University of Massachusetts Amherst

Digital Image Processing ECE 566

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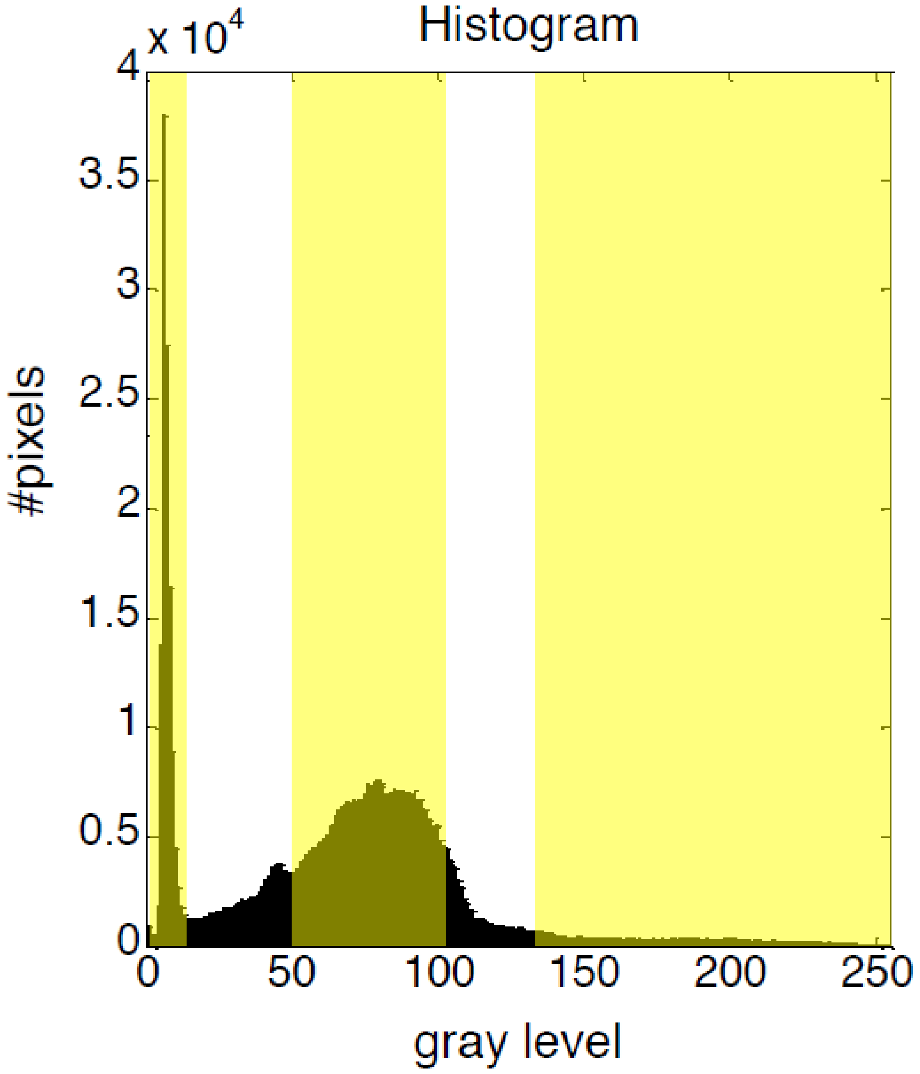
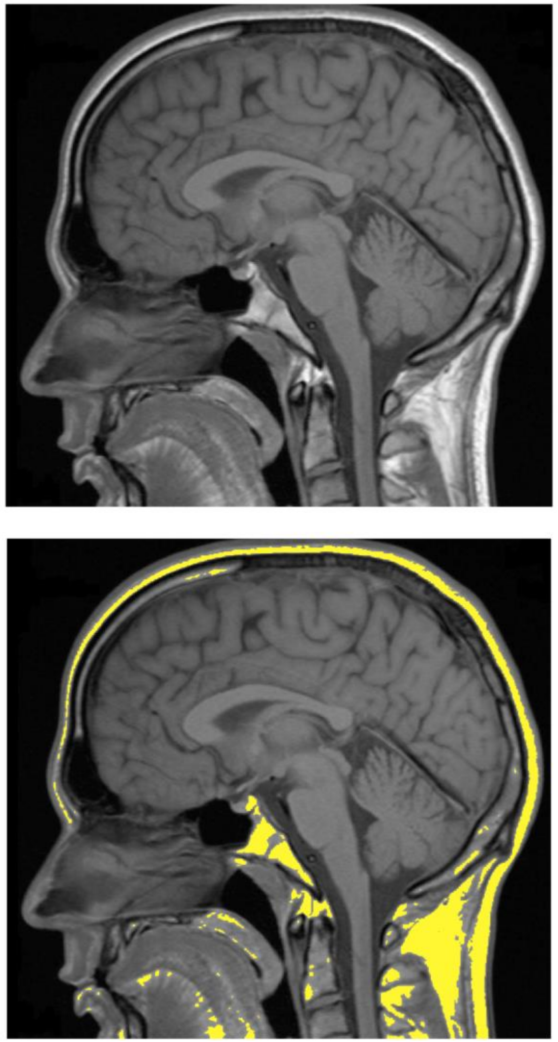
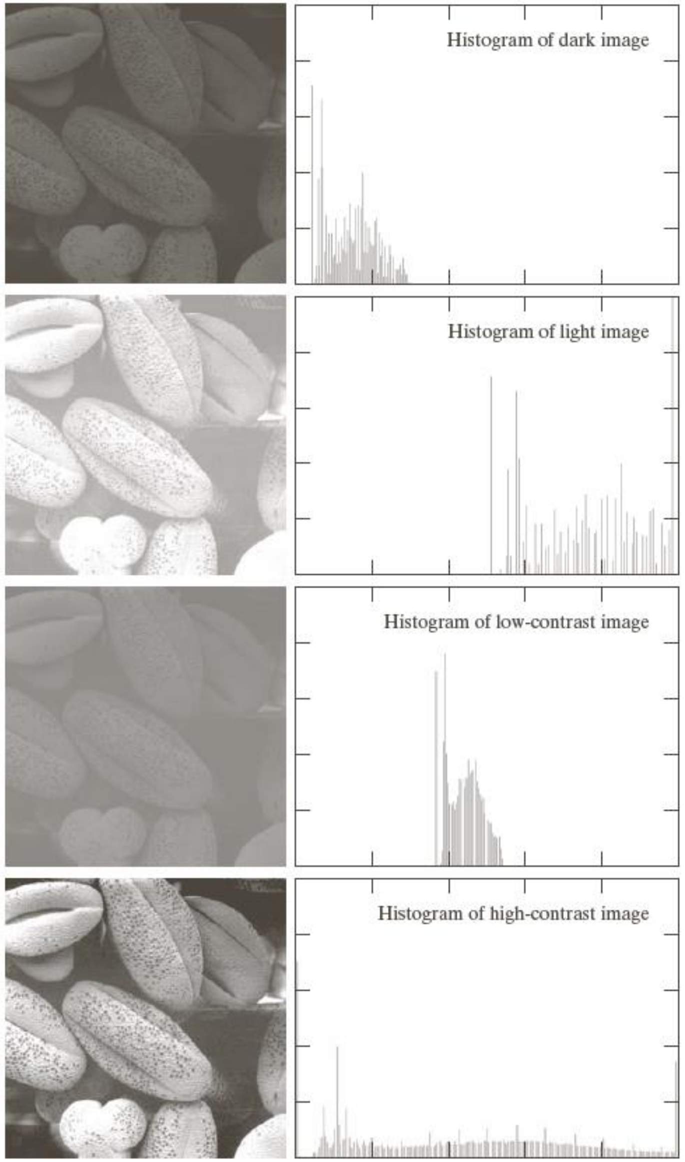
Gray Level Histograms



