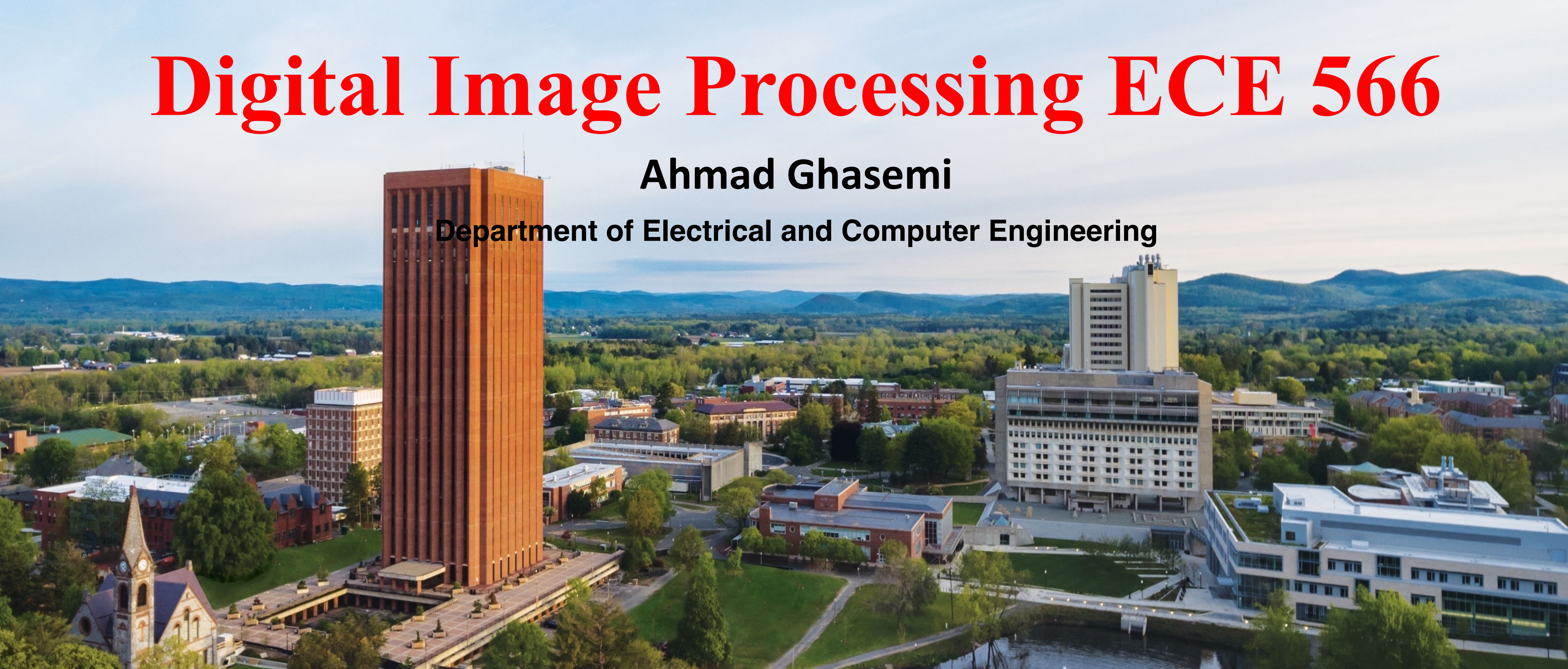
University of Massachusetts Amherst



Digital Image Processing ECE 566

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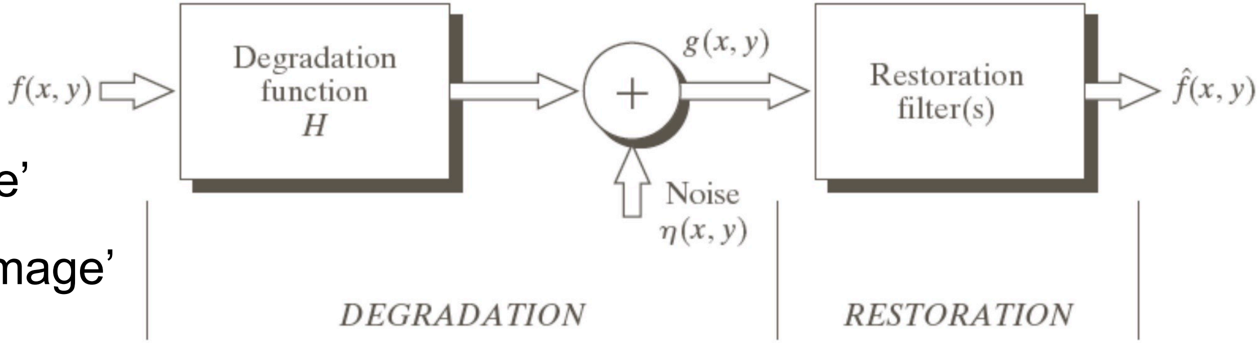
Model of Image Degradation/Restoration



- $f(x,y)$ – image before degradation, i.e., ‘true image’
- $g(x,y)$ – image after degradation, i.e., ‘observed image’
- $h(x,y)$ – degradation filter
- $f(x,y)$ – estimate of $f(x,y)$ computed from $g(x,y)$
- $n(x,y)$ – additive noise

$$g(x, y) = h(x, y) \star f(x, y) + \eta(x, y)$$

$$G(u, v) = H(u, v)F(u, v) + N(u, v)$$

Degradations

- original

- optical blur

- motion blur

- spatial quantization (discrete pixels)

