

Tight Bounds on 3-Neighbor Bootstrap Percolation

by

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We acknowledge with respect the Lekwungen peoples on whose traditional territory the university stands, and the Songhees, Esquimalt, and WSÁNEĆ peoples whose historical relationships with the land continue to this day.

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ABSTRACT

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DEDICATION

Chapter 1

Introduction

1.1 Problem Overview

1.2 Literature Review

1.3 Outline of ...

Chapter 2

A Tight Bound on Grids of Size ≥ 7

2.1 Introduction and Definitions

Let the ordered tuple (a, b, c) represent the $a \times b \times c$ grid G where $a \geq b \geq c$. We refer to c as the “thickness” of G . For example, the tuple $(5, 3, 3)$ represents a $5 \times 3 \times 3$ grid of thickness 3. We refer to a tuple as “divisible”, or a “divisibility case”, if and only if $ab + bc + ca \equiv 0 \pmod{3}$. Observe that the divisibility cases are precisely those grids with integral lower bounds. The divisibility cases of thicknesses belonging to the three residue classes modulo 3 are illustrated in {Figure something}.

In the following lemmas, we use the notation $(a, b, c) + (x, y, z) = (a + x, b + y, c + z)$ to represent respective increases of x , y , and z to the side lengths a , b , and c of G . We note the following:

Remark 2.1. By applying the recursion, $(a, b, c) + (x, y, z)$ percolates at the lower bound when either:

1. $(a, b, c), (a, y, z), (x, b, z), (x, y, c)$ all percolate at the lower bound, or;
2. $(x, y, z), (x, b, c), (a, y, c), (a, b, z)$ all percolate at the lower bound.

We shall call a thickness “complete” if it can be shown that all divisibility cases in that thickness percolate at the lower bound. In this section, we demonstrate that thickness 5, thickness 6 and thickness 7 are all complete. As these belong to the residue classes 2, 0, and 1 modulo 3, respectively, we then use a recursive construction to show that all larger grids are also complete.

2.2 Completeness of Thickness 5

Leveraging {lemmas from earlier chapters yet to be written}, we show that all divisibility cases in thickness 5 percolate at the lower bound.

Lemma 2.2. *Thickness 5 is complete.*

Chapter 3

Chapter on the Next Thing