returned by this function and their meanings are listed below.

Error Value	Meaning
CUBLAS_STATUS_SUCCESS	the operation completed successfully

Error Value	Meaning
CUBLAS_STATUS_NOT_INITIALIZED	the library was not initialized
CUBLAS_STATUS_INVALID_VALUE	the parameters n,k<0
CUBLAS_STATUS_ARCH_MISMATCH	the device does not support double-precision
CUBLAS_STATUS_EXECUTION_FAILED	the function failed to launch on the GPU

For references please refer to:

cherk, zherk

## 7.12. cublas<t>her2k()

This function performs the Hermitian rank- 2k update

$$C = \alpha \operatorname{op}(A) \operatorname{op}(B)^{H} + \bar{\alpha} \operatorname{op}(B) \operatorname{op}(A)^{H} + \beta C$$

where  $\alpha$  and  $\beta$  are scalars, C is a Hermitian matrix stored in lower or upper mode, and A and B are matrices with dimensions op(A)  $n \times k$  and op(B)  $n \times k$ , respectively. Also, for matrix A and B

$$op(A)$$
 and  $op(B) = \begin{cases} A \text{ and } B & \text{if trans} == CUBLAS\_OP\_N \\ A^H \text{ and } B^H & \text{if trans} == CUBLAS\_OP\_C \end{cases}$ 

Param.	Memory	In/out	Meaning
handle		input	handle to the CUBLAS library context.
uplo		input	indicates if matrix A lower or upper part is stored, the other Hermitian part is not referenced and is inferred from the stored elements.
trans		input	operation op(A) that is non- or (conj.) transpose.
n		input	number of rows of matrix op(A), op(B) and C.
k		input	number of columns of matrix op(A) and op(B).
alpha	host or device	input	<type> scalar used for multiplication.</type>

Param.	Memory	In/out	Meaning
A	device	input	<pre><type> array of dimension lda x k with lda&gt;=max(1,n) if transa == CUBLAS_OP_N and lda x n with lda&gt;=max(1,k) otherwise.</type></pre>
lda		input	leading dimension of two-dimensional array used to store matrix A.
В	device	input	<pre><type> array of dimension ldb x k with ldb&gt;=max(1,n) if transa == CUBLAS_OP_N and ldb x n with ldb&gt;=max(1,k) otherwise.</type></pre>
ldb		input	leading dimension of two-dimensional array used to store matrix в.
beta	host or device	input	<pre><type> scalar used for multiplication, if beta==0 then c does not have to be a valid input.</type></pre>
С	device	in/out	<pre><type> array of dimension ldc x n, with ldc&gt;=max(1,n). The imaginary parts of the diagonal elements are assumed and set to zero.</type></pre>
ldc		input	leading dimension of two-dimensional array used to store matrix c.

Error Value	Meaning
CUBLAS_STATUS_SUCCESS	the operation completed successfully
CUBLAS_STATUS_NOT_INITIALIZED	the library was not initialized
CUBLAS_STATUS_INVALID_VALUE	the parameters n,k<0
CUBLAS_STATUS_ARCH_MISMATCH	the device does not support double-precision
CUBLAS_STATUS_EXECUTION_FAILED	the function failed to launch on the GPU

For references please refer to:

cher2k, zher2k

## 7.13. cublas<t>herkx()

```
const cuDoubleComplex *B, int ldb,
const double *beta,
cuDoubleComplex *C, int ldc)
```

This function performs a variation of the Hermitian rank- *k* update

$$C = \alpha \operatorname{op}(A) \operatorname{op}(B)^H + \beta C$$

where  $\alpha$  and  $\beta$  are scalars, C is a Hermitian matrix stored in lower or upper mode, and A and B are matrices with dimensions op(A)  $n \times k$  and op(B)  $n \times k$ , respectively. Also, for matrix A and B

$$op(A)$$
 and  $op(B) = \begin{cases} A \text{ and } B & \text{if trans} == CUBLAS\_OP\_N \\ A^H \text{ and } B^H & \text{if trans} == CUBLAS\_OP\_C \end{cases}$ 

This routine can be used when the matrix B is in such way that the result is garanteed to be hermitian. An usual example is when the matrix B is a scaled form of the matrix A: this is equivalent to B being the product of the matrix A and a diagonal matrix. For an efficient computation of the product of a regular matrix with a diagonal matrix, refer to the routine cublas<t>dgmm.

Param.	Memory	In/out	Meaning
handle		input	handle to the CUBLAS library context.
uplo		input	indicates if matrix <b>A</b> lower or upper part is stored, the other Hermitian part is not referenced and is inferred from the stored elements.
trans		input	operation op(A) that is non- or (conj.) transpose.
n		input	number of rows of matrix $op(A)$ , $op(B)$ and $C$ .
k		input	number of columns of matrix op(A) and op(B).
alpha	host or device	input	<type> scalar used for multiplication.</type>
Α	device	input	<pre><type> array of dimension lda x k with lda&gt;=max(1,n) if transa == CUBLAS_OP_N and lda x n with lda&gt;=max(1,k) otherwise.</type></pre>
lda		input	leading dimension of two-dimensional array used to store matrix A.
В	device	input	<pre><type> array of dimension ldb x k with ldb&gt;=max(1,n) if transa == CUBLAS_OP_N and ldb x n with ldb&gt;=max(1,k) otherwise.</type></pre>
ldb		input	leading dimension of two-dimensional array used to store matrix B.
beta	host or device	input	real scalar used for multiplication, if beta==0 then c does not have to be a valid input.
С	device	in/out	<pre><type> array of dimension ldc x n, with ldc&gt;=max(1,n). The imaginary parts of the diagonal elements are assumed and set to zero.</type></pre>
ldc		input	leading dimension of two-dimensional array used to store matrix c.