- additive intensity noise

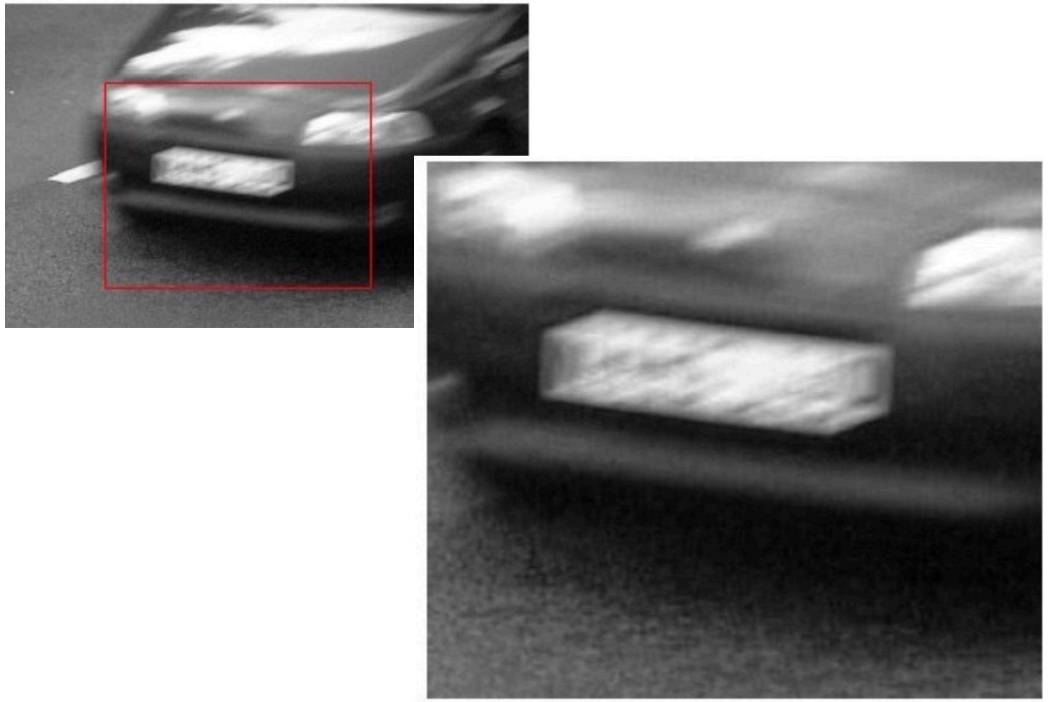
Degradations: Optical Blur

- optical blur

for out of focus blurring, model $h(x, y)$ as a Gaussian

$$H(u, v) = \mathrm{e}^{-k(u^2+v^2)^{5/6}}$$

Degradations: Uniform Linear Motion Blur



T - duration of the exposure

blurred output:

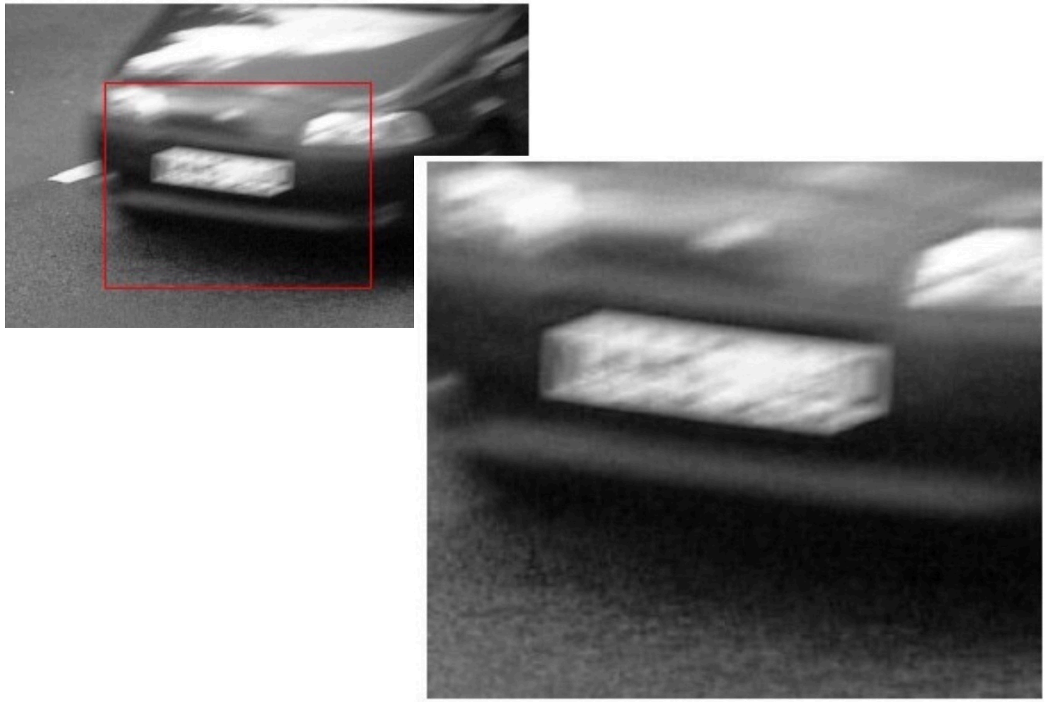
$$g(x,y) = \int_0^T f(x - x_0(t), y - y_0(t))dt$$

