Benchmark Performance of GTA Test Suite

***Abstract*—Recently, GTA test suite is proposed for assessing dynamic multiobjective evolutionary algorithm. This techincal report aims to provide some benchmark performances of two well-known multiobjective evolutionary algorithms, which are NSGA-II and MOEA/D, on solving the GTA benchmark problems. Four performance metrics are used to evaluate the performances of algorithms, namely inverted generational distance (IGD), generational distance (GD), expected value of β ($\mathbb{E}[\beta, t]$) and inverted generational distance of α (IGD$_\alpha$).**

This technical report provides the benchmark performance of the GTA dynamic multiobjective optimization test suite. The formulation of the benchmark problems can be found in Table I. In this report, Non-dominated Sorting Genetic Algorithm (NSGA-II) and Decomposition-based Multiobjective Evolutionary Algorithm (MOEA/D) are used to optimize the proposed benchmark problems due to the effectiveness and popularity of these two algorithms. Random re-initialization strategy with 20% of random solutions is used after change in fitness landscape is detected. Parameter settings of these two algorithms are shown in Table II.

To evaluate the performance of a given dynamic multiobjective evolutionary algorithm, four types of performance metrics are used, namely inverted generational distance (IGD), generational distance (GD), expected value of β ($\mathbb{E}[\beta, t]$), and inverted generational distance of α (IGD$_\alpha$). For two-objective benchmark problems, 1000 points are uniformly sampled in the interval of $[0, 1]$ for generating the x_1 values. To obtain the Pareto optimal solutions, the procedure is shown as follows:

1) Uniformly sample point x in the interval $[0, 1]$, calculate $\alpha_{A,2}(x, t)$ for each point and form $(x, \alpha_A(x, t))$ pair.
2) For all the pairs generated from Step 1, perform non-dominated filtering to remove dominated pairs from the set. The filtered set P is the Pareto optimal front (POF) of the problem at time instance t.
3) For all the x in the set P, form the decision vector $[x, g_A(x, t), \ldots, g_A(x, t)] \in \Omega$ for each x. The resulting vector set is the Pareto optimal set (POS) of the problem at time instance t.

For GTA7m and GTA8m, their $\alpha_{M,j}$ functions (counterpart of $\alpha_{A,j}$ in multiplicative form) may become zero at certain time instants. When $\alpha_{M,j}$ becomes zero, POS of these problems are $[0, x_2, x_3, \ldots]$ where $x_i \in [a_i, b_i]$. For three-objective problem, similar procedure are used as follows:

1) Uniformly sample point x_1 and x_2 in the interval $[0, 1]$. Perform cartesian product on x_1 and x_2 sets to form $x_1 - x_2$ pairs. Compute α_A for each $x_1 - x_2$ pair and form $((x_1, x_2), \alpha_A)$ pairs.
2) Peform non-dominated filtering on α_A to remove dominated solutions. The set of α_A from the remaining solutions form the POF of the problem.
3) For each of the remaining solutions, form the decision vector $[x_1, x_2, g_A(x, t), \ldots, g_A(x, t)]$ set which is the POS of the problem at time instance t.

By following the above-mentioned procedures, user should be able to generate the POFs. To ease the process of performance assessment, we have provided the Python script to generate the POFs in the Github repository.

TABLE II: Parameter Settings for Experiments

Parameters	**Values**
Population size, N	100
Total number of generations	500, 1000, 2000 (for different τ_T)
Total number of fitness evaluations	5×10^4, 1.0×10^5, 2.0×10^5
Neighbourhood size, T	20
Recombination operator	DE
Scaling factor, F	0.5
Crossover rate, C_r	1.0
Distribution index in mutation, η_m	20
Mutation rate, p_m	$1/n$ (n: num. of decision variables)

I. Cross-problem Comparison

Fig. 1 and Fig. 2 present the performance of NSGA-II and MOEA/D over GTA1-GTA3 test pairs and with different τ_T. The purpose of the cross-problem comparison is to show the sensitivity of the algorithm to different type of fitness modality. From the performance metric trend charts, it can be observed that the two algorithms are more sensitive to mixed multi-modal problem as the performances of GTA3 are more erratic than GTA1 and GTA2. Similar observation can be found in the comparison of GD and IGD performance of GTA5 and GTA6 (as shown in Fig. 3 and Fig. 4). Mixed modality introduced in GTA6 has more negative effects on algorithm optimization performance than multi-modality introduced in GTA5.

