# ME 597 Lab 2 Report

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# 1 Extended Kalman Filter Design

## 1.1 Motion Model

The robot motion model has the following state:

$$\begin{bmatrix} v_f \\ \psi \\ x \\ y \end{bmatrix} \tag{1}$$

Where $v_f$ is forward velocity, $\psi$ is heading, and $x$ and $y$ are position in the horizontal plane. The inputs to the system are:

$$\begin{bmatrix} V_m \\ \delta \end{bmatrix} \tag{2}$$

Where $V_m$ is motor power, and $\delta$ is steering angle.

Due to the presence of a delay, velocity is most conveniently modelled in discrete-time, and has the following transfer function:

$$G_v(z) = (\frac{KT\alpha}{\alpha T + 2})(\frac{z+1}{z + \frac{\alpha T - 2}{\alpha T + 2}})(\frac{1}{z^4}) \tag{3}$$

Where $K$ and $\alpha$ are constants which may be found experimentally, and $T$ is the sampling period of 50 ms.

The following differential equations define $\psi$, $x$, and $y$:

$$\dot{\psi} = \frac{v_f}{L} sin(\delta) \tag{4}$$

Where $L$ is wheelbase length,

$$\dot{x} = v_f cos(\psi) \tag{5}$$

and

$$\dot{y} = v_f sin(\psi) \tag{6}$$

## 1.2 Measurement Models

A simple measurement model where all states are directly measured is used. Encoder readings are differentiated to yield direct forward velocity measurements. The local position system yields direct measurements of heading and planar position. GPS can be considered to have the same model as the LPS, with heading directly estimated from successive position measurements, so long as the distance travelled between measurements is large relative to position measurement error.

The measurement model is trivially defined as follows:

$$y_k = \begin{bmatrix} v_f \\ \psi \\ x \\ y \end{bmatrix}_k \tag{7}$$

## 1.3 Measurement and Process Covariance

Placeholder.

## 1.4 Prediction

The a priori state expectation is computed from the motion model by

$$\bar{x}_k = f(\hat{x}_{k-1}, u_{k-1}) \tag{8}$$

Where $f$ may be trivially derived from equations 3 - 6. Note that, technically, equation 3 violates the Markov assumption. This could be corrected by incorporating a few input delay buffer cells in the sytem state. This has no practical impact on filter behaviour, so it is ignored for the sake of simplicity.

The a priori covariance is computed by linearizing about the current state:

$$\bar{P}_k = A_k P_{k-1} A_k^T + W_k Q_{k-1} W_k^T \tag{9}$$

Where $A_k$ is the Jacobian, $\frac{\delta f}{\delta x}|_{\hat{x}_{k-1},u_{k-1}}$:

$$\begin{bmatrix} -\frac{\alpha T-2}{\alpha T+2} & 0 & 0 & 0 \\ \frac{T}{L}sin(\delta_{k-1}) & 1 & 0 & 0 \\ Tcos(\psi_{k-1}) & -v_{f,k-1}Tsin(\psi_{k-1}) & 1 & 0 \\ Tsin(\psi_{k-1}) & v_{f,k-1}Tcos(\psi_{k-1}) & 0 & 1 \end{bmatrix} \tag{10}$$

And $W_k$ is the Jacobian of $x_k$ with respect to process noise, which is simply $I$ in our motion model.

### 1.5 Correction

The a priori state estimate is corrected by sensor measurements. First, the Kalman gain is computed:

$$K_k = \bar{P}_k H_k^T (H_k \bar{P}_k H_k^T + V_k R_k V_k^T)^{-1} \tag{11}$$

Where $H_k$ is the Jacobian of $y_k$ with respect to $x_k$, and $V_k$ is the Jacobian of $y_k$ with respect to measurement noise. For our measurement model, $H_k$ and $V_k$ are both $I$.

The a posteriori state expectation and covariance can then be computed:

$$\hat{x}_k = \bar{x}_k + K_k(y_k - \bar{x}_k) \tag{12}$$

and

$$P_k = (I - K_k H_k)\bar{P}_k \tag{13}$$

### 1.6 Simulation Results

Placeholder.

### 1.7 Empirical Results

Placeholder.

## 2 Mapping Design

As it moves, the ro model.

## 3 LIDAR Measurement Model

The LIDAR provides a sequence of range measurements $r_i$, each of which is made in a different direction, $\phi_i$:

$$r = \{r_1 \dots r_k\} \quad r_i \in [r_{min}, r_{max}] \tag{14}$$

$$\phi = \{\phi_1 \dots \phi_k\} \quad \phi_i \in [\phi_{min}, \phi_{max}] \tag{15}$$

## 4 Inverse LIDAR Measurement Model

In order to compute an occupancy grid, we must use LIDAR to determine whether or not a particular grid cell is occupied. For any given cell, there are three possibilities:

1. At least one scanline passes through the cell, but no range measurements fall within the cell.

2. At least one range measurement falls within the cell.

3. No scanlines pass through the cell.

In the first case, it can be inferred with confidence that the cell is unoccupied. In the second case, it is clear that the cell contains an object. The third case arises either when a cell is obscured by a nearby object, or when the cell is outside of the area swept by the LIDAR. In the third case, no information is about the status of the cell.

In order to evaluate the state of a cell, we find the scanline which passes closest to the centre of the cell:

$$i_{nearest} = \underset{i}{\operatorname{argmin}} \quad |\phi_i - tan^{-1}(\frac{y_c - y_r}{x_c - x_r})| \tag{16}$$