University of Massachusetts Amherst

Digital Image Processing ECE 566

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Last Lecture

Linear Spatial Filtering

Convolution and Correlation

Last Lecture

Linear Spatial Filtering

Convolution and Correlation

Separable Kernels

Smoothing Filter

Spatial Sharpening

Separable Kernels

Spatial Sharpening

Smoothing Filter

Separable Kernels

Separable Kernels

$w(x, y)$ is separable if $w(x, y) = w_1(x)w_2(y)$

$$w = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \end{bmatrix} = w_1 w_2 = w_1 \bigstar w_2$$

Separable Kernels

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In general, if $w = w_1 \star w_2$, then

$$w \star f = (w_1 \star w_2) \star f = (w_2 \star w_1) \star f = w_2 \star (w_1 \star f) = (w_1 \star f) \star w_2$$

Separable Kernels

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$$w \star f = (w_1 \star w_2) \star f = (w_2 \star w_1) \star f = w_2 \star (w_1 \star f) = (w_1 \star f) \star w_2$$

Note: A kernel is separable if its rank is 1.

$$w = \frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \star \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \star \frac{1}{3}\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$ and $$\mathrm{w} = \frac{1}{9}\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}\begin{bmatrix} 1 & 1 & 1 \end{bmatrix}$$

Smoothing Filter

Smoothing Spatial Filters

- ✓ Low pass filtering/smoothing/blurring, is a technique used to remove noise and other unwanted artifacts from images.

- ✓ Applied to various types of noise, e.g., salt-and-pepper noise and Gaussian noise.

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Smoothing Spatial Filters

Smoothing Filter

- ✓ Low pass filtering/smoothing/blurring, is a technique used to remove noise and other unwanted artifacts from images.

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  - ➢ Average/Mean Smoothing
  - ➢ Gaussian Smoothing
  - ➢ Median Smoothing

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Smoothing Filter

Average/Mean Smoothing

- ✓ Determines the mean of the pixel values within an $n \times n$ kernel.
- ✓ The mean then replaces the pixel intensity of the center element.
- ✓ It is a low-pass uniform filter.
- ✓ Eliminating some of the noise in the image and smoothening the edges of the image.

Average/Mean Smoothing (Example)

Gaussian Smoothing

✓ Involves a weighted average of the surrounding pixels and has sigma σ as a parameter.

$$w(x, y) = \exp(-\frac{x^2 + y^2}{2\sigma^2})$$

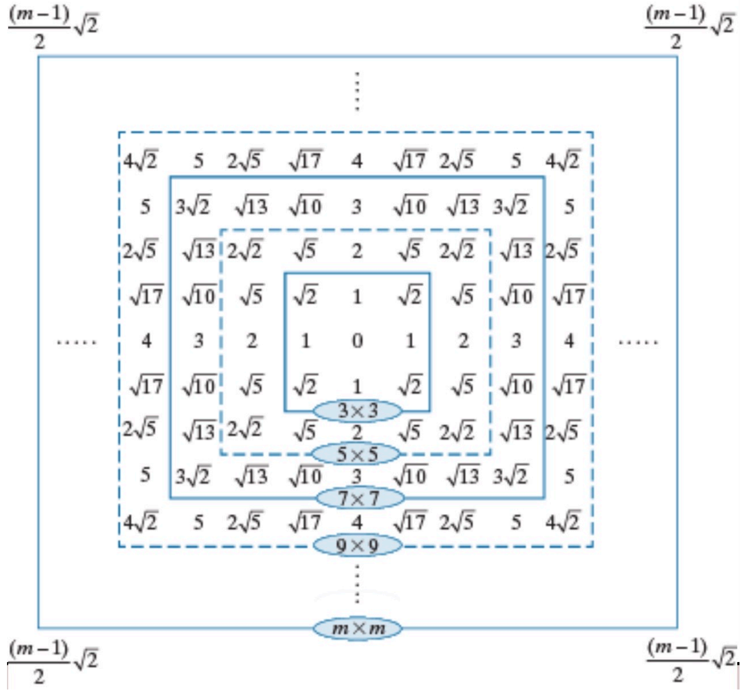
$w(-1,-1)$	$w(-1,0)$	$w(-1,1)$
$w(0,-1)$	$w(0,0)$	$w(0,1)$
$w(1,-1)$	$w(1,0)$	$w(1,1)$

3×3 filter

Gaussian Smoothing (Cont.)

- ✓ The kernel used in Gaussian blurring represents a discrete approximation of a Gaussian distribution.
- ✓ The coefficients of the kernel decrease as the distance from the center increases.
- ✓ It is a low-pass non-uniform filter.

Distance from the center



Gaussian Smoothing (Example)

✓ The image's brightness may not be preserved after applying a Gaussian filter.

Median Smoothing

- ✓ The median filter calculates the median of the pixel intensities surrounded by the center pixel in an $n \times n$ kernel.
- ✓ The intensity of the center pixel is then replaced with a median.
- ✓ It helps in removing salt and pepper noise from the image.
- ✓ This preserves the edges of an image.
- ✓ It is a non-linear digital filtering technique.

Median Smoothing (Example)

Smoothing Challenges

✓ Trade-off between smoothing and detail preservation:

Finding the right balance between reducing noise and preserving the important details is challenging.
Over-smoothing ➔ loss of important details and edges.
Under-smoothing ➔ leaves noise and artifacts in the image.

✓ Computational complexity:

Some smoothing techniques are computationally expensive and take a long time to process large images or video streams.

