Digital Image Processing ECE 566

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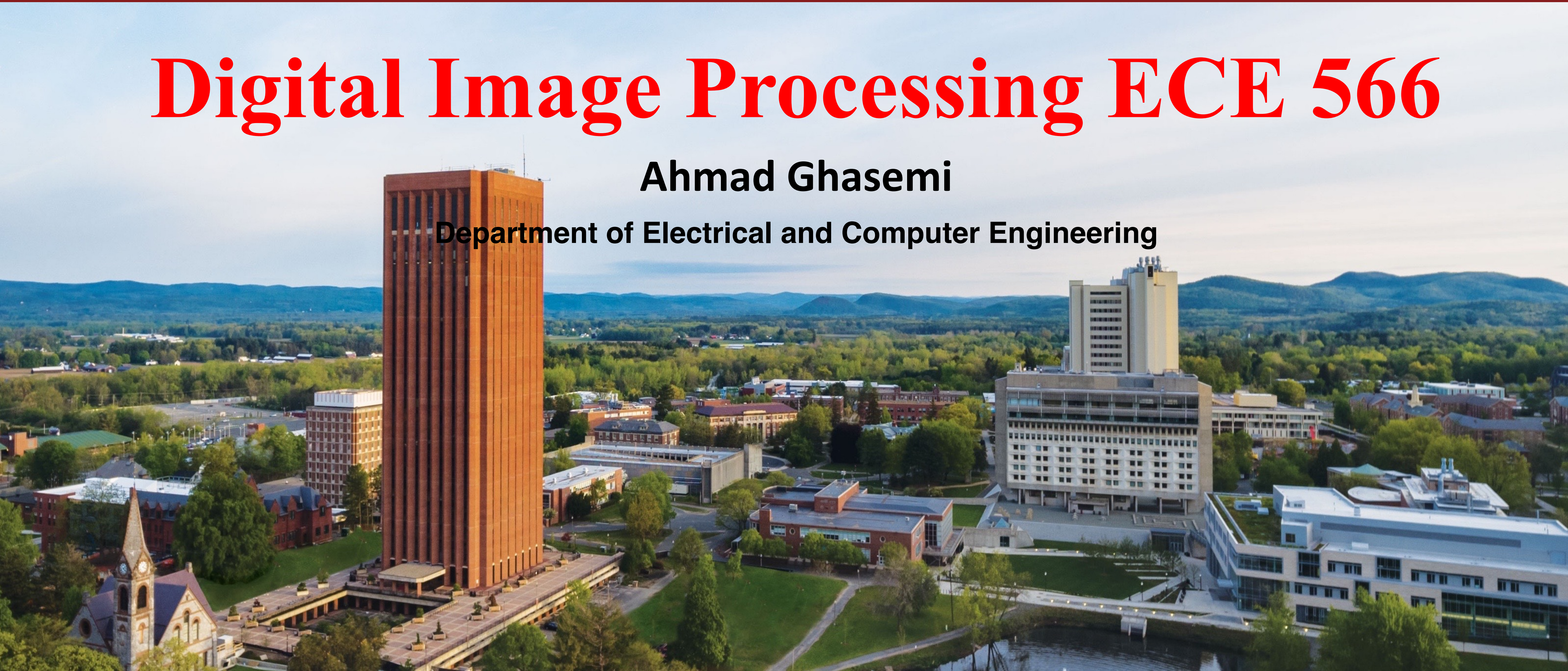