Error Value	Meaning
CUBLAS_STATUS_SUCCESS	the operation completed successfully
CUBLAS_STATUS_NOT_INITIALIZED	the library was not initialized
CUBLAS_STATUS_INVALID_VALUE	the parameters n,k<0
CUBLAS_STATUS_ARCH_MISMATCH	the device does not support double-precision
CUBLAS_STATUS_EXECUTION_FAILED	the function failed to launch on the GPU

For references please refer to:

cherk, zherk and

cher2k, zher2k

# Chapter 8. BLAS-LIKE EXTENSION

In this chapter we describe the BLAS-extension functions that perform matrix-matrix operations.

## 8.1. cublas<t>geam()

```
cublasStatus_t cublasSgeam(cublasHandle_t handle,
                                                                                          cublasOperation t transa, cublasOperation t transb,
                                                                                       int m, int n,
                                                                                                                                                                      *alpha,
                                                                                                                                                                      *A, int lda,
                                                                                                                                                                      *B, int ldb,
cublasStatus t cublasDgeam(cublasHandle t handle,
                                                                                      cublasOperation_t transa, cublasOperation_t transb,
                                                                                                                                                                      *alpha,
                                                                                                                                                                    *A, int lda,
                                                                                                                                                                    *B, int ldb,
cublasStatus_t cublasCgeam(cublasHandle_t handle,
                                                                                         cublasOperation t transa, cublasOperation t transb,
                                                                                        int m, int n,
const cuComplex
*alpha,
*A, int lda,
*beta,
*beta,
*B, int ldb,
**Int ldb,
**
                                                                                         int m, int n,
                                                                                         const cuComplex *B, in cuComplex *C, int ldc)
cublasStatus_t cublasZgeam(cublasHandle_t handle,
                                                                                         cublasOperation t transa, cublasOperation t transb,
                                                                                          int m, int n,
                                                                                          const cuDoubleComplex *alpha,
                                                                                          const cuDoubleComplex *A, int lda,
                                                                                          const cuDoubleComplex *beta,
                                                                                          const cuDoubleComplex *B, int ldb,
                                                                                          cuDoubleComplex *C, int ldc)
```

This function performs the matrix-matrix addition/transposition

```
C = \alpha \operatorname{op}(A) + \beta \operatorname{op}(B)
```

where  $\alpha$  and  $\beta$  are scalars, and A, B and C are matrices stored in column-major format with dimensions op(A)  $m \times n$ , op(B)  $m \times n$  and C  $m \times n$ , respectively. Also, for matrix A

$$op(A) = \begin{cases} A & \text{if transa} == \text{CUBLAS\_OP\_N} \\ A^T & \text{if transa} == \text{CUBLAS\_OP\_T} \\ A^H & \text{if transa} == \text{CUBLAS\_OP\_C} \end{cases}$$

and op(B) is defined similarly for matrix B.

The operation is out-of-place and CUBLAS would not check range of pointer A, B and C. If C overlaps A or B, then behaviour is undefined. The operation includes the following special cases:

the user can reset matrix C to zero by setting **\*alpha=\*beta=0**.

the user can transpose matrix A by setting \*alpha=1 and \*beta=0.

Param.	Memory	In/out	Meaning
handle		input	handle to the CUBLAS library context.
transa		input	operation op(A) that is non- or (conj.) transpose.
transb		input	operation op(B) that is non- or (conj.) transpose.
m		input	number of rows of matrix op(A) and c.
n		input	number of columns of matrix op(B) and c.
alpha	host or device	input	<pre><type> scalar used for multiplication. If *alpha == 0, A does not have to be a valid input.</type></pre>
A	device	input	<pre><type> array of dimensions lda x n With lda&gt;=max(1,m) if transa == CUBLAS_OP_N and lda x m With lda&gt;=max(1,n) otherwise.</type></pre>
lda		input	leading dimension of two-dimensional array used to store the matrix A.
В	device	input	<pre><type> array of dimension ldb x n with ldb&gt;=max(1,m) if transa == CUBLAS_OP_N and ldb x m with ldb&gt;=max(1,n) otherwise.</type></pre>
ldb		input	leading dimension of two-dimensional array used to store matrix B.
beta	host or device	input	<type> scalar used for multiplication. If *beta == 0, в does not have to be a valid input.</type>
С	device	output	<pre><type> array of dimensions ldc x n With ldc&gt;=max(1,m).</type></pre>
ldc		input	leading dimension of a two-dimensional array used to store the matrix c.

The possible error values returned by this function and their meanings are listed below.

