University of Massachusetts Amherst

Digital Image Processing ECE 566

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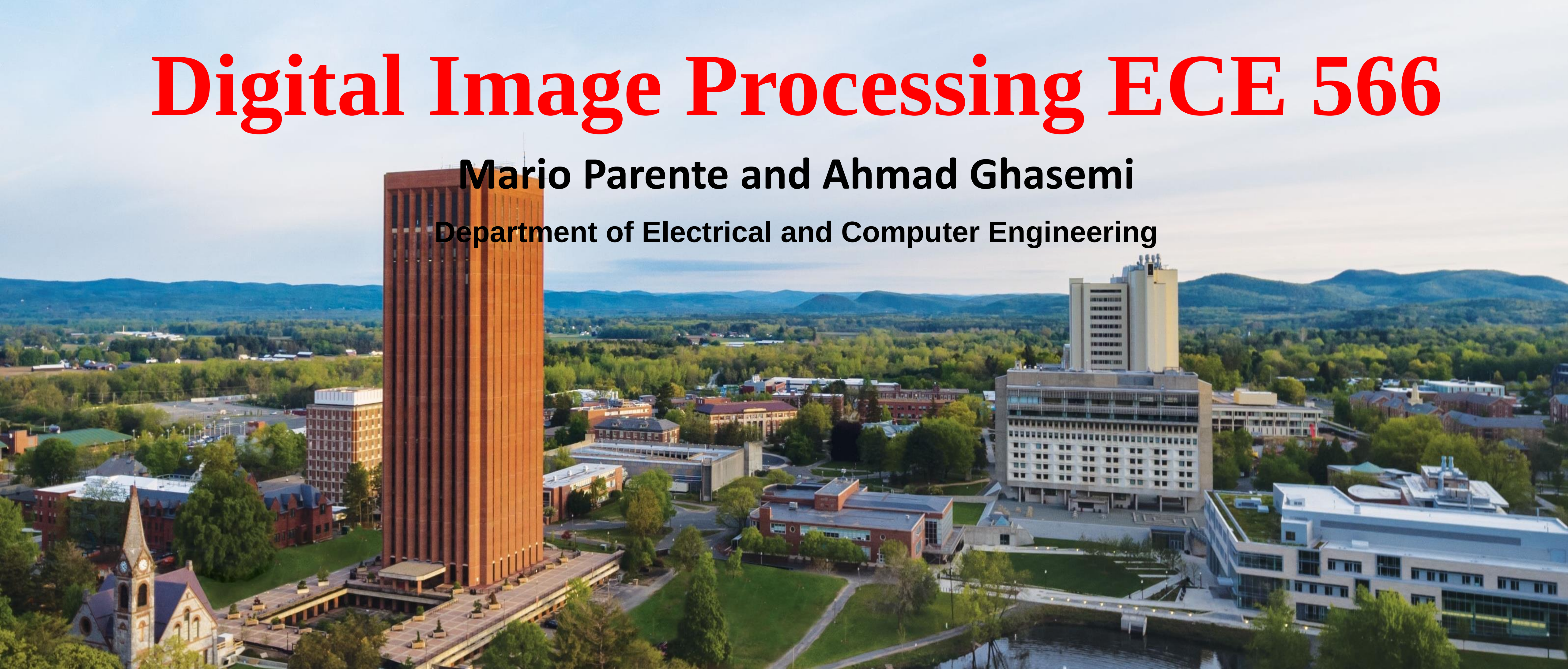
Department of Electrical and Computer Engineering



Spatial Image Transformations

We talked about image transformation.

Question:

How to find the ideal affine transformation between given pair of images?

A

B

Composite Affine Transformations

Suppose a composite transformation:

First translation T, next scaling S, and then rotation R.

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$$\mathbf{H} = \boldsymbol{RST} = \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} s_x & 0 & 0 \\ 0 & s_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} s_x\cos\alpha & s_y\sin\alpha & s_x t_x\cos\alpha + s_y t_y\sin\alpha \\ -s_x\sin\alpha & s_y\cos\alpha & s_y t_y\cos\alpha - s_x t_x\sin\alpha \\ 0 & 0 & 1 \end{bmatrix} \qquad (1)$$

Given $\mathbf{H}$, five parameters should be determined: α, s_x, s_y, t_x, and t_y.

Composite Affine Transformations

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First translation T, next scaling S, and then rotation R.

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Given $\mathbf{H}$, five parameters should be determined: α, s_x, s_y, t_x, and t_y.

