Lab 2: Wave-Particle Duality

PHYS4321 – Advanced Lab I, Spring 2025

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Abstract

This report chronicles the application of experiments described in three manuals [1, 2, 3], which demonstrate the wave-particle duality of electrons. These experiments, by allowing the measurement of properties of electrons that meet both particle and wave characteristics, demonstrate that even massive particles can experience this duality, and change the way that physics can be applied today.

1 Introduction

Wave-particle duality is a fundamental concept in quantum mechanics that details the ability of an entity to possess both wavelike and particle-like characteristics. Albert Einstein first introduced the concept of light as discretized packets of energy – *photons* – whereas it had previously been thought of as only an electromagnetic wave. This was later supported by the first observations of the Compton effect in 1922. In 1924, however, wave-particle duality was extended to non-massless bodies when French physicist Louis de Broglie proposed that electrons and similar discrete matter particles could have wavelike properties [4], and that they could even have a wavelength that depends on momentum [3].

The series of experiments we describe here demonstrate the wavelike properties of small particles, seeking to demonstrate that electrons exhibit both particle behavior and light behavior. The first experiment, developed by J.J. Thompson and one of the first experiments to verify de Broglie's proposition, consists of two parts: measuring the charge-to-mass ratio (e/m) of an electron and demonstrating electron diffraction to confirm wave behavior. The second, carried out by Davisson and Germer simultaneously with Thompson's activities, measures electron diffraction patterns and gives a definitive value for the de Broglie wavelength. While the original experiment uses crystalline nickel, ours uses gold. The last experiment continues with measuring electron diffraction, but does this for a 2-dimensional material, graphite. To carry out these experiments, we have followed the procedure outlined in the **manuals** [1, 2, 3]. Mentions of our manual(s) refer to these procedures and the theoretical considerations derived therein.

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2 Theory

Particle Nature of Electron.

In the first experiment, the goal is to demonstrate the *particle* nature of the electron by using the trajectory of electrons through a magnetic field. When electrons accelerate through this magnetic field, one can obtain the charge to mass ratio (e/m), demonstrating that the electron has the particle-like characteristics of charge and mass. We do so by measuring the radius of the beam of the electrons when accelerated from different voltages. Using Newton's second law for a circular orbit and the magnetic force on the electron, as well as relating electron kinetic energy to the electrostatic potential through which electrons are accelerated, one can find the value of e/m.

The magnitude of the magnetic field can be calculated as seen in section 7.2, but it is also given as we are equipped with a Gaussmeter to make our measurements. By combining the equations as derived in the lab manual (see Section 7.2), we can equate centripetal and Lorentz forces:

$$\frac{mv^2}{R} = evB \tag{1}$$

Here m is the mass of the electron, v is its velocity, R is the beam radius, and e is the electron charge. This can be arranged to isolate e/m:

$$\frac{e}{m} = \frac{v}{BR} \tag{2}$$

To eliminate the velocity term, we can simply use the kinetic energy equation (again, see Section 7.2):

$$\frac{1}{2}mv^2 = eV \rightarrow v = \sqrt{\frac{2eV}{m}} \tag{3}$$

This allows us to find our final equation, in terms of measurable quantities:

$$\frac{e^2}{m^2} = \frac{2eV}{B^2R^2m} \Rightarrow \boxed{\frac{e}{m} = \frac{2V}{B^2R^2}} \tag{4}$$

By plotting X versus Y, where $X = B^2R^2$ and $Y = 2V$, we can obtain a slope that gives us e/m.

Wave Nature of Electron.

In the next two experiments, we obtain values that allow us to verify the wave nature of the electron. The purpose of having two experiments is to see that these results can be obtained from different kinds of crystal lattices, knowing different lattice constants - in our case, the constant for gold, for a three-dimensional material, and in two dimensions the constant for graphite. In the first experiment, we can also reverse this process to calculate an experimental value for the lattice constant of polycrystalline gold.

