3 Interference and Diffraction

Abstract: In this lab, you will observe the effects of diffraction of visible light through a narrow slit and of interference of light passing through multiple slits. You will use the interference patterns to measure the wavelength of the light from a laser.

3.1 Pre-lab preparation

Interference is a property of waves. Put simply, waves superimpose - meaning that the amplitude at any position and time is the sum of all waves at that position at that time. If the superimposing waves are **periodic** and **coherent**, they combine in ways that lead to striking phenomena, which you've learned about in lecture courses: standing waves on a string, diffraction of water waves around a corner, and interference of sound waves from a pair of speakers. In this lab, you will examine diffraction and interference using light waves, and then use these phenomena to determine the wavelength of the light from a laser. But first, you may find the following re-cap of the underlying physics useful.

Consider two point sources of light that are completely **coherent** (*i.e.*, they emit light of the same intensity and wavelength, and do so in phase with each other). The light from each point source propagates in all directions, but for simplicity we will consider only a single plane. Suppose the point sources are separated by a distance d. We'll use the (x, y) plane and place the light sources at points $(0, 0)$ and $(0, d)$ (see Figure 9a). We can then determine the intensity of the light at different points along a screen, placed a distance L away on the x-axis, by considering the superposition of the two light waves at an arbitrary point, (L, y), on the screen (see Figure 9b).

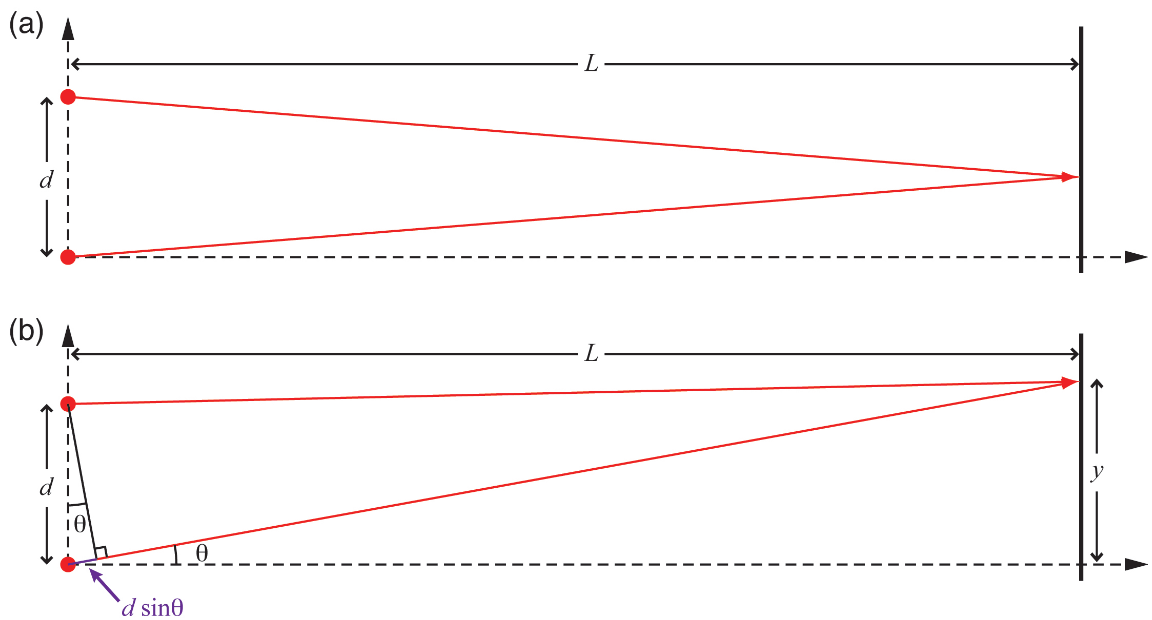

Figure 9: Diagram of two point sources (red dots) used for calculating the interference pattern. The (x, y) coordinate system is shown by the dashed lines. The point sources are separated by a distance d along the y-axis and the interference pattern is calculated considering the light projected onto a screen a distance L away along the x axis. (a) Light rays of equal pathlength reaching a point on the screen; because they traveled the same distance, the light from the two sources are in phase with each other and constructive interference occurs. (b) Light rays reach a point on the screen where their pathlengths differ by an amount $d \sin \theta$, with consequences discussed in the text.