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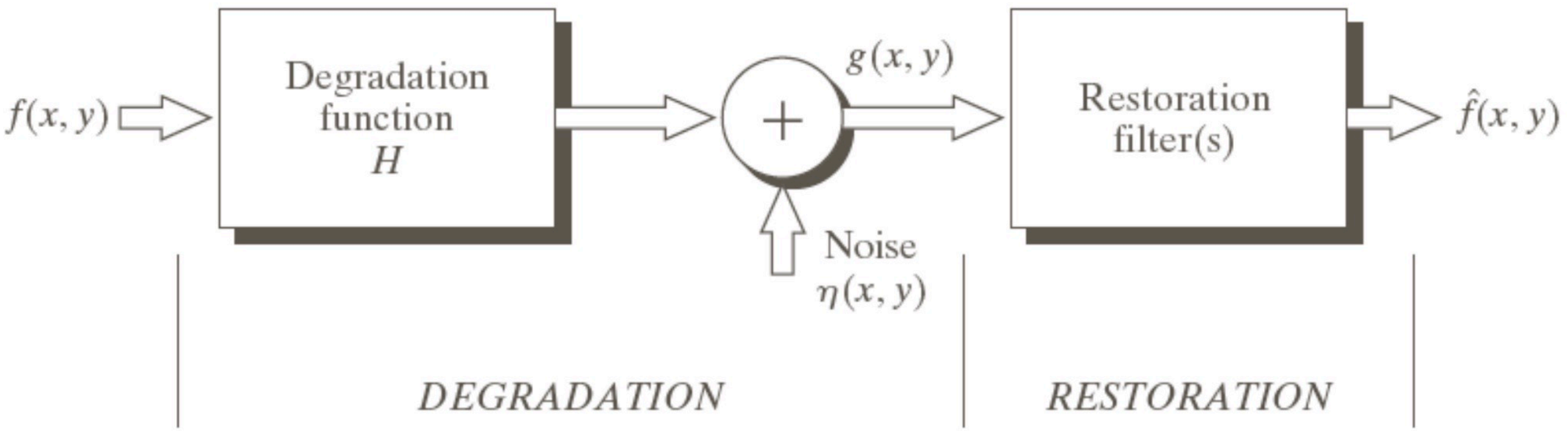
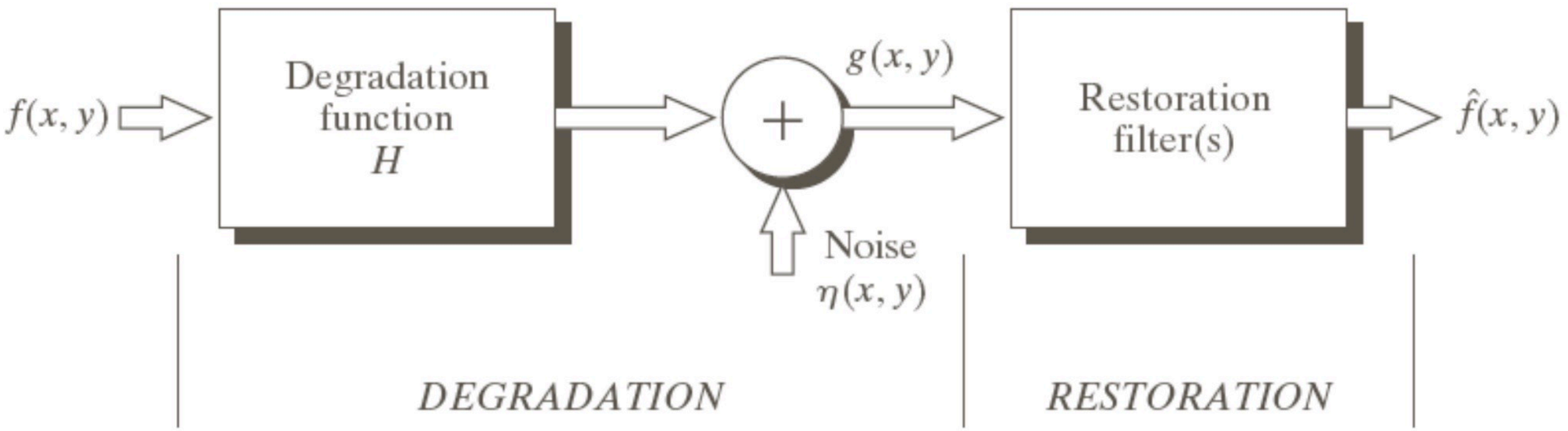
Image Restoration

Image Restoration by inverse filtering



$$g(x,y) = h(x,y) * f(x,y) + n(x,y) \quad \Leftrightarrow \quad G(u,v) = H(u,v)\, F(u,v) + N(u,v)$$