The theoretical value for the de Broglie wavelength of an electron is given as follows, using h for the Planck constant and p for the momentum:

$$\lambda = \frac{h}{p} \tag{5}$$

We can use the kinetic energy of the electron (see Section 7.2) to rearrange this and obtain the following equation:

$$\lambda = \frac{h}{\sqrt{2meV}} \tag{6}$$

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When a beam of electrons is aimed at a material with a crystal lattice, the electrons interfere with each other according to Bragg's law (once again, see Section 7.2). With the use of an apparatus for measuring this diffraction, we obtain interference rings whose radius r can be measured. By using the known lattice constant of polycrystalline gold (a=0.40786 nm), we can substitute this value for the spacing between the parallel planes of the atoms in the crystal to obtain the Bragg diffraction angle:

$$\sin\theta \approx \frac{r}{2D} \tag{7}$$

Where D is the distance between the target and the screen.

For our graphite sample, the crystal structure possesses two different lattice planes, with respective distances $d_1 = 123$ pm and $d_2 = 213$ pm. In the first-order $n = 1$, we can then measure the Bragg diffraction angle as follows, in terms of the diameter of the diffraction ring D and the distance from the graphite to the screen L:

$$\theta = \tan^{-1}\left(\frac{D}{2L}\right) \tag{8}$$

From this equation, we rearrange the equation to obtain the electron wavelength by substituting the Bragg condition $\lambda = 2d\sin\theta$:

$$\lambda = \frac{2d}{D}\sin\left(\frac{D}{2L}\right) \tag{9}$$

We can then make a similar plot to that in the corresponding manual to verify our agreement with theoretical values of the de Broglie wavelength.

We can invert the analysis then, and also calculate certain important constants with our measurements - for example, the gold lattice constant, and Planck's constant.

The gold lattice constant can be obtained by assigning diffraction rings to their proper (hkl) planes. Unfortunately, the experiment we use does not have a very bright screen due to age, and we can only see at most two diffraction rings in the film. By comparing with an image shown in the lab manual with each of the appropriate diffraction planes, we can assign diffraction planes to our own image and use this to calculate the lattice constant.

Planck's constant is a fundamental value in physics; a good exercise is to measure it using the values obtained in our experiment and to compare this with the accepted value. This is obtained by deriving Planck's constant. Starting from the de Broglie relation in equation (6), we rearrange to isolate Planck's constant, and use this in our calculations:

$$\lambda^2 = \frac{h^2}{2me}\frac{1}{V} \rightarrow h = \lambda\sqrt{2meV} \tag{10}$$

By plotting λ^2 and the inverse of V, the slope we obtain gives us the value of h.

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3 Methods

A crucial portion of all experiments is to perform them inside a dark room. For this, we have a dark curtain pulled over a separate section of the lab room, allowing us to work in darkness.

3.1 Electron Charge/Mass Ratio Experiment

3.1.1 Apparatus and Setup

The apparatus used in this experiment consists of the following:

1. Klinger e/m tube, Helmholtz coils, power supply box for each
   (a) 6.3 V alternating current for gun filament of e/m tube, and an adjustable 0-300 V direct current supply to accelerate the electrons.
   (b) Fragile bulb with scale attached behind to measure electron trajectory diameter.
2. Voltmeter to measure accelerating voltage
3. Compass
   (a) We did not anticipate that the direction of the Earth's magnetic field would have significant impact on our results, but we took note of the field's direction just in case it could correct any large errors.
4. Phone for data collection
   (a) We set up a phone on a tripod to take pictures of the electron trajectories, in order to measure their diameter.

We also measure the bulb diameter for our calibrations (see 3.1.2). Our measured value matches the manual at 6.1 inches (2.54 cm).

3.1.2 Procedure

Preparation

Beginning by turning on the system (the filament of the e/m tube has a slight glow at this point), we allow it to warm up for a little over a minute. The filament current is supplied by a 6.3V alternating current supply that we do not modify. Once this is done, we turn up the voltage to 200V. A blue electron beam begins to appear as we increase voltage, but for our particular system it does not start to connect (forming a loop between one end of the filament to the other) until we raise the voltage past 250V to excite the beam. While this is not generally recommended, there is a discrete jump–an energy gap–that the electrons must hop, and this is facilitated by adding this excessive voltage.

