- Measure the uncertainty in the thickness of the absorbers. You can do this by measuring the count rate with several different absorbers that are listed as having the same density thickness; use the spread in count rate and the measured λ to deduce the variation in thickness.
- Do you see twice as many gammas as betas, as predicted in Fig. 3(a)? If not, why not? (Hint: where is the source material?)
- Measure the total activity of the source.
- If the source is in the bottom slot, is there any difference between putting a plastic absorber right above it *vs* in the top slot? If so, why?
- Carefully investigate whether there is a difference in the count rate when the source is placed label up or label down. Suggest an explanation for this based on what else you observed in the lab. Can you test that explanation somehow?
- Measure the count rate for any object of your choice, like a dollar bill. Is it significantly higher than the background count rate? Why or why not?

3 Interference and Diffraction

Abstract: In this lab, you will observe the effects of diffraction of visible light through a narrow slit and of interference of light passing through multiple slits. You will use the interference patterns to measure the wavelength of the light from a laser.

3.1 Pre-lab preparation

Interference is a property of waves. Put simply, waves superimpose - meaning that the amplitude at any position and time is the sum of all waves at that position at that time. If the superimposing waves are **periodic** and **coherent**, they combine in ways that lead to striking phenomena, which you've learned about in lecture courses: standing waves on a string, diffraction of water waves around a corner, and interference of sound waves from a pair of speakers. In this lab, you will examine diffraction and interference using light waves, and then use these phenomena to determine the wavelength of the light from a laser. But first, you may find the following re-cap of the underlying physics useful.

Consider two point sources of light that are completely **coherent** (*i.e.*, they emit light of the same intensity and wavelength, and do so in phase with each other). The light from each point source propagates in all directions, but for simplicity we will consider only a single plane. Suppose the point sources are separated by a distance d. We'll use the (x, y) plane and place the light sources at points $(0, 0)$ and $(0, d)$ (see Figure 9a). We can then determine the intensity of the light at different points along a screen, placed a distance L away on the x-axis, by considering the superposition of the two light waves at an arbitrary point, (L, y), on the screen (see Figure 9b).