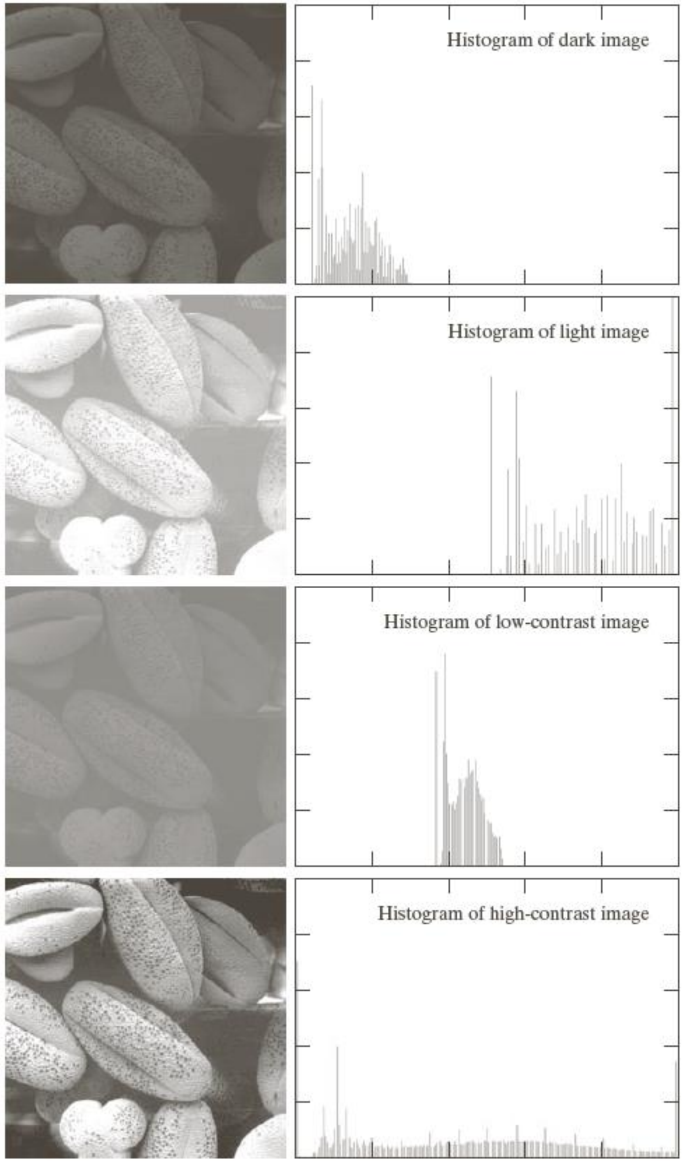


Gray Level Histograms



- ✓ To measure a histogram:
  - For B-bit image (2^{B} intensity values), initialize 2^{B} counters with 0
  - Loop over all pixels x, y
  - When encountering gray level $f\ [x, y] = i$, increment counter # i.

Gray Level Histograms



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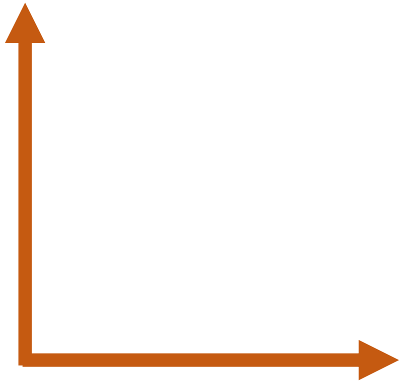
✓ Normalized histogram can be thought of as an estimate of the probability distribution of the continuous signal amplitude



$$p(r_k) = \frac{n_k}{MN}$$

pixels at k-th intensity value

Total # pixels

k-th intensity/gray value

Histogram ignores spatial information

Histogram Equalization

Increases local contrast by spreading out the intensity histogram

Idea: Find a non-linear transformation $s = T(r)$ that is applied to each pixel of the input image $f\,[x, y]$, such that a uniform distribution of gray levels results for the output image $g[x, y]$.

$$\boxed{s = T(r)}$$

Output intensity/gray value

Input intensity/gray value

Flat histogram of intensities $\longleftrightarrow$ Narrow histogram of intensities

Histogram Equalization

Continuous case first ...

Assume

- ✓ Normalized input values $0 \leq r \leq 1$ and output values $0 \leq s \leq 1$
- ✓ $T(r)$ is differentiable, increasing, and invertible, i.e., there exists

$$r = T^{-1}(s)$$

Goal: pdf $p_s\,(s) = 1$ over the entire range $0 \leq s \leq 1$

Histogram Equalization for continuous case

✓ From basic probability theory

$$p_r(r) = 1 \xrightarrow{\;r\;} \boxed{T(r)} \xrightarrow{\;s\;} \qquad p_s(s) = \left[p(r)\frac{dr}{ds}\right]_{r=T^{-1}(s)}$$

✓ Consider the transformation function

$$\boxed{s = T(r) = \int_0^r p_r(\alpha)d\alpha \qquad 0 \le r \le 1}$$

$$\Rightarrow \quad p_s(s) = \left[p_r(r)\frac{dr}{ds}\right]_{r=T^{-1}(s)} = \left[p_r(r)\frac{1}{p_r(r)}\right]_{r=T^{-1}(s)} = 1 \qquad 0 \le s \le 1$$

$$\frac{ds}{dr} = p_r(r)$$

Histogram Equalization for discrete case

- ✓ Now, r only assumes discrete amplitude values $r_0, r_1, \ldots, r_{L-1}$ with empirical probabilities

$$p_r(r_0) = \frac{n_0}{MN}, p_r(r_1) = \frac{n_1}{MN}, \ldots, p_r(r_{L-1}) = \frac{n_{L-1}}{MN}$$

pixels within bin i

Total # pixels

- ✓ Discrete approximation of s $= T(r) = \int_0^r p_r\,(\alpha)d\alpha$

$$\boxed{s_k = T(r_k) = \sum_{i=0}^{k} p_r(r_i) \quad \text{for } k = 0,1,\ldots,L-1}$$

