Digital Image Processing ECE 566

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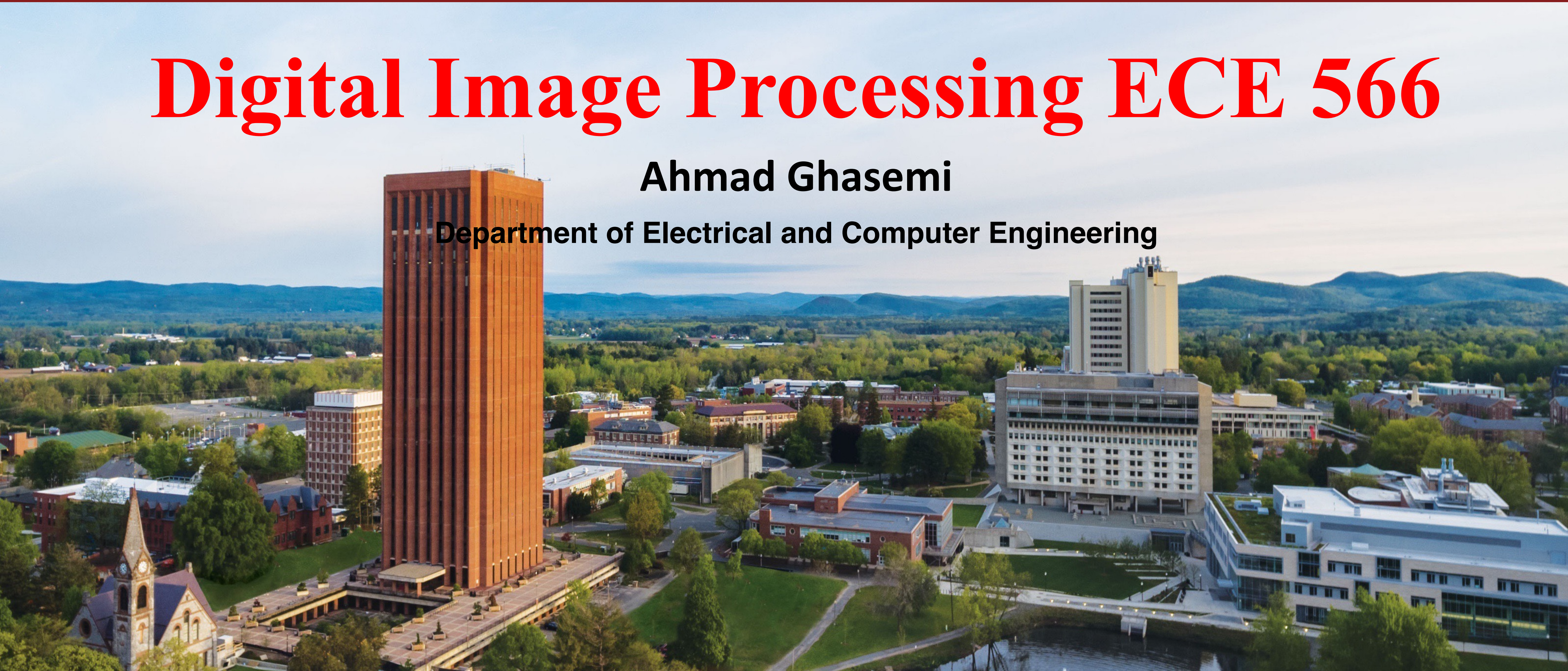
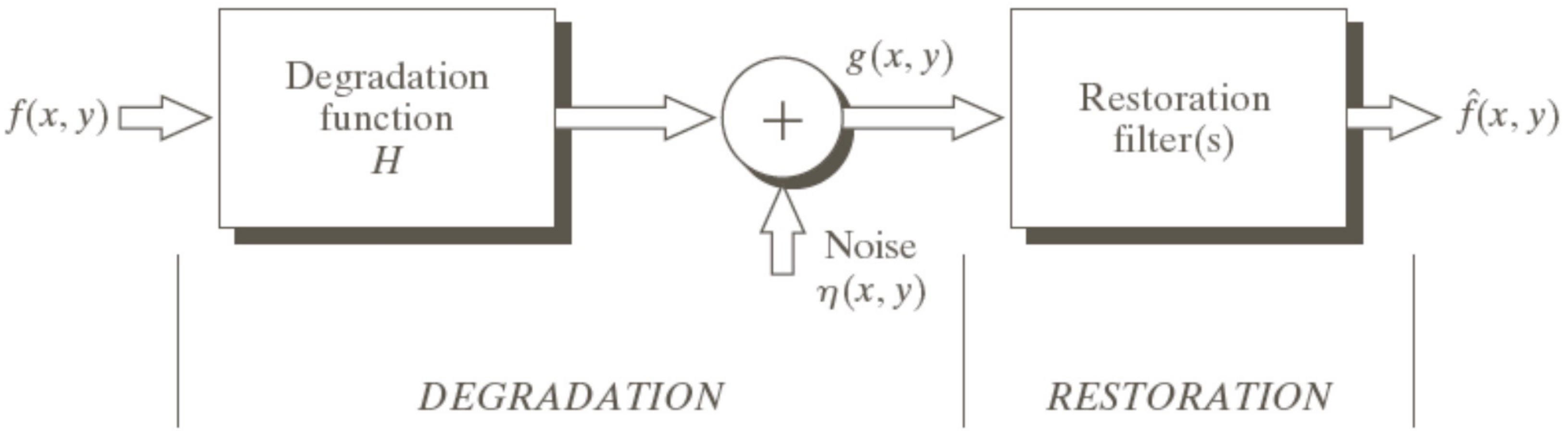