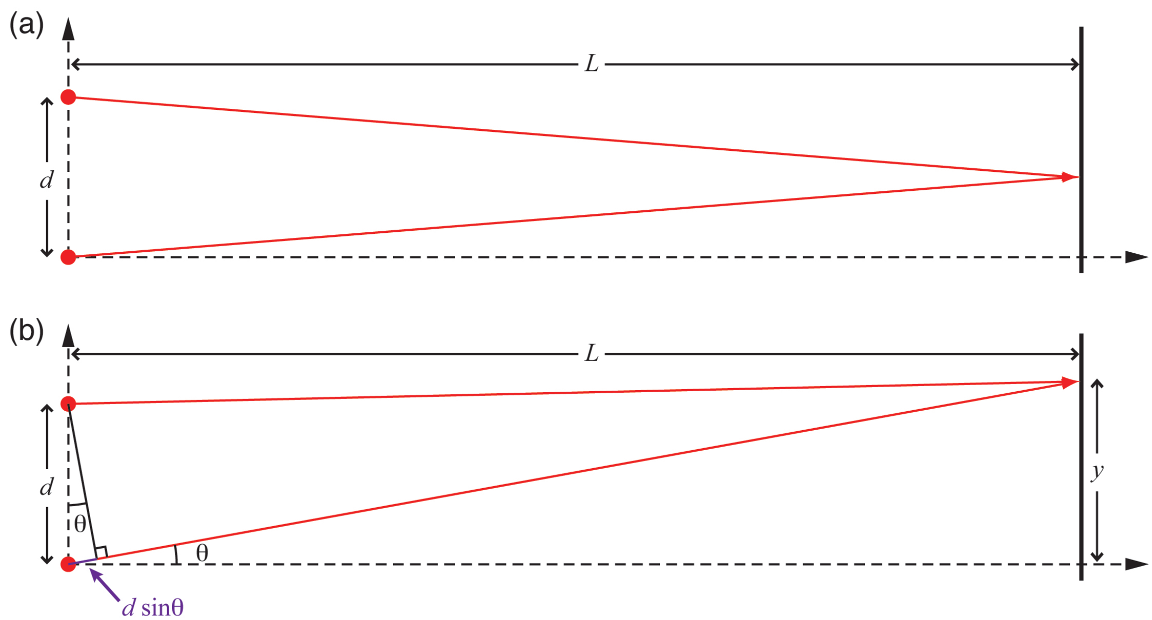

Figure 9: Diagram of two point sources (red dots) used for calculating the interference pattern. The (x, y) coordinate system is shown by the dashed lines. The point sources are separated by a distance d along the y-axis and the interference pattern is calculated considering the light projected onto a screen a distance L away along the x axis. (a) Light rays of equal pathlength reaching a point on the screen; because they traveled the same distance, the light from the two sources are in phase with each other and constructive interference occurs. (b) Light rays reach a point on the screen where their pathlengths differ by an amount $d \sin \theta$, with consequences discussed in the text.

As shown in Figure 9(a), at the point $(L, d/2)$, the path length for each of the two light waves is equal. That leaves the two waves in phase with each other, so they add (*i.e.*, interfere) constructively, and make a bright spot at that point on the screen.

At any other point on the screen, the path lengths of the two light waves differ, as shown in Figure 9(b), where the light that comes from the lower point travels an additional length $d \sin\theta$. This light arrives at the screen with a phase that is shifted by $\Delta\phi = 2\pi d \sin\theta/\lambda$ with respect to the light from the upper point. If the additional path length d is such that $d \sin\theta = \lambda/2$ then the phase shift is $\Delta\phi = \pi$. That makes the wave from the lower point exactly the negative of the wave from the upper point, so they interfere destructively and that point on the screen is dark.

We can calculate the y-coordinate, $y = L \tan\theta$, of the point on the screen where this destructive interference occurs.

If θ is small (*i.e.*, if $L \gg d$), then we can approximate $\tan\theta \approx \sin\theta$, so $y_{\text{destr}} \approx L \sin\theta = L\lambda/2d$.

Another special point on the screen, (L, y_{constr}), is where the path lengths of the two light waves differ by exactly one wavelength, *i.e.*, $d \sin\theta = \lambda$. qAt this point, the phase of the light that travels the longer path is shifted with respect to that of the light that travels the shorter path by $\Delta\phi = 2\pi d \sin\theta/\lambda = 2\pi$, which has the same effect as no phase shift at all, so the two waves interfere constructively. Mathematically, if $y_{\text{constr}} = L \sin\theta = L\lambda/d$, the point on the screen at (L, y_{constr}) will be bright.

The pattern repeats for larger values of y: destructive interference occurs at any point for which $\Delta\phi$ is an odd multiple of π and constructive interference occurs at any point where the phase difference is an even multiple of π. The generalized relations for y_{constr} and y_{destr} are key to what you'll do in this lab, so I'll write them here in an easily findable and concise mathematical format

$$y_{\text{destr}} = \quad nL\lambda/2d, \quad \text{where } n = \pm1, \pm3, \pm5, \ldots \tag{10}$$
$$y_{\text{constr}} = \quad nL\lambda/d, \quad \text{where } n = 0, \pm1, \pm2, \pm3, \ldots \tag{11}$$

(The drawing in Fig. 9 includes a coordinate system offset of $d/2$, but this is negligible since we are working in the limit where $L \gg d$.)

An experimentally direct way to create two coherent light sources is to shine a single laser on something opaque that has two very small holes poked in it. If the holes are small enough, they will each behave as a point source from which spherical light waves emanate. Because the incident laser light is coherent, the holes will be coherent light sources.

This is basically the approach that we'll use in this lab, but we'll use *slits* rather than holes. We are only looking in two dimensions, (x, y). A slit in the z-direction is the same as a hole for that purpose, with the added benefit that we can illuminate a slit without having to aim the laser as carefully!

A possible problem with this approach is that our slits are not necessarily narrow enough to be truly "point" sources in the plane. So, let's consider the effect of passing light through a finite slit width. We'll do that by considering a single slit of width w as shown in Fig. 10. We'll again consider only the xy-plane, and following Huygen's principle, we can consider the wavefront at the slit as a series of coherent point sources, illustrated by the red dots in the figure.