One possible approach is **Point Matching.**

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Point Matching

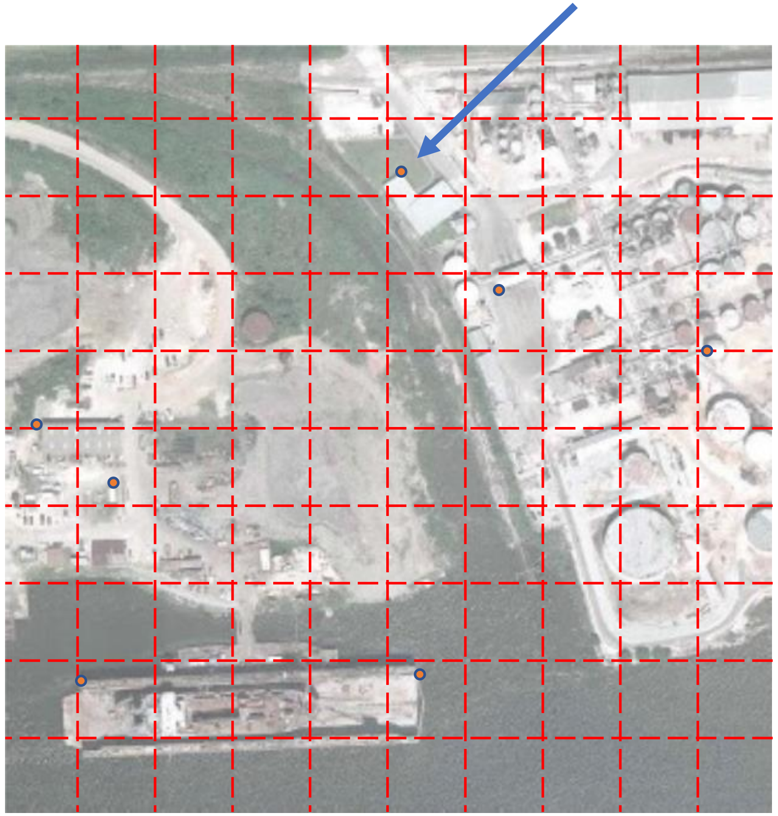
Step 1: Randomly select few matching points in both images.

Step 2: Determine matrix $\mathbf{H}$ that transform the selected points in image A to the points in image B.

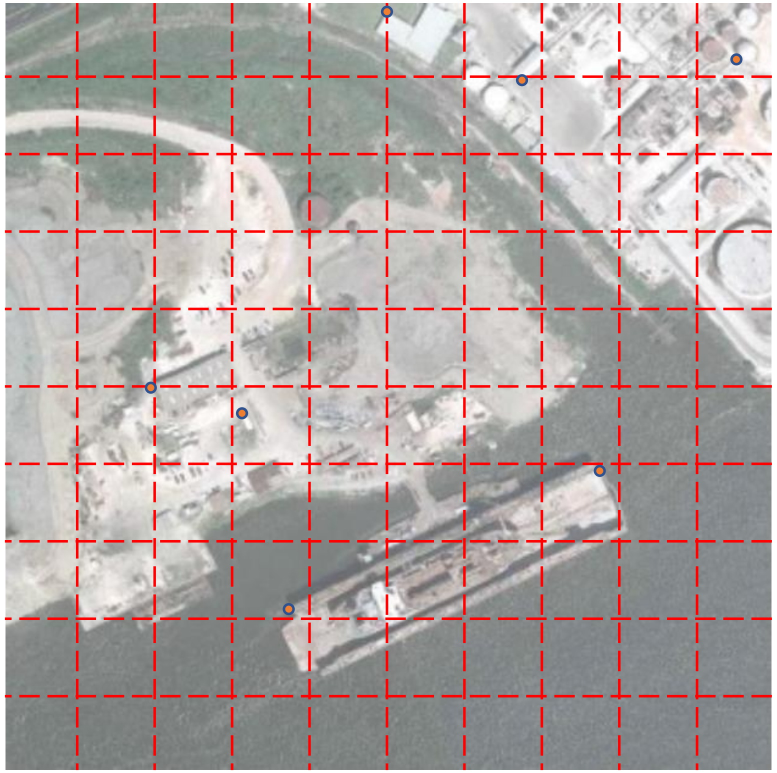
Step 3: Determine parameters α, s_x, s_y, t_x, and t_y using determined $\mathbf{H}$ in Step 2 and equation (1) in previous slide.

Point Matching (Cont.)

Step 1: Randomly select few matching points in both images.



A



B

Point Matching (Cont.)

Step 2: Determine matrix $\mathbf{H}$ that transform the selected points in image A to the points in image B.