- ✓ The resulting values s_k are in the range [0,1] and might have to be scaled and rounded appropriately

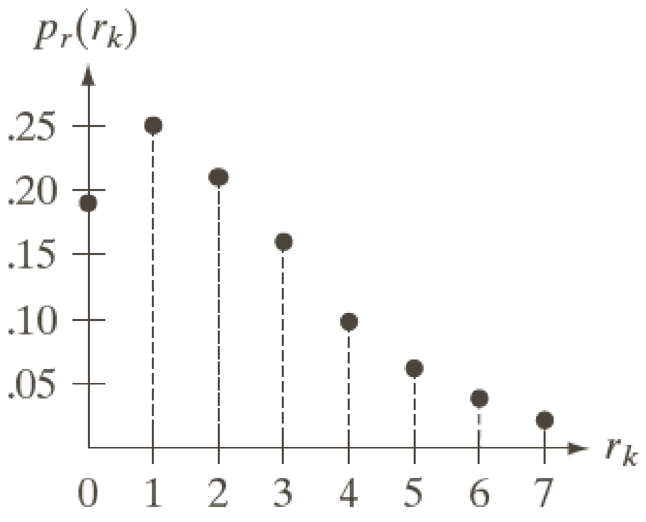
Histogram Equalization

If input values r and output values s are not normalized.
They are in L intensity levels.

$$s = T(r) = (L-1)\int_0^r p_r(\alpha)d\alpha$$

$$s_k = T(r_k) = (L-1)\sum_{i=0}^{k} p_r(r_i) = (L-1)\sum_{i=0}^{k}\frac{n_i}{MN}$$

Example: Histogram Equalization



Original histogram of a 3-bit ($2^3 = 8$ intensity levels) image, 64×64 digital

r_k	n_k	$p_r(r_k) = n_k/MN$
$r_0 = 0$	790	0.19
$r_1 = 1$	1023	0.25
$r_2 = 2$	850	0.21
$r_3 = 3$	656	0.16
$r_4 = 4$	329	0.08
$r_5 = 5$	245	0.06
$r_6 = 6$	122	0.03
$r_7 = 7$	81	0.02

Intensity distribution and histogram values

Example: Histogram Equalization

$$s_k = T(r_k) = (L-1)\sum_{i=0}^{k}\frac{n_i}{MN} = 7\sum_{i=0}^{7}\frac{n_i}{64\times 64}$$

r_k	n_k	$p_r(r_k) = n_k/MN$
$r_0 = 0$	790	0.19
$r_1 = 1$	1023	0.25
$r_2 = 2$	850	0.21
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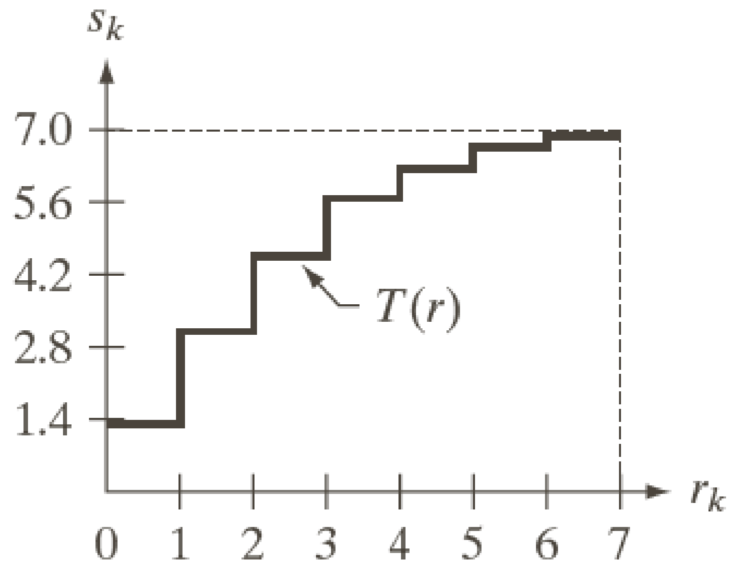
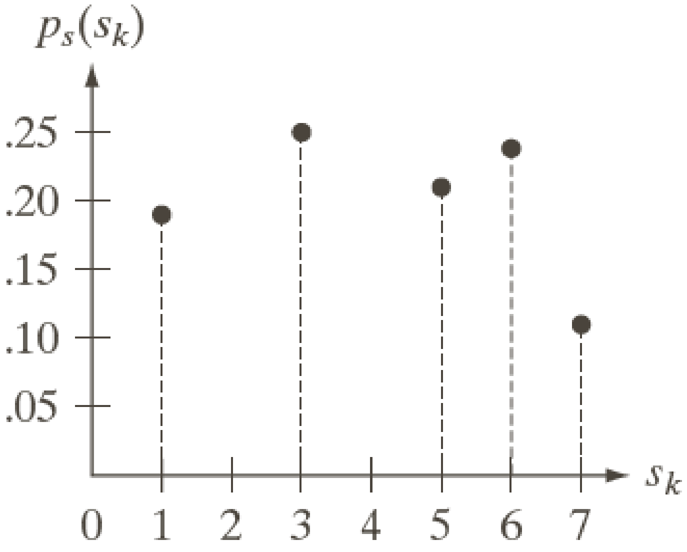