II. Convergence & Diversity Comparison

The $\mathbb{E}[\beta, t]$ and IGD$_\alpha$ are test-problem-specific performance metrics which are used to assess the diversity and convergence performances of a given algorithm. The $\mathbb{E}[\beta, t]$ can be calculated by using following formulas:

$$\mathbb{E}[\beta, t] = \frac{1}{m|P|} \sum_{i=1}^{m} \sum_{\{\mathbf{x}_\mathrm{I}, \mathbf{x}_\mathrm{II}\} \in P} \beta_{A,i}(\mathbf{x}_\mathrm{II} - g_A(\mathbf{x}_\mathrm{I}, t), t) \quad (1)$$

for the additive form test problem and

$$\mathbb{E}[\beta, t] = \frac{1}{m|P|} \sum_{i=1}^{m} \sum_{\{\mathbf{x}_\mathrm{I}, \mathbf{x}_\mathrm{II}\} \in P} [\beta_{M,i}(\mathbf{x}_\mathrm{II} - g_M(\mathbf{x}_\mathrm{I}, t), t) - 1] \quad (2)$$

for the multiplicative test problem. For the IGD$_\alpha$, following formula is used:

$$\mathrm{IGD}_\alpha(P^*, A) = \frac{\sum_{v \in P^*} d(v, A)}{|P^*|} \quad (3)$$

where P^* denotes the POF; A represents the set of α_A (additive form) or α_M (multiplicative form); and d is the