Suppose N selected points in images A and B are $\{\mathbf{p}_0, \mathbf{p}_1, \dots, \mathbf{p}_{N-1}\}$ and $\{\mathbf{q}_0, \mathbf{q}_1, \dots, \mathbf{q}_{N-1}\}$, successfully. Use the homogeneous coordinate representation of these points as columns of matrices $\mathbf{P}$ and $\mathbf{Q}$.

$$\mathbf{P} = \begin{bmatrix} x_0 & x_1 & \cdots & x_{N-1} \\ y_0 & y_1 & \cdots & y_{N-1} \\ 1 & 1 & \cdots & 1 \end{bmatrix} = [\mathbf{p}_0 \quad \mathbf{p}_1 \quad \cdots \quad \mathbf{p}_{N-1}]$$

$$\mathbf{Q} = \begin{bmatrix} u_0 & u_1 & \cdots & u_{N-1} \\ v_0 & v_1 & \cdots & v_{N-1} \\ 1 & 1 & \cdots & 1 \end{bmatrix} = [\mathbf{q}_0 \quad \mathbf{q}_1 \quad \cdots \quad \mathbf{q}_{N-1}]$$

$$\mathbf{Q} = \mathbf{HP} \quad \rightarrow \quad \mathbf{H} = \mathbf{Q}\underbrace{\mathbf{P}^{\mathrm{T}}(\mathbf{P}\mathbf{P}^{\mathrm{T}})^{-1}}_{\text{Right pseudo-inverse of } \mathbf{P}}$$

Point Matching (Cont.)

The first two columns of the table below represent the coordinates of matching points in image A and the next two the point coordinates of image B. The corresponding matrix $\mathbf{H}$ is

$$\mathbf{H} = \mathbf{QP}^{\mathrm{T}}(\mathbf{PP}^{\mathrm{T}})^{-\mathbf{1}} = \begin{bmatrix} 0.92 & -0.39 & 224.17 \\ 0.39 & 0.92 & 10.93 \\ 0 & 0 & 1 \end{bmatrix}$$

Given this matrix $\mathbf{H}$, the approximation of the selected point in image A are shown in the last two columns. These estimated points are very close to the actual points of image B in columns 3 and 4, i.e., small estimation error.

Matching Points					
X_a	Y_a	X_b	Y_b	X'_a	Y'_a
30.5	325.3	125.8	322.5	126	322.8
86.8	271.3	199.3	295.5	198.7	294.9
330.3	534	320	632	320.5	632.2
62	110.3	238	137	238.4	136.8
342	115	494	250	493.9	250.4
412	437	434.3	574.8	433.3	574.7
584.5	384.8	611.8	594	612.2	593.8

Point Matching (Cont.)

Step 3: Determine parameters α, s_x, s_y, t_x, and t_y using determined $\mathbf{H}$ in Step 2 and equation (1) in previous slide.

$$\mathbf{H} = \boldsymbol{RST} = \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix}$$

$$= \begin{bmatrix} s_x \cos\alpha & s_y \sin\alpha & s_x t_x \cos\alpha + s_y t_y \sin\alpha \\ -s_x \sin\alpha & s_y \cos\alpha & s_y t_y \cos\alpha - s_x t_x \sin\alpha \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0.92 & -0.39 & 224.17 \\ 0.39 & 0.92 & 10.93 \\ 0 & 0 & 1 \end{bmatrix}$$

$$s_x = \sqrt{h_{11}^2 + h_{21}^2} = 0.999$$

$$s_x = \sqrt{h_{11}^2 + h_{21}^2} = 1.001$$

$$\alpha = -\arctan(h_{21}, h_{11}) = -23°$$

$$t_x = \frac{(h_{13}\cos\alpha - h_{23}\sin\alpha)}{s_x}$$

$$t_y = \frac{(h_{13}\sin\alpha + h_{23}\cos\alpha)}{s_y}$$

Point Matching (Cont.)

- ✓ The dark area in "Transformed A" is the region of B that is not contained in image A.
- ✓ The gray image values of "Transformed A" were computed by an **interpolation** of the gray values of image A.

Transformed A

B

Point Matching (Cont.)