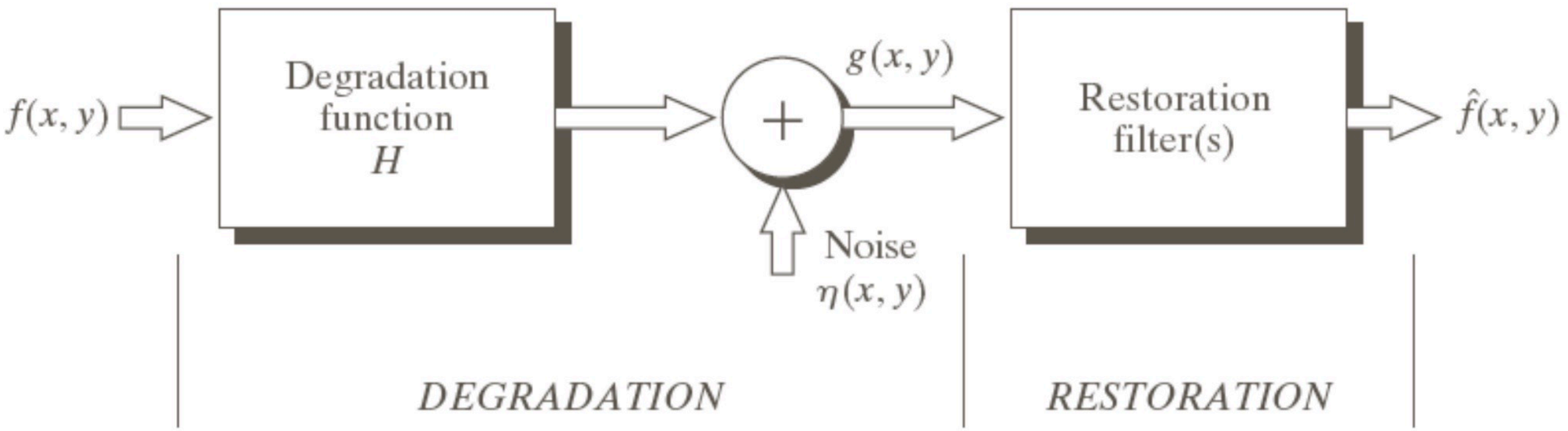
Image Restoration by inverse filtering



$$G(u,v) = H(u,v)F(u,v) + N(u,v)$$

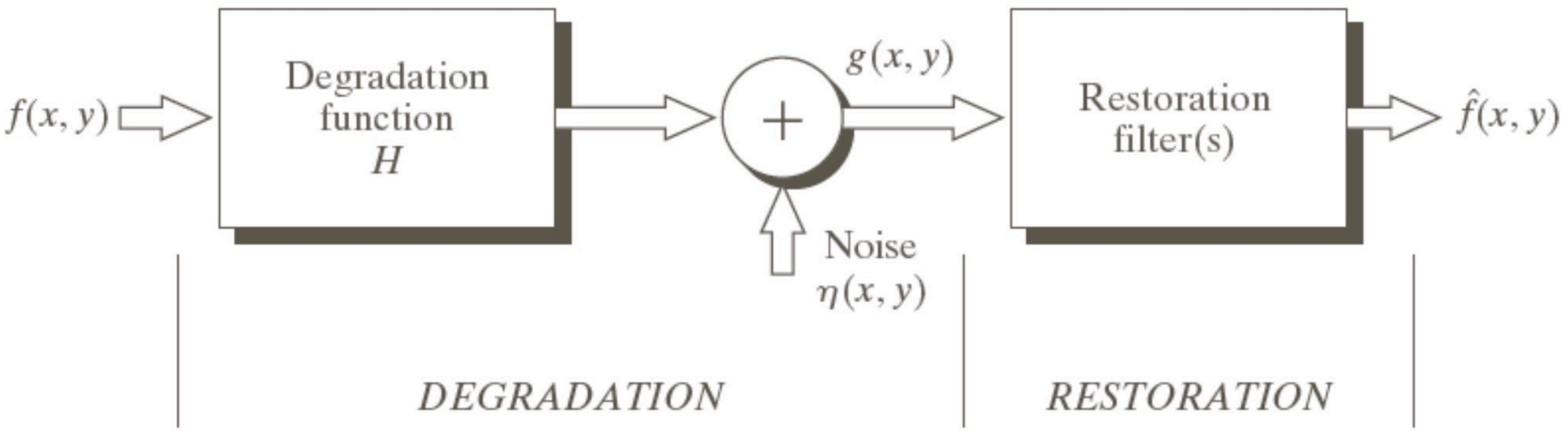
If no noise ➔ $$\hat{F}(u,v) = \frac{G(u,v)}{H(u,v)}$$

Image Restoration by inverse filtering



$$G(u, v) = H(u, v)F(u, v) + N(u, v)$$

Image Restoration by inverse filtering



$$G(u,v) = H(u,v)F(u,v) + N(u,v)$$

$$\hat{F}(u,v) = F(u,v) + \frac{N(u,v)}{H(u,v)}$$

Image Restoration by inverse filtering

$$\hat{F}(u,v) = F(u,v) + \frac{N(u,v)}{H(u,v)}$$

Bad news:

- Even when H(u,v) is known, there is always unknown noise
- Often H(u,v) has values close to zero