Error Value	Meaning
CUBLAS_STATUS_SUCCESS	the operation completed successfully

Error Value	Meaning
CUBLAS_STATUS_NOT_INITIALIZED	the library was not initialized
CUBLAS_STATUS_INVALID_VALUE	the parameters m,n<0 or alpha,beta=NULL
CUBLAS_STATUS_ARCH_MISMATCH	the device does not support double-precision
CUBLAS_STATUS_EXECUTION_FAILED	the function failed to launch on the GPU

## 8.2. cublas<t>dgmm()

This function performs the matrix-matrix multiplication

$$C = \begin{cases} A \times diag(X) & \text{if mode} == \text{CUBLAS\_SIDE\_RIGHT} \\ diag(X) \times A & \text{if mode} == \text{CUBLAS\_SIDE\_LEFT} \end{cases}$$

where A and C are matrices stored in column-major format with dimensions  $m \times n$ . X is a vector of size n if  $mode == CUBLAS_SIDE_RIGHT$  and of size m if  $mode == CUBLAS_SIDE_LEFT$ . X is gathered from one-dimensional array x with stride incx. The absolute value of incx is the stride and the sign of incx is direction of the stride. If incx is positive, then we forward x from the first element. Otherwise, we backward x from the last element. The formula of X is

$$X[j] = \begin{cases} x[j \times incx] & \text{if } incx \ge 0 \\ x[(\chi - 1) \times |incx| - j \times |incx|] & \text{if } incx < 0 \end{cases}$$

where  $\chi = m$  if mode == CUBLAS\_SIDE\_LEFT and  $\chi = n$  if mode == CUBLAS SIDE RIGHT.

Example 1: if the user wants to perform  $diag(diag(B)) \times A$ , then incx = ldb + 1 where ldb is leading dimension of matrix **B**, either row-major or column-major.

Example 2: if the user wants to perform  $\alpha \times A$ , then there are two choices, either cublasgeam with \*beta=0 and transa == CUBLAS\_OP\_N or cublasdgmm with incx=0 and x[0]=alpha.

The operation is out-of-place. The in-place only works if <b>lda</b> = <b>ldc</b> .
---

Param.	Memory	In/out	Meaning
handle		input	handle to the CUBLAS library context.
mode		input	left multiply if mode == CUBLAS_SIDE_LEFT or right multiply if mode == CUBLAS_SIDE_RIGHT
m		input	number of rows of matrix A and C.
n		input	number of columns of matrix ${f A}$ and ${f C}$ .
А	device	input	<type> array of dimensions lda x n with lda&gt;=max(1,m)</type>
lda		input	leading dimension of two-dimensional array used to store the matrix ${\bf A}$ .
х	device	input	one-dimensional <type> array of size <math> inc  \times m</math> if mode == CUBLAS_SIDE_LEFT and <math> inc  \times n</math> if mode == CUBLAS_SIDE_RIGHT</type>
incx		input	stride of one-dimensional array x.
С	device	in/out	<type> array of dimensions ldc x n with ldc&gt;=max(1,m).</type>
ldc		input	leading dimension of a two-dimensional array used to store the matrix c.

Error Value	Meaning
CUBLAS_STATUS_SUCCESS	the operation completed successfully
CUBLAS_STATUS_NOT_INITIALIZED	the library was not initialized
CUBLAS_STATUS_INVALID_VALUE	the parameters m, n<0 or mode != CUBLAS_SIDE_LEFT, CUBLAS_SIDE_RIGHT
CUBLAS_STATUS_ARCH_MISMATCH	the device does not support double-precision
CUBLAS_STATUS_EXECUTION_FAILED	the function failed to launch on the GPU

## 8.3. cublas<t>getrfBatched()

**Aarray** is an array of pointers to matrices stored in column-major format with dimensions **nxn** and leading dimension **lda**.

This function performs the LU factorization of each Aarray[i] for i = 0, ..., batchSize-1 by the following equation

```
P*Aarray[i] = L*U
```

where  $\mathbf{P}$  is a permutation matrix which represents partial pivoting with row interchanges.  $\mathbf{L}$  is a lower triangular matrix with unit diagonal and  $\mathbf{U}$  is an upper triangular matrix.

Formally P is written by a product of permutation matrices Pj, for j = 1,2,...,n, say P = P1 \* P2 \* P3 \* .... \* Pn. Pj is a permutation matrix which interchanges two rows of vector x when performing Pj\*x. Pj can be constructed by j element of PivotArray[i] by the following matlab code

```
// In Matlab PivotArray[i] is an array of base-1.
// In C, PivotArray[i] is base-0.
Pj = eye(n);
swap Pj(j,:) and Pj(PivotArray[i][j] ,:)
```

 $\mathbf{L}$  and  $\mathbf{U}$  are written back to original matrix  $\mathbf{A}$ , and diagonal elements of  $\mathbf{L}$  are discarded. The  $\mathbf{L}$  and  $\mathbf{U}$  can be constructed by the following matlab code

```
// A is a matrix of nxn after getrf.
L = eye(n);
for j = 1:n
    L(:,j+1:n) = A(:,j+1:n)
end
U = zeros(n);
for i = 1:n
    U(i,i:n) = A(i,i:n)
end
```

If matrix A (=Aarray[i]) is singular, getrf still works and the value of info(=infoArray[i]) reports first row index that LU factorization cannot proceed. If info is k, U(k,k) is zero. The equation P\*A=L\*U still holds, however L and U are from the following matlab code

```
// A is a matrix of nxn after getrf.
// info is k, which means U(k,k) is zero.
```

```
L = eye(n);
for j = 1:k-1
    L(:,j+1:n) = A(:,j+1:n)
end
U = zeros(n);
for i = 1:k-1
    U(i,i:n) = A(i,i:n)
end
for i = k:n
    U(i,k:n) = A(i,k:n)
end
```

This function is intended to be used for matrices of small sizes where the launch overhead is a significant factor.