As shown in Figure 9(a), at the point $(L, d/2)$, the path length for each of the two light waves is equal. That leaves the two waves in phase with each other, so they add (*i.e.*, interfere) constructively, and make a bright spot at that point on the screen.

At any other point on the screen, the path lengths of the two light waves differ, as shown in Figure 9(b), where the light that comes from the lower point travels an additional length $d\sin\theta$. This light arrives at the screen with a phase that is shifted by $\Delta\phi = 2\pi d \sin\theta/\lambda$ with respect to the light from the upper point. If the additional path length d is such that $d\sin\theta = \lambda/2$ then the phase shift is $\Delta\phi = \pi$. That makes the wave from the lower point exactly the negative of the wave from the upper point, so they interfere destructively and that point on the screen is dark.

We can calculate the y-coordinate, $y = L\tan\theta$, of the point on the screen where this destructive interference occurs.

If θ is small (*i.e.*, if $L \gg d$), then we can approximate $\tan\theta \approx \sin\theta$, so $y_{\mathrm{destr}} \approx L\sin\theta = L\lambda/2d$.

Another special point on the screen, (L, y_{constr}), is where the path lengths of the two light waves differ by exactly one wavelength, *i.e.*, $d\sin\theta = \lambda$. qAt this point, the phase of the light that travels the longer path is shifted with respect to that of the light that travels the shorter path by $\Delta\phi = 2\pi d\sin\theta/\lambda = 2\pi$, which has the same effect as no phase shift at all, so the two waves interfere constructively. Mathematically, if $y_{\mathrm{constr}} = L\sin\theta = L\lambda/d$, the point on the screen at (L, y_{constr}) will be bright.

The pattern repeats for larger values of y: destructive interference occurs at any point for which $\Delta\phi$ is an odd multiple of π and constructive interference occurs at any point where the phase difference is an even multiple of π. The generalized relations for y_{constr} and y_{destr} are key to what you'll do in this lab, so I'll write them here in an easily findable and concise mathematical format

$$y_{\mathrm{destr}} = nL\lambda/2d, \quad \text{where } n = \pm1, \pm3, \pm5, \dots \tag{10}$$
$$y_{\mathrm{constr}} = nL\lambda/d, \quad \text{where } n = 0, \pm1, \pm2, \pm3, \dots \tag{11}$$

(The drawing in Fig. 9 includes a coordinate system offset of $d/2$, but this is negligible since we are working in the limit where $L \gg d$.)

An experimentally direct way to create two coherent light sources is to shine a single laser on something opaque that has two very small holes poked in it. If the holes are small enough, they will each behave as a point source from which spherical light waves emanate. Because the incident laser light is coherent, the holes will be coherent light sources.

This is basically the approach that we'll use in this lab, but we'll use *slits* rather than holes. We are only looking in two dimensions, (x, y). A slit in the z-direction is the same as a hole for that purpose, with the added benefit that we can illuminate a slit without having to aim the laser as carefully!

A possible problem with this approach is that our slits are not necessarily narrow enough to be truly "point" sources in the plane. So, let's consider the effect of passing light through a finite slit width. We'll do that by considering a single slit of width w as shown in Fig. 10. We'll again consider only the xy-plane, and following Huygen's principle, we can consider the wavefront at the slit as a series of coherent point sources, illustrated by the red dots in the figure.

It is easy to see that there is constructive interference at the point $(L, 0)$ because for every point source in the slit, at a position of say $(0, y_1)$, there is another point source at $(0, -y_1)$, symmetrically across the y-axis, which therefore hßas the same pathlength to the point $(L, 0)$. As a result, there is constructive interference, and hence a bright spot, at $(L, 0)$.