Image Restoration by inverse filtering

$$\hat{F}(u,v) = F(u,v) + \frac{N(u,v)}{H(u,v)}$$

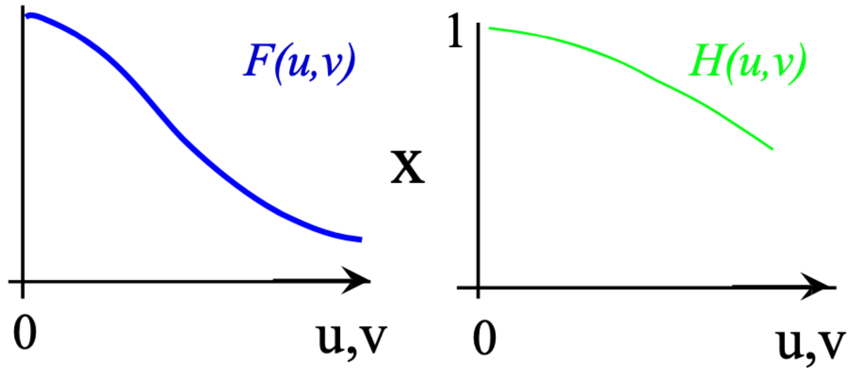


Image Restoration by inverse filtering

$$\hat{F}(u,v) = F(u,v) + \frac{N(u,v)}{H(u,v)}$$

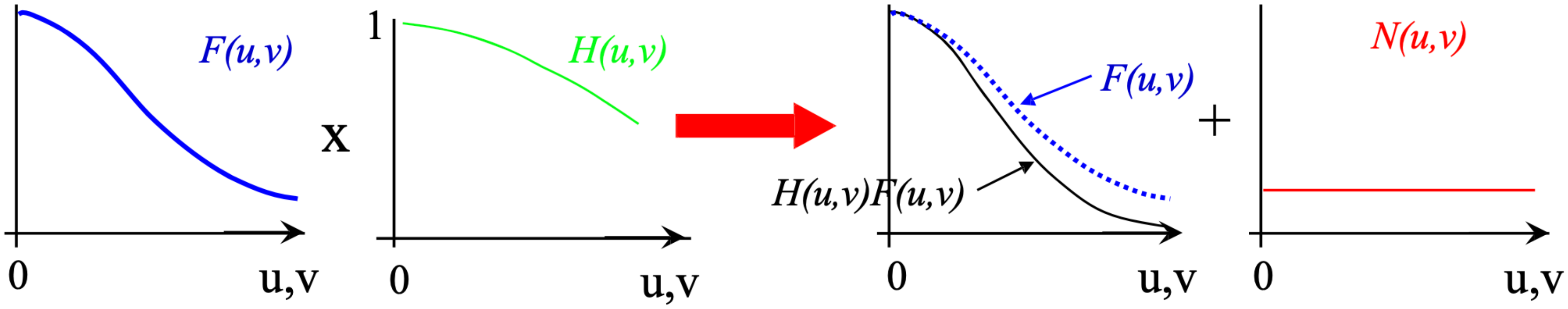


Image Restoration by inverse filtering

$$\hat{F}(u,v) = F(u,v) + \frac{N(u,v)}{H(u,v)}$$

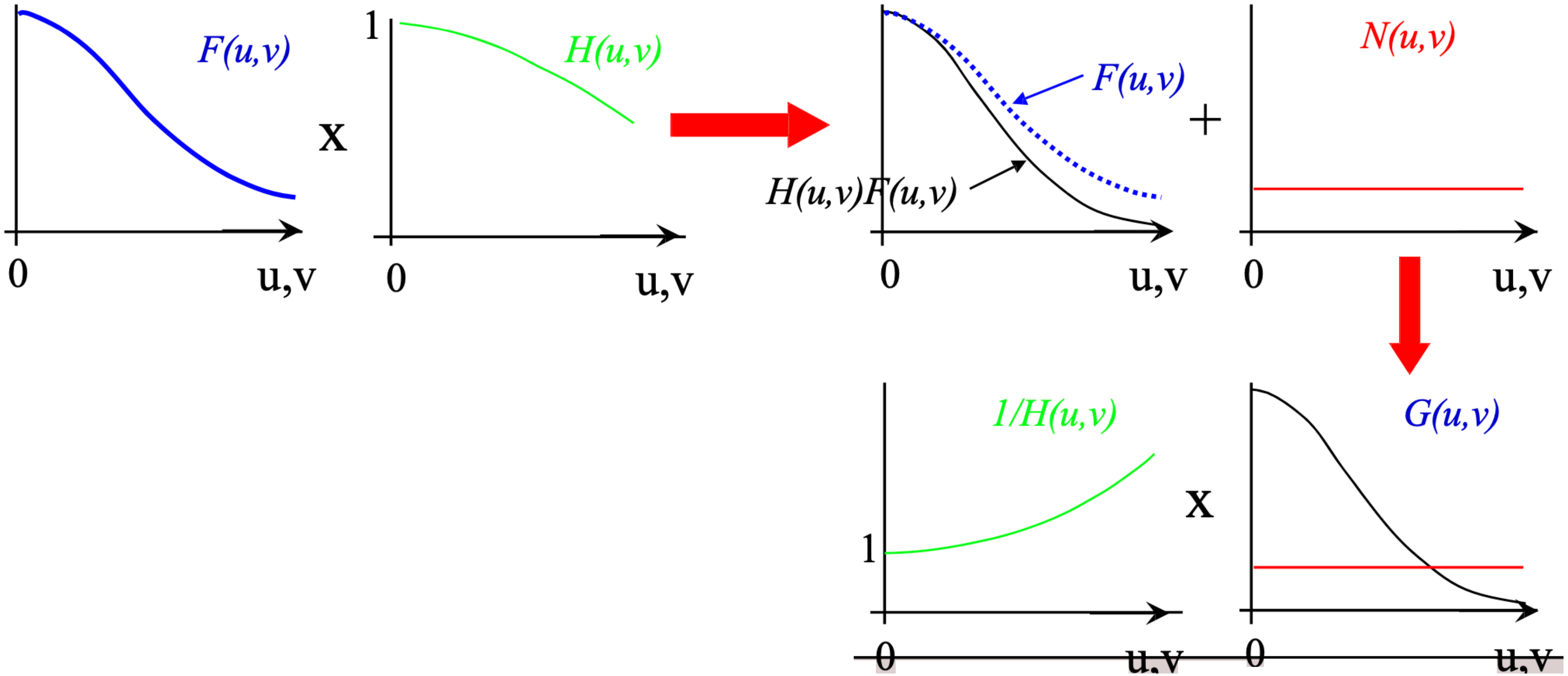