It is easy to see that there is constructive interference at the point $(L, 0)$ because for every point source in the slit, at a position of say $(0, y_1)$, there is another point source at $(0, -y_1)$, symmetrically across the y-axis, which therefore hßas the same pathlength to the point $(L, 0)$. As a result, there is constructive interference, and hence a bright spot, at $(L, 0)$.

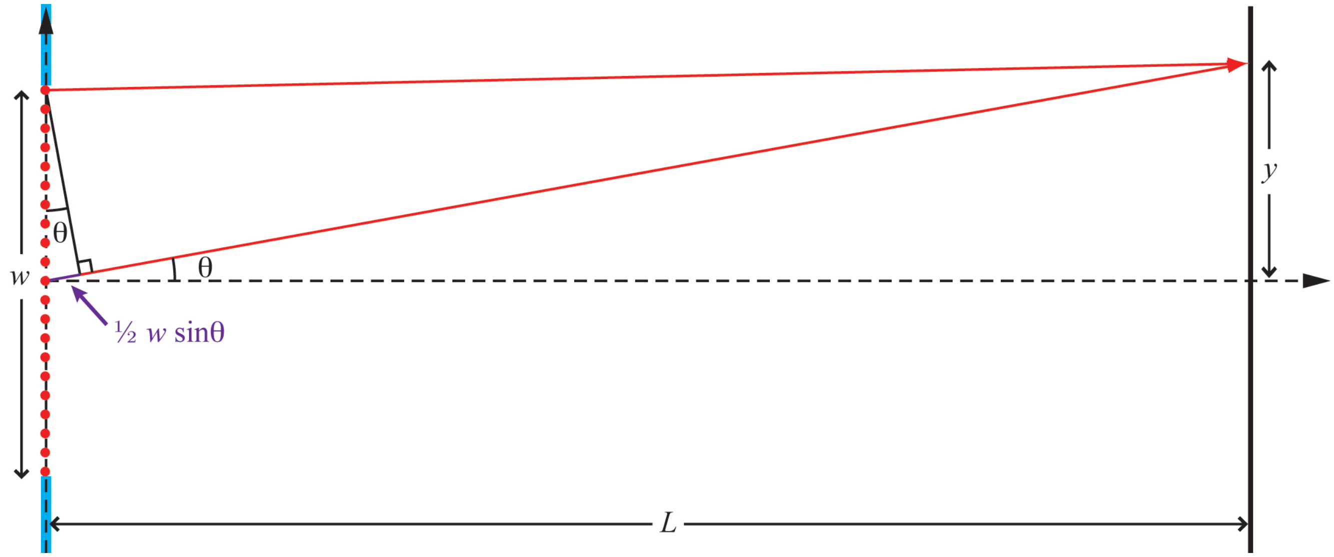

Figure 10: Diagram of multiple point sources (red dots) used for calculating the diffraction pattern of a thin slit. The blue lines are a material blocking all light except for the thin slit of width w, which is situated symmetrically across the y-axis. Interference of light from two points separated by $w/2$, one above and one below the y-axis, have pathlength difference of $\frac{w}{2}\sin\theta$ when reaching the screen placed at $x = L$, with consequences discussed in the text.

Using a similar argument, we can see that destructive interference will occur at some point (L, y_{destr}). This happens because light from the point at $(0, w/2)$ and light from the point at $(0, 0)$ have a pathlength difference of $(w/2)\sin\theta$. If that pathlength difference equals $\lambda/2$, then the light rays will combine out of phase and destructively interfere.[6] Given that the light from the point $(0, w/2)$ is canceled by the light from $(0, 0)$, it follows that light from the point $(0, w/2 - \Delta)$ is canceled by light from $(0, -\Delta)$. In fact, at $y_{\text{destr}} = L\lambda/w$, the light from any point in the upper half of the slit is canceled by a point that lies a distance $w/2$ below it, in the lower half of the slit. As a result, all of the light from the slit undergoes destructive interference at the point (L, y_{destr}); the point appears dark. The same argument applies for any point with a y-coordinate that is an odd integer multiple of $\lambda/2$, so there are dark spots for

$$y_{\text{destr}} = n\lambda L/w, \quad \text{where } n = \pm 1, \pm 2, \pm 3, ... \tag{12}$$

This is called a *diffraction pattern*; it is a special type of interference pattern that results from light passing through different parts of a finite width slit. The light is said to "diffract" through the slit. (Notice that as the slit width w goes to zero, the point of destructive interference moves out to infinity along the y-aixs and we get the spherical wave pattern of a single point. In the opposite extreme, as w goes to infinity, the spacing between dark points goes to zero so no pattern is discernible.)