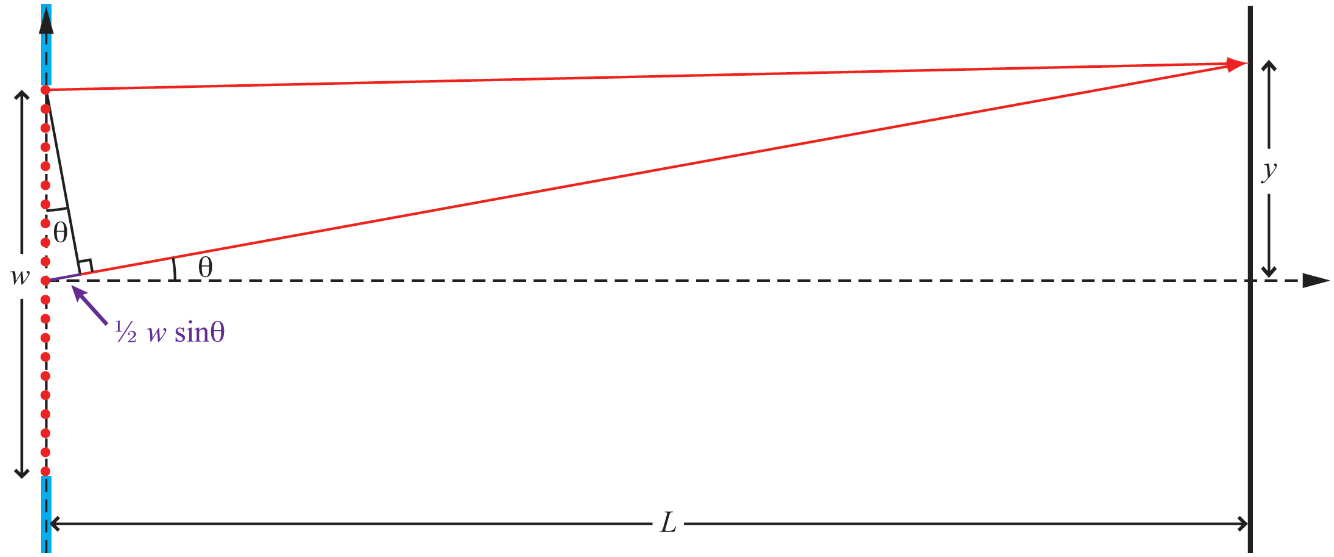

Figure 10: Diagram of multiple point sources (red dots) used for calculating the diffraction pattern of a thin slit. The blue lines are a material blocking all light except for the thin slit of width w, which is situated symmetrically across the y-axis. Interference of light from two points separated by $w/2$, one above and one below the y-axis, have pathlength difference of $\frac{w}{2}\sin\theta$ when reaching the screen placed at $x = L$, with consequences discussed in the text.

Using a similar argument, we can see that destructive interference will occur at some point (L, y_{destr}). This happens because light from the point at $(0, w/2)$ and light from the point at $(0, 0)$ have a pathlength difference of $(w/2)\sin\theta$. If that pathlength difference equals $\lambda/2$, then the light rays will combine out of phase and destructively interfere.[6] Given that the light from the point $(0, w/2)$ is canceled by the light from $(0, 0)$, it follows that light from the point $(0, w/2 - \Delta)$ is canceled by light from $(0, -\Delta)$. In fact, at $y_{\mathrm{destr}} = L\lambda/w$, the light from any point in the upper half of the slit is canceled by a point that lies a distance $w/2$ below it, in the lower half of the slit. As a result, all of the light from the slit undergoes destructive interference at the point (L, y_{destr}); the point appears dark. The same argument applies for any point with a y-coordinate that is an odd integer multiple of $\lambda/2$, so there are dark spots for

$$y_{\mathrm{destr}} = n\lambda L/w, \quad \text{where } n = \pm1, \pm2, \pm3, ... \tag{12}$$

This is called a *diffraction pattern*; it is a special type of interference pattern that results from light passing through different parts of a finite width slit. The light is said to "diffract" through the slit. (Notice that as the slit width w goes to zero, the point of destructive interference moves out to infinity along the y-aixs and we get the spherical wave pattern of a single point. In the opposite extreme, as w goes to infinity, the spacing between dark points goes to zero so no pattern is discernible.)

In this lab, you will look at both the diffraction pattern from a single slit and the interference pattern from two, or more, slits. The pattern from two slits will be a convolution of the interference pattern from the two slits and the diffraction from each slit. A typical two-slit pattern is sketched in Figure 11. The number of interference maxima within each diffraction peak depends on the ratio of the slit width to the slit spacing, so the pattern you observe may be substantially different in this respect.