TABLE I: Dynamic Multiobjective Test Problems

Type / Function	Setting	Type / Function	Setting
Search Space $\mathbf{x}$	$\mathbf{x} = (\mathbf{x}_{\mathrm{I}}, \mathbf{x}_{\mathrm{II}})$, $\mathbf{x}_{\mathrm{I}} = (x_1, \ldots, x_{m-1}) \in [0,1]$, $\mathbf{x}_{\mathrm{II}} = (x_m, x_{m+1}, \ldots, x_n) \in [-1,1]$	Variable Subset $\mathbf{x}_{\mathrm{II},i}$	$\mathbf{x}_{\mathrm{II},i} = (x_{i+am}, x_{i+(a+1)m}, \ldots, x_{i+bm}) \subset \mathbf{x}_{\mathrm{II}}$, where a is the smallest non-negative value which satisfies $i + am > 1$, b is a value which satisfies $i + bm \leq n < i + (b+1)m$ and $i \in 1, \ldots, m$.
$\alpha_{\text{conv}}(\mathbf{x}_{\mathrm{I}}, t)$	$\alpha_1 = x_1$, $\alpha_2 = 1 - \sqrt{x_1}$	$\alpha_{\text{disc}}(\mathbf{x}_{\mathrm{I}}, t)$	$\alpha_1 = x_1$, $\alpha_2 = 1.5 - \sqrt{x_1} - 0.5 \sin 10\pi x_1$
$\alpha_{\text{mix}}(\mathbf{x}_{\mathrm{I}}, t)$	$\alpha_1 = x_1$, $\alpha_2 = 1 - \sqrt{x_1} + 0.1k(t)(1 + \sin 10\pi x_1)$	$\alpha_{\text{conf,2}}(\mathbf{x}_{\mathrm{I}}, t)$	$\alpha_1 = x_1$, $\alpha_2 = 1 - x_1^{p(t)}$
$\alpha_{\text{conf,3a}}(\mathbf{x}_{\mathrm{I}}, t)$	$\alpha_1 = \cos(0.5x_1\pi)\cos(0.5x_2\pi)$, $\alpha_2 = \cos(0.5x_1\pi)\sin(0.5x_2\pi)$, $\alpha_3 = \lvert \sin(0.5\pi x_1 + \frac{\pi k(t)}{4k_{\max}}) \rvert$	$\alpha_{\text{conf,3b}}(\mathbf{x}_{\mathrm{I}}, t)$	$\alpha_1 = \cos 0.5\pi x_1 \cos 0.5\pi x_2 k_r$, $\alpha_2 = \cos 0.5\pi x_1 \sin 0.5\pi x_2 k_r$, $\alpha_3 = \sin(0.5\pi x_1)$, where $k_r = (k_{\max} - k)/k_{\max}$
$\beta_{\text{uni}}(\mathbf{x}_{\mathrm{II}}, t)$	$\beta_i = \frac{2}{\lvert \mathbf{x}_{\mathrm{II},i} \rvert} \sum_{x_j \in \mathbf{x}_{\mathrm{II},i}} (x_j - \sin 0.5\pi(t - x_1))^2$	$\beta_{\text{multi}}(\mathbf{x}_{\mathrm{II}}, t)$	$\beta_i = \frac{2}{\lvert \mathbf{x}_{\mathrm{II},i} \rvert} \sum_{x_j \in \mathbf{x}_{\mathrm{II},i}} h^2(x_j, t)[1 + \lvert \sin 4\pi h(x_j, t) \rvert]$
$\beta_{\text{mix}}(\mathbf{x}_{\mathrm{II}}, t)$	$\beta_i = \frac{2}{\lvert \mathbf{x}_{\mathrm{II},i} \rvert} \sum_{x_j \in \mathbf{x}_{\mathrm{II},i}} [1 + h^2(x_j, t) + \cos 2\pi k(t) h(x_j, t)]$	$k(t)$	$k(t) = \lfloor \lvert 5(\lfloor t/5 \rfloor \bmod 2) - t \bmod 5 \rvert \rfloor$
$h(x, t)$	$h(x, t) = x - \sin 0.5\pi(t - x)$	$p(t)$	$p(t) = \log(1 - 0.1k(t))/\log(0.1k(t) + \epsilon)$
GTA1a	$\mathbf{f}(\mathbf{x}, t) = \alpha_{\text{conv}}(\mathbf{x}_{\mathrm{I}}, t) + \beta_{\text{uni}}(\mathbf{x}_{\mathrm{II}}, t)$	GTA1m	$\mathbf{f}(\mathbf{x}, t) = \alpha_{\text{conv}}(\mathbf{x}_{\mathrm{I}}, t) \times (\mathbf{1} + \beta_{\text{uni}}(\mathbf{x}_{\mathrm{II}}, t))$
GTA2a	$\mathbf{f}(\mathbf{x}, t) = \alpha_{\text{conv}}(\mathbf{x}_{\mathrm{I}}, t) + \beta_{\text{multi}}(\mathbf{x}_{\mathrm{II}}, t)$	GTA2m	$\mathbf{f}(\mathbf{x}, t) = \alpha_{\text{conv}}(\mathbf{x}_{\mathrm{I}}, t) \times (\mathbf{1} + \beta_{\text{multi}}(\mathbf{x}_{\mathrm{II}}, t))$