In this lab, you will look at both the diffraction pattern from a single slit and the interference pattern from two, or more, slits. The pattern from two slits will be a convolution of the interference pattern from the two slits and the diffraction from each slit. A typical two-slit pattern is sketched in Figure 11. The number of interference maxima within each diffraction peak depends on the ratio of the slit width to the slit spacing, so the pattern you observe may be substantially different in this respect.

[6]Note that, when solving for $y_{\text{destr}} = L\lambda/w$, the two factors of $1/2$ cancel and the "small angle approximation" allows us to set $\tan\theta \approx \sin\theta$.

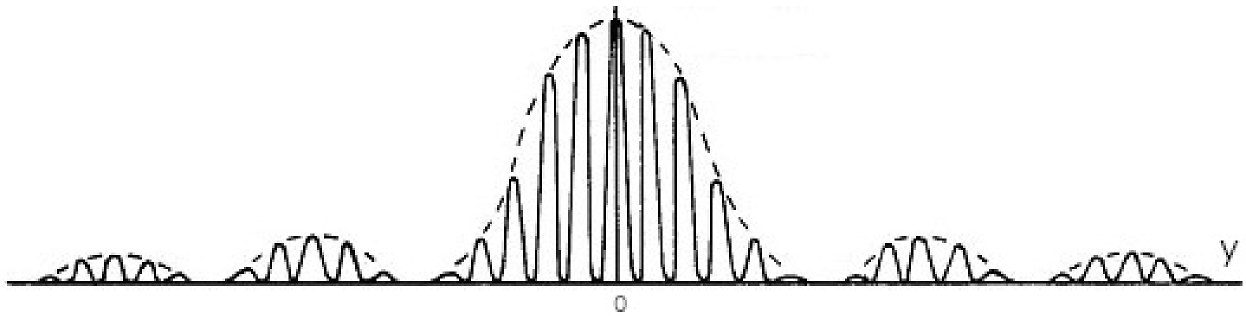

Figure 11: Cartoon of the pattern expected from two thin slits. The diffraction pattern (dashed line), defined by the slit width, is convolved with the interference pattern, defined by the slit spacing, to give the intensity *vs* position shown by the solid line.

3.1.1 Experimental planning

In this lab you will *observe* the qualitative features of interference and diffraction patterns and then use them to *measure* the wavelength of the laser. Since the wavelength of visible light is less than a micron, you will need to use some clever experimental "tricks" to extract a precision measurement.

You will want to leverage the fact that an interference pattern scales with the wavelength times L/d. You will use small slit spacings, d, and a long optical path, L, to make the pattern visible. Even so, the pattern will still only be a few centimeters across. How might you precisely measure the positions of the maxima and minima? Think for a minute or two before reading further to see if you can come up with an idea.

3.1.2 Apparatus

The apparatus you will use is an **optics bench** with a fixed laser mount, a pair of rotatable slit wheels and a movable screen mount, as shown in Fig. 12(a). The "bench" provides a robust way to set and measure the positions of the mounts.

You will access and control the equipment remotely through a webpage and observe the results via the live-stream of a webcam. The setup is shown in Fig. 13. From this remote interface, you can select which slit you wish to view, adjust the alignment of the slit and screen position, turn on the ambient lights and laser, and turn on the opacity of the screen. There are three camera views available. The overview camera looks down the optical bench from just behind the slit wheels. It allows you to see when the slit wheels are turning as and when the unobstructed laser light is falling on the screen at the far end. The position camera is rigidly affixed to the screen and looks at a tape measure along the rail, allowing you to measure the change in position when you move the screen. The screen camera looks down the optical axis. It is aimed directly at the laser and slit wheels, which can be seen by clicking the closed eye symbol in the lower left of the webpage, making the screen itself transparent. Clicking again on the (now) open eye symbol makes the screen opaque again, allowing you to observe the laser light intensity patterns that result from the different slits. Clicking on the "crosshair" button creates a green crosshair on the screen camera image and lets you view the pixel position of your cursor. There is a ruler taped to the screen which you will use to convert pixels into centimeters or millimeters.

