ME 597 Lab 2 Report

Iain Peet Andrei Danaila Matthew Uehara
Abdel Hamid Ahmed Salam

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1 Extended Kalman Filter Design

1.1 Motion Model

The robot motion model has the following state:

$$\begin{bmatrix} v_f \\ \psi \\ x \\ y \end{bmatrix} \tag{1}$$

Where v_f is forward velocity, ψ is heading, and x and y are position in the horizontal plane. The inputs to the system are:

$$\begin{bmatrix} V_m \\ \delta \end{bmatrix} \tag{2}$$

Where V_m is motor power, and δ is steering angle.

Due to the presence of a delay, velocity is most conveniently modelled in discrete-time, and has the following transfer function:

$$G_v(z) = (\frac{KT\alpha}{\alpha T + 2})(\frac{z+1}{z + \frac{\alpha T - 2}{\alpha T + 2}})(\frac{1}{z^4}) \tag{3}$$

Where K and α are constants which may be found experimentally, and T is the sampling period of 50 ms.

The following differential equations define ϕ, x, and y:

$$\dot{\psi} = \frac{v_f}{L} sin(\delta) \tag{4}$$

Where L is wheelbase length,

$$\dot{x} = v_f cos(\psi) \tag{5}$$

and

$$\dot{y} = v_f sin(\psi) \tag{6}$$

1.2 Measurement Model

A simple measurement model where all states are directly measured is used. Encoder readings are differentiated to yield direct forward velocity measurements, and the local position system yields direct measurements of heading and planar position. This measurement can also be reasonably used with GPS by estimating heading from successive position measurements, so long as the distance travelled between measurements is large relative to position measurement error.

The measurement model is defined as follows:

$$y_k = \begin{bmatrix} v_f \\ \psi \\ x \\ y \end{bmatrix}_k \tag{7}$$

1.3 Prediction

The a priori state expectation is computed from the motion model by

$$\bar{x}_k = f(\hat{x}_{k-1}, u_{k-1}) \tag{8}$$

Where f may be trivially derived from equations 3 - 6. Note that, technically, equation 3 violates the Markov assumption. This could be corrected by incorporating a few input delay buffer cells in the sytem state. This has no actual impact on filter behaviour, so it is ignored for the sake of simplicity.

The a priori covariance is computed by linearizing about the current state:

$$\bar{P}_k = A_k P_{k-1} A_k^T + W_k Q_{k-1} W_k^T \tag{9}$$

Where A_k is the Jacobian, $\frac{\delta f}{\delta x}|_{\hat{x}_{k-1}, u_{k-1}}$:

$$\begin{bmatrix} -\frac{\alpha T-2}{\alpha T+2} & 0 & 0 & 0 \\ \frac{T}{L} sin(\delta_{k-1}) & 1 & 0 & 0 \\ T cos(\psi_{k-1}) & -v_{f,k-1} T sin(\psi_{k-1}) & 1 & 0 \\ T sin(\psi_{k-1}) & v_{f,k-1} T cos(\psi_{k-1}) & 0 & 1 \end{bmatrix} \tag{10}$$

And W_k is the Jacobian of x_k with respect to process noise, which is simply I in our motion model.

1.4 Correction

The a priori state estimate is corrected by sensor measurements. First, the Kalman gain is computed:

$$K_k = \bar{P}_k H_k^T (H_k \bar{P}_k H_k^T + V_k R_k V_k^T)^{-1} \tag{11}$$

Where H_k is the Jacobian of y_k with respect to x_k, and V_k is the Jacobian of y_k with respect to measurement noise. For our measurement model, H_k and V_k are both I.

The a posteriori state expectation and covariance can then be computed:

$$\hat{x}_k = \bar{x}_k + K_k(y_k - \bar{x}_k) \tag{12}$$

and

$$P_k = (I - K_k H_k)\bar{P}_k \tag{13}$$