ASTR 400B Homework 7: Orbit Integration

Due: March 29th 2018

For this assignment you are going to write a code that will predict the future trajectory of M33 as it orbits M31. You will need the COM orbit files for M33 and M31 that you computed in Homework 6 so you can plot their positions and velocities as a function of time and compare them to the analytic solutions. See Patel, Besla, Sohn 2017a (`https://arxiv.org/abs/1609.04823`) for more details and background on M33's past orbital trajectory.

Note that G = 4.498768e-6 in units of $\mathrm{kpc}^3/\mathrm{M}_\odot/\mathrm{Gyr}$ (define this as a global variable, but let's forget about storing the units.)

1 M33AnalyticOrbit

Create a Class called *M33AnalyticOrbit*.

In this class we will create a series of functions that will determine the acceleration M33 feels from M31 and integrate its current position and velocity forwards in time.

Initialize the class (*def __init__*), taking as input a filename for the file in which you will store the integrated orbit. At the beginning of the class, also initialize the following quantities:

- self.x, self.y, self.z: the COM position vector of M33 relative to M31 calculated using the disk particles of both galaxies at Snapshot 0 (location of M33 today) from Assignment 4
- self.vx, self,vy, self.vz: the COM velocity vector of M33 relative to M31 as calculated in Assignment 4
- self.rd and self.Mdisk: the disk scale length and the disk mass (set self.rd=5 kpc and use the disk mass computed in Assignment 3
- self.rbulge and self.Mbulge: set self.rbulge=1 kpc and use the bulge mass from Assignment 3
- self.rhalo and self.Mhalo: for self.rhalo, use the Hernquist scale length (a) computed in Assignment 5 and use the halo mass from Assignment 3

2 Define the Acceleration Terms

Define functions that will compute the gravitational acceleration from 3 components of the M31 galaxy: Halo, Bulge and Disk.

2.1 Halo and Bulge Acceleration

1. The gravitational acceleration induced by a Hernquist profile in the x direction is given by:

$$a_x = -\frac{GM}{r(r_a + r)^2}\mathbf{x} \tag{1}$$

   where r_a is the scale length and M is the total halo or bulge mass. If you wanted the **y** direction, the bold faced x would be replaced with y.

2. Define a function **HenquistAccel** that takes 7 inputs: self, M, r_a, x,y,z coordinates and a dummy variable that indicates which component of the acceleration (x,y,z) you are after.

3. This function returns the acceleration from a Hernquist potential in the direction of the input dummy variable.

4. This function will be used for both the halo and the bulge of M31 (set by the input M and r_a

2.2 Disk Acceleration

1. For the disk of M31 we will use an approximation that mimics the exponential disk profile at distances far from the disk. It is called a Miyamoto-Nagai 1975 profile. The potential for this profile is:

$$\Phi(r) = \frac{-GM_{disk}}{\sqrt{R^2 + (r_d + \sqrt{z^2 + z_d^2})^2}} \tag{2}$$

   The acceleration for the x or y component is thus given by:

$$a_x = -\frac{GM_{\text{disk}}}{(R^2 + B^2)^{1.5}}\mathbf{x} \qquad B = r_d + \sqrt{z^2 + z_d^2} \tag{3}$$

   Where $R = \sqrt{x^2 + y^2}$ and r_d is the disk scale length and z_d is the disk scale height. For the y component of the acceleration, the form is the same, just replacing **x** with **y**. The acceleration in the z component is different:

$$a_z = -\frac{GM_{\text{disk}}B}{(R^2 + B^2)^{1.5}\sqrt{z^2 + z_d^2}}\mathbf{z} \tag{4}$$

2. Define a function **MiyamotoNagaiAccel** that takes 7 inputs: self, M,rd,x,y,z coordinates and a dummy variable that indicates which component of the acceleration (x,y,z) you are after.

3. Note that you will have to define $z_d = self.r_d/5$

4. You will need to direct the code to compute and return the appropriate component of the acceleration, depending on the dummy variable given.

2.3 M31Acceleration

Define a new function **M31Accel** that sums all acceleration terms from each galaxy component. The function takes as input the 3D position (x,y,z) and a dummy variable that indicates the component of the acceleration you are interested in (x,y, or z).

3 Build an Integrator

We need to solve the orbit of M33 by integrating the equation of motion forwards in time.

$$\ddot{\mathbf{r}} = \frac{d}{d\mathbf{r}}\Big[\Phi_{halo}(\mathbf{r}) + \Phi_{bulge}(\mathbf{r}) + \Phi_{disk}\mathbf{r}\Big] \tag{5}$$

Where Φ represents the potential for each galaxy component. To do this we will adopt a variant of the "Leap Frog" integration scheme, treating M33 as a point mass. Define a function **LeapFrog** that takes as input:

- a time interval for integration (Δt)
- a starting position vector x,y,z for the M33 COM position
- a starting velocity vector vx, vy, vz

Update the positions and velocities using standard kinematic equations, according to the following:

1. Predict the 3D position of M33's center of mass (COM) at the middle of the timestep Δt using the current COM velocity and position according to

$$\mathbf{x}_{n+\frac{1}{2}} = \mathbf{x}_n + \mathbf{v}_n\frac{\Delta t}{2} \tag{6}$$

   Do this for each component of the position (x,y,z) using the appropriate velocity component.

2. Then the COM position and velocity is advanced a full time step using the acceleration at the 1/2 timestep according to

$$\mathbf{v}_{n+1} = \mathbf{v}_n + a_{n+\frac{1}{2}}\Delta t \tag{7}$$

$$\mathbf{r}_{n+1} = r_n + \frac{1}{2}\Big[\mathbf{v}_n + \mathbf{v}_{n+1}\Big]\Delta t \tag{8}$$

Where the acceleration (a_n) is determined by calling *self.M31Accel*. Note that Δt can be positive or negative because Leap Frog integrators are symplectic, meaning they can be used for calculations that run both forward and backward in time. For this assignment, you'll want positive values since we are calculating future orbits.

4 Integrate the Orbit

Now we will loop over the LeapFrog integrator to solve the equations of motion and compute the future orbit of M33 for 10 Gyr into the future.

- Define a function **OrbitIntegrator** that takes as input: self, a starting time t_o, a time interval Δt and a final time t_{max}.
- Supply the starting COM position and velocities of M33 **relative to M31**.You will want to define new variables x,y,z,vx,vy,vz and set them equal to self.x, self.y, self.z, etc. at the top of the function so they can be updated as you integrate.
- Define a vairable t and start the integration at t_o. Continue looping (i.e. a while loop) over **LeapFrog** until you reach t_{max}, updating the time, positions and velocities along the way. Store the results in an array that you initialize outside the loop (as you did when you calculated the COM orbits from the simulations). Don't forget to store the initial positions and velocities before you begin the loop.
- Once the loop is complete, store the array into a file, like in Homework 6.

5 Analysis

1. Create a plot of your predicted M33 orbit from $t_o = 0$ Gyr to $t_{max} = 10$ Gyr. Start with 0.5 Gyr intervals for Δt and refine once you know the code is working. Overplot the solution to Assignment 6 for M33's orbit with respect to M31 from the simulation. Do this for both the total position and total velocity as a function of time.
2. How do the plots compare?
3. What missing physics could make the difference?
4. The MW is missing in these calculations. How might you include its effects?