The remote setup includes camera controls at the bottom to change the exposure time, brightness, and contrast of the screen image. These parameters can be adjusted to improve the visibility of peaks.

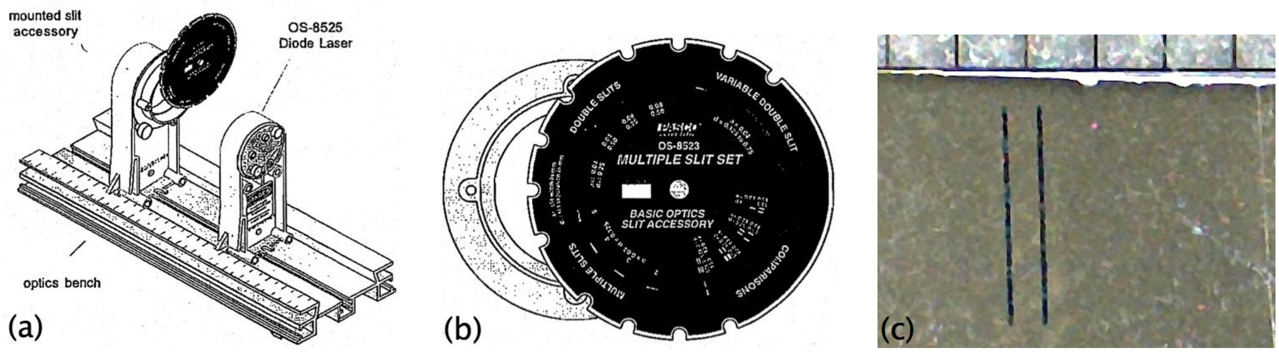

Figure 12: (a) The PASCO OS-8515 Basic Optics System with a OS-8525 Laser Diode assembly and OS-8523 slit accessory mounted. (b) A close-up of the Multiple Slit Accessory. There is a similar Single Slit Accessory; each accessory allows you to change which slit (or set of slits) is in the laser beam by rotating part of the device. (c) A close-up of one pair of slits, taken through a microscope; a ruler with 1 mm divisions is placed at the top of the field of view to calibrate the scale.