Image Restoration

Restoration: reconstruct/recover a degraded image

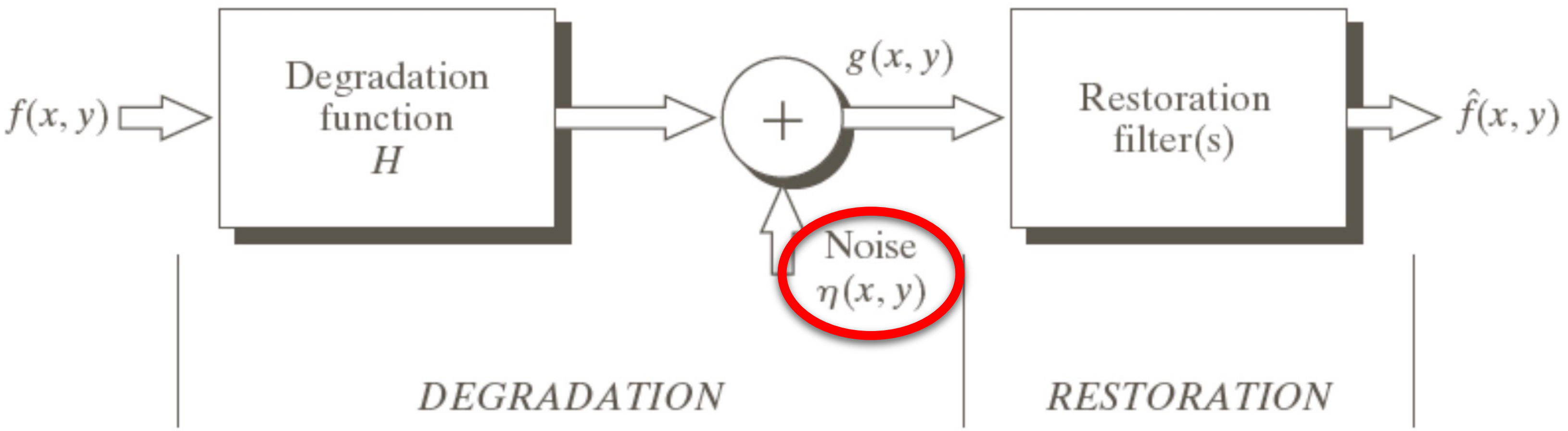
Model of Image Degradation/Restoration



$$g(x, y) = h(x, y) \star f(x, y) + \eta(x, y)$$

$$G(u, v) = H(u, v)F(u, v) + N(u, v)$$

Model of Image Degradation/Restoration



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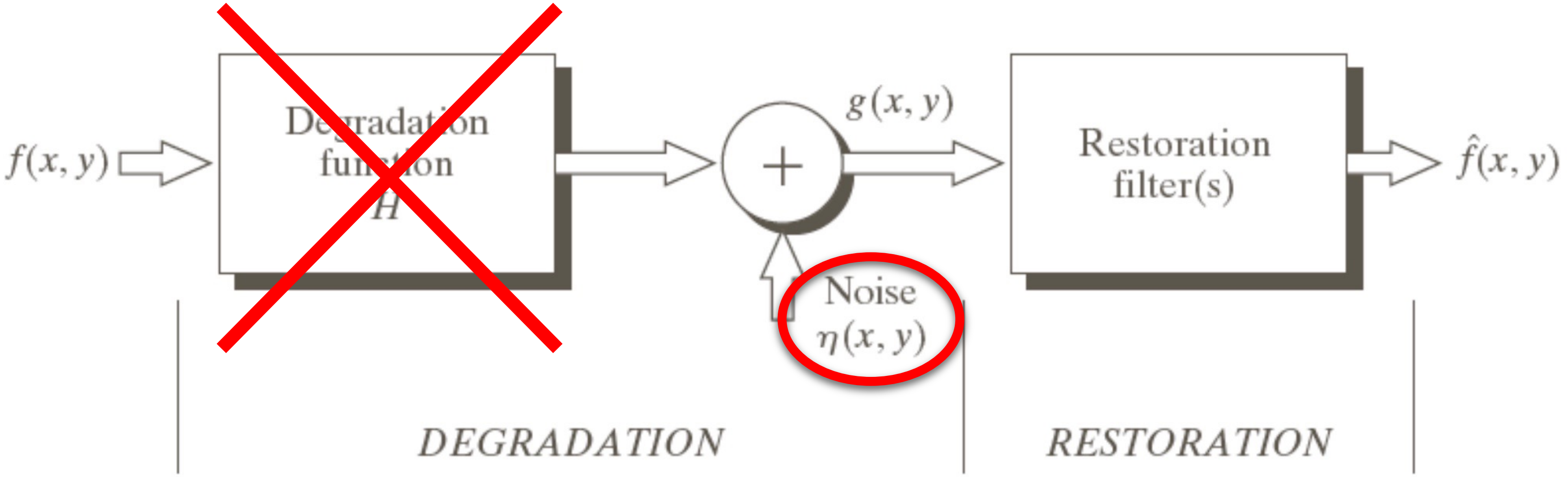
$$G(u,v) = H(u,v)F(u,v) + N(u,v)$$

Model of Image Degradation/Restoration



$$g(x,y) = \underbrace{h(x,y)}_{=\delta(x,y)} \star f(x,y) + \eta(x,y)$$

