3d MFS Elastance, recap Th. ① 11/9/23.

$K = \#$ bodies.

Want $(I-L)\vec{1}_N = 0$ where $L = \alpha 1 1^T$

what's α? $I1 - \alpha 1 \cancel{1^T 1}^{N} = 0$ $\alpha = \frac{1}{N}$.

pick $\alpha_0 = \frac{q_k}{N}\vec{1}$ so $S\alpha_0$ is "completion pot."

* use Dir BVP for $u|_{\Gamma_k} = k$ (sphere #) const potentials test case; use charges q_k as input data.

$\boxed{K=1}$

Rep $u = S'(I-L)\overset{\text{unkn.}}{\alpha} + S'\alpha_0$ (R)

solve $[\underbrace{\underset{M\times N}{S}(I-L) + \underset{M\times N}{L_r}}_{A_L,\text{ modified from } A = S',\ M\times N.}]\alpha = -S\alpha_0$ (*) where $L_r := \frac{1}{N}1_M 1_N^T$

i) let α solve (*), then eval (R) on Γ: $u_\Gamma = S(I-L)\alpha + S\alpha_0 \overset{(*)}{=} -L_r\alpha = \text{const.}$ (property of L_r) $(=-$

ii) By (R), flux $\int_\Gamma u_n \underset{\text{since } \Sigma(I-L)\alpha = 0}{=} \int_\Gamma n\cdot\nabla(S'\alpha_0) = \Sigma\alpha_0 = q_1$ by construc. of α_0.

iii) Finally, by (R), $\Delta u = 0$ in ext.

i) & ii) & iii) $\Rightarrow$ u solves elastance BVP.

Multibody $K>1$:

$\alpha = \begin{bmatrix}\alpha_1\\ \vdots \\ \alpha_K\end{bmatrix}$ stack vecs.

$u = \sum_k S'_k[(I-L)\overset{\text{unknowns}}{\alpha_k} + \overset{\text{completion.}}{\alpha_{0k}}]$ where $\alpha_{0k} := \frac{q_k}{N}1_N$. (R) $\Rightarrow$ ii), iii) by construc.

Solve ($K=2$): $\begin{bmatrix} S(1-L)+L_r & S_{12}(1-L) \\ S_{21}(1-L) & S(1-L)+L_r\end{bmatrix}\begin{bmatrix}\alpha_1\\ \alpha_2\end{bmatrix} = -\underbrace{\begin{bmatrix} S & S_{12} \\ S_{21} & S\end{bmatrix}\begin{bmatrix}\alpha_{01}\\ \alpha_{02}\end{bmatrix}}_{\text{evals completion pot.}}$ (*)

$S = $ 1-body, $M\times N$.

$= -\begin{bmatrix}u_{01}\\ u_{02}\end{bmatrix}$, $u_0 := \sum_k S'_k\alpha_{0k}$.

$u_{0k} := u_0|_{\Gamma_k}$.

Lemma: i) holds:

Eval $u|_{\Gamma_1} \overset{(R)}{=} S(1-L)\alpha_1 + \underbrace{S\alpha_{01}} + S_{12}(1-L)\alpha_2 + \underbrace{S_{12}\alpha_{02}}_{\to u_{01}}$

$\overset{(*)}{=} -L_r\alpha_1$ which is const $\Rightarrow$ QED.

$\hookrightarrow$ can read off voltage answers $V_k = -\frac{1}{N}\Sigma\alpha_k$.

R-Precond: let $\gamma_k = (S(1-L)+L_r)\alpha_k =: A_L\alpha_k$.

then $\underbrace{\begin{bmatrix} I & S_{12}(1-L)A_L^+ \\ S_{21}(1-L)A_L^+ & I\end{bmatrix}}\begin{bmatrix}\gamma_1\\ \gamma_2\end{bmatrix} = \begin{bmatrix}-u_{01}\\ -u_{02}\end{bmatrix}$

$\text{matvec}(\gamma)_k = \underset{(\text{Id part})}{\gamma_k} + \sum_{k'\neq k} S_{kk'}(I-L)\overbrace{A_L^+\gamma_{k'}}^{\alpha_{k'}}$

Works! 7:29pm.

Can send in Eqn? output from Dir BVP with known $V_k = k$, $k = 1\ldots K$.

Check elastance recovers the same V_k.

(Sign err? $a>0$. $G = \frac{1}{4\pi r}$, $\hat{r}\cdot\nabla G = \frac{-1}{4\pi r^2}$ $q = CV$ (1-body) so $q>0 \Rightarrow V>0$.

* show translucent contour 2d slice?