[6]Note that, when solving for $y_{\mathrm{destr}} = L\lambda/w$, the two factors of $1/2$ cancel and the "small angle approximation" allows us to set $\tan\theta \approx \sin\theta$.

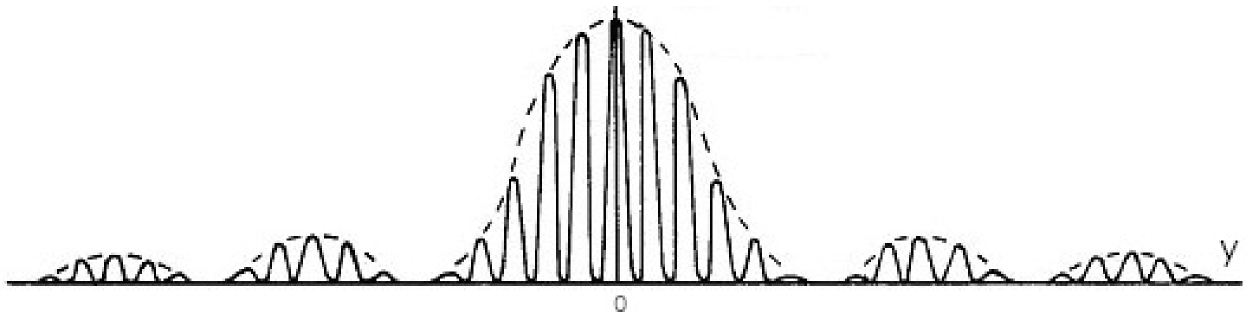

Figure 11: Cartoon of the pattern expected from two thin slits. The diffraction pattern (dashed line), defined by the slit width, is convolved with the interference pattern, defined by the slit spacing, to give the intensity *vs* position shown by the solid line.

3.1.1 Experimental planning

In this lab you will *observe* the qualitative features of interference and diffraction patterns and then use them to *measure* the wavelength of the laser. Since the wavelength of visible light is less than a micron, you will need to use some clever experimental "tricks" to extract a precision measurement.

You will want to leverage the fact that an interference pattern scales with the wavelength times L/d. You will use small slit spacings, d, and a long optical path, L, to make the pattern visible. Even so, the pattern will still only be a few centimeters across. How might you precisely measure the positions of the maxima and minima? Think for a minute or two before reading further to see if you can come up with an idea.

3.1.2 Apparatus

The apparatus you will use is an **optics bench** with a fixed laser mount, a pair of rotatable slit wheels and a movable screen mount, as shown in Fig. 12(a). The "bench" provides a robust way to set and measure the positions of the mounts.

You will access and control the equipment remotely through a webpage and observe the results via the live-stream of a webcam. The setup is shown in Fig. 13. From this remote interface, you can select which slit you wish to view, adjust the alignment of the slit and screen position, turn on the ambient lights and laser, and turn on the opacity of the screen. There are three camera views available. The overview camera looks down the optical bench from just behind the slit wheels. It allows you to see when the slit wheels are turning as and when the unobstructed laser light is falling on the screen at the far end. The position camera is rigidly affixed to the screen and looks at a tape measure along the rail, allowing you to measure the change in position when you move the screen. The screen camera looks down the optical axis. It is aimed directly at the laser and slit wheels, which can be seen by clicking the closed eye symbol in the lower left of the webpage, making the screen itself transparent. Clicking again on the (now) open eye symbol makes the screen opaque again, allowing you to observe the laser light intensity patterns that result from the different slits. Clicking on the "crosshair" button creates a green crosshair on the screen camera image and lets you view the pixel position of your cursor. There is a ruler taped to the screen which you will use to convert pixels into centimeters or millimeters.

The remote setup includes camera controls at the bottom to change the exposure time, brightness, and contrast of the screen image. These parameters can be adjusted to improve the visibility of peaks.