$$G(u,v) = \underbrace{H(u,v)}_{=1} F(u,v) + N(u,v)$$

Noise Models

- ✓ Spatial noise models are based on the statistical behavior of gray-level values

- ✓ Gray-level values are considered as random variables characterized by a probability density function (PDF)

  - ✓ Gaussian (normal)
  - ✓ Impulse (salt-and-pepper)
  - ✓ Uniform
  - ✓ Rayleigh
  - ✓ Gamma (Erlang)
  - ✓ Exponential

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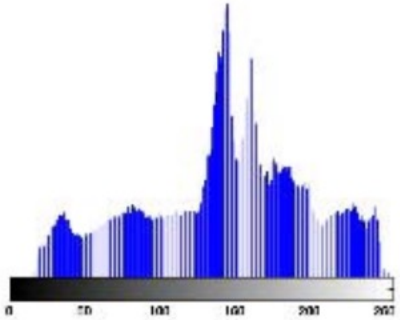
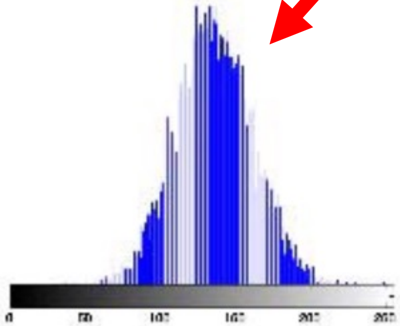
Noise Estimation