Image Restoration by inverse filtering

$$\hat{F}(u,v) = F(u,v) + \frac{N(u,v)}{H(u,v)}$$

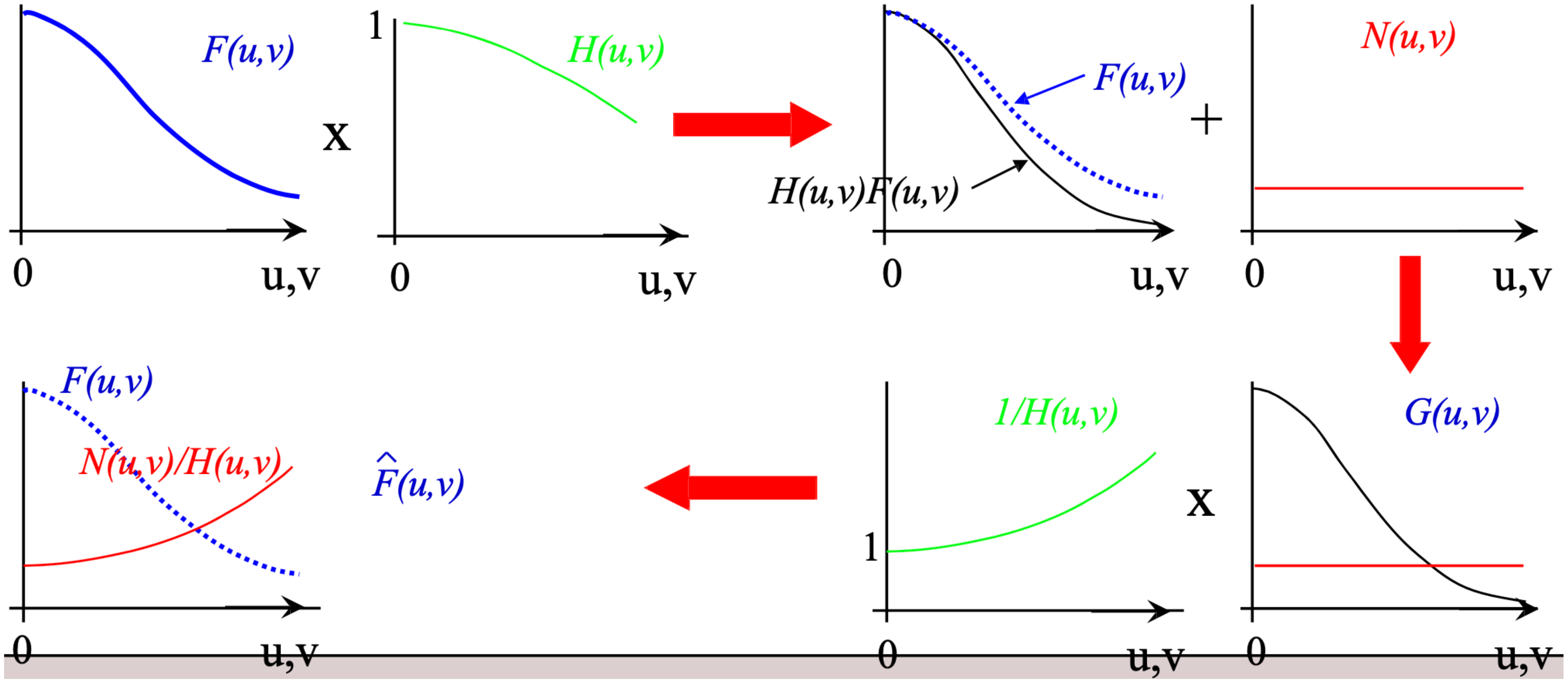


Image Restoration by inverse filtering

To mitigate the effect of zeros in the degradation function

Inverse filter with cut-off:

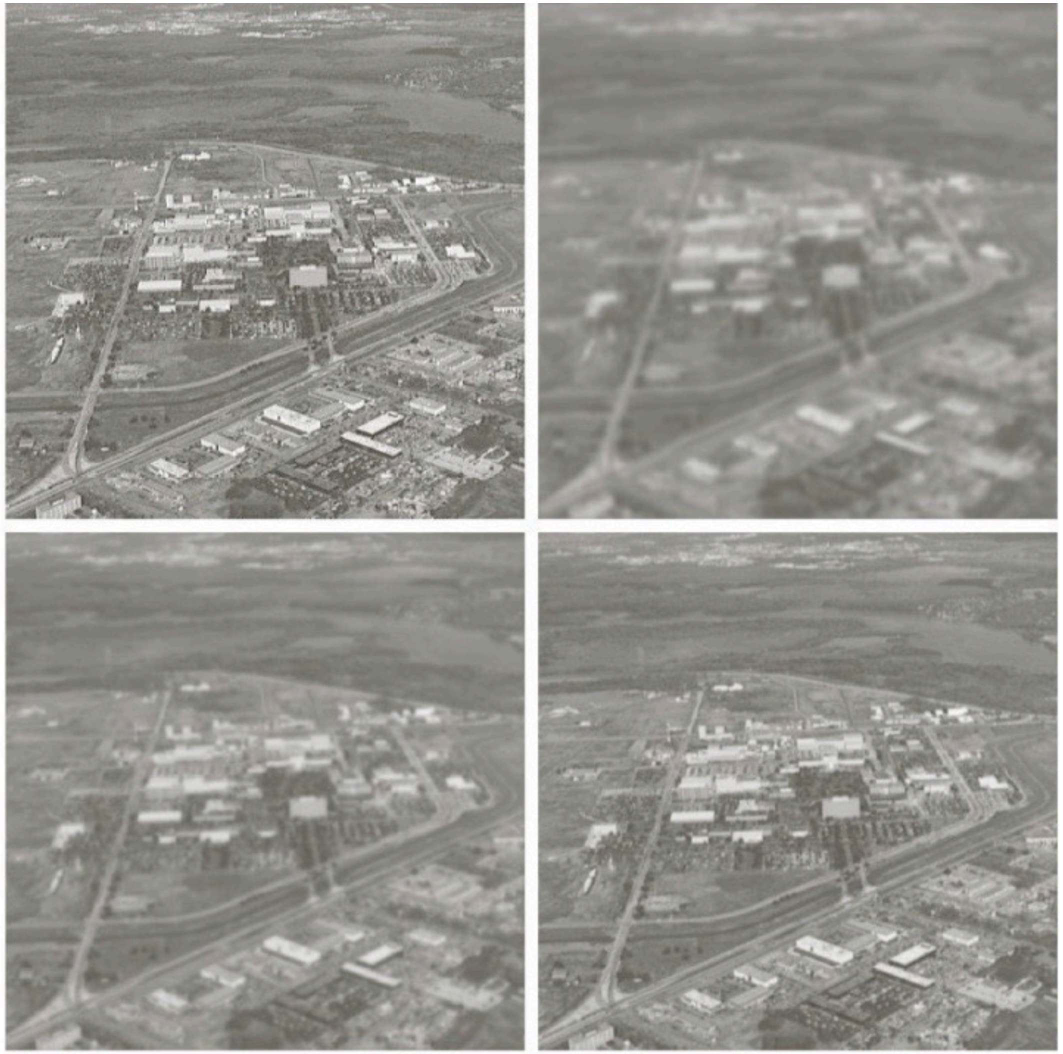
$$\hat{H}(u,v) = \begin{cases} 1/H(u,v), & |u^2+v^2| \leq \eta \\ 0, & |u^2+v^2| > \eta \end{cases}$$