Param.	Memory	In/out	Meaning
handle		input	handle to the CUBLAS library context.
n		input	number of rows and columns of Aarray[i].
Aarray	device	input	array of pointers to <type> array, with each array of dim. <math>n \times n</math> with lda&gt;=max(1,n).</type>
lda		input	leading dimension of two-dimensional array used to store each matrix Aarray[i].
PivotArray	device	output	array of size $n \times batchSize$ that contains the pivoting sequence of each factorization of $Aarray[i]$ stored in a linear fashion.
infoArray	device	output	array of size batchSize that info(=infoArray[i]) contains the information of factorization of Aarray[i].
			If info=0, the execution is successful.
			If info = -j, the j-th parameter had an illegal value.
			If info = $k$ , $U(k,k)$ is 0. The factorization has been completed, but $U$ is exactly singular.
batchSize		input	number of pointers contained in A

The possible error values returned by this function and their meanings are listed below.

Error Value	Meaning
CUBLAS_STATUS_SUCCESS	the operation completed successfully
CUBLAS_STATUS_NOT_INITIALIZED	the library was not initialized
CUBLAS_STATUS_INVALID_VALUE	the parameters $n$ , batchSize, lda <0
CUBLAS_STATUS_ARCH_MISMATCH	the device has a compute capability < 200 or the device does not support double-precision
CUBLAS_STATUS_EXECUTION_FAILED	the function failed to launch on the GPU

## 8.4. cublas<t>getriBatched()

```
int lda,
                                   int *PivotArray,
                                   float *Carray[],
                                   int ldc,
                                   int *infoArray,
                                   int batchSize);
cublasStatus t cublasDgetriBatched(cublasHandle t handle,
                                   double *Aarray[],
                                   int lda,
                                   int *PivotArray,
                                   double *Carray[],
                                   int ldc,
                                   int *infoArray,
                                   int batchSize);
cublasStatus t cublasCgetriBatched(cublasHandle t handle,
                                   cuComplex *Aarray[],
                                   int lda,
                                   int *PivotArray,
                                   cuComplex *Carray[],
                                   int ldc,
                                   int *infoArray,
                                   int batchSize);
cublasStatus t cublasZgetriBatched(cublasHandle t handle,
                                   int n,
                                   cuDoubleComplex *Aarray[],
                                   int lda,
                                   int *PivotArray,
                                   cuDoubleComplex *Carray[],
                                   int ldc,
                                   int *infoArray,
                                   int batchSize);
```

**Aarray** and **Carray** are arrays of pointers to matrices stored in column-major format with dimensions **n**\***n** and leading dimension **1da** and **1dc** respectively.

This function performs the inversion of matrices A[i] for i = 0, ..., batchSize-1.

Prior to calling cublas<t>getriBatched, the matrix A[i] must be factorized first using the routine cublas<t>getrfBatched. After the call of cublas<t>getrfBatched, the matrix pointing by Aarray[i] will contain the LU factors of the matrix A[i] and the vector pointing by (PivotArray+i) will contain the pivoting sequence.

Following the LU factorization, cublas<t>getriBatched uses forward and backward triangular solvers to complete inversion of matrices A[i] for i = 0, ..., batchSize-1. The inversion is out-of-place, so memory space of Carray[i] cannot overlap memory space of Array[i].

Typically all parameters in cublas<t>getrfBatched would be passed into cublas<t>getriBatched. For example,

```
// step 1: perform in-place LU decomposition, P*A = L*U.
// Aarray[i] is n*n matrix A[i]
  cublasDgetrfBatched(handle, n, Aarray, lda, PivotArray, infoArray,
batchSize);
// check infoArray[i] to see if factorization of A[i] is successful or not.
// Array[i] contains LU factorization of A[i]
```

```
// step 2: perform out-of-place inversion, Carray[i] = inv(A[i])
    cublasDgetriBatched(handle, n, Aarray, lda, PivotArray, Carray, ldc,
    infoArray, batchSize);
// check infoArray[i] to see if inversion of A[i] is successful or not.
```

The user can check singularity from either cublas<t>getrfBatched or cublas<t>getriBatched.

This function is intended to be used for matrices of small sizes where the launch overhead is a significant factor.

Param.	Memory	In/out	Meaning
handle		input	handle to the CUBLAS library context.
n		input	number of rows and columns of Aarray[i].
Aarray	device	input	array of pointers to <type> array, with each array of dimension <math>n*n</math> with <math>1da&gt;=max(1,n)</math>.</type>
lda		input	leading dimension of two-dimensional array used to store each matrix Aarray[i].
PivotArray	device	output	array of size n*batchSize that contains the pivoting sequence of each factorization of Aarray[i] stored in a linear fashion.
Carray	device	output	array of pointers to <type> array, with each array of dimension n*n with ldc&gt;=max(1,n).</type>
ldc		input	leading dimension of two-dimensional array used to store each matrix Carray[i].
infoArray	device	output	array of size batchSize that info(=infoArray[i]) contains the information of inversion of A[i].
			If info=0, the execution is successful.
			If info = $k$ , $U(k,k)$ is 0. The $U$ is exactly singular and the inversion failed.
batchSize		input	number of pointers contained in A

The possible error values returned by this function and their meanings are listed below.