Measurement of Magnetic Field

We are able to measure the magnetic field using the current through the Helmholtz coils, which is one of the quantities we are controlling. There are 130 turns in the coils, so we use this to calculate

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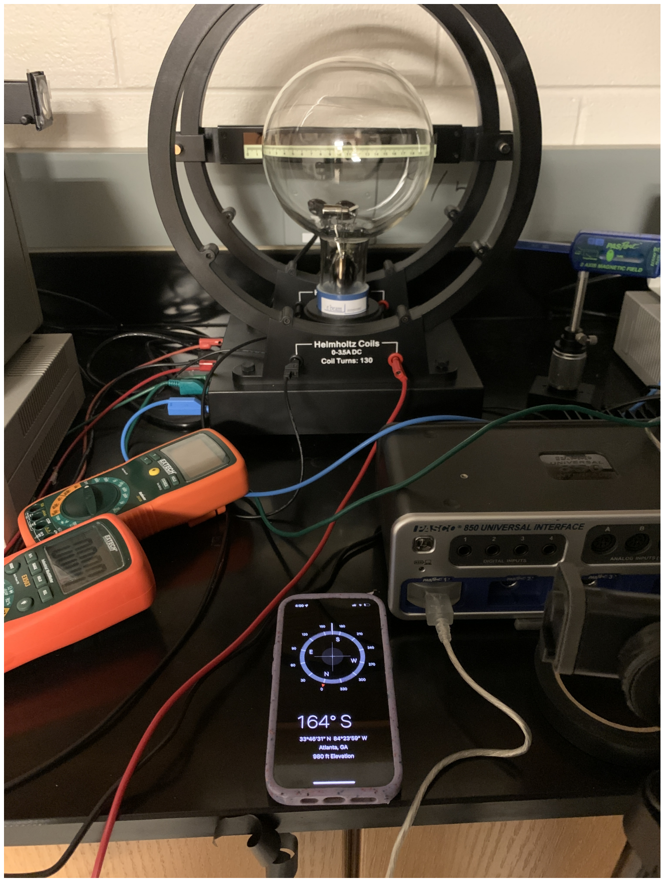

Figure 1: Setup for Experiment 1. Helmholtz coils are oriented so that their parallel side faces 164° S. The center shows Helmholtz coils with the e/m tube between them and scale behind, which is slightly distorted by the bulb. To the left of the setup are the two boxes which route power supply to the tube and to the coils.

our field. We also make use of a provided gaussmeter to measure the magnetic field corresponding to each current; this also provides a useful calibration, as the quantity should have a linear relation to the current in the low-current range. As shown in Figure 2, this was indeed to the satisfactory calibration.

Measuring Beam Diameter

Going back down to 200V so that the beam is disconnected again, we can now slowly increase voltage until the electron trajectory closes back in. We then measure the diameter of the electron trajectory for several combinations of accelerating voltage; in order to get at least 4 combinations of voltage and current, we start with 200.5V, which for us is the minimum voltage for a clear electron trajectory, then measure diameters for two different currents each for the voltages 225.5V and 250V. For one of the 250V measurements, we also include a measurement with 2.81 A, the maximum that the coil supply allows for us.

In order to make our measurements, we install a phone on a tripod, relying on the phone camera and its ability to take better photos in the darkness in order to make accurate measurements. While the bulb is equipped with a scale to measure this diameter, the scale is behind the bulb and becomes slightly distorted when viewed head-on through both sides of the glass bulb. To combat