Degradations: Uniform Linear Motion Blur

T - duration of the exposure

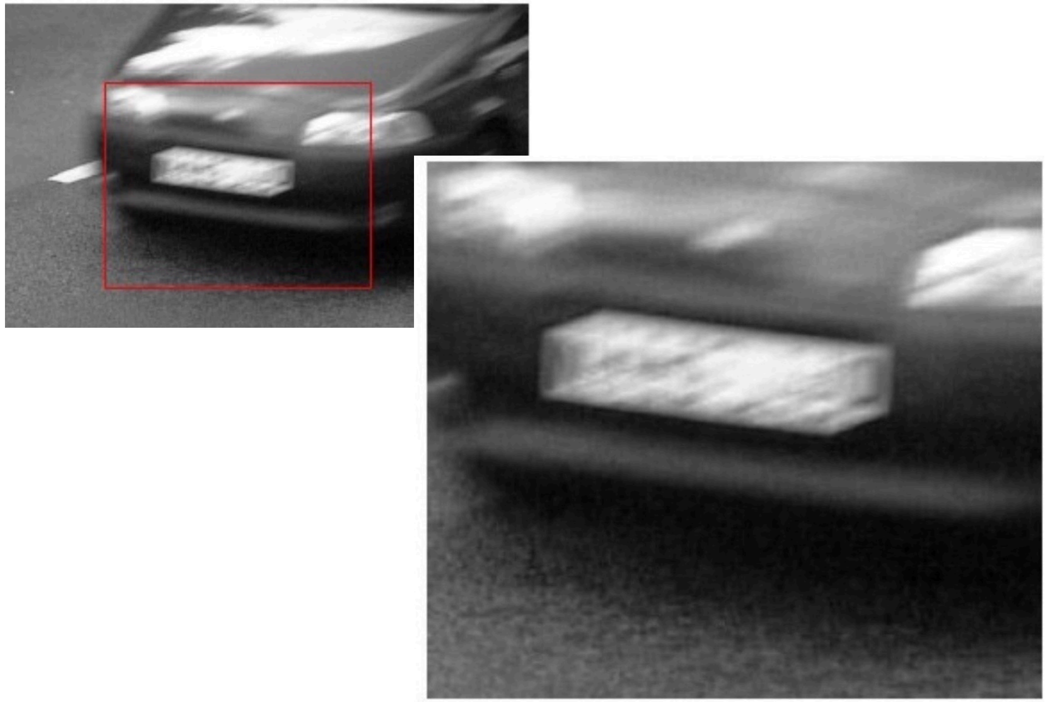
blurred output:

$$g(x, y) = \int_0^T f(x - x_0(t), y - y_0(t))dt$$



$$G(u, v) = F(u, v) \int_0^T \mathrm{e}^{-2j\pi[ux_0(t)+vy_0(t)]}dt$$

Degradations: Uniform Linear Motion Blur



T - duration of the exposure

blurred output:

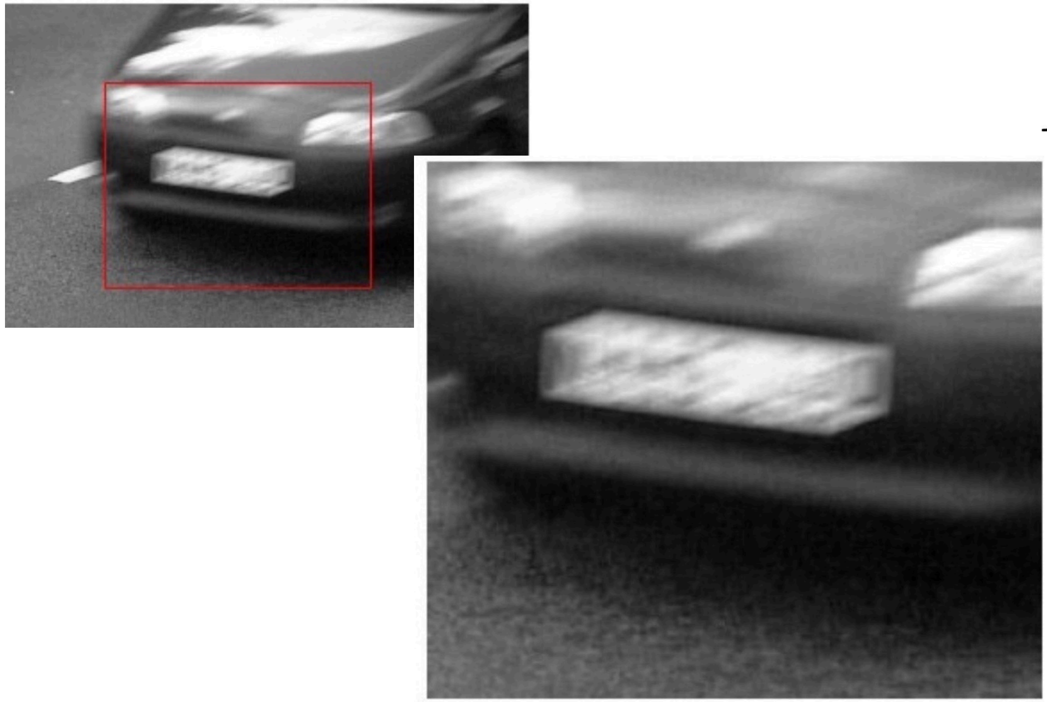
$$g(x,y)=\int_0^T f(x-x_0(t),y-y_0(t))dt$$