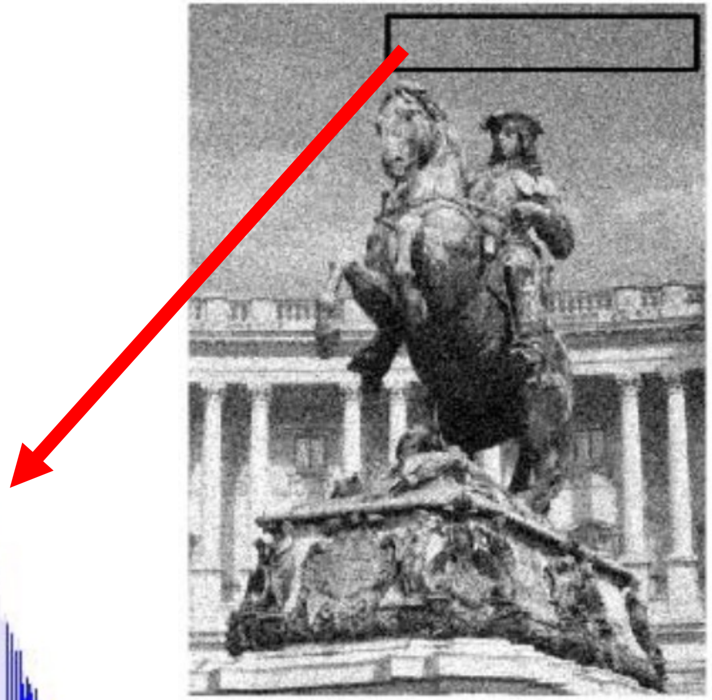
Histogram of

Original image

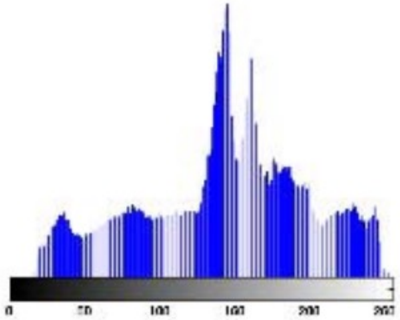


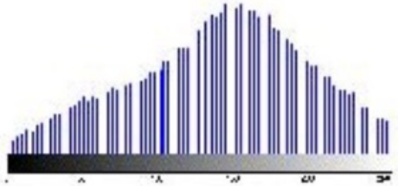
Noise Estimation





Histogram of Original image





Histogram of Noisy image

Restoration of Noise-only Degradation

Filters to be considered

Mean filters:

- Arithmetic mean filter
- Geometric mean filter
- Harmonic mean filter
- Contraharmonic mean filter

Order-statistics filters:

- Median filter
- Max and min filters
- Midpoint filter
- Alpha-trimmed mean filter

Mean filters: Arithmetic Mean Filters

- Smooth local variations in an image
- Noise is reduced as a result of blurring
- $g(s,t)$: degraded image
- S_{xy}: set of coordinates in a rectangular subimage window of size $m \times n$

$$\hat{f}(x,y) = \frac{1}{mn} \sum_{(s,t)\in S_{xy}} g(s,t)$$

- Causes a certain amount of blurring (proportional to the window size) to the image, thereby reducing the effects of noise.
- Can be used to reduce noise of different types, but works best for Gaussian, uniform, or Erlang noise.

Mean filters: Arithmetic Mean Filters (Example)

$$\begin{bmatrix} 1 & 2 & \cdots & 5 \\ \vdots & & \ddots & \vdots \\ 1 & 2 & \cdots & 5 \end{bmatrix}_{5\times 5}$$

Mean filters: Geometric Mean Filters

- Comparable to the arithmetic mean filter→ achieve smoothing; lose less image detail in the process

$$\hat{f}(x,y) = \left[\prod_{(s,t)\in S_{xy}} g(s,t) \right]^{\frac{1}{mn}}$$

- A variation of the arithmetic mean filter
- Primarily used on images with Gaussian noise
- Retains image detail better than the arithmetic mean

Mean filters: Geometric Mean Filters (Example)

$$\begin{bmatrix} 1 & 2 & \cdots & 5 \\ \vdots & & \ddots & \vdots \\ 1 & 2 & \cdots & 5 \end{bmatrix}_{5\times 5}$$

Mean filters: Harmonic Mean Filters

$$\hat{f}(x,y) = \frac{mn}{\displaystyle\sum_{(s,t)\in S_{xy}} \frac{1}{g(s,t)}}$$

- Another variation of the arithmetic mean filter
- Useful for images with Gaussian or salt noise
- Black pixels (pepper noise) are not filtered

Mean filters: Harmonic Mean Filters (Example)

$$\begin{bmatrix} 1 & 2 & \cdots & 5 \\ \vdots & & \ddots & \vdots \\ 1 & 2 & \cdots & 5 \end{bmatrix}_{5\times 5}$$

Mean filters: Contra-harmonic Mean Filters

- Reduce the effects of salt-and-pepper noise
- Q>0→ eliminate pepper noise (Q: order of the filter)
- Q<0→ eliminate salt noise
- Q=0→ arithmetic mean filter
- Q=–1→ harmonic mean filter
- Cannot eliminate both the salt noise and pepper noise simultaneously

$$\hat{f}(x,y) = \frac{\sum_{(s,t)\in S_{xy}} g^{Q+1}(s,t)}{\sum_{(s,t)\in S_{xy}} g^{Q}(s,t)}$$

Mean filters: Contra-harmonic Mean Filters (Example)

$$\begin{bmatrix} 1 & 2 & \cdots & 5 \\ \vdots & & \ddots & \vdots \\ 1 & 2 & \cdots & 5 \end{bmatrix}_{5\times 5}$$