GTA3a	$\mathbf{f}(\mathbf{x}, t) = \alpha_{\text{conv}}(\mathbf{x}_{\mathrm{I}}, t) + \beta_{\text{mix}}(\mathbf{x}_{\mathrm{II}}, t)$	GTA3m	$\mathbf{f}(\mathbf{x}, t) = \alpha_{\text{conv}}(\mathbf{x}_{\mathrm{I}}, t) \times (\mathbf{1} + \beta_{\text{mix}}(\mathbf{x}_{\mathrm{II}}, t))$
GTA4a	$\mathbf{f}(\mathbf{x}, t) = \alpha_{\text{disc}}(\mathbf{x}_{\mathrm{I}}, t) + \beta_{\text{mix}}(\mathbf{x}_{\mathrm{II}}, t)$	GTA4m	$\mathbf{f}(\mathbf{x}, t) = \alpha_{\text{disc}}(\mathbf{x}_{\mathrm{I}}, t) \times (\mathbf{1} + \beta_{\text{mix}}(\mathbf{x}_{\mathrm{II}}, t))$
GTA5a	$\mathbf{f}(\mathbf{x}, t) = \alpha_{\text{mix}}(\mathbf{x}_{\mathrm{I}}, t) + \beta_{\text{multi}}(\mathbf{x}_{\mathrm{II}}, t)$	GTA5m	$\mathbf{f}(\mathbf{x}, t) = \alpha_{\text{mix}}(\mathbf{x}_{\mathrm{I}}, t) \times (\mathbf{1} + \beta_{\text{multi}}(\mathbf{x}_{\mathrm{II}}, t))$
GTA6a	$\mathbf{f}(\mathbf{x}, t) = \alpha_{\text{mix}}(\mathbf{x}_{\mathrm{I}}, t) + \beta_{\text{mix}}(\mathbf{x}_{\mathrm{II}}, t)$	GTA6m	$\mathbf{f}(\mathbf{x}, t) = \alpha_{\text{mix}}(\mathbf{x}_{\mathrm{I}}, t) \times (\mathbf{1} + \beta_{\text{mix}}(\mathbf{x}_{\mathrm{II}}, t))$
GTA7a	$\mathbf{f}(\mathbf{x}, t) = \alpha_{\text{conf,2}}(\mathbf{x}_{\mathrm{I}}, t) + \beta_{\text{multi}}(\mathbf{x}_{\mathrm{II}}, t)$	GTA7m	$\mathbf{f}(\mathbf{x}, t) = \alpha_{\text{conf,2}}(\mathbf{x}_{\mathrm{I}}, t) \times (\mathbf{1} + \beta_{\text{multi}}(\mathbf{x}_{\mathrm{II}}, t))$
GTA8a	$\mathbf{f}(\mathbf{x}, t) = \alpha_{\text{conf,2}}(\mathbf{x}_{\mathrm{I}}, t) + \beta_{\text{mix}}(\mathbf{x}_{\mathrm{II}}, t)$	GTA8m	$\mathbf{f}(\mathbf{x}, t) = \alpha_{\text{conf,2}}(\mathbf{x}_{\mathrm{I}}, t) \times (\mathbf{1} + \beta_{\text{mix}}(\mathbf{x}_{\mathrm{II}}, t))$
GTA9a	$\mathbf{f}(\mathbf{x}, t) = \alpha_{\text{conf,3a}}(\mathbf{x}_{\mathrm{I}}, t) + \beta_{\text{multi}}(\mathbf{x}_{\mathrm{II}}, t)$	GTA9m	$\mathbf{f}(\mathbf{x}, t) = \alpha_{\text{conf,3a}}(\mathbf{x}_{\mathrm{I}}, t) \times (\mathbf{1} + \beta_{\text{multi}}(\mathbf{x}_{\mathrm{II}}, t))$
GTA10a	$\mathbf{f}(\mathbf{x}, t) = \alpha_{\text{conf,3a}}(\mathbf{x}_{\mathrm{I}}, t) + \beta_{\text{mix}}(\mathbf{x}_{\mathrm{II}}, t)$	GTA10m	$\mathbf{f}(\mathbf{x}, t) = \alpha_{\text{conf,3a}}(\mathbf{x}_{\mathrm{I}}, t) \times (\mathbf{1} + \beta_{\text{mix}}(\mathbf{x}_{\mathrm{II}}, t))$
GTA11a	$\mathbf{f}(\mathbf{x}, t) = \alpha_{\text{conf,3b}}(\mathbf{x}_{\mathrm{I}}, t) + \beta_{\text{multi}}(\mathbf{x}_{\mathrm{II}}, t)$	GTA11m	$\mathbf{f}(\mathbf{x}, t) = \alpha_{\text{conf,3b}}(\mathbf{x}_{\mathrm{I}}, t) \times (\mathbf{1} + \beta_{\text{multi}}(\mathbf{x}_{\mathrm{II}}, t))$
GTA12a	$\mathbf{f}(\mathbf{x}, t) = \alpha_{\text{conf,3b}}(\mathbf{x}_{\mathrm{I}}, t) + \beta_{\text{mix}}(\mathbf{x}_{\mathrm{II}}, t)$	GTA12m	$\mathbf{f}(\mathbf{x}, t) = \alpha_{\text{conf,3b}}(\mathbf{x}_{\mathrm{I}}, t) \times (\mathbf{1} + \beta_{\text{mix}}(\mathbf{x}_{\mathrm{II}}, t))$