$$G(u,v)=F(u,v)\int_0^T \mathrm{e}^{-2j\pi[ux_0(t)+vy_0(t)]}dt$$

$$\Rightarrow \quad H(u,v)=\int_0^T \mathrm{e}^{-2j\pi[ux_0(t)+vy_0(t)]}dt$$

Degradations: Uniform Linear Motion Blur

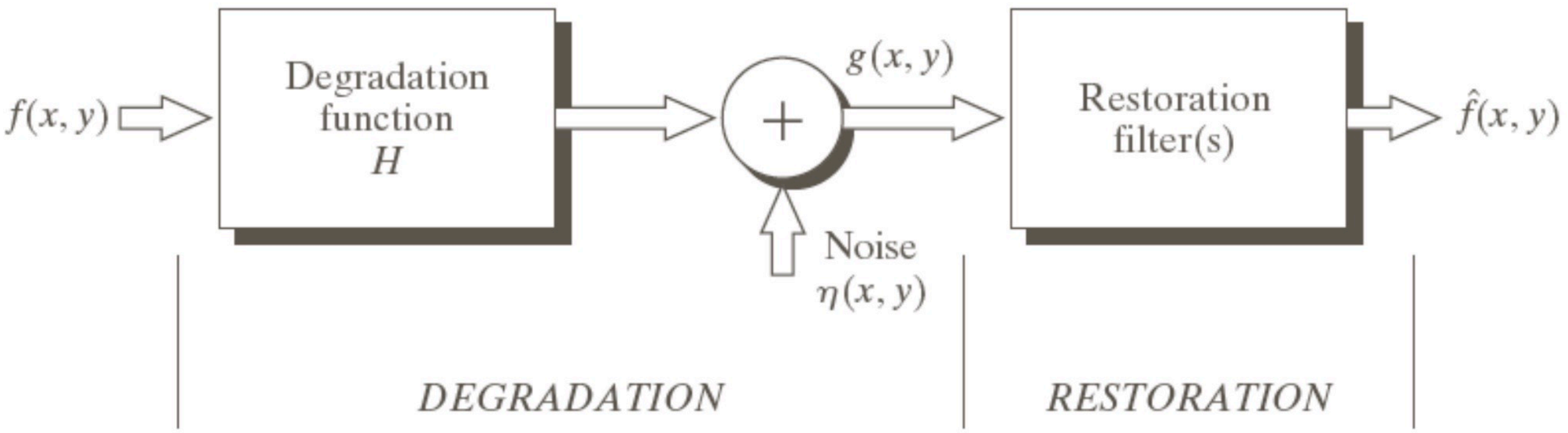
$$x_0(t) = \frac{at}{T} \qquad y_0(t) = \frac{bt}{T}$$



$$\begin{aligned} H(u,v) &= \int_0^T \mathrm{e}^{-2j\pi[ux_0(t)+vy_0(t)]}dt \\ &= \frac{T}{\pi(ua+vb)}\sin[\pi(ua+vb)]\mathrm{e}^{-j\pi(ua+ub)} \end{aligned}$$

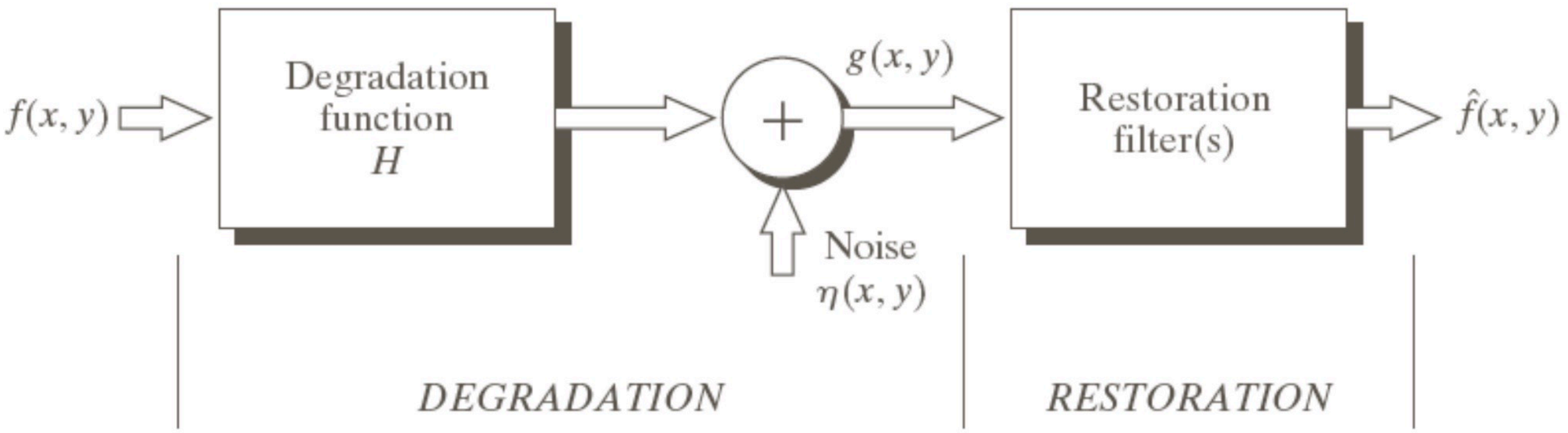
Image Restoration

Image Restoration by inverse filtering



$$g(x,y) = h(x,y) * f(x,y) + n(x,y) \;\; \Leftrightarrow \;\; G(u,v) = H(u,v)\, F(u,v) + N(u,v)$$