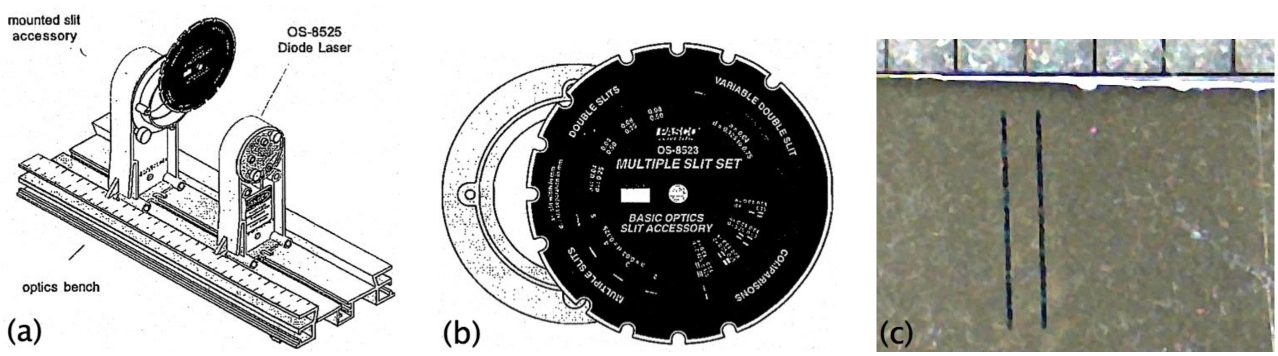

Figure 12: (a) The PASCO OS-8515 Basic Optics System with a OS-8525 Laser Diode assembly and OS-8523 slit accessory mounted. (b) A close-up of the Multiple Slit Accessory. There is a similar Single Slit Accessory; each accessory allows you to change which slit (or set of slits) is in the laser beam by rotating part of the device. (c) A close-up of one pair of slits, taken through a microscope; a ruler with 1 mm divisions is placed at the top of the field of view to calibrate the scale.

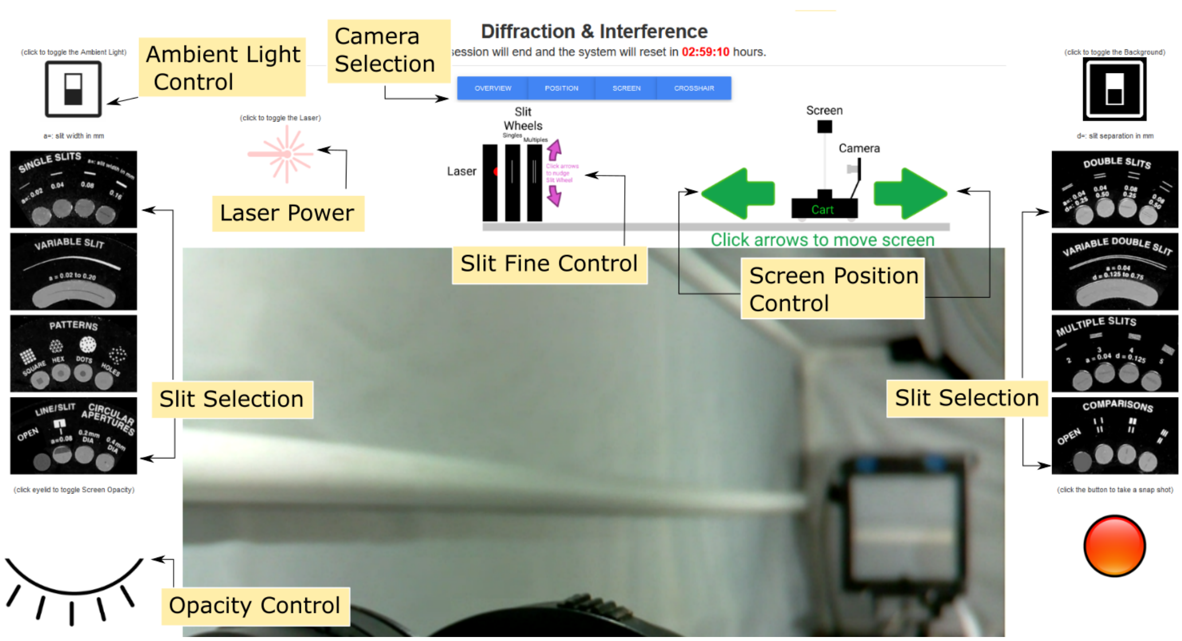

Figure 13: The remote interface includes a variety of controls to turn on the lights, select slit, adjust position of the screen and slit alignment, and switch camera view.