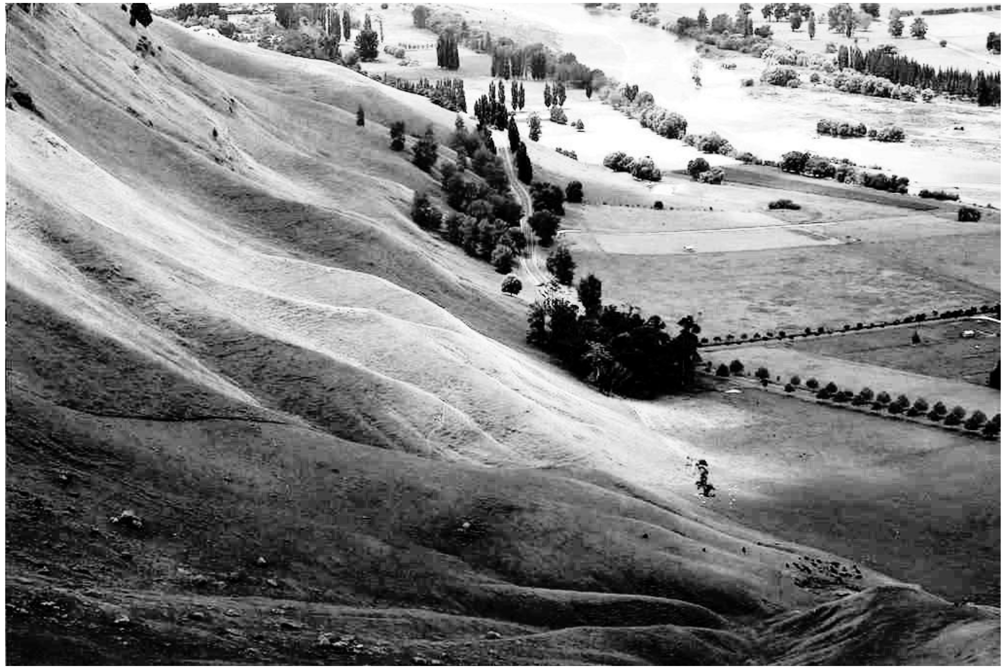
Transformation Function



Equalized Histogram

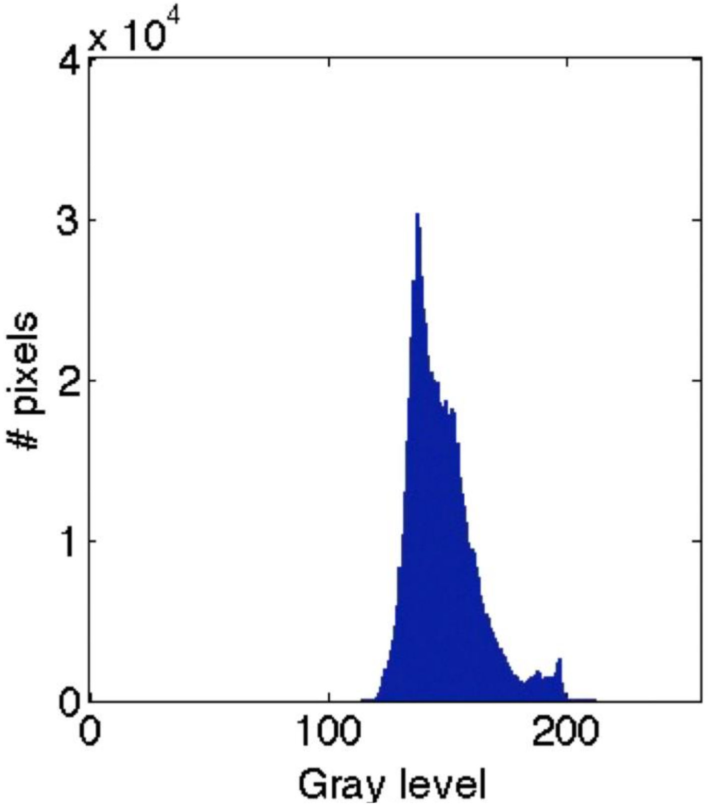
Histogram Equalization Example

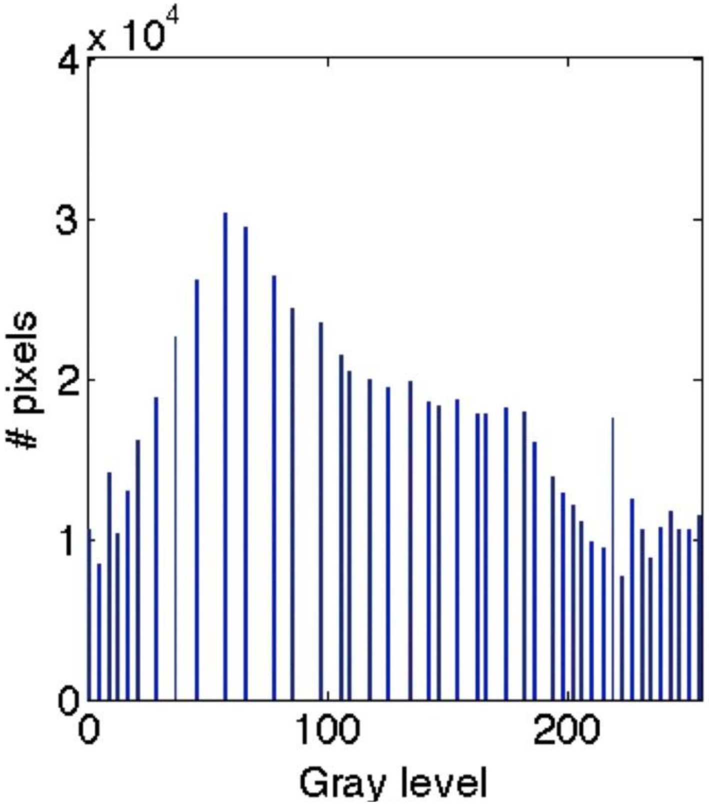
Original image

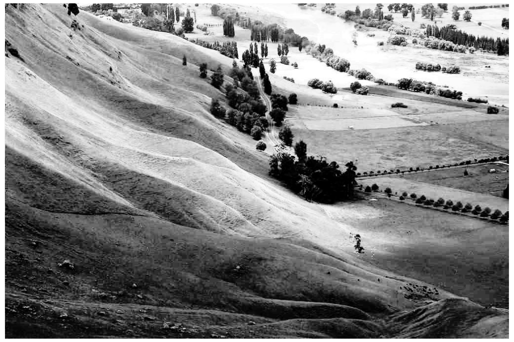


After histogram equalization

Histogram Equalization Example