✓ Parameter tuning:

Smoothing techniques have several parameters that need to be tuned for optimal performance.
E.g., kernel size, filter type, and sigma values.
Time-consuming and requires expertise in image processing.

Spatial Sharpening

Spatial Sharpening

Original Blurred image

After Sharpening

Spatial Sharpening

✓ Blur/Smooth the original image f to obtain image f'

✓ Subtract/add the Laplacian of the blurred image f' to f

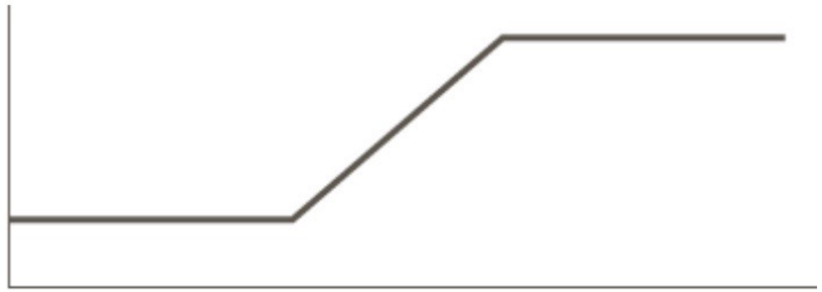
$$g(x, y) = f'(x, y) \mp c.\nabla^2 f'(x, y)$$

Intuitively?

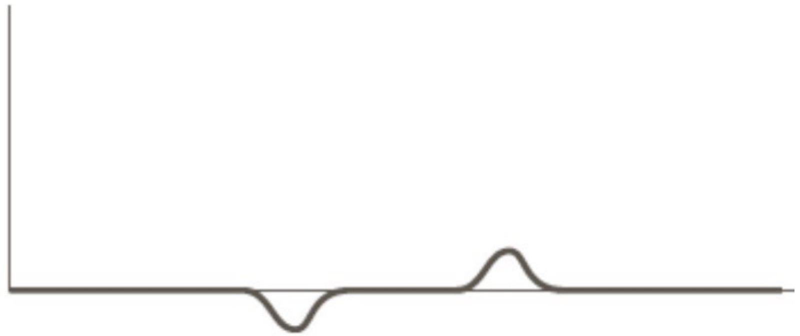
Laplacian of an image?

Spatial Sharpening (Signals)

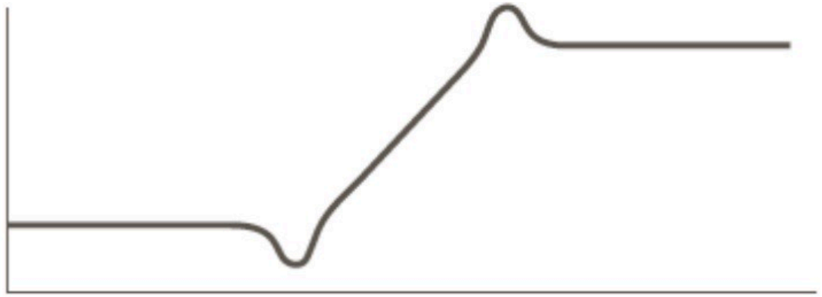
- ✓ Original signal



- ✓ Second derivative



- ✓ Original signal - Second derivative



First Image Derivatives

$$\frac{\partial f(x,y)}{\partial x} \approx f(x+1,y) - f(x,y)$$

$$\frac{\partial f(x,y)}{\partial y} \approx f(x,y+1) - f(x,y)$$

Second Image Derivatives

$$\frac{\partial^2 f(x,y)}{\partial^2 x} \approx f(x+1,y) + f(x-1,y) - 2f(x,y)$$

$$\frac{\partial^2 f(x,y)}{\partial^2 y} \approx f(x,y+1) + f(x,y-1) - 2f(x,y)$$

Laplacian

$$\nabla^2 f(x,y) = \frac{\partial^2 f(x,y)}{\partial^2 x} + \frac{\partial^2 f(x,y)}{\partial^2 y}$$

$$\nabla^2 f(x,y) \approx f(x+1,y) + f(x-1,y)$$
$$+f(x,y+1) + f(x,y-1) - 4f(x,y)$$

0	1	0
1	−4	1
0	1	0

Sharpening Spatial Filters

Variants of the Laplacian filter

0	1	0
1	−4	1
0	1	0

1	1	1
1	−8	1
1	1	1

0	−1	0
−1	4	−1
0	−1	0

−1	−1	−1
−1	8	−1
−1	−1	−1

Sharpening Spatial Filters

Variants of the Laplacian filter

0	1	0
1	−4	1
0	1	0

1	1	1
1	−8	1
1	1	1

use $c = -1$ for

0	−1	0
−1	4	−1
0	−1	0

−1	−1	−1
−1	8	−1
−1	−1	−1

use $c = +1$ for

$$g(x,y) = f'(x,y) + c.\nabla^2 f'(x,y)$$

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Spatial Sharpening

Sharpening Spatial Filters (Example)

Original image f

blurred image f'

Laplacian $\nabla^2 f'(x,y)$

$f'(x,y) - 1.\nabla^2 f'(x,y)$

$f'(x,y) - 3.\nabla^2 f'(x,y)$

Unsharp Masking

- ✓ Blur/Smooth the original image
- ✓ Subtract the blurred image from the original

$$f(x,y) - \overline{f(x,y)}$$

- ✓ Add the resulting mask to the original

$$g(x,y) = f(x,y) + k(f(x,y) - \overline{f(x,y)})$$

$k = 1$ unsharp masking
$k > 1$ highboost filtering

QUESTIONS & ANSWERS

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