3.1.3 Experimental planning

Measuring positions of maxima and minima: One way to measure the positions of maxima and minima in the interference pattern is to take a photo of the pattern with a ruler placed in the field of view. You can then use image analysis software to measure the spacing of the maxima and minima in units of pixels within the image and scale that by the number of pixels per mm using the ruler.

You can process and analyze the images you take using Fiji, a free software developed and maintained by the National Institutes of Health that works on Macs as well as PCs. (But if you are already familiar with a different image processing software, feel free to use that instead.)

3.2 Getting started and gaining familiarity

The first thing you should do in lab is play around with the remote setup to understand how it works. Rotate the slit wheels, move the screen and look at the different camera views. If you like, take a screenshot of the setup for your logbook. You don't need to immediately insert the photo into your logbook, but it is a good habit to note the time you took it. Alternatively, you could draw a schematic of the setup. Honestly, a schematic is often a lot more useful. Photos capture tons of information that can obscure, or just distract from, the essential. A schematic captures only what you want it to and is easily labeled. Either way, you should note the make and model of the various components in your setup as best you can.

3.3 Qualitative observations of interference and diffraction

Use the Single Slit Accessory, view the diffraction pattern using *at least* three different slit widths. Take screenshots of the patterns for your logbook. Write your name, the slit width and any other variables that you suspect effect the pattern on a post-it note and include that writing in the photograph's field of view. In general, it is a good habit to include such "metadata" directly in your experimental data whenever possible because doing so can resolve ambiguities that might arise later on down the line.

Write a brief qualitative summary of your observations. For example, you might conclude that the separation of the maxima and minima increases as the slit width decreases. Of course, that is what you expect from equation (12), but the point of doing an experiment is to record what you observe in a clear and concise way. Imagine that the theory of the diffraction pattern where not yet understood, and write your conclusions with the intention of defining the experimental facts that a developing theory must explain.

3.4 Measurement of the wavelength of a laser

The observations you just completed were a qualitative way to get familiar with the phenomenon of interference. Now, it's time to make a measurement. Your goal is to measure the laser's wavelength as precisely as possible. You will do this using the the Multiple Slit Accessory and the relations in equations (10) and (11), where you can measure L, y, and d and extract λ.

Start by thinking about the uncertainties. In your logbook, calculate how the uncertainty on λ depends on the uncertainties in the things you will measure. Then make some estimates. What do you think the uncertainties will be in each of the three parameters, i.e., δL, δy, and δd? Which of the three do you expect to dominate the total uncertainty in λ?

Next, you should think about possible systematic errors. While the fractional uncertainty in L is probably going to be smaller than the other uncertainties, it is something that could easily have a systematic bias. For example, while the ruler provides a nice way to measure the position the screen cart, that isn't necessarily the distance from the slit to the screen, due to offsets in the placement of the ruler. The difference between the distance and measured position will be a systematic bias on L that will need to be corrected.

There is, however, a trick to avoid that systematic uncertainty in L: measure y as a function of L and fit the results to a straight line. The slope of that line gives λ (after correcting for d). The systematic uncertainty from the difference between the mount's position and the actual slit position changes the intercept of the line, but not its slope. Measuring the dependence of two observables and extracting a desired quantity from a fit is a common trick because of its ability to do away with certain systematic errors, and because it is a very good way to validate the data. For example, the goodness of the fit (the χ^2 per degrees of freedom) shows whether the measurements are behaving properly within the determined uncertainties.[7] If the χ^2 is too small, the uncertainties may be overestimated. If the χ^2 is too big, either the uncertainties are underestimated or there is something else going on beyond what is assumed in the model (*i.e.*, the equation to which the data is being fit).

You should make measurements for several different values of L and use a fit to extract λ. As discussed in section 3.1.2, you can obtain a more precise measurement of the positions of the maxima and minima by photographing the pattern and using image analysis software to determine the positions in pixels, and calibrating pixels to distance. Think a bit about how you will do this. Write your thoughts in your notes.