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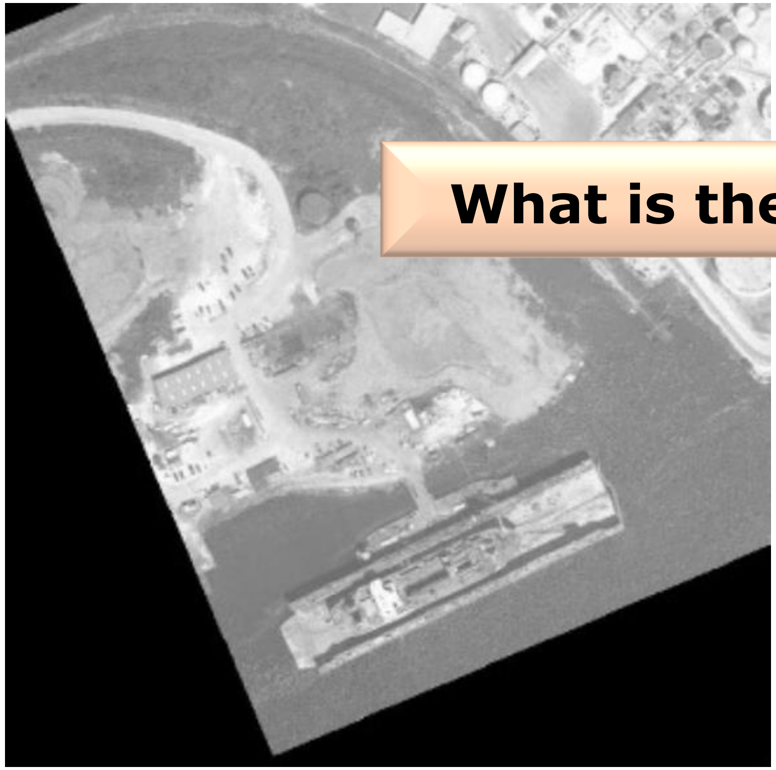
- ✓ The gray image values of "Transformed A" were computed by an **interpolation** of the gray values of image A.

What is the image interpolation?



Transformed A

B

Interpolation

- ✓ Image interpolation refers to the “guess” or “approximating” of intensity values at missing points/locations given the value of intensity in points around (neighboring) those points.
- ✓ Note that it is just a guess (Note that all sensors have finite sampling distance).

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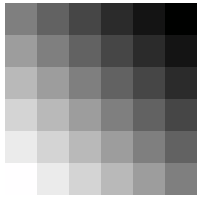
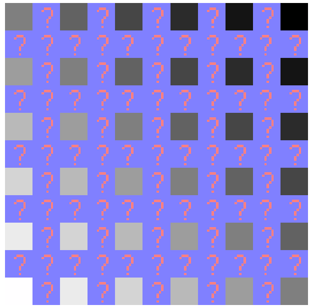
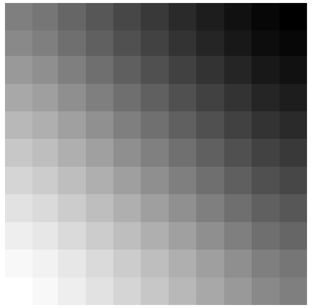