Pseudo-inverse filter:

$$\hat{H}(u,v) = \begin{cases} 1/H(u,v), & |H(u,v)| \geq \epsilon \\ 0, & |H(u,v)| < \epsilon \end{cases}$$

Image Restoration by inverse filtering (Example)

Atmospheric turbulence effect



a b
c d

FIGURE 5.25 Illustration of the atmospheric turbulence model. (a) Negligible turbulence. (b) Severe turbulence, $k = 0.0025$. (c) Mild turbulence, $k = 0.001$. (d) Low turbulence, $k = 0.00025$. (Original image courtesy of NASA.)

Image Restoration by inverse filtering (Example)

$$\hat{F}(u,v) = G(u,v)\hat{H}(u,v)$$

$$\hat{H}(u,v) = \begin{cases} 1/H(u,v), & |u^2+v^2| \le \eta \\ 0, & |u^2+v^2| > \eta \end{cases}$$

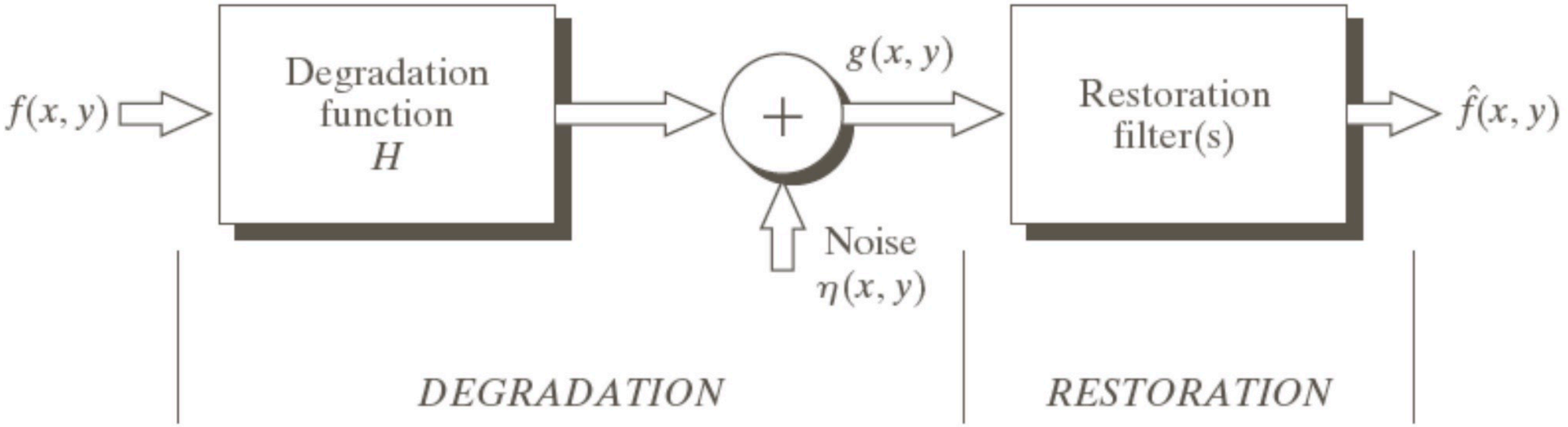
$$H(u,v) = e^{-k(u^2+v^2)}$$

a b
c d

FIGURE 5.27 Restoring Fig. 5.25(b) with Eq. (5.7-1). (a) Result of using the full filter. (b) Result with H cut off outside a radius of 40; (c) outside a radius of 70; and (d) outside a radius of 85.