It is, in general, a good idea to plan your experimental protocol before you begin. Here are some tips that might help you plan:

- Make a table in your logbook where you record information about each measurement. It should include a unique identifier for each measurement (*e.g.*, a number, letter, or combination thereof).
- Write that identifier on a piece of paper that you can attach to the screen so it is visible in the photos as metadata.
- Note the filename (or equally identifying information) for each photo in your logbook.
- Take more than one photo for each value of L. This will allow you to select the best one, *e.g.*, if some have focus, lighting or perspective problems. It will also allow you to compare the measured y values between two or more good photos to assess whether the uncertainties are as you estimate.
- Think about how to keep your screenshots equally sized, so that you can properly calibrate distances to pixel widths.
- It is more precise to measure the difference between the +1 and -1 maxima and then divide by two than it is to just measure the difference between +1 and 0. Similarly, you might find it more precise to determine the position of the minima than the maxima.
- Leave enough space in your table to add comments or other information later.
- Using a larger number of slits gives narrower, more sharply defined maxima.
- Too high of an exposure setting will saturate the brightest peaks and make the center position difficult to locate, but too low will make the dim peaks hard to see. Adjusting the contrast can have a similar effect. Take the same image with different settings and/or try to find an optimal balance for optimal analyzis of the images.

Once you have thought through your plan, summarize what you have decided to do and why in your logbook. Now you are ready to start taking data. (Of course, your actual procedure may differ from your plan as you have ideas, make mistakes, gather new information. If it does, just note how and why - you don't have to re-write the whole plan unless you want to (*e.g.*, to make it easier to reconstruct later). As President Eisenhower famously said, "Plans are useless, but planning is essential!"

Ideally you would analyze each photo as you take it, but given your limited time with the equipment, doing so would not allow you to make enough measurements to get good precision. Still, you should promptly analyze at least the first photo, *i.e.*, copy it into your image processing software and extract a

[7] See Chapter 1, Section 1.1.3

distance to make sure the photo has all the qualities that will make it easy to analyze. This is a good habit because it is best to find problems at the beginning rather than after you've spent 30 minutes collecting data.

3.4.1 Image analysis

You are free to choose whatever method you prefer to measure the positions in your images. As mentioned above, I recommend the NIH ImageJ-based package called Fiji because it is platform independent, has an intuitive user interface, a complete suite of tools, and good documentation. I encourage you to download it onto your own computer so you can play around with it any time.

Within Fiji you can draw a line anywhere on your image, in any direction, and of any thickness (double-click on the line-selection tool in the toolbar to set the thickness, *a.k.a.* line width) Once you've drawn a line, you can use `Analyze` ⟩ `Plot Profile` to create a plot of intensity values (averaged across the width of the line) versus position along the length of the line. Pushing the List button gives a list of the intensity values used to create the plot. These values can be copied and pasted into your favorite graphing program for further plotting and fitting. Read more about this and other commands available in the `Analyze` menu in the on-line documentation.

Feel free to brainstorm with your classmates about how to best get at the information you want. If you get a particularly good idea from a classmate, feel free to use it for your measurement, but you should cite them in your logbook as the source of the idea.

3.5 Going beyond

There are several opportunities for you to go beyond the minimal requirements, and I encourage you to do so. This class will prepare you for an experimental research position or internship and ultimately for experimental research in graduate school. I'm often asked by students of lab courses to write letters of recommendation for their graduate school applications. Saying that they did everything required of them is underwhelming, but it is great when I can point out where they have pursued additional ideas and/or tried their own methods. Make doing so your habit.

Here are a few additional things you could measure or explore if time allows.

- Measure the positions of the minima in the photos of your single slit diffraction patterns and plot them vs the slit width. Does it fit to $1/w$?
- Measure the spacing between the first, the second, and (maybe) the third minima in either the diffraction or the interference patterns. Are they equal?
- Observe the patterns from other features on the Single Slit Accessory, *e.g.*, there is an opaque line that can be compared to a transparent slit. There is also a circular aperture. Circular diffraction can be described by the Airy pattern which uses the Bessel function. Try plotting the intensity profile and fitting this function.
- A CD or DVD consists of circular tracks with a small spacing. Light passing through (or reflecting from) those tracks will generate an interference pattern. If you have a laser pointer, the red light will have a wavelength of 630-670 nm. Now that you know the wavelength of your laser, determine the track spacing for a CD or DVD.
- Measure the thickness of your hair (again using a laser pointer whose wavelength you know).