The GTA functions are written in vector form. Operator '$\times$' and $\mathbf{1}$ denote element-wise multiplication operator and vector of ones, respectively.

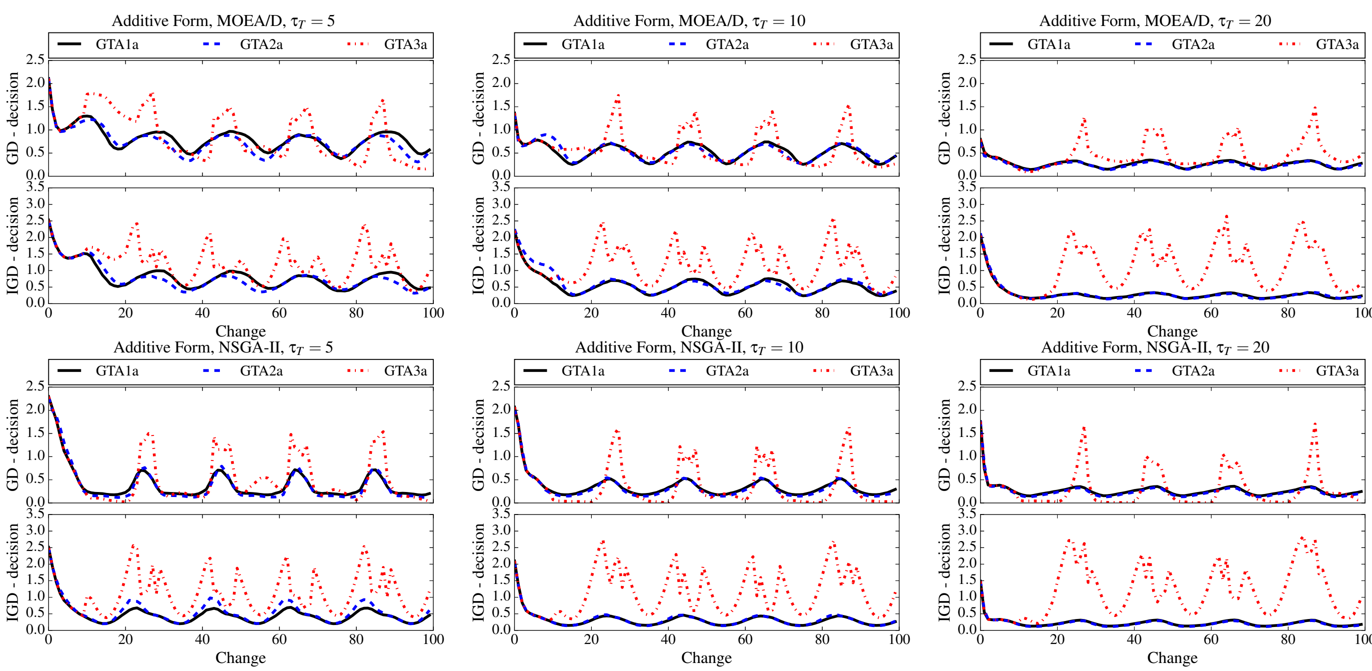

Fig. 1: These plots show the cross-problem comparative study for GTA1–GTA3 in additive form. The GD and IGD values are calculated based on optimization performance of a given algorithm in decision space.

function to calculate the minimum distance between v and the points in A. IGD$_\alpha$ has the same properties as normal IGD value but it is only used to represent the diversity performance of the given algorithm. Lower value of IGD$_\alpha$ is desirable as it denotes higher similarity between the POS approximation set and the α values of the algorithm's output. Fig. 5 shows