Original

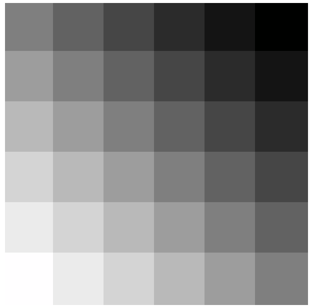
Enlarge



Before interpolation



After interpolation



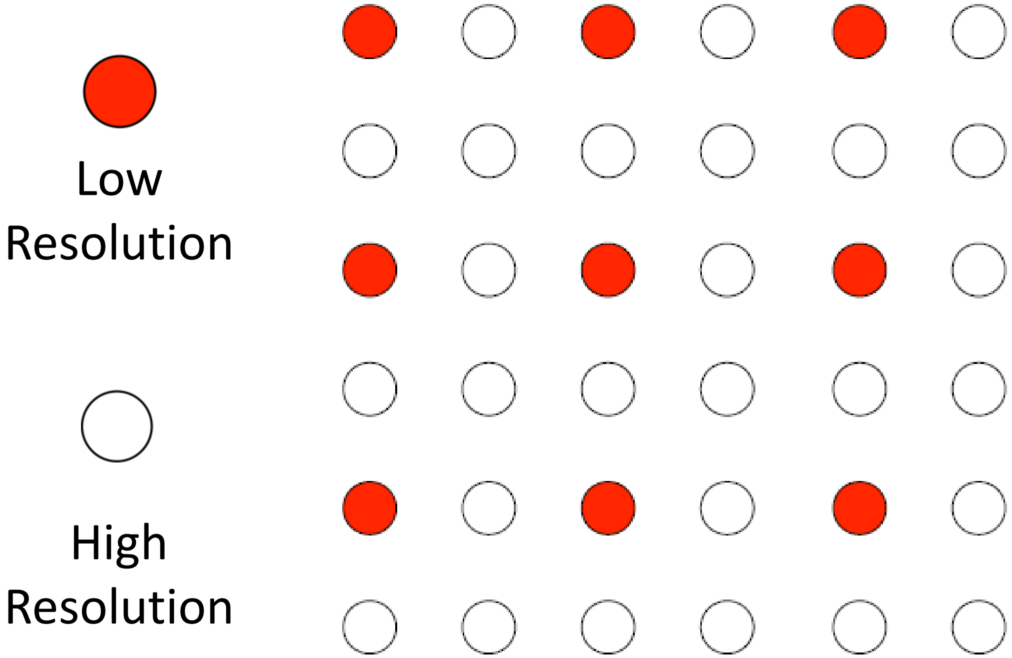
No interpolation

Engineering Motivations

Resolution Enhancement

- ✓ We want BIG images

  When we see a video clip on a PC, we like to see it in the full screen mode



Engineering Motivations

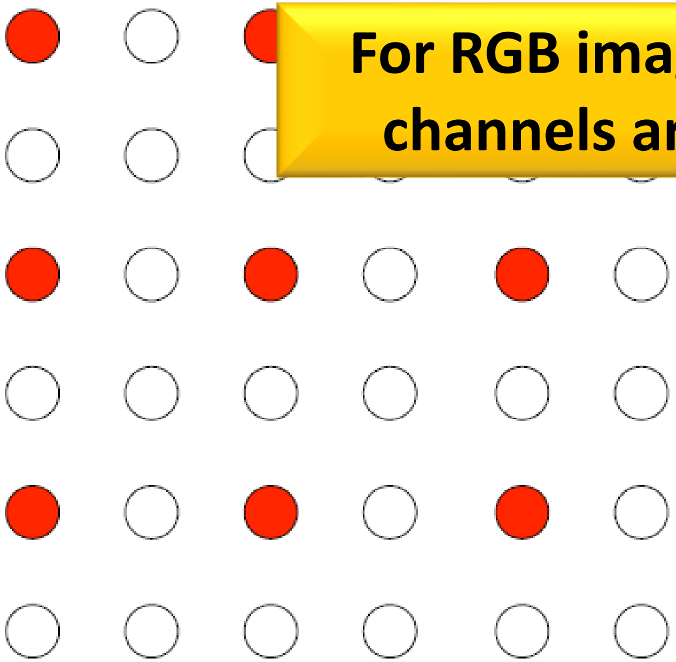
Resolution Enhancement

✓ We want BIG images

When we see a video clip on a PC, we like to see it in the full screen mode

Low Resolution

High Resolution



For RGB image, the Red, Green and Blue channels are interpolated separately.

Engineering Motivations (Cont.)

✓ We want GOOD images

If some block of an image gets damaged during the transmission, we want to repair it.

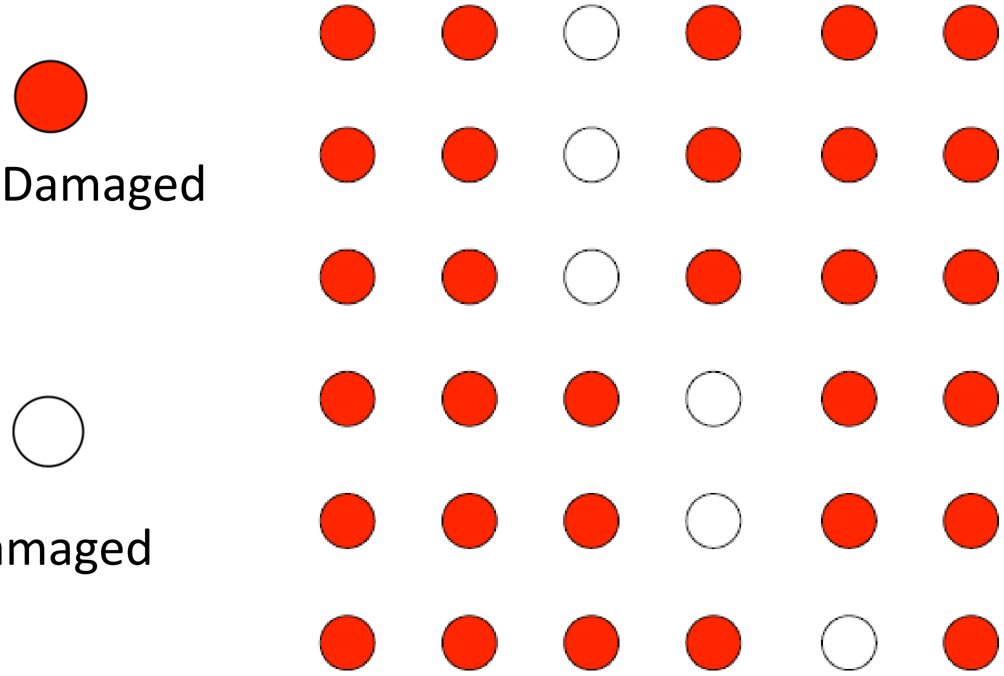
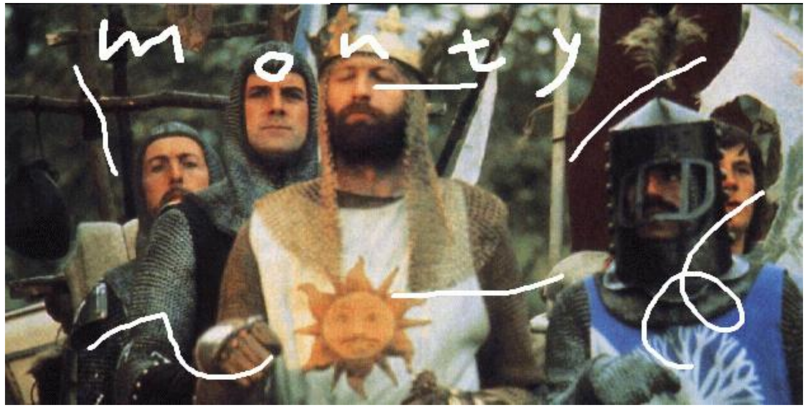