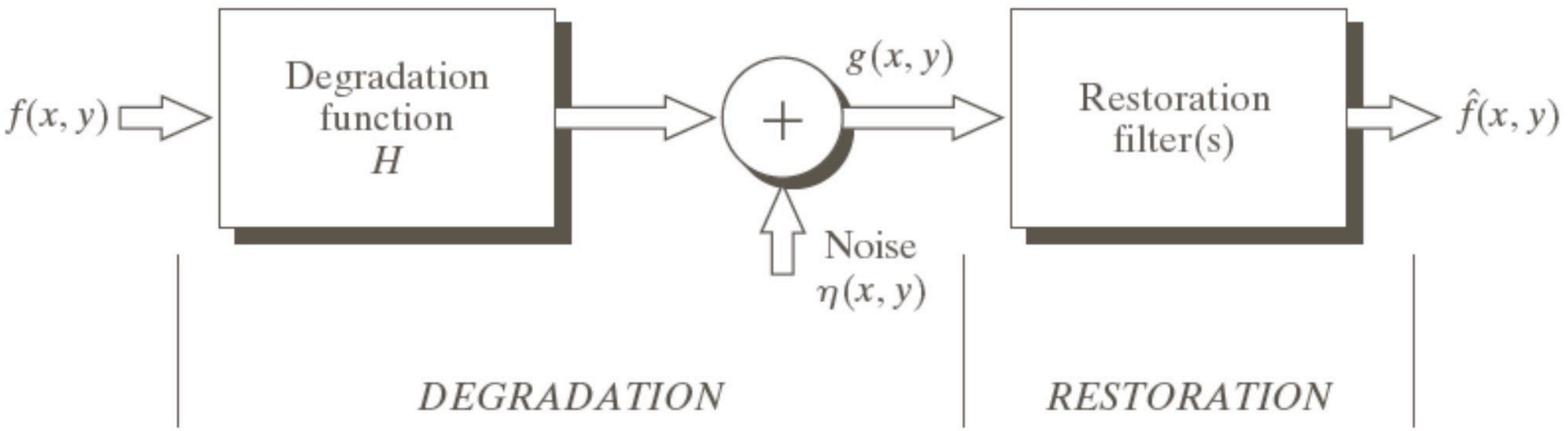
Image Restoration by inverse filtering



$$G(u,v) = H(u,v)F(u,v) + N(u,v)$$

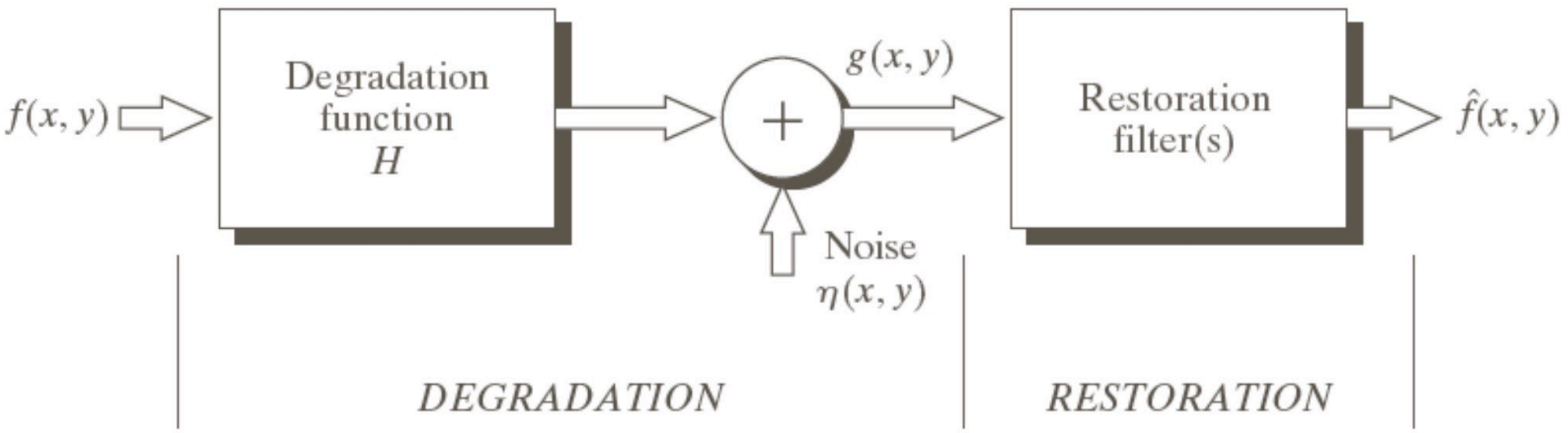
If no noise → $$\hat{F}(u,v) = \frac{G(u,v)}{H(u,v)}$$