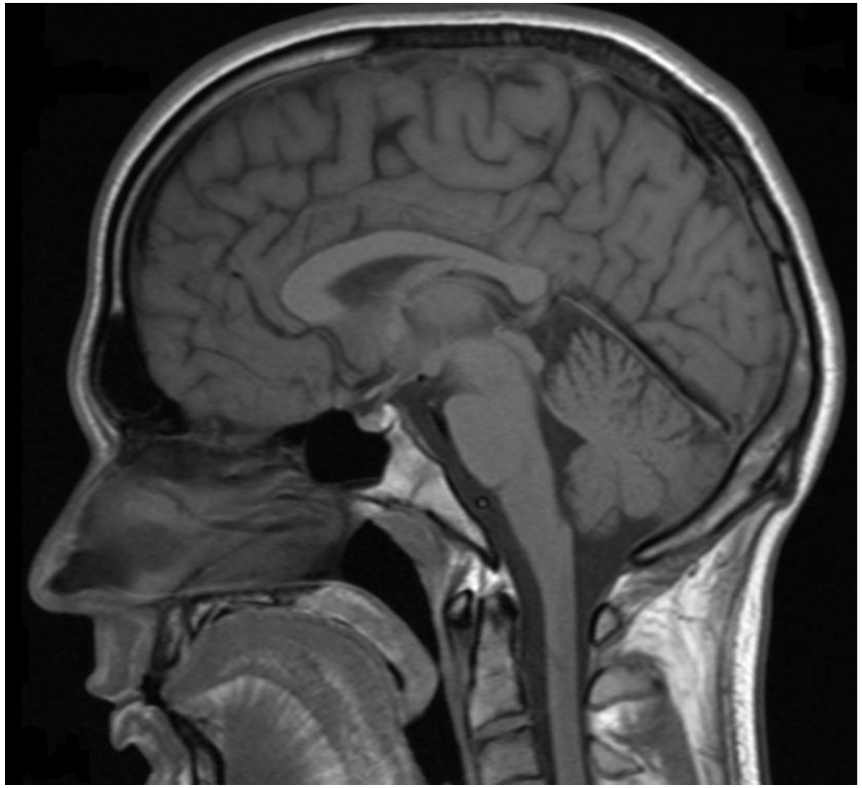




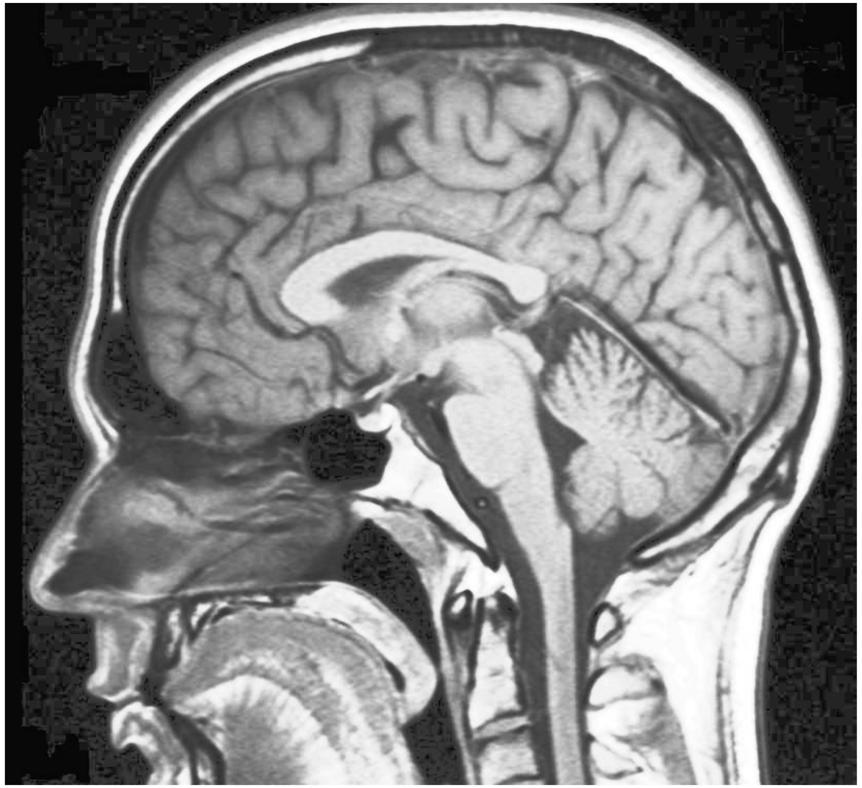
Original image

After histogram equalization

Histogram Equalization Example

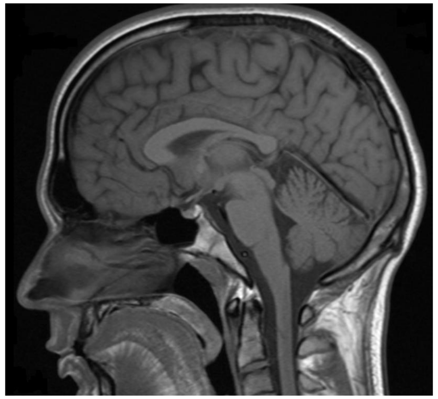


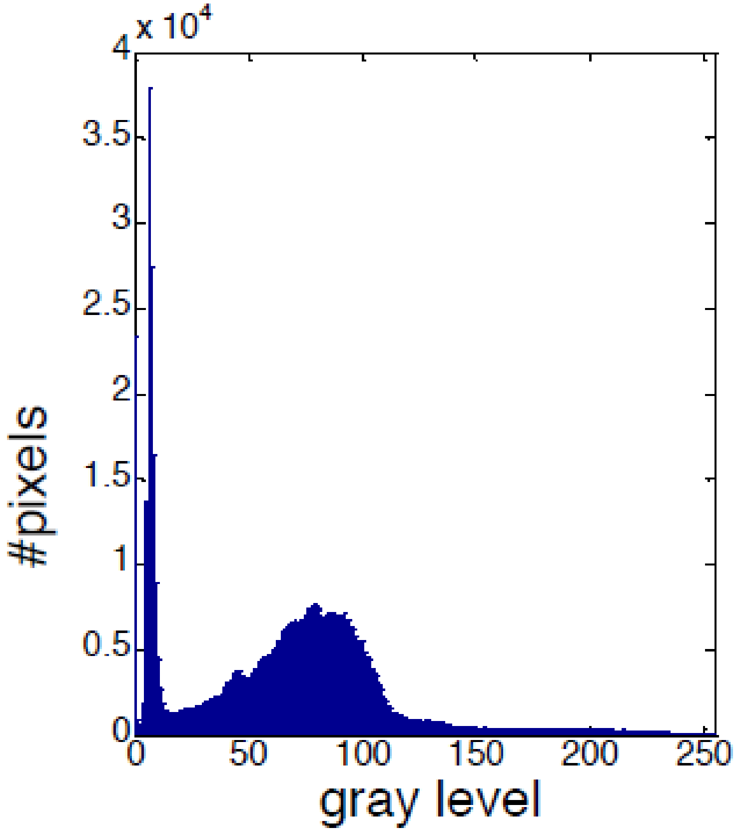
Original image



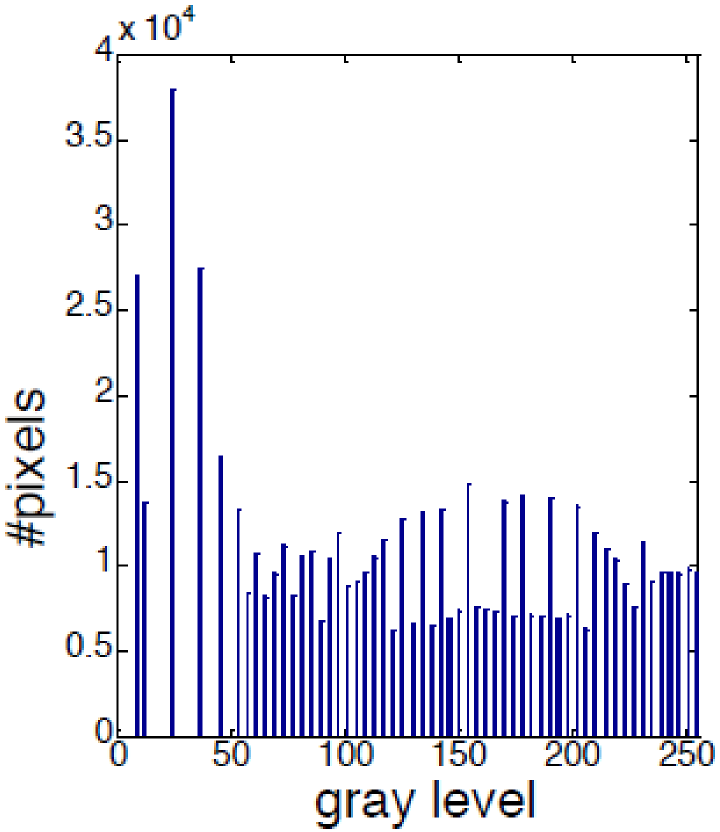
After histogram equalization

Histogram Equalization Example