$$\frac{G(u,v)}{H(u,v)}$$

Wiener Filter

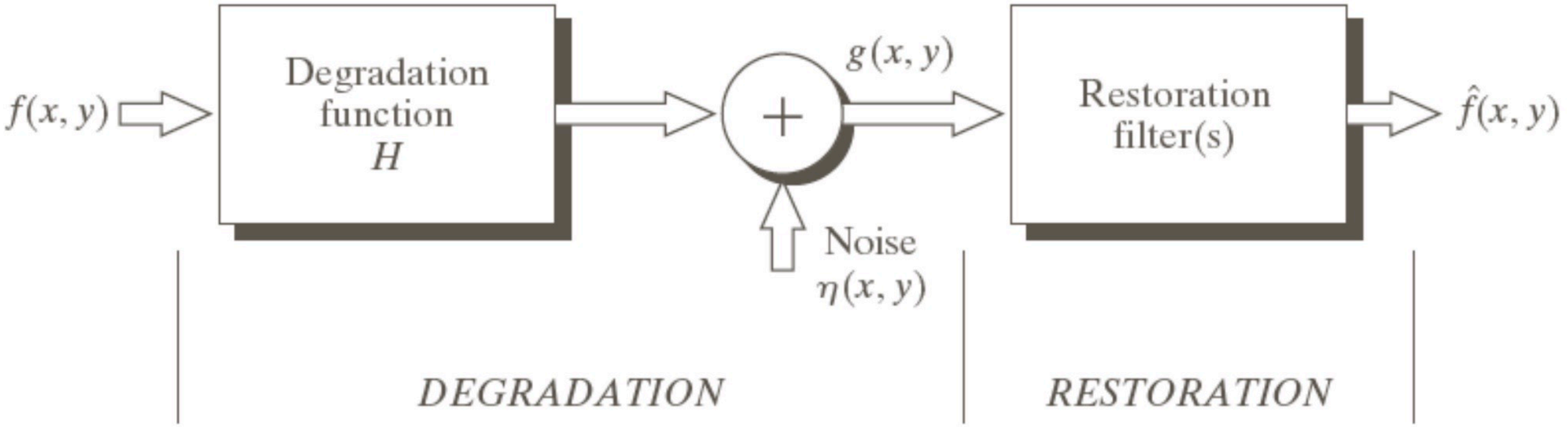


$$g(x, y) = f(x, y) * h(x, y) + n(x, y)$$

Goal: restoration with minimum expected mean-square error (MSE)

$$\min \; e = \sum_x \sum_y (f(x, y) - \hat{f}(x, y))^2$$

Wiener Filter

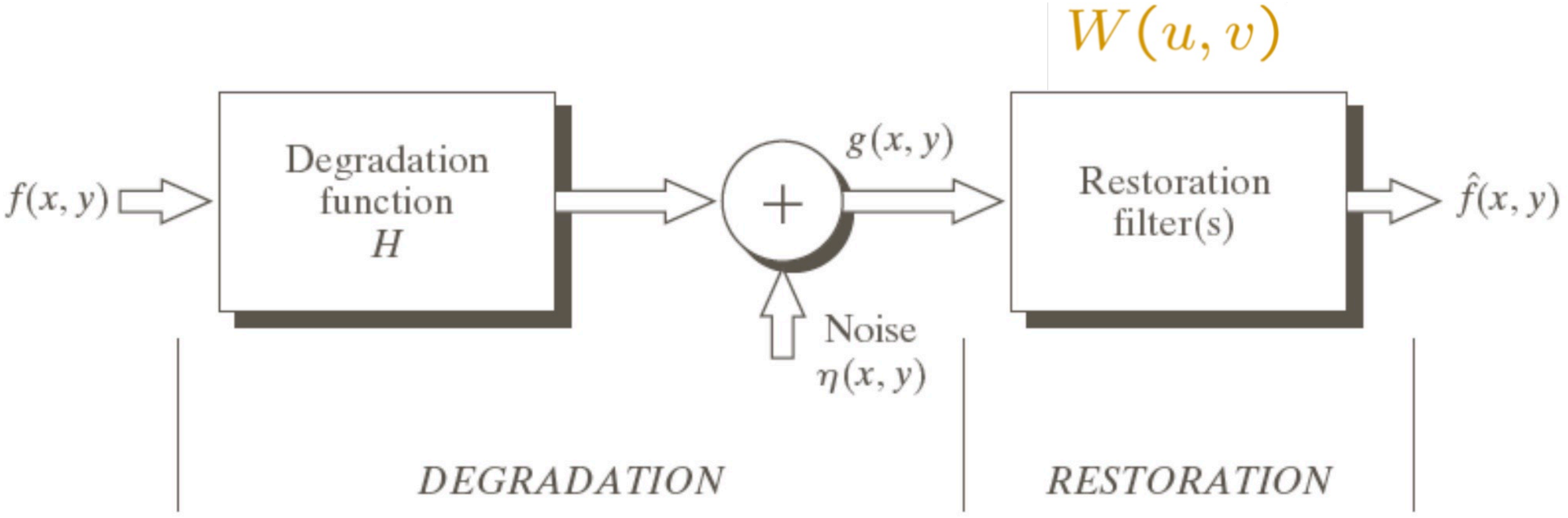


$$g(x,y) = f(x,y) * h(x,y) + n(x,y)$$

$$\min \; e = \sum_x \sum_y (f(x,y) - \hat{f}(x,y))^2$$

- Optimal solution (nonlinear)
- Restrict to linear space-invariant filter

Wiener Filter



$$g(x,y) = f(x,y) * h(x,y) + n(x,y)$$

$$\hat{f}(x,y) = w(x,y) * g(x,y)$$

- Optimal solution (nonlinear)
- Restrict to linear space-invariant filter
- Find "optimal" linear filter $W(u,v)$ with minimum MSE
- Assumption: noise and the image are uncorrelated

$$\min\ e = \sum_x \sum_y \left(f(x,y) - \hat{f}(x,y)\right)^2$$

Wiener Filter: Derivation

unknown original

after Wiener filtering

$$e = MN \sum_x \sum_y |f(x,y) - \hat{f}(x,y)|^2$$

Wiener Filter: Derivation

unknown original

after Wiener filtering

$$e = MN \sum_x \sum_y |f(x,y) - \hat{f}(x,y)|^2$$

$$= \sum_u \sum_v |F(u,v) - \hat{F}(u,v)|^2$$

Parseval's Theorem

Wiener Filter: Derivation

$$
\begin{aligned}
e &= MN \textstyle\sum_x \sum_y |f(x,y) - \hat{f}(x,y)|^2 \\
&= \textstyle\sum_u \sum_v |F(u,v) - \hat{F}(u,v)|^2 \\
&= \textstyle\sum_u \sum_v |F(u,v) - [F(u,v)H(u,v) + N(u,v)]W(u,v))|^2
\end{aligned}
$$