Error Value	Meaning
CUBLAS_STATUS_SUCCESS	the operation completed successfully
CUBLAS_STATUS_NOT_INITIALIZED	the library was not initialized
CUBLAS_STATUS_INVALID_VALUE	the parameters n,batchSize,lda,ldc <0
CUBLAS_STATUS_ARCH_MISMATCH	the device has a compute capability < 200
CUBLAS_STATUS_EXECUTION_FAILED	the function failed to launch on the GPU

# Appendix A. USING THE CUBLAS LEGACY API

This appendix does not provide a full reference of each Legacy API datatype and entry point. Instead, it describes how to use the API, especially where this is different from the regular CUBLAS API.

Note that in this section, all references to the "CUBLAS Library" refer to the Legacy CUBLAS API only.

#### A.1. Error Status

The cublasStatus type is used for function status returns. The CUBLAS Library helper functions return status directly, while the status of core functions can be retrieved using cublasGetError(). Notice that reading the error status via cublasGetError(), resets the internal error state to CUBLAS\_STATUS\_SUCCESS. Currently, the following values for are defined:

Value	Meaning
CUBLAS_STATUS_SUCCESS	the operation completed successfully
CUBLAS_STATUS_NOT_INITIALIZED	the library was not initialized
CUBLAS_STATUS_ALLOC_FAILED	the resource allocation failed
CUBLAS_STATUS_INVALID_VALUE	an invalid numerical value was used as an argument
CUBLAS_STATUS_ARCH_MISMATCH	an absent device architectural feature is required
CUBLAS_STATUS_MAPPING_ERROR	an access to GPU memory space failed
CUBLAS_STATUS_EXECUTION_FAILED	the GPU program failed to execute
CUBLAS_STATUS_INTERNAL_ERROR	an internal operation failed

This legacy type corresponds to type **cublasStatus\_t** in the CUBLAS library API.

#### A.2. Initialization and Shutdown

The functions <code>cublasInit()</code> and <code>cublasShutdown()</code> are used to initialize and shutdown the CUBLAS library. It is recommended for <code>cublasInit()</code> to be called before any other function is invoked. It allocates hardware resources on the GPU device that is currently bound to the host thread from which it was invoked.

The legacy initialization and shutdown functions are similar to the CUBLAS library API routines cublasCreate() and cublasDestroy().

### A.3. Thread Safety

The legacy API is not thread safe when used with multiple host threads and devices. It is recommended to be used only when utmost compatibility with Fortran is required and when a single host thread is used to setup the library and make all the functions calls.

### A.4. Memory Management

The memory used by the legacy CUBLAS library API is allocated and released using functions **cublasAlloc()** and **cublasFree()**, respectively. These functions create and destroy an object in the GPU memory space capable of holding an array of **n** elements, where each element requires **elemSize** bytes of storage. Please see the legacy CUBLAS API header file "cublas.h" for the prototypes of these functions.

The function **cublasAlloc()** is a wrapper around the function **cudaMalloc()**, therefore device pointers returned by **cublasAlloc()** can be passed to any CUDA<sup>TM</sup> device kernel functions. However, these device pointers can not be dereferenced in the host code. The function **cublasFree()** is a wrapper around the function **cudaFree()**.

#### A.5. Scalar Parameters

In the CUBLAS API the scalar parameters  $\alpha$  and  $\beta$  can be passed by reference on the host or the device

Also, the few functions that return a scalar result, such as <code>amax()</code>, <code>amin</code>, <code>asum()</code>, <code>rotg()</code>, <code>rotmg()</code>, <code>dot()</code> and <code>nrm2()</code>, return the resulting value by reference on the host or the device. Notice that even though these functions return immediately, similarly to matrix and vector results, the scalar result is ready only when execution of the routine on the GPU completes. This requires proper synchronization in order to read the result from the host.

These changes allow the library functions to execute completely asynchronously using streams even when  $\alpha$  and  $\beta$  are generated by a previous kernel. For example, this situation can arise when iterative methods for solution of linear systems and eigenvalue problems are implemented using the CUBLAS library.

## A.6. Helper Functions

In this section we list the helper functions provided by the legacy CUBLAS API and their functionality. For the exact prototypes of these functions please refer to the legacy CUBLAS API header file "cublas.h".