Image Restoration by inverse filtering



$$G(u,v) = H(u,v)F(u,v) + N(u,v)$$

Image Restoration by inverse filtering



$$G(u,v) = H(u,v)F(u,v) + N(u,v)$$

$$\hat{F}(u,v) = F(u,v) + \frac{N(u,v)}{H(u,v)}$$

Image Restoration by inverse filtering

$$\hat{F}(u,v) = F(u,v) + \frac{N(u,v)}{H(u,v)}$$

Bad news:

- Even when H(u,v) is known, there is always unknown noise
- Often H(u,v) has values close to zero

Image Restoration by inverse filtering

$$\hat{F}(u,v) = F(u,v) + \frac{N(u,v)}{H(u,v)}$$

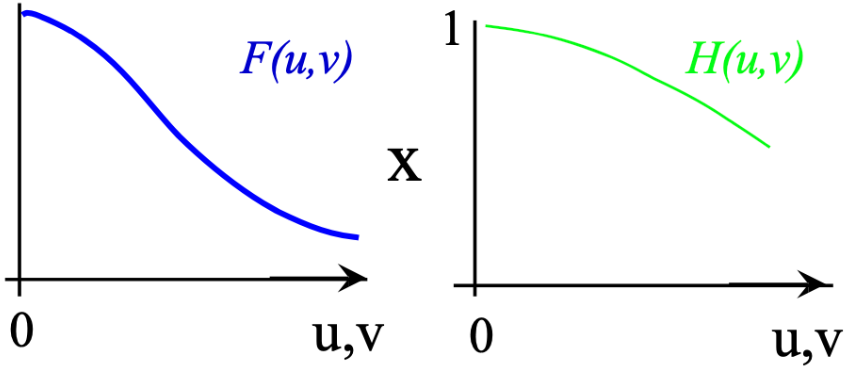


Image Restoration by inverse filtering

