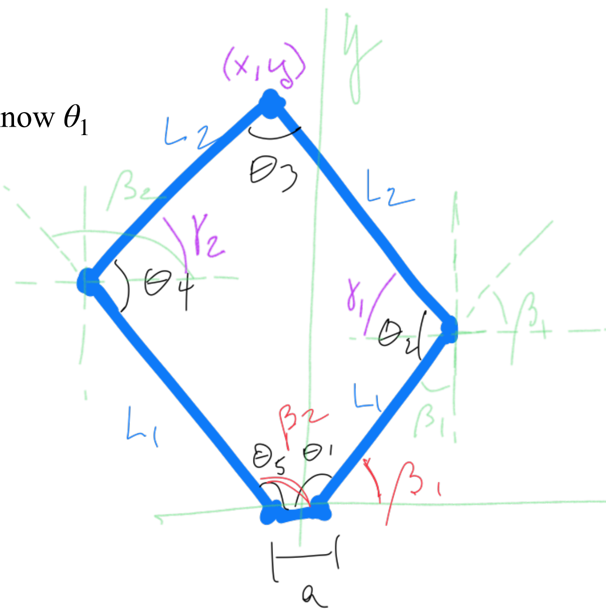
Some hints on doing the forward kinematics

2/19/21

So you can do this in Onshape (see link), or SimScape in Matlab or in LabVIEW if you have the Robotics plugin (which we do). This one is also easy enough that you can do it analytically as well.

Note that you have two sets of equal length beams (L_1 and L_2) and you control the two motor angles (β_1 and β_2). Everything else you should be able to derive from simple geometry and remembering that the angles in a polygon add up to:

$$\sum \theta_i = (n-2)\pi$$



Where n is the number of sides (5 in our case). So clearly you know θ_1 and θ_5 (they are related to (β_1 and β_2). So that is three equations and 5 unknowns. We need two more equations and those were the ones I was showing in class. We know that the position (x,y) must be the same if I get to it from the left or the right arm. So I know, for instance,

$$x = \frac{a}{2} + L_1 cos(\beta_1) - L_2 cos(\gamma_1)$$

And

$$y = L_1 sin(\beta_1) + L_2 sin(\gamma_1)$$

I can do the same thing on the left side as well with $\hat{\beta}_2 = \pi - \beta_2$:

$$x = -\frac{a}{2} - L_1 cos(\hat{\beta}_2) + L_2 cos(\gamma_2)$$

$$y = L_1 sin(\hat{\beta}_2) + L_2 sin(\gamma_2)$$

I now can combine the two equations for x and y to get:

$$x = \frac{a}{2} + L_1 cos(\beta_1) - L_2 cos(\gamma_1) = -\frac{a}{2} - L_1 cos(\hat{\beta}_2) + L_2 cos(\gamma_2)$$

$$y = L_1 sin(\beta_1) + L_2 sin(\gamma_1) = L_1 sin(\hat{\beta}_2) + L_2 sin(\gamma_2)$$

Getting everything you know over to one side….

$$\frac{a}{L_2} + \frac{L_1}{L_2}(cos\beta_1 + cos\hat{\beta}_2) = cos\gamma_1 + cos\gamma_2 = B$$

$$\frac{L_1}{L_2}(sin\beta_1 - sin\hat{\beta}_2) = sin\gamma_2 - sin\gamma_1 = A$$

Or if we assume $\hat{\gamma}_1 = -\gamma_1$

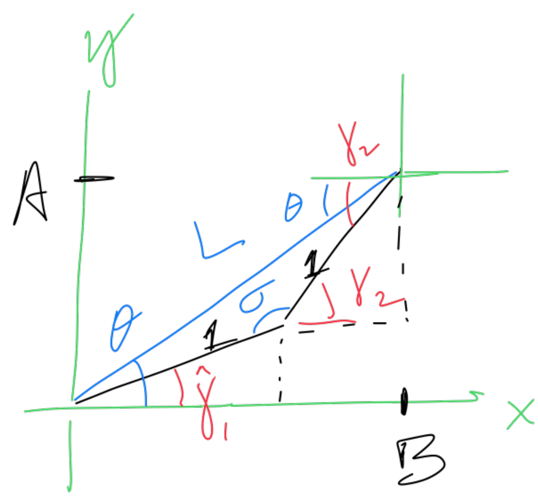
$$sin\hat{\gamma}_1 + sin\gamma_2 = A$$

$$cos\hat{\gamma}_1 + cos\gamma_2 = B$$

Which we can solve through a fun little graphical trick by building two unit triangles.

$$L^2 = A^2 + B^2$$

$$\theta = atan2(A/B)$$



$$L^2 = 1^2 + 1^2 - 2cos(\sigma)$$

So, solving for the angle

$$\sigma = atan2(\frac{\sqrt{1-C^2}}{C})\ with\ \ C = \frac{2-L^2}{2}$$

And we know the sum of all internal angles must equal pi:

$$(\theta - \hat{\gamma}_1) + \sigma + (\gamma_2 - \theta) = \pi \quad or \quad \gamma_1 + \sigma + \gamma_2 = \pi$$

(Remembering that $\hat{\gamma}_1 = -\gamma_1$). And we also know that since both sides of the triangle are 1, then the corresponding angles must be equal:

$$\theta - \hat{\gamma}_1 = \gamma_2 - \theta\ or\ \gamma_2 + \hat{\gamma}_1 = 2\theta\ or\ \gamma_2 - \gamma_1 = 2\theta$$

Combining gives me:

$$\gamma_2 - 2\theta + \sigma + \gamma_2 = \pi \quad or \quad \gamma_2 = \frac{\pi + 2\theta - \sigma}{2}$$

And

$$\gamma_1 = \gamma_2 - 2\theta$$

Gives us all the angles we need to position the arms.