Image Inpainting



Engineering Motivations (Cont.)

✓ We want COOL images

Manipulate images digitally can render fancy artistic effects as we often see in movies.

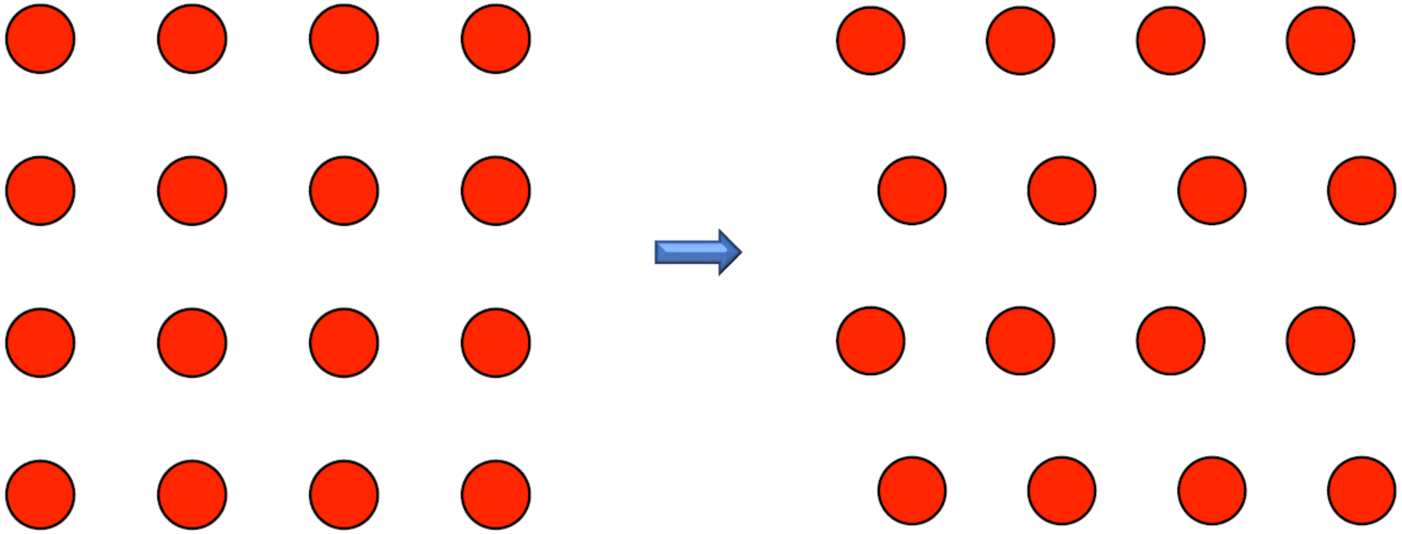


Image Warping

Interpolation

Three common 2D interpolation:

