module *SlidingPuzzles*

extends *Integers*

variable *board*

$W \triangleq 4$
$H \triangleq 5$
$Pos \triangleq (0 \,..\, W-1) \times (0 \,..\, H-1)$
$Piece \triangleq$ subset Pos

$Klotski \triangleq \{\{\langle 0, 0\rangle, \langle 0, 1\rangle\},$
$\{\langle 1, 0\rangle, \langle 2, 0\rangle, \langle 1, 1\rangle, \langle 2, 1\rangle\},$
$\{\langle 3, 0\rangle, \langle 3, 1\rangle\}, \{\langle 0, 2\rangle, \langle 0, 3\rangle\},$
$\{\langle 1, 2\rangle, \langle 2, 2\rangle\}, \{\langle 3, 2\rangle, \langle 3, 3\rangle\},$
$\{\langle 1, 3\rangle\}, \{\langle 2, 3\rangle\}, \{\langle 0, 4\rangle\}, \{\langle 3, 4\rangle\}\}$

$KlotskiGoal \triangleq \{\langle 1, 3\rangle, \langle 1, 4\rangle, \langle 2, 3\rangle, \langle 2, 4\rangle\} \in board$

$ChooseOne(S, P(\_)) \triangleq$ choose $x \in S : P(x) \land \forall\, y \in S : P(y) \Rightarrow y = x$

$TypeOK \triangleq board \in$ subset $Piece$

Given a position and a set of empty positions return a set of appropriately filtered von *Neumann* neighborhood points

$dir(p, es) \triangleq$ let $dir \triangleq \{\langle 1, 0\rangle, \langle 0, 1\rangle, \langle -1, 0\rangle, \langle 0, -1\rangle\}$
in $\{d \in dir : \land\, \langle p[1] + d[1], p[2] + d[2]\rangle \in Pos$
$\land\, \langle p[1] + d[1], p[2] + d[2]\rangle \notin es\}$

Given a position and a unit translation vector return a pair of pieces, before and after translation in opposite this vector direction

$move(p, d) \triangleq$ let $s \triangleq \langle p[1] + d[1], p[2] + d[2]\rangle$
$pc \triangleq ChooseOne(board,$ lambda $pc : s \in pc)$
in $\langle pc, \{\langle q[1] - d[1], q[2] - d[2]\rangle : q \in pc\}\rangle$

Given specific free position and a set of all free positions return a set of boards updated by moving appropriate pieces to that free position

$update(e, es) \triangleq$ let $dirs \triangleq dir(e, es)$
$moved \triangleq \{move(e, d) : d \in dirs\}$
$free \triangleq \{\langle pc, m\rangle \in moved :$
$\land\, m \cap$ (union $(board \setminus \{pc\})) = \{\}$
$\land\, \forall\, p \in m : p \in Pos\}$
in $\{(board \setminus \{pc\}) \cup \{m\} : \langle pc, m\rangle \in free\}$

$Init \triangleq board = Klotski$

$Next \triangleq$ let $empty \triangleq Pos \setminus$ union $board$
in $\exists\, e \in empty : board' \in update(e, empty)$