$$\hat{F}(u,v) = F(u,v) + \frac{N(u,v)}{H(u,v)}$$

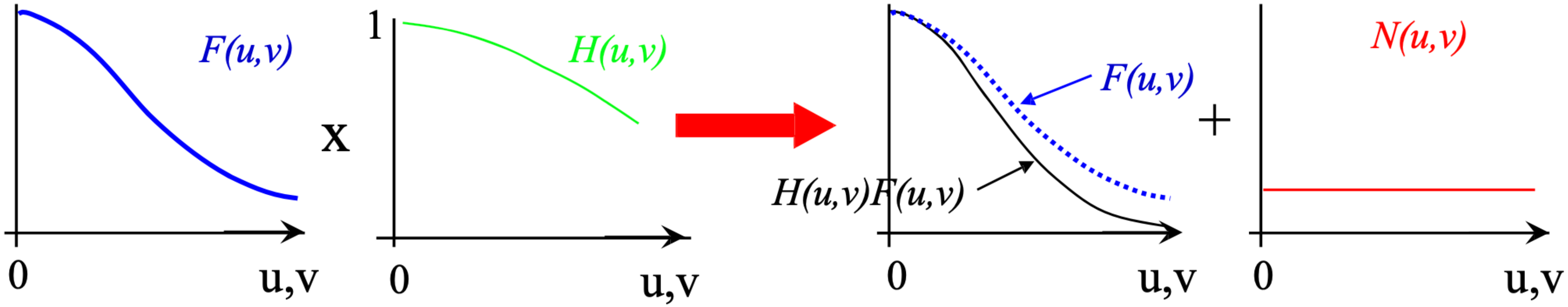


Image Restoration by inverse filtering

$$\hat{F}(u,v) = F(u,v) + \frac{N(u,v)}{H(u,v)}$$

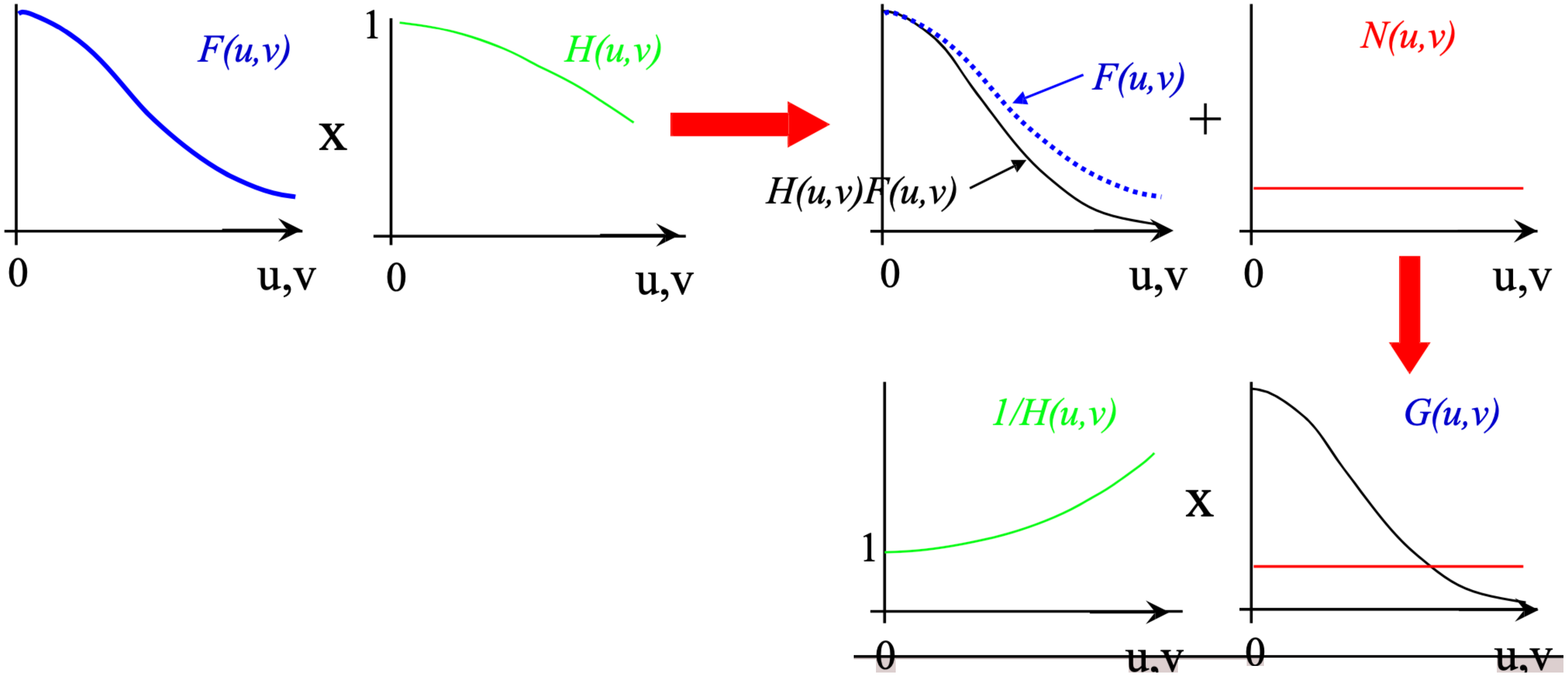


Image Restoration by inverse filtering

$$\hat{F}(u,v) = F(u,v) + \frac{N(u,v)}{H(u,v)}$$

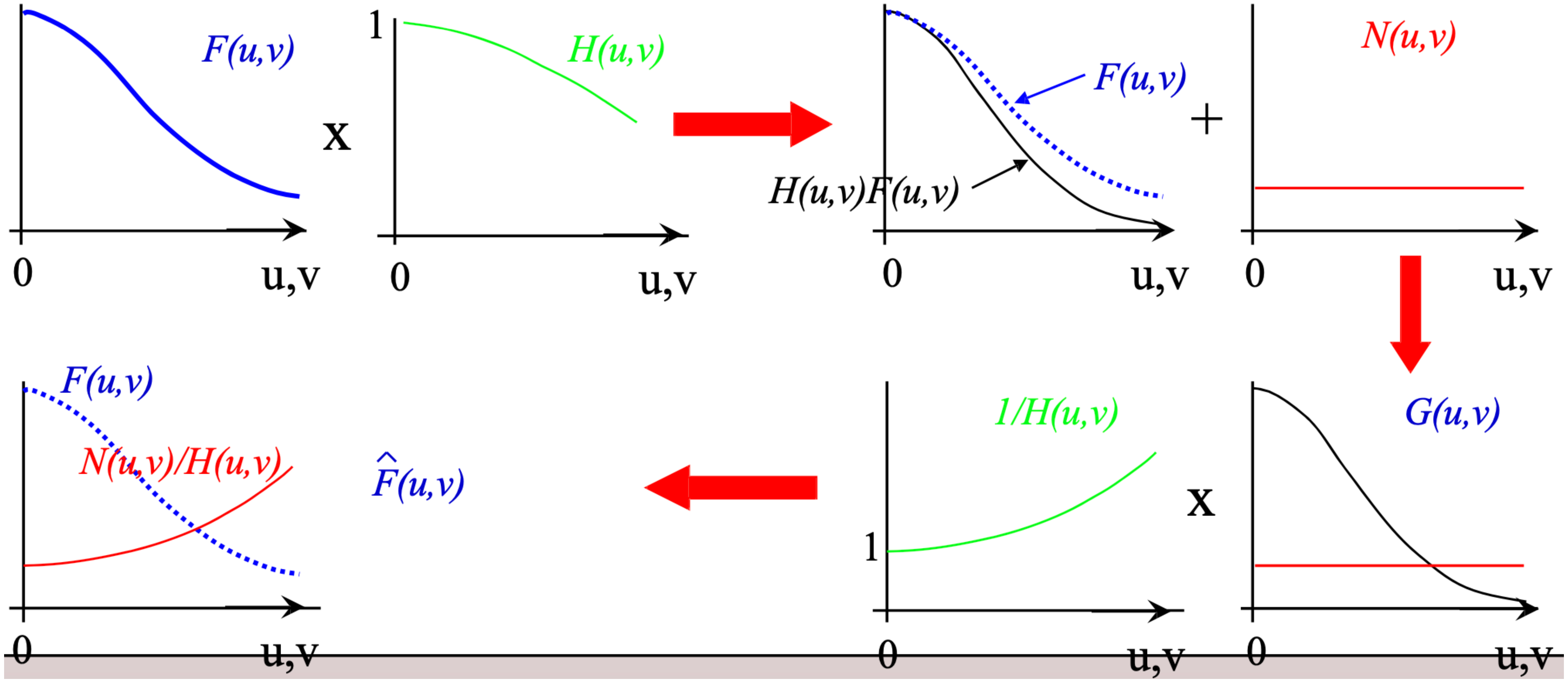


Image Restoration by inverse filtering

To mitigate the effect of zeros in the degradation function

Inverse filter with cut-off:

$$\hat{H}(u,v) = \begin{cases} 1/H(u,v), & |u^2+v^2| \leq \eta \\ 0, & |u^2+v^2| > \eta \end{cases}$$