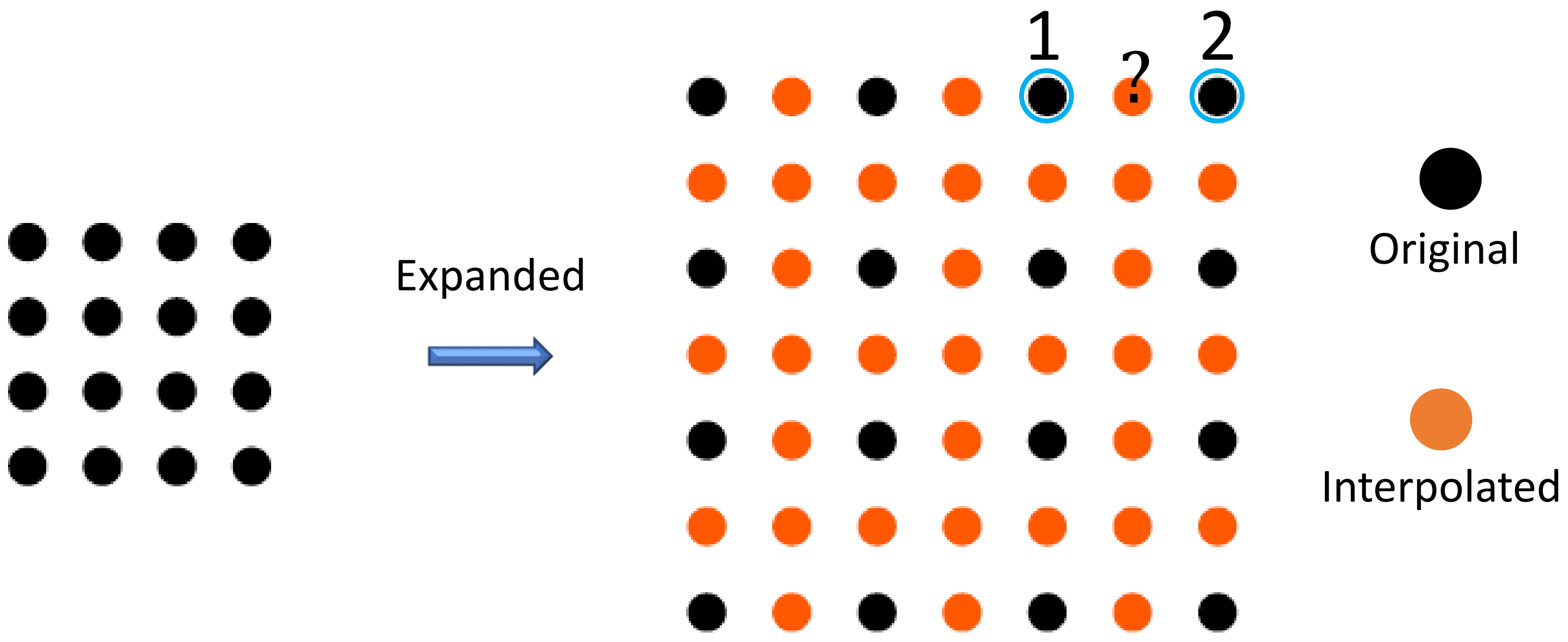
Original image:  X 8

Nearest--neighbor interpolation

Bilinear interpolation

Bicubic interpolation

Interpolation: Nearest neighbor interpolation



$$\textcolor{red}{f(x,y)} = \underbrace{f(x_1,y_1) \text{ or } f(x_2,y_2)}_{\text{nearest neighbor}} \text{ or } \underbrace{\text{floor}\left(\frac{f(x_1,y_1)+f(x_2,y_2)}{2}\right)}_{\text{Neighbors are at the same distance}}$$