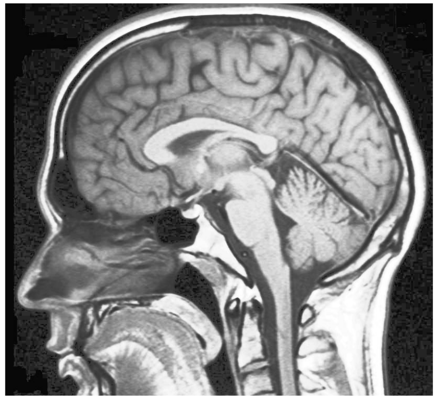




Original image

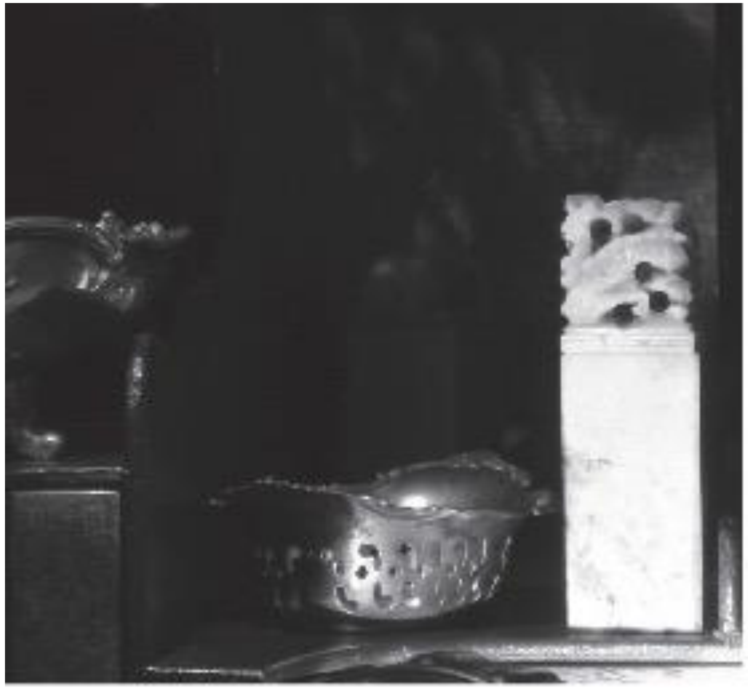


After histogram equalization

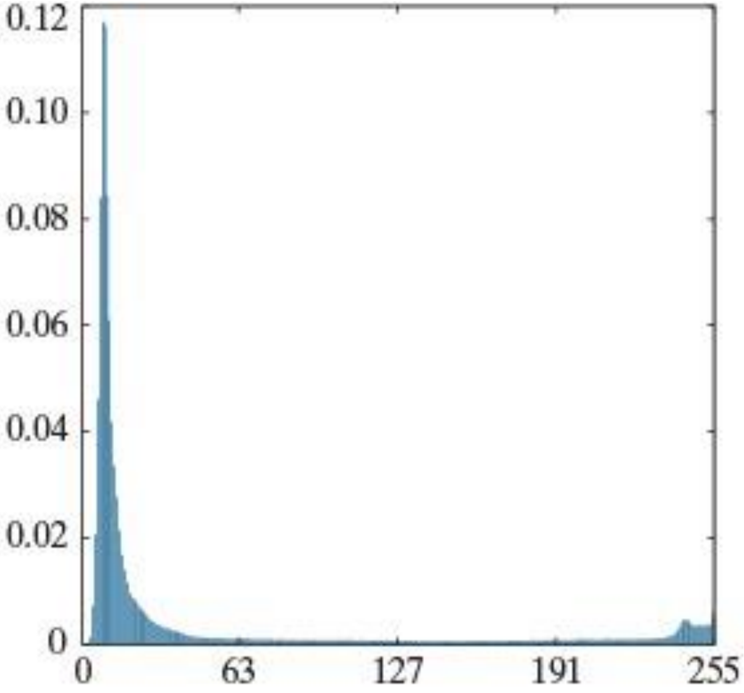


Histogram Equalization vs Specification

- ✓ Large concentration of dark pixels
- ✓ Want to make dark pixels more visible (object in the background not visible now)



