MODULE *CigaretteSmokers*

A specification of the cigarette smokers problem, originally described in 1971 by *Suhas Patil*. https://*en.wikipedia.org*/wiki/*Cigarette_smokers_problem*

EXTENDS *Integers*, *FiniteSets*

CONSTANT *Ingredients*, *Offers*
VARIABLE *smokers*, *dealer*

'Ingredients' is a set of ingredients, originally {*matches*, *paper*, *tobacco*}. 'Offers' is a subset of subsets of ingredients, each missing just one ingriedent

ASSUME $\land\ Offers \subseteq (\text{SUBSET } Ingredients)$
$\land\ \forall\, n \in Offers : Cardinality(n) = Cardinality(Ingredients) - 1$

'smokers' is a function from the ingredient the smoker has infinite supply of, to a BOOLEAN flag signifying smoker's state (smoking/not smoking) 'dealer' is an element of 'Offers', or an empty set

$$\begin{aligned} TypeOK \triangleq\ & \land\ smokers \in [Ingredients \to [smoking : \text{BOOLEAN }]] \\ & \land\ dealer \quad \in Offers \lor dealer = \{\} \end{aligned}$$

$$vars \triangleq \langle smokers,\ dealer \rangle$$

$$ChooseOne(S,\ P(\_)) \triangleq \text{CHOOSE } x \in S : P(x) \land \forall\, y \in S : P(y) \Rightarrow y = x$$

$$\begin{aligned} Init \triangleq\ & \land\ smokers = [r \in Ingredients \mapsto [smoking \mapsto \text{FALSE}]] \\ & \land\ dealer \in Offers \end{aligned}$$

$$\begin{aligned} startSmoking \triangleq\ & \land\ dealer \neq \{\} \\ & \land\ smokers' = [r \in Ingredients \mapsto [smoking \mapsto \{r\} \cup \\ & \qquad\qquad\qquad\qquad dealer = Ingredients]] \\ & \land\ dealer' = \{\} \end{aligned}$$

$$\begin{aligned} stopSmoking \triangleq\ & \land\ dealer = \{\} \\ & \land \text{LET } r \triangleq ChooseOne(Ingredients, \\ & \qquad\qquad\qquad\qquad \text{LAMBDA } x : smokers[x].smoking) \\ & \quad \text{IN} \quad smokers' = [smokers \text{ EXCEPT } ![r].smoking = \text{FALSE}] \\ & \land\ dealer' \in Offers \end{aligned}$$

$$Next \triangleq startSmoking \lor stopSmoking$$

$$Spec \triangleq Init \land \Box[Next]_{vars}$$
$$FairSpec \triangleq Spec \land \text{WF}_{vars}(Next)$$

An invariant checking that at most one smoker smokes at any particular moment

$$AtMostOne \triangleq Cardinality(\{r \in Ingredients : smokers[r].smoking\}) \leq 1$$