Interpolation: Bilinear interpolation

$$f(x,y) = ax + by + cxy + d$$

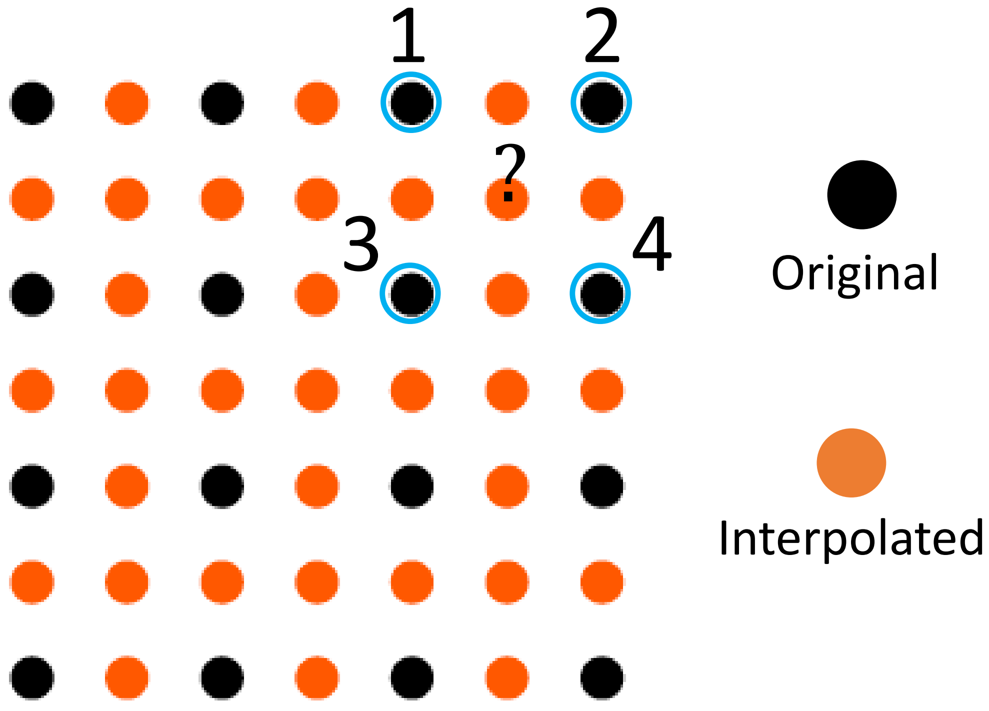
coefficients to be estimated: a,b,c,d

$$ax_1 + by_1 + cx_1y_1 + d = f(x_1,y_1)$$

$$ax_2 + by_2 + cx_2y_2 + d = f(x_2,y_2)$$

$$ax_3 + by_3 + cx_3y_3 + d = f(x_3,y_3)$$

$$ax_4 + by_4 + cx_4y_4 + d = f(x_4,y_4)$$



$$f(x,y) = \sum_{i=0}^{1}\sum_{j=0}^{1} a_{ij}x_iy_j$$

Interpolation: Bicubic interpolation

$$\color{red}{f(x,y)} = ax + by + cxy + d$$

coefficients to be estimated: a, b, c, d

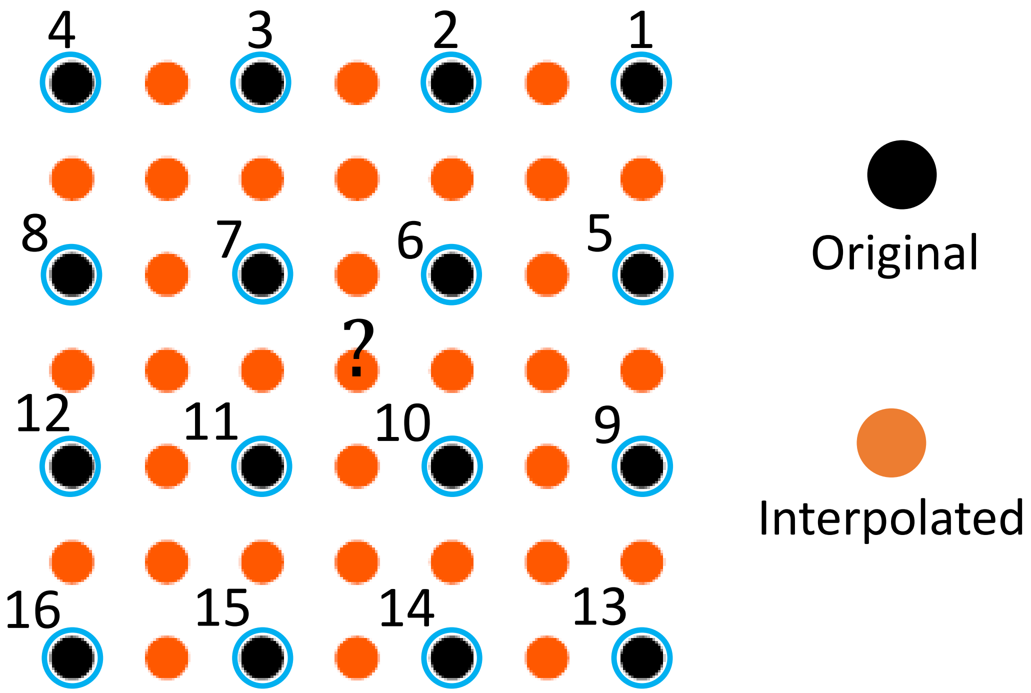
$$ax_1 + by_1 + cx_1y_1 + d = \color{red}{f(x_1, y_1)}$$

$$ax_2 + by_2 + cx_2y_2 + d = \color{red}{f(x_2, y_2)}$$

$$\vdots$$

$$ax_3 + by_3 + cx_3y_3 + d = \color{red}{f(x_3, y_3)}$$

$$ax_4 + by_4 + cx_4y_4 + d = \color{red}{f(x_4, y_4)}$$



$$\color{red}{f(x,y)} = \sum_{i=0}^{3}\sum_{j=0}^{3} a_{ij} x_i y_j$$

QUESTIONS & ANSWERS

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