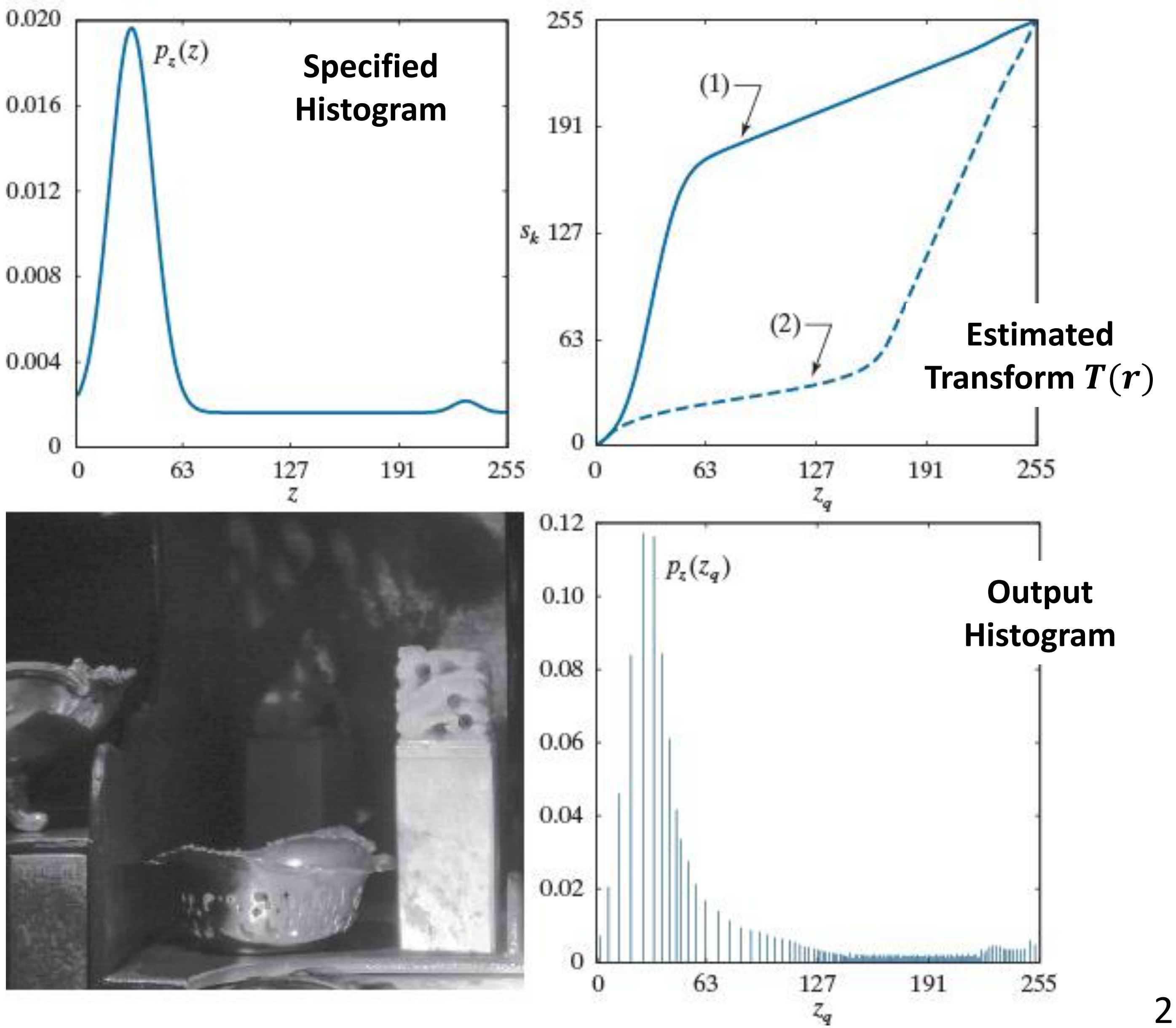
Original image



Histogram

Histogram Equalization vs Specification

- ✓ Manually specified T
- ✓ Preserves general shape of histogram but smoother transition of levels in dark region
- ✓ Tonality of image more even and noise level reduced
- ✓ Less saturation



Specification Output

Histogram Specification

$$s = T(r) = ?$$

Solving two equalization problems

Equalize input r to z

$$z = T_1(r)$$

Equalize output s to z'

$$z' = T_2(s)$$

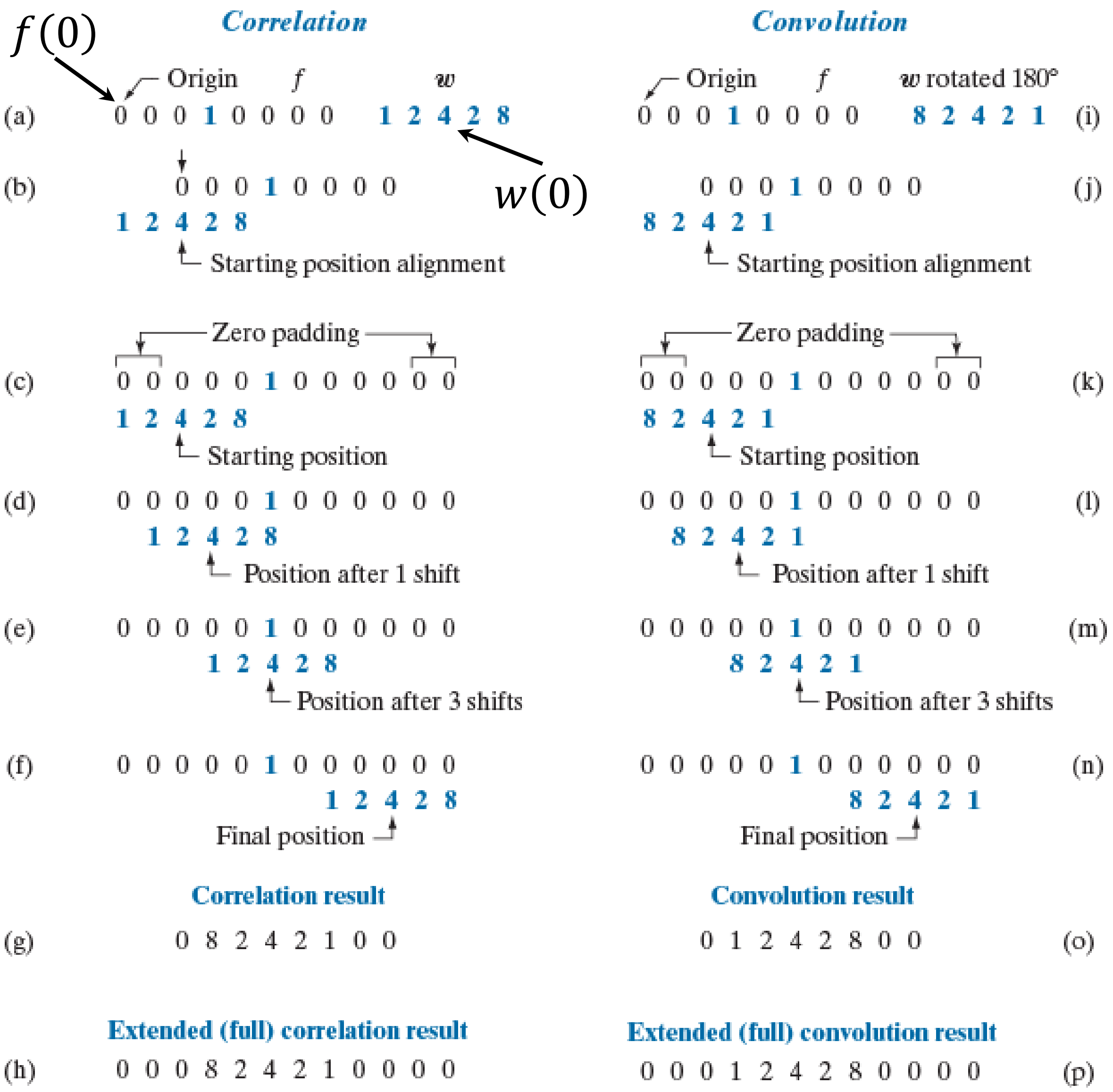
It must be that: $z = z'$

$$\Rightarrow \quad s = T_2^{-1}(T_1(r))$$

1D convolution and Correlation

$$g(x) = \sum_{s=-a}^{a} w(s)f(x+s)$$

- ✓ 1D correlation and convolution of kernel w with function f.
- ✓ They are function of variable x, which acts to displace one function with respect to other.