Mean filters features

- The positive-order filter → effectively reduce the pepper noise, at the expense of blurring the dark areas
- The negative-order filter → effectively reduce the salt noise, at the expense of blurring the bright areas
- Arithmetic and geometric mean filters → suit the Gaussian or uniform noise (particularly the geometric mean filter)
- Contraharmonic filter → suit the impulse noise, yet, with the information of dark or light noise to select the proper sign for Q

Order-Statistics filters

- Known as Rank filters, Order filters OR Order Statistics filters

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- Operate on a neighborhood around a reference pixel by ordering (ranking) the pixel values and then performing an operation on those ordered values to obtain the new value for the reference pixel

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- Known as Rank filters, Order filters OR Order Statistics filters
- Operate on a neighborhood around a reference pixel by ordering (ranking) the pixel values and then performing an operation on those ordered values to obtain the new value for the reference pixel
- They perform very well in the presence of salt and pepper noise but are more computationally expensive as compared to mean filters

Order-Statistics filters: Median Filter

- Output is based on ordering (ranking) the pixels in a subimage
- Replace the value of a pixel by the median of the gray levels in the neighborhood of that pixel (a specified mask)
- Excellent for removing both bipolar and unipolar impulse noise

$$\hat{f}(x,y) = \underset{(s,t)\in S_{xy}}{median}\{g(s,t)\}$$

Order-Statistics filters: Median Filter

- Most popular and useful of the rank filters.
- It works by selecting the middle pixel value from the ordered set of values within the m x n neighborhood (W) around the reference pixel.
- mn is an even number, the arithmetic average of the two values closest to the middle of the ordered set is used instead.

Order-Statistics filters: Median Filter (Example)

$$\begin{bmatrix} 1 & 2 & \cdots & 5 \\ \vdots & & \ddots & \vdots \\ 1 & 2 & \cdots & 5 \end{bmatrix}_{5\times 5}$$

Order-Statistics filters: Max and Min Filter

- Max filter→ Replace the value of a pixel by the maximum of the gray levels (the brightest point) in the neighborhood of that pixel
- Min filter→ Replace the value of a pixel by the minimum of the gray levels (the darkest point) in the neighborhood of that pixel

$$\hat{f}(x,y) = \max_{(s,t)\in S_{xy}} \{g(s,t)\}$$

$$\hat{f}(x,y) = \min_{(s,t)\in S_{xy}} \{g(s,t)\}$$

Order-Statistics filters: Max and Min Filter

- Max filter also known as 100th percentile filter
- Min filter also known as zeroth percentile filter
- Max filter helps in removing pepper noise
- Min filter helps in removing salt noise

Order-Statistics filters: Max and Min Filter (Example)

$$\begin{bmatrix} 1 & 2 & \cdots & 5 \\ \vdots & & \ddots & \vdots \\ 1 & 2 & \cdots & 5 \end{bmatrix}_{5\times 5}$$

Order-Statistics filters: Midpoint Filter

- Filter's output→ the midpoint between the maximum and minimum values of the gray levels in the mask
- Combine order statistics and averaging
- Midpoint filter works best for randomly distributed noise (Gaussian or uniform)

$$\hat{f}(x,y) = \frac{1}{2}\left[\max_{(s,t)\in S_{xy}}\{g(s,t)\} + \min_{(s,t)\in S_{xy}}\{g(s,t)\}\right]$$

Order-Statistics filters: Midpoint Filter (Example)

$$\begin{bmatrix} 1 & 2 & \cdots & 5 \\ \vdots & & \ddots & \vdots \\ 1 & 2 & \cdots & 5 \end{bmatrix}_{5\times 5}$$

Order-Statistics filters: Alpha-Trimmed Mean Filter

- Filter's output→ average of gray levels of the remaining (mn−d) pixels ($g_r(s,t)$) in the mask after removing the d/2 lowest and the d/2 highest gray levels in S_{xy}
- $0 \leq d \leq (mn-1)$
- d=0 → arithmetic mean filter
- d=(mn-1)/2 → median filter
- Alpha-trimmed filter can be used to solve the multi-type noise problem (e.g., combination of salt-and pepper and Gaussian)

$$\hat{f}(x,y) = \frac{1}{mn-d} \sum_{(s,t)\in S_{xy}} g_r(s,t)$$

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Order-Statistics filters: Alpha-Trimmed Mean Filter (Example)

$$\begin{bmatrix} 1 & 2 & \cdots & 5 \\ \vdots & & \ddots & \vdots \\ 1 & 2 & \cdots & 5 \end{bmatrix}_{5\times 5}$$