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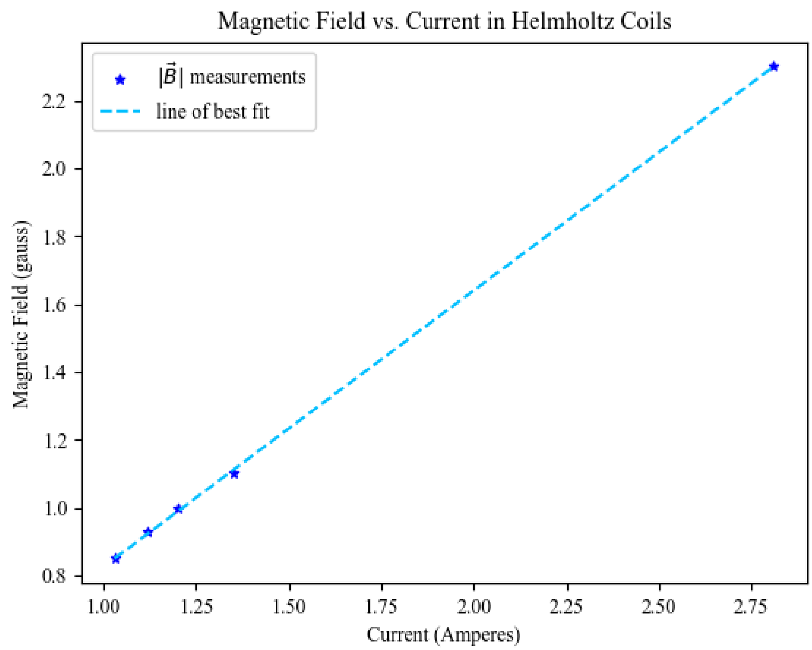

Figure 2: Plot of magnetic field vs. current.

this issue, we instead use the measured bulb diameter as a calibration tool; taking the ratio of the electron beam diameter and the bulb diameter in pixels within each photo, we can use the bulb's diameter to calculate the actual diameter of the electron beam in meters. These measurements, along with final calculated values, can be seen in Table 3 in the Appendix (section 7.1).

3.2 Electron Diffraction Experiment

3.2.1 Apparatus and Setup

For this experiment, we use an electron diffraction apparatus, complete with a diffraction tube and a high-voltage source. The high voltage between the cathode and the crystal thin film is 0-20 kV, which we can adjust. The diffraction sample used in this experiment is gold, and the distance between the target and the screen is given on the device as 255 ± 3 mm.

3.2.2 Procedure

Preparation

Before beginning this experiment, we allow the apparatus to heat up, turning on power and leaving it on for 30 minutes. Once this is done, we set the voltage to a higher amount - around 6.0 kV - and adjust the brightness, focus, auxiliary focus, and X and Y position controls to move the diffraction pattern to the center of the screen.

To obtain our data, we once again position a tripod with phone camera before the screen, and obtain images. In this case, however, we also make use of a simple ruler to measure the ring radius, since there is no distortion as there is with the bulb in the previous experiment. These measurements can be recorded manually, but we take photos simply to preserve exact data for later analysis.

Part A: Electron Wavelength and de Broglie Relation

For this portion of the experiment, multiple measurements of diffraction ring radius are obtained for multiple accelerating voltages at the same order. These measurements provide a value for the de Broglie wavelength of the electron, verifying the wavelike nature of the electron. We compare to the actual accepted value to verify our results. The lattice constant of gold is needed, and is given as $a = 0.40786$ nm.

Part B: Crystal Lattice Constant

For the previous portion of the experiment, the lattice constant of gold is *given*, but now we measure the constant ourselves. Once again measuring at multiple voltages, we use the radii of diffraction rings to obtain the lattice constant. In our case, however, because the apparatus is older, we are unable to get the multiple rings that are visible in the manual, but good values can still be obtained with what we have.

3.3 Electron Diffraction in Graphite

This experiment works similarly to the previous diffraction-focused experiment, except that the diffraction is caused by polycrystalline graphite, a two-dimensional material. The diffraction tool is an evacuated glass tube, which also demonstrates diffraction rings that are then observed and recorded. Similarly to the previous experiment, the ring diameter depends on the acceleration voltage, which we vary in order to compare results and use to derive de Broglie wavelengths anew.