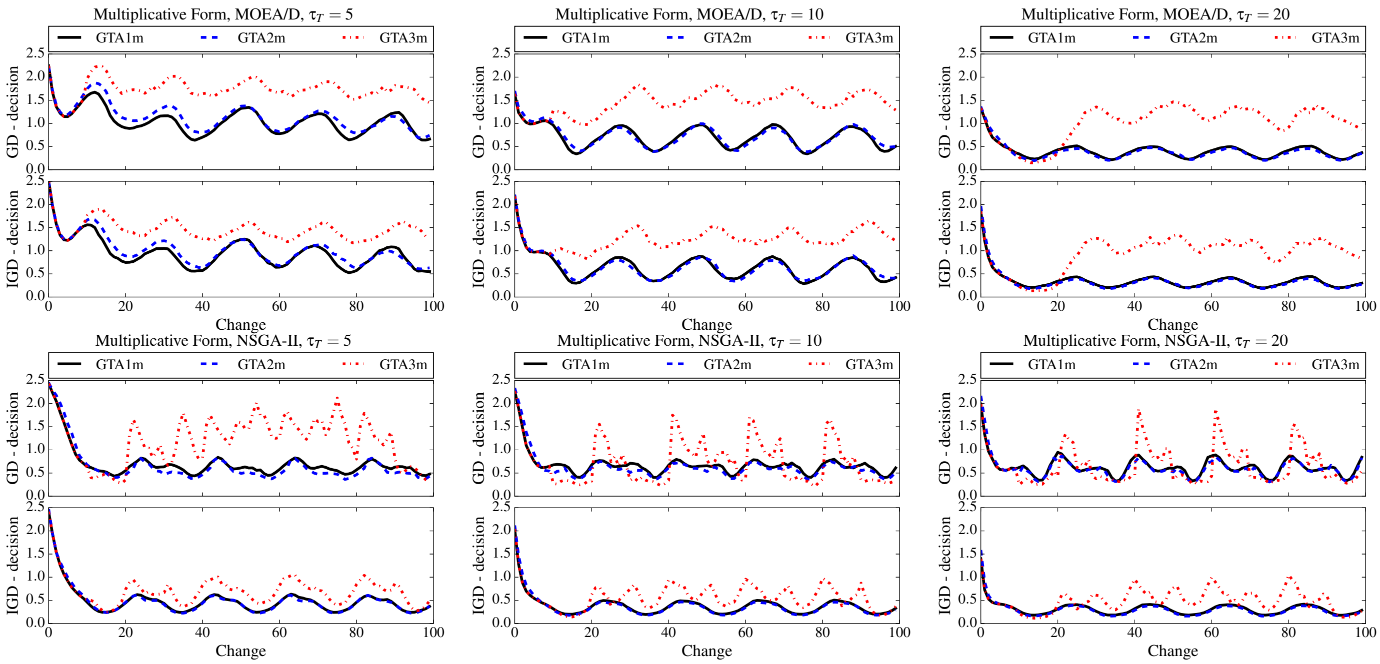

Fig. 2: These plots show the cross-problem comparative study for GTA1–GTA3 in multiplicative form. The GD and IGD values are calculated based on optimization performance of a given algorithm in decision space.

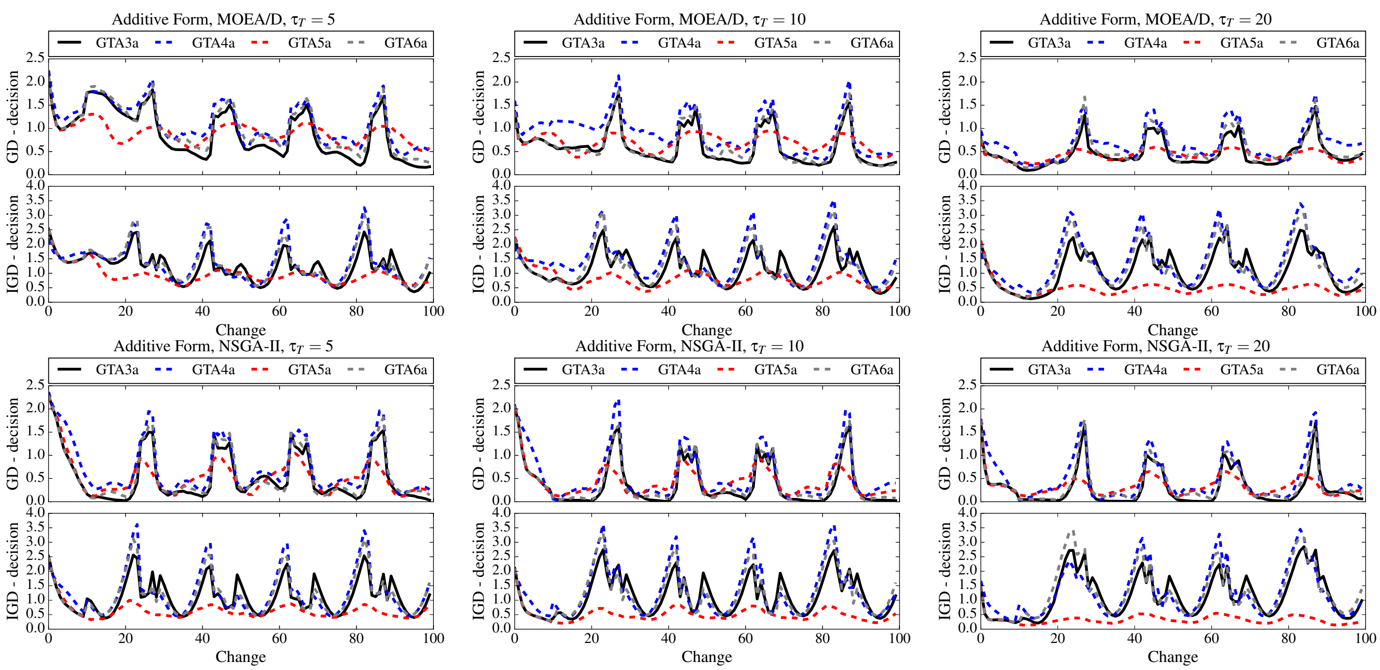

Fig. 3: These plots show the cross-problem comparative study for GTA3-GTA6 in additive form. The GD and IGD values are calculated based on optimization performance of a given algorithm in decision space.