Pseudo-inverse filter:

$$\hat{H}(u,v) = \begin{cases} 1/H(u,v), & |H(u,v)| \geq \epsilon \\ 0, & |H(u,v)| < \epsilon \end{cases}$$

Image Restoration by inverse filtering (Example)

Atmospheric turbulence effect

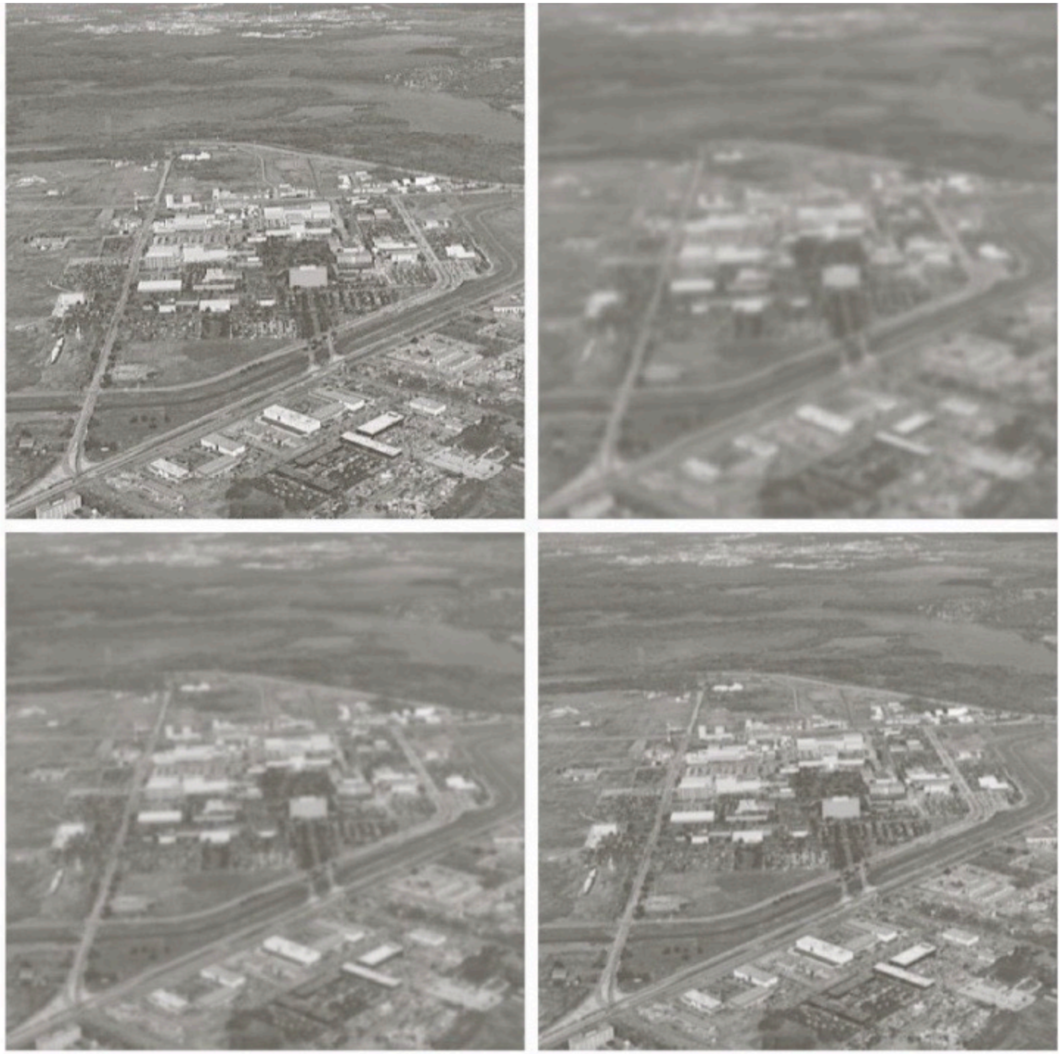


FIGURE 5.25 Illustration of the atmospheric turbulence model. (a) Negligible turbulence. (b) Severe turbulence, $k = 0.0025$. (c) Mild turbulence, $k = 0.001$. (d) Low turbulence, $k = 0.00025$. (Original image courtesy of NASA.)

Image Restoration by inverse filtering (Example)

$$\hat{F}(u,v) = G(u,v)\hat{H}(u,v)$$

$$\hat{H}(u,v) = \begin{cases} 1/H(u,v), & |u^2+v^2| \leq \eta \\ 0, & |u^2+v^2| > \eta \end{cases}$$

$$H(u,v) = e^{-k(u^2+v^2)}$$

a b
c d

FIGURE 5.27 Restoring Fig. 5.25(b) with Eq. (5.7-1). (a) Result of using the full filter. (b) Result with *H* cut off outside a radius of 40; (c) outside a radius of 70; and (d) outside a radius of 85.

$$\frac{G(u,v)}{H(u,v)}$$