Helper function	Meaning
cublasInit()	initialize the library
cublasShutdown()	shuts down the library
cublasGetError()	retrieves the error status of the library
cublasSetKernelStream()	sets the stream to be used by the library
cublasAlloc()	allocates the device memory for the library
cublasFree()	releases the device memory allocated for the library
cublasSetVector()	copies a vector <b>x</b> on the host to a vector on the GPU
cublasGetVector()	copies a vector <b>x</b> on the GPU to a vector on the host
cublasSetMatrix()	copies a $m \times n$ tile from a matrix on the host to the GPU
cublasGetMatrix()	copies a $m \times n$ tile from a matrix on the GPU to the host
cublasSetVectorAsync()	similar to cublasSetVector(), but the copy is asynchronous
cublasGetVectorAsync()	similar to cublasGetVector(), but the copy is asynchronous
cublasSetMatrixAsync()	similar to cublasSetMatrix(), but the copy is asynchronous
cublasGetMatrixAsync()	similar to cublasGetMatrix(), but the copy is asynchronous

## A.7. Level-1,2,3 Functions

The Level-1,2,3 CUBLAS functions (also called core functions) have the same name and behavior as the ones listed in the chapters 3, 4 and 5 in this document. Please refer to the legacy CUBLAS API header file "cublas.h" for their exact prototype. Also, the next section talks a bit more about the differences between the legacy and the CUBLAS API prototypes, more specifically how to convert the function calls from one API to another.

### A.8. Converting Legacy to the CUBLAS API

There are a few general rules that can be used to convert from legacy to the CUBLAS API.

Exchange the header file "cublas.h" for "cublas\_v2.h".

Exchange the type cublasStatus for cublasStatus\_t.

Exchange the function cublasSetKernelStream() for cublasSetStream().

Exchange the function cublasAlloc() and cublasFree() for cudaMalloc() and cudaFree(), respectively. Notice that cudaMalloc() expects the size of the allocated memory to be provided in bytes (usually simply provide n x elemSize to allocate n elements, each of size elemSize bytes).

Declare the **cublasHandle\_t** CUBLAS library handle.

Initialize the handle using **cublasCreate()**. Also, release the handle once finished using **cublasDestroy()**.

Add the handle as the first parameter to all the CUBLAS library function calls.

Change the scalar parameters to be passed by reference, instead of by value (usually simply adding "&" symbol in C/C++ is enough, because the parameters are passed by reference on the host by *default*). However, note that if the routine is running asynchronously, then the variable holding the scalar parameter cannot be changed until the kernels that the routine dispatches are completed. See the CUDA C Programming Guide for a detailed discussion of how to use streams.

Change the parameter characters 'N' or 'n' (non-transpose operation), 'T' or 't' (transpose operation) and 'C' or 'c' (conjugate transpose operation) to CUBLAS\_OP\_N, CUBLAS\_OP\_T and CUBLAS\_OP\_C, respectively.

Change the parameter characters 'L' or 'l' (lower part filled) and 'U' or 'u' (upper part filled) to CUBLAS\_FILL\_MODE\_LOWER and CUBLAS\_FILL\_MODE\_UPPER, respectively.

Change the parameter characters 'N' or 'n' (non-unit diagonal) and 'U' or 'u' (unit diagonal) to CUBLAS\_DIAG\_NON\_UNIT and CUBLAS\_DIAG\_UNIT, respectively.

Change the parameter characters 'L' or 'l' (left side) and 'R' or 'r' (right side) to **CUBLAS SIDE LEFT** and **CUBLAS SIDE RIGHT**, respectively.

If the legacy API function returns a scalar value, add an extra scalar parameter of the same type passed by reference, as the last parameter to the same function.

Instead of using **cublasGetError**, use the return value of the function itself to check for errors.

Finally, please use the function prototypes in the header files "cublas.h" and "cublas v2.h" to check the code for correctness.