$$g(x) = \sum_{s=-a}^{a} w(s)f(x-s)$$

(g) or (o): center of w visited every pixel in f

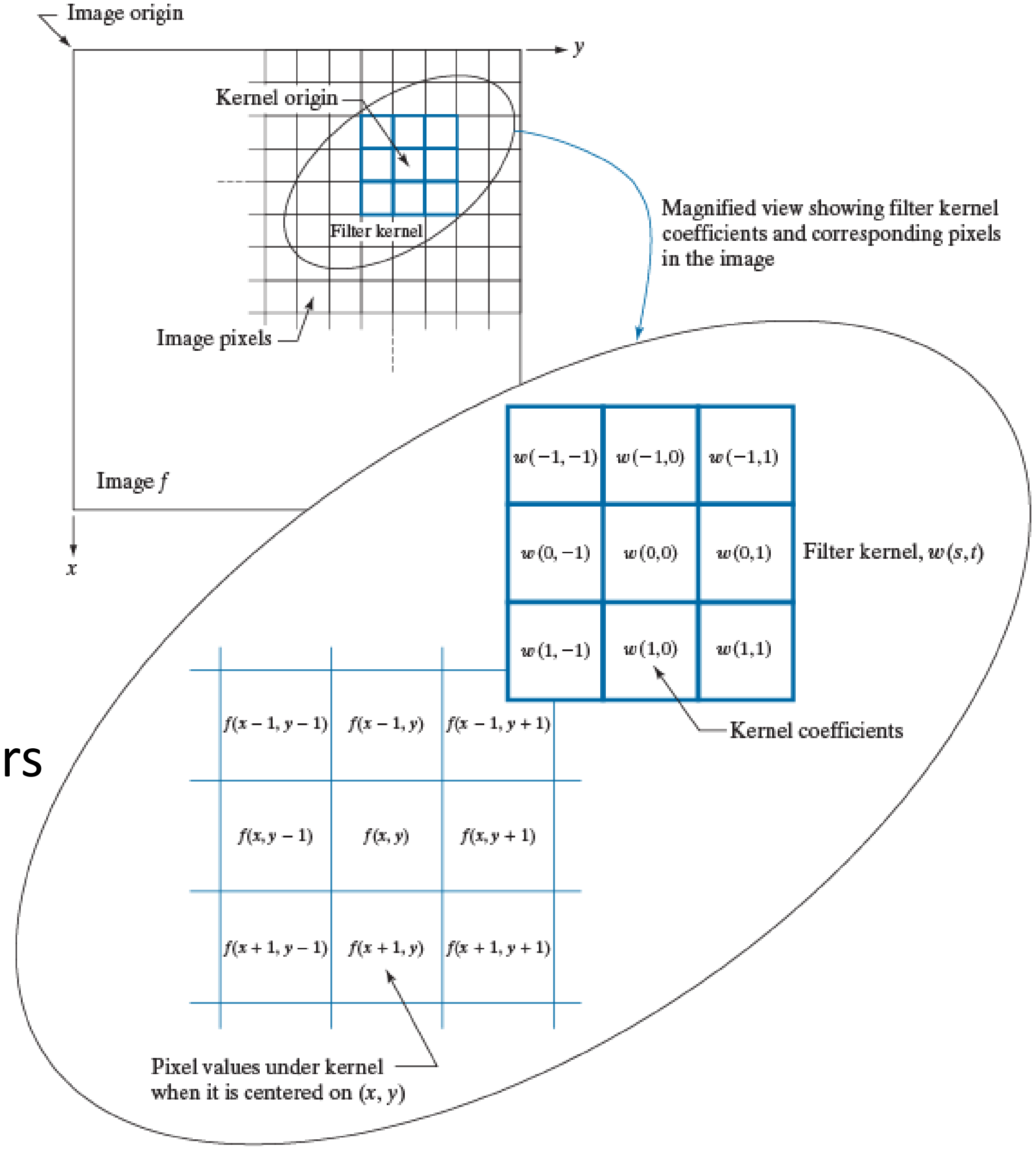
(h) or (p): every element of w visited every pixel in f (need to start with last element of w coinciding with first of f and end with last of w to last of f

Linear Spatial Filtering

$$g(x,y) = \sum_{i=-a}^{a} \sum_{j=-b}^{b} w(i,j) f(x+i, y+j)$$

$w(0,0)$ aligns with $f(x,y)$

$m = 2a + 1$ and $n = 2b + 1$ are odd integers



Convolution and Correlation

- Correlation:

  1. Move the filter mask to a location

  2. Compute the sum of products

  3. Go to 1.

$$w(x,y) \star f(x,y) = \sum_{i=-a}^{a} \sum_{j=-b}^{b} w(i,j) f(x+i, y+j)$$

- Convolution:

  1. Rotate the filter mask by 180 degrees

  2. Correlation

$$w(x,y) \ast f(x,y) = \sum_{i=-a}^{a} \sum_{j=-b}^{b} w(i,j) f(x-i, y-j)$$

Example: Convolution and Correlation

Origin $f(x, y)$

0	0	0	0	0
0	0	0	0	0
0	0	1	0	0
0	0	0	0	0
0	0	0	0	0

$w(x, y)$

1	2	3
4	5	6
7	8	9

(a)

Padded f

0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	1	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0

(b)

Initial position for w

1	2	3	0	0	0	0	0	0
4	5	6	0	0	0	0	0	0
7	8	9	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	1	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0

(c)

Full correlation result

0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	9	8	7	0	0	0
0	0	0	6	5	4	0	0	0
0	0	0	3	2	1	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0

(d)

Example: Convolution and Correlation

Full correlation result								
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	9	8	7	0	0	0
0	0	0	6	5	4	0	0	0
0	0	0	3	2	1	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0

(d)

Cropped correlation result				
0	0	0	0	0
0	9	8	7	0
0	6	5	4	0
0	3	2	1	0
0	0	0	0	0

(e)

Example: Convolution and Correlation

Origin $f(x, y)$

0	0	0	0	0
0	0	0	0	0
0	0	1	0	0
0	0	0	0	0
0	0	0	0	0

$w(x, y)$

1	2	3
4	5	6
7	8	9

(a)

Rotated w

9	8	7	0	0	0	0	0	0
6	5	4	0	0	0	0	0	0
3	2	1	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	1	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0

(f)

Full convolution result

0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	1	2	3	0	0	0
0	0	0	4	5	6	0	0	0
0	0	0	7	8	9	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0

(g)

Example: Convolution and Correlation

Cropped correlation result

0	0	0	0	0
0	9	8	7	0
0	6	5	4	0
0	3	2	1	0
0	0	0	0	0

Cropped convolution result

0	0	0	0	0
0	1	2	3	0
0	4	5	6	0
0	7	8	9	0
0	0	0	0	0

Quiz (time: 2 min)

1. DPI is defined as

   a) Number of gray levels per image

   b) Number of pixels per image

   c) Number of pixels per inch

   d) Number of gray levels per image

2. How many bits do we need for 16 gray level images:

   a) 16 b) 8 c) 4 d) 2

QUESTIONS & ANSWERS

University of Massachusetts Amherst