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4 Results and Analysis

4.1 Electron Charge and Mass Ratio

Our final obtained values are given in Section 7.1. In the end, using calculated values for the magnetic field, we obtain an average value for e/m of 2.06 $\times 10^{11} \pm$ 0.05 C/kg. If e/m is obtained using the slope of a plot of X versus Y as described in the theoretical considerations (Section 2), we obtain a value of 2.11 $\times 10^{11}$ C/kg, as shown below. Both of these values are well within our tolerances for the accepted value, known to be 1.76 $\times 10^{11}$ C/kg - the values have an error of 17% and 19%, respectively. With a standard deviation of 0.05×10^{11} C/kg, this is within the tolerance (calculated to be 24% using the standard deviation). However, this is quite a large error, larger than we expected. Some issues that may have contributed to this large error are distortions from the bulb that we did not take into account, a loss of accuracy when comparing pixels rather than making physical measurements of the beam radius, calibration issues (our calculated magnetic field is different than that given by the gaussmeter, for example), and the direction of the Earth's magnetic field. We also lose some accuracy because the electrons move in a relativistic regime, and our equations do not make allowances for this, preferring to approximate instead that there are no Lorentz boost effects.

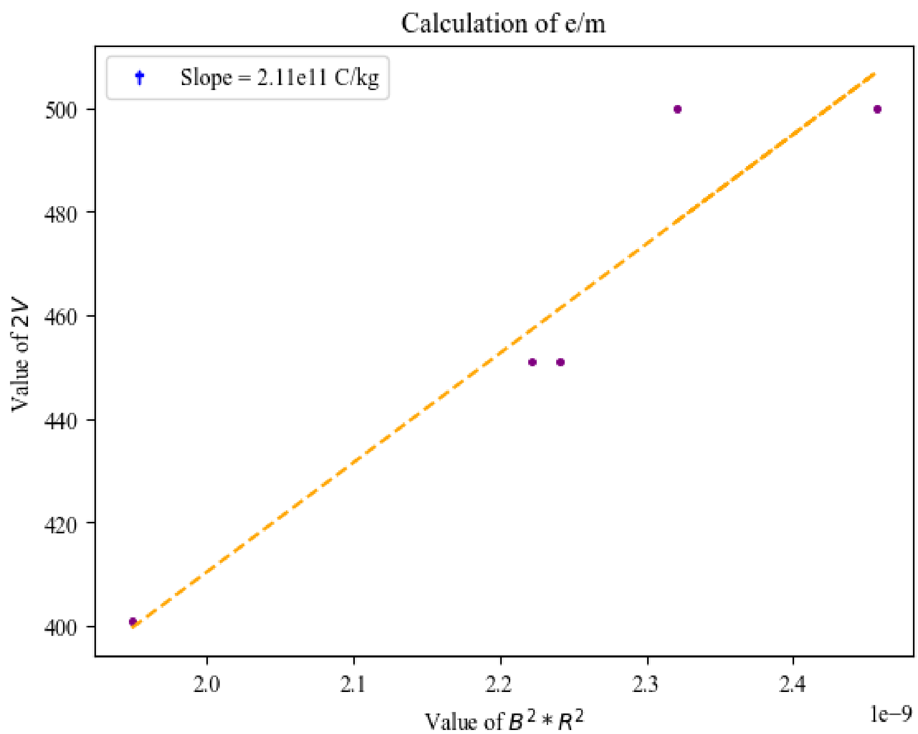

Figure 3: Plot for calculation of e/m.

4.2 Measurement of de Broglie Wavelength

Our average value for the de Broglie wavelength is found to be 9.92 $\pm 0.097 \times 10^{-11}$ m, which is quite close to the theoretical values obtained using Equation 4 in Section 7.2.2, which uses the lattice constant of gold to obtain an approximate value for what the de Broglie wavelength should be; the average error between experimental and theoretical values is 1.23%.

For experiment 3, the results become a little bit more bizarre. Here we obtain fairly accurate values for the de Broglie wavelength from the outer ring, with λ_2, on average $1.73 \pm 0.21 \times 10^{-11}$m, being in good agreement with the theoretical values, which average 2.04×10^{-11}m. These values agree with an error of 15.25%.