## A.9. Examples

For sample code references that use the legacy CUBLAS API please see the two examples below. They show an application written in C using the legacy CUBLAS library API with two indexing styles (Example A.1. "Application Using C and CUBLAS: 1-based indexing" and Example A.2. "Application Using C and CUBLAS: 0-based Indexing"). This application is analogous to the one using the CUBLAS library API that is shown in the Introduction chapter.

```
//Example A.1. Application Using C and CUBLAS: 1-based indexing
#include <stdio.h>
#include <stdlib.h>
#include <math.h>
#include "cublas.h"
#define M 6
#define N 5
#define IDX2F(i,j,ld) ((((j)-1)*(ld))+((i)-1))
static __inline__ void modify (float *m, int ldm, int n, int p, int q, float
alpha, float beta) {
   cublasSscal (n-p+1, alpha, &m[IDX2F(p,q,ldm)], ldm);
   cublasSscal (ldm-p+1, beta, &m[IDX2F(p,q,ldm)], 1);
int main (void) {
    int i, j;
   cublasStatus stat;
   float* devPtrA;
   float* a = 0;
   a = (float *) malloc (M * N * sizeof (*a));
    if (!a) {
       printf ("host memory allocation failed");
       return EXIT FAILURE;
    for (j = 1; j <= N; j++) {
        for (i = 1; i <= M; i++) {</pre>
            a[IDX2F(i,j,M)] = (float)((i-1) * M + j);
   cublasInit();
   stat = cublasAlloc (M*N, sizeof(*a), (void**)&devPtrA);
    if (stat != CUBLAS STATUS SUCCESS) {
       printf ("device memory allocation failed");
       cublasShutdown();
       return EXIT FAILURE;
   stat = cublasSetMatrix (M, N, sizeof(*a), a, M, devPtrA, M);
    if (stat != CUBLAS STATUS SUCCESS) {
       printf ("data download failed");
       cublasFree (devPtrA);
       cublasShutdown();
       return EXIT FAILURE;
   modify (devPtrA, M, N, 2, 3, 16.0f, 12.0f);
   stat = cublasGetMatrix (M, N, sizeof(*a), devPtrA, M, a, M);
   if (stat != CUBLAS STATUS SUCCESS) {
       printf ("data upload failed");
       cublasFree (devPtrA);
       cublasShutdown();
       return EXIT FAILURE;
```

```
cublasFree (devPtrA);
cublasShutdown();
for (j = 1; j <= N; j++) {
    for (i = 1; i <= M; i++) {
        printf ("%7.0f", a[IDX2F(i,j,M)]);
    }
    printf ("\n");
}
free(a);
return EXIT_SUCCESS;
}</pre>
```

```
//Example A.2. Application Using C and CUBLAS: 0-based indexing
#include <stdio.h>
#include <stdlib.h>
#include <math.h>
#include "cublas.h"
#define M 6
#define N 5
#define IDX2C(i,j,ld) (((j)*(ld))+(i))
static inline void modify (float *m, int ldm, int n, int p, int q, float
alpha, float beta) {
   cublasSscal (n-p, alpha, &m[IDX2C(p,q,ldm)], ldm);
    cublasSscal (ldm-p, beta, &m[IDX2C(p,q,ldm)], 1);
int main (void) {
   int i, j;
   cublasStatus stat;
    float* devPtrA;
   float* a = 0;
   a = (float *) malloc (M * N * sizeof (*a));
   if (!a) {
       printf ("host memory allocation failed");
       return EXIT_FAILURE;
    for (j = 0; j < N; j++) {
       for (i = 0; i < M; i++) {</pre>
           a[IDX2C(i,j,M)] = (float)(i * M + j + 1);
   }
   cublasInit();
    stat = cublasAlloc (M*N, sizeof(*a), (void**)&devPtrA);
    if (stat != CUBLAS_STATUS_SUCCESS) {
        printf ("device memory allocation failed");
        cublasShutdown();
       return EXIT FAILURE;
   stat = cublasSetMatrix (M, N, sizeof(*a), a, M, devPtrA, M);
    if (stat != CUBLAS STATUS SUCCESS) {
       printf ("data download failed");
       cublasFree (devPtrA);
       cublasShutdown();
       return EXIT FAILURE;
   modify (devPtrA, M, N, 1, 2, 16.0f, 12.0f);
   stat = cublasGetMatrix (M, N, sizeof(*a), devPtrA, M, a, M);
    if (stat != CUBLAS STATUS SUCCESS) {
       printf ("data upload failed");
        cublasFree (devPtrA);
        cublasShutdown();
       return EXIT FAILURE;
   cublasFree (devPtrA);
   cublasShutdown();
```

```
for (j = 0; j < N; j++) {
    for (i = 0; i < M; i++) {
        printf ("%7.0f", a[IDX2C(i,j,M)]);
    }
    printf ("\n");
}
free(a);
return EXIT_SUCCESS;
}</pre>
```

## Appendix B. CUBLAS FORTRAN BINDINGS

The CUBLAS library is implemented using the C-based CUDA toolchain, and thus provides a C-style API. This makes interfacing to applications written in C and C++ trivial, but the library can also be used by applications written in Fortran. In particular, the CUBLAS library uses 1-based indexing and Fortran-style column-major storage for multidimensional data to simplify interfacing to Fortran applications. Unfortunately, Fortran-to-C calling conventions are not standardized and differ by platform and toolchain. In particular, differences may exist in the following areas:

- symbol names (capitalization, name decoration)
- argument passing (by value or reference)
- passing of string arguments (length information)
- passing of pointer arguments (size of the pointer)
- returning floating-point or compound data types (for example single-precision or complex data types)

To provide maximum flexibility in addressing those differences, the CUBLAS Fortran interface is provided in the form of wrapper functions and is part of the Toolkit delivery. The C source code of those wrapper functions is located in the **src** directory and provided in two different forms:

- the thunking wrapper interface located in the file fortran\_thunking.c
- the direct wrapper interface located in the file fortran.c

The code of one of those 2 files needs to be compiled into an application for it to call the CUBLAS API functions. Providing source code allows users to make any changes necessary for a particular platform and toolchain.

The code in those two C files has been used to demonstrate interoperability with the compilers g77 3.2.3 and g95 0.91 on 32-bit Linux, g77 3.4.5 and g95 0.91 on 64-bit Linux, Intel Fortran 9.0 and Intel Fortran 10.0 on 32-bit and 64-bit Microsoft Windows XP, and g77 3.4.0 and g95 0.92 on Mac OS X.

Note that for g77, use of the compiler flag -fno-second-underscore is required to use these wrappers as provided. Also, the use of the default calling conventions with regard to argument and return value passing is expected. Using the flag -fno-f2c changes the default calling convention with respect to these two items.