the performance of these two metrics on GTA1-GTA3 for $\tau_T = 5$. Each plot in the figure consists of IGD, IGD$_\alpha$ and $\mathbb{E}[\beta]$ performance over the change of fitness landscape. For GTA1 and GTA2 test pairs, it can be observed that NSGA-II generally has lower IGD value over number of change than MOEA/D when τ is less than 20. Inspecting the performance of $\mathbb{E}[\beta]$ of these two test pairs, it can be revealed that NSGA-II performs superior to MOEA/D on $\mathbb{E}[\beta]$ value. This implies that NSGA-II converge faster on these two test pairs when τ_T is less than 20. Another observation from the figure is that the diversity performance of MOEA/D on GTA1m and GTA2m improves significantly when τ_T increases from 5 to 20. Although the performance analysis of IGD$_\alpha$ and $\mathbb{E}[\beta]$ is useful to identify the contributions of diversity and convergence information to the IGD value, it may not provide significantly helpful information when the performance trends is erratic.

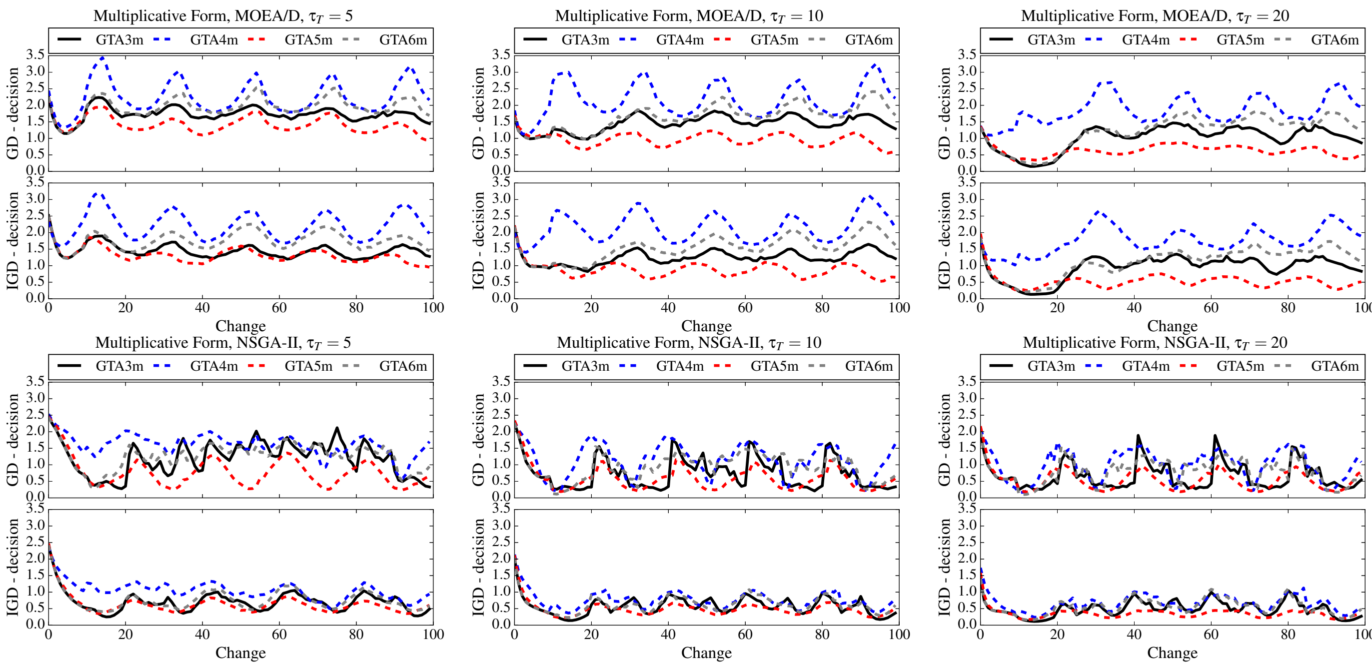

Fig. 4: These plots show the cross-problem comparative study for GTA3-GTA6 in multiplicative form. The GD and IGD values are calculated based on optimization performance of a given algorithm in decision pace.