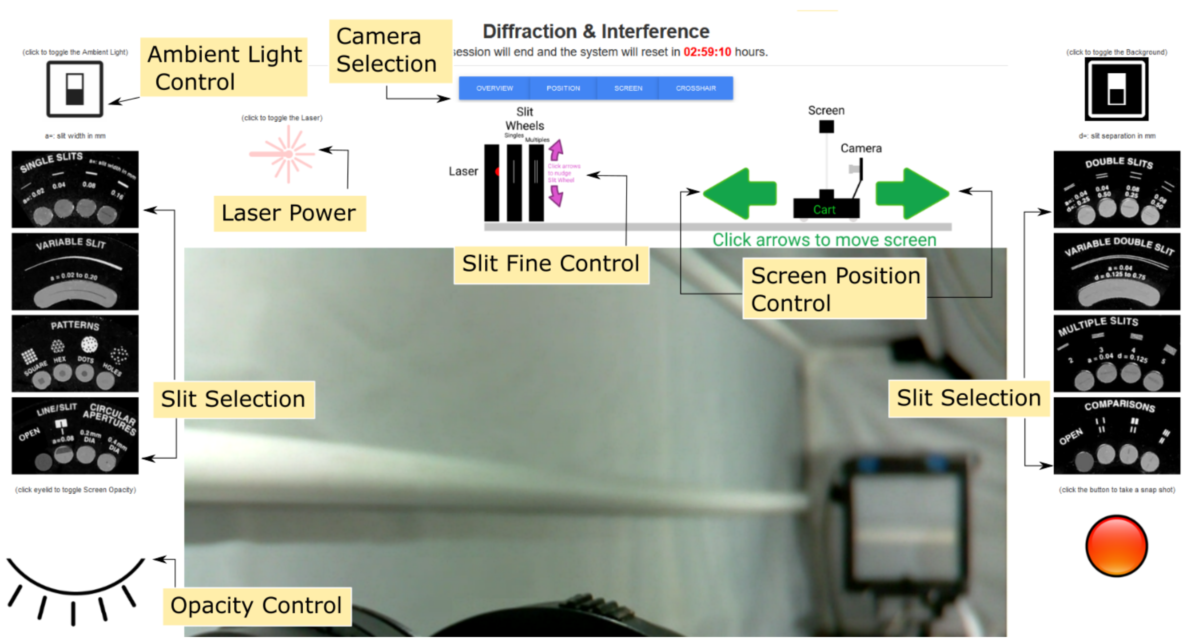

Figure 13: The remote interface includes a variety of controls to turn on the lights, select slit, adjust position of the screen and slit alignment, and switch camera view.

3.1.3 Experimental planning

Measuring positions of maxima and minima: One way to measure the positions of maxima and minima in the interference pattern is to take a photo of the pattern with a ruler placed in the field of view. You can then use image analysis software to measure the spacing of the maxima and minima in units of pixels within the image and scale that by the number of pixels per mm using the ruler.

You can process and analyze the images you take using Fiji, a free software developed and maintained by the National Institutes of Health that works on Macs as well as PCs. (But if you are already familiar with a different image processing software, feel free to use that instead.)

3.2 Getting started and gaining familiarity

The first thing you should do in lab is play around with the remote setup to understand how it works. Rotate the slit wheels, move the screen and look at the different camera views. If you like, take a screenshot of the setup for your logbook. You don't need to immediately insert the photo into your logbook, but it is a good habit to note the time you took it. Alternatively, you could draw a schematic of the setup. Honestly, a schematic is often a lot more useful. Photos capture tons of information that can obscure, or just distract from, the essential. A schematic captures only what you want it to and is easily labeled. Either way, you should note the make and model of the various components in your setup as best you can.

3.3 Qualitative observations of interference and diffraction

Use the Single Slit Accessory, view the diffraction pattern using *at least* three different slit widths. Take screenshots of the patterns for your logbook. Write your name, the slit width and any other variables that you suspect effect the pattern on a post-it note and include that writing in the photograph's field of view. In general, it is a good habit to include such "metadata" directly in your experimental data whenever possible because doing so can resolve ambiguities that might arise later on down the line.

Write a brief qualitative summary of your observations. For example, you might conclude that the separation of the maxima and minima increases as the slit width decreases. Of course, that is what you expect from equation (12), but the point of doing an experiment is to record what you observe in a clear and concise way. Imagine that the theory of the diffraction pattern where not yet understood, and write your conclusions with the intention of defining the experimental facts that a developing theory must explain.

3.4 Measurement of the wavelength of a laser

The observations you just completed were a qualitative way to get familiar with the phenomenon of interference. Now, it's time to make a measurement. Your goal is to measure the laser's wavelength as precisely as possible. You will do this using the the Multiple Slit Accessory and the relations in equations (10) and (11), where you can measure L, y, and d and extract λ.

Start by thinking about the uncertainties. In your logbook, calculate how the uncertainty on λ depends on the uncertainties in the things you will measure. Then make some estimates. What do you think the uncertainties will be in each of the three parameters, i.e., δL, δy, and δd? Which of the three do you expect to dominate the total uncertainty in λ?

Next, you should think about possible systematic errors. While the fractional uncertainty in L is probably going to be smaller than the other uncertainties, it is something that could easily have a systematic bias. For example, while the ruler provides a nice way to measure the position the screen cart, that isn't necessarily the distance from the slit to the screen, due to offsets in the placement of the ruler. The difference between the distance and measured position will be a systematic bias on L that will need to be corrected.

There is, however, a trick to avoid that systematic uncertainty in L: measure y as a function of L and fit the results to a straight line. The slope of that line gives λ (after correcting for d). The systematic uncertainty from the difference between the mount's position and the actual slit position changes the intercept of the line, but not its slope. Measuring the dependence of two observables and extracting a