The thunking wrappers allow interfacing to existing Fortran applications without any changes to the application. During each call, the wrappers allocate GPU memory, copy source data from CPU memory space to GPU memory space, call CUBLAS, and finally copy back the results to CPU memory space and deallocate the GPU memory. As this process causes very significant call overhead, these wrappers are intended for light testing, not for production code. To use the thunking wrappers, the application needs to be compiled with the file fortran\_thunking.c

The direct wrappers, intended for production code, substitute device pointers for vector and matrix arguments in all BLAS functions. To use these interfaces, existing applications need to be modified slightly to allocate and deallocate data structures in GPU memory space (using CUBLAS\_ALLOC and CUBLAS\_FREE) and to copy data between GPU and CPU memory spaces (using CUBLAS\_SET\_VECTOR, CUBLAS\_GET\_VECTOR, CUBLAS\_SET\_MATRIX, and CUBLAS\_GET\_MATRIX). The sample wrappers provided in fortran.c map device pointers to the OS-dependent type size\_t, which is 32-bit wide on 32-bit platforms and 64-bit wide on a 64-bit platforms.

One approach to deal with index arithmetic on device pointers in Fortran code is to use C-style macros, and use the C preprocessor to expand these, as shown in the example below. On Linux and Mac OS X, one way of pre-processing is to use the option '-E -x f77-cpp-input' when using g77 compiler, or simply the option '-cpp' when using g95 or gfortran. On Windows platforms with Microsoft Visual C/C++, using 'cl -EP' achieves similar results.

```
! Example B.1. Fortran 77 Application Executing on the Host
· -----
   subroutine modify ( m, ldm, n, p, q, alpha, beta )
   implicit none
   integer ldm, n, p, q
   real*4 m (ldm, *) , alpha , beta
   external cublas sscal
   call cublas sscal (n-p+1, alpha , m(p,q), ldm)
   call cublas sscal (ldm-p+1, beta, m(p,q), 1)
   return
   end
   program matrixmod
   implicit none
   integer M, N
   parameter (M=6, N=5)
   real*4 a(M,N)
   integer i, j
   external cublas init
   external cublas shutdown
   do j = 1, N
      do i = 1, M
          a(i, j) = (i-1)*M + j
       enddo
   enddo
   call cublas init
   call modify (a, M, N, 2, 3, 16.0, 12.0)
   call cublas shutdown
   do j = 1 , N
      do i = 1 , M
          write(*,"(F7.0$)") a(i,j)
       enddo
       write (*,*) ""
   enddo
   stop
```

end

When traditional fixed-form Fortran 77 code is ported to use the CUBLAS library, line length often increases when the BLAS calls are exchanged for CUBLAS calls. Longer function names and possible macro expansion are contributing factors. Inadvertently exceeding the maximum line length can lead to run-time errors that are difficult to find, so care should be taken not to exceed the 72-column limit if fixed form is retained.

The examples in this chapter show a small application implemented in Fortran 77 on the host and the same application with the non-thunking wrappers after it has been ported to use the CUBLAS library.

The second example should be compiled with ARCH\_64 defined as 1 on 64-bit OS system and as 0 on 32-bit OS system. For example for g95 or gfortran, this can be done directly on the command line by using the option '-cpp -DARCH\_64=1'.

```
! Example B.2. Same Application Using Non-thunking CUBLAS Calls
#define IDX2F (i,j,ld) ((((j)-1)*(ld))+((i)-1))
   subroutine modify ( devPtrM, ldm, n, p, q, alpha, beta )
   implicit none
   integer sizeof real
   parameter (sizeof real=4)
   integer ldm, n, p, q
#if ARCH 64
   integer*8 devPtrM
#else
   integer*4 devPtrM
#endif
   real*4 alpha, beta
   call cublas sscal ( n-p+1, alpha,
                      devPtrM+IDX2F(p, q, ldm)*sizeof_real,
                        ldm)
   call cublas_sscal(ldm-p+1, beta,
         devPtrM+IDX2F(p, q, ldm)*sizeof real,
   return
   end
   program matrixmod
   implicit none
   integer M, N, sizeof real
#if ARCH 64
   integer*8 devPtrA
   integer*4 devPtrA
   parameter(M=6,N=5,sizeof real=4)
   real*4 a(M,N)
   integer i, j, stat
   external cublas_init, cublas_set_matrix, cublas get matrix
   external cublas shutdown, cublas alloc
   integer cublas_alloc, cublas_set_matrix, cublas_get_matrix
   do j=1,N
       do i=1, M
           a(i,j) = (i-1) *M+j
       enddo
   enddo
   call cublas init
   stat= cublas alloc(M*N, sizeof real, devPtrA)
   if (stat.NE.\overline{0}) then
       write(*,*) "device memory allocation failed"
       call cublas shutdown
       stop
   endif
```

```
stat = cublas_set_matrix(M,N,sizeof_real,a,M,devPtrA,M)
 if (stat.NE.0) then
      call cublas_free( devPtrA )
write(*,*) "data download failed"
call cublas_shutdown
  endif
 call modify(devPtrA, M, N, 2, 3, 16.0, 12.0)
stat = cublas_get_matrix(M, N, sizeof_real, devPtrA, M, a, M)
  if (stat.NE.0) then
  call cublas free ( devPtrA )
  write(*,*) "data upload failed"
  call cublas_shutdown
  stop
 endif
 call cublas free ( devPtrA )
 call cublas shutdown
 do j = 1 , \overline{N}
      do i = 1 , M
write (*,"(F7.0$)") a(i,j)
      write (*,*) ""
  enddo
  stop
  end
```

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