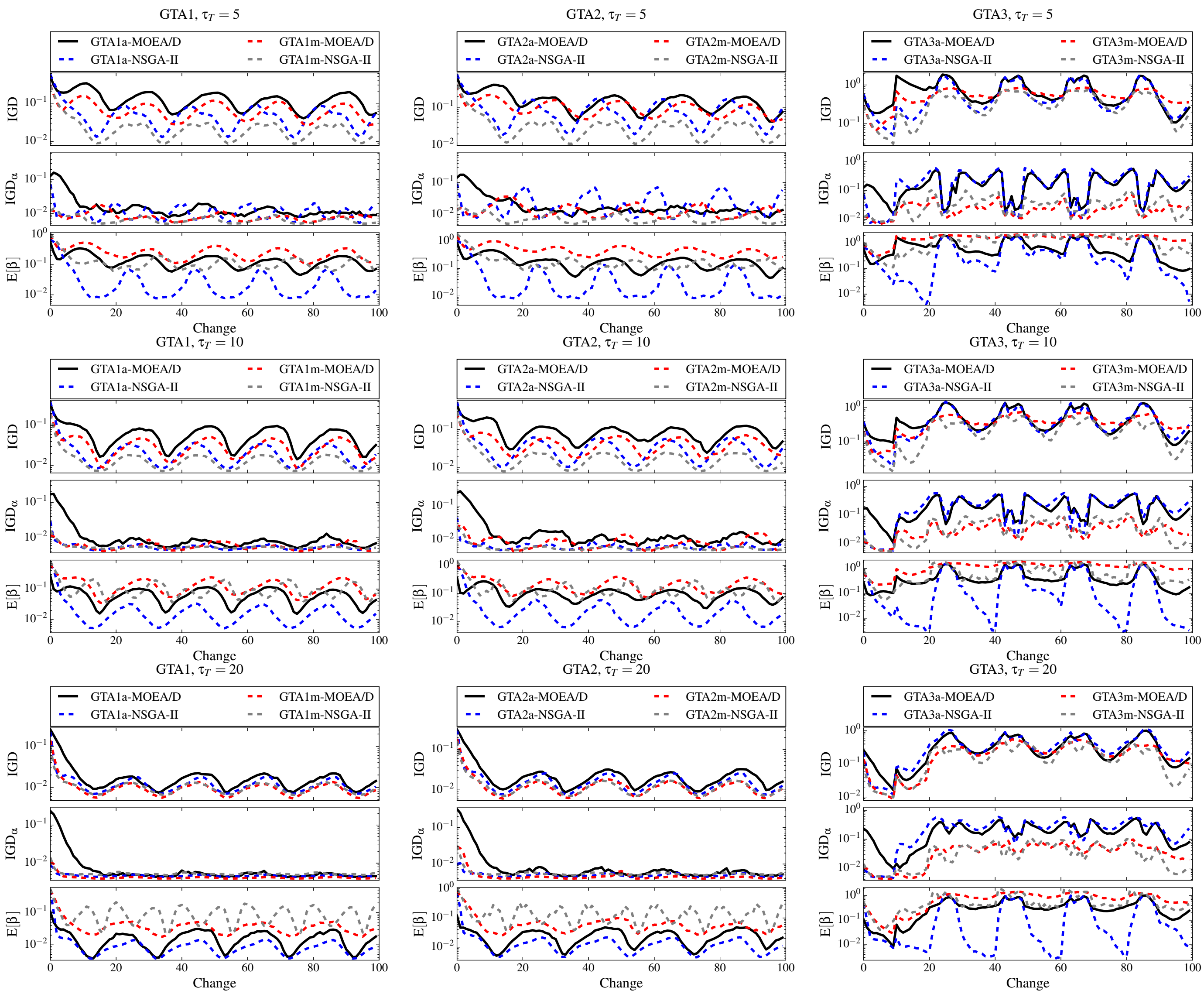

Fig. 5: These plots show the IGD, IGD_α and β performance of GTA1–GTA3 test pairs for τ_T= 5.