Unknown original

Corrupted original

Wiener filter

Wiener Filter: Derivation

independent signals

$$e = \sum_u \sum_v |F(u,v)[1 - H(u,v)W(u,v)] - N(u,v)W(u,v)|^2$$

$$= \sum_u \sum_v |F(u,v)[1 - H(u,v)W(u,v)]|^2 + |N(u,v)W(u,v)|^2$$

$$= \sum_u \sum_v |F(u,v)|^2 |1 - H(u,v)W(u,v)|^2 + |N(u,v)|^2 |W(u,v)|^2$$

Wiener Filter: Derivation

independent signals

$$e = \sum_u \sum_v |F(u,v)[1 - H(u,v)W(u,v)] - N(u,v)W(u,v)|^2$$

$$= \sum_u \sum_v |F(u,v)[1 - H(u,v)W(u,v)]|^2 + |N(u,v)W(u,v)|^2$$

$$= \sum_u \sum_v |F(u,v)|^2 |1 - H(u,v)W(u,v)|^2 + |N(u,v)|^2 |W(u,v)|^2$$

$$\boxed{\frac{\partial e}{\partial W(u,v)} = 0 \quad \Rightarrow \quad W(u,v)}$$

Wiener Filter: Derivation

$$e = \sum_u \sum_v |F(u,v)[1 - H(u,v)W(u,v)] - N(u,v)W(u,v)|^2$$

$$= \sum_u \sum_v |F(u,v)[1 - H(u,v)W(u,v)]|^2 + |N(u,v)W(u,v)|^2$$

$$= \sum_u \sum_v |F(u,v)|^2 |1 - H(u,v)W(u,v)|^2 + |N(u,v)|^2 |W(u,v)|^2$$

Since $\frac{\partial}{\partial z}(zz^*) = 2z^*$

$$\frac{\partial e}{\partial W(u,v)} = |F|^2[2(1 - W^* H^*)(-H)] + |N|^2[2W^*]$$

Wiener Filter: Derivation

$$\frac{\partial e}{\partial W(u,v)} = 0 \quad \Rightarrow \quad W^*(u,v) = \frac{|F(u,v)|^2 H(u,v)}{|H(u,v)|^2 |F(u,v)|^2 + |N(u,v)|^2}$$

$$\Downarrow$$

$$W(u,v) = \frac{H^*(u,v)}{|H(u,v)|^2 + \frac{|N(u,v)|^2}{|F(u,v)|^2}} = \frac{1}{H(u,v)} \frac{|H(u,v)|^2}{|H(u,v)|^2 + \frac{|N(u,v)|^2}{|F(u,v)|^2}}$$

Wiener Filter: Derivation

$$\frac{\partial e}{\partial W(u,v)} = 0 \quad \Rightarrow \quad W^*(u,v) = \frac{|F(u,v)|^2 H(u,v)}{|H(u,v)|^2 |F(u,v)|^2 + |N(u,v)|^2}$$

$$W(u,v) = \frac{H^*(u,v)}{|H(u,v)|^2 + \frac{|N(u,v)|^2}{|F(u,v)|^2}} = \boxed{\frac{1}{H(u,v)}} \frac{|H(u,v)|^2}{|H(u,v)|^2 + \boxed{\frac{|N(u,v)|^2}{|F(u,v)|^2}}}$$

Inverse Filter

$\frac{1}{\text{SNR}}$

SNR: signal-to-noise ratio

Wiener Filter: Approximation

$$H_W(u,v) \approx \frac{1}{H(u,v)} \frac{|H(u,v)|^2}{|H(u,v)|^2 + \boxed{\frac{1}{\mathrm{SNR}}}}$$

A constant chosen according to our knowledge of the noise level.

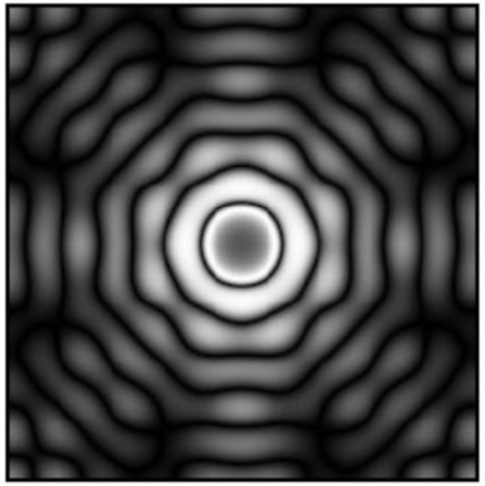
Wiener Filter: Example

1. Compute the FT of the blurred image
2. Multiply the FT by the Wiener filter
3. Compute the inverse FT

$$\hat{F}(u,v) = W(u,v)\, G(u,v)$$

Wiener Filter: Example

image 'blurr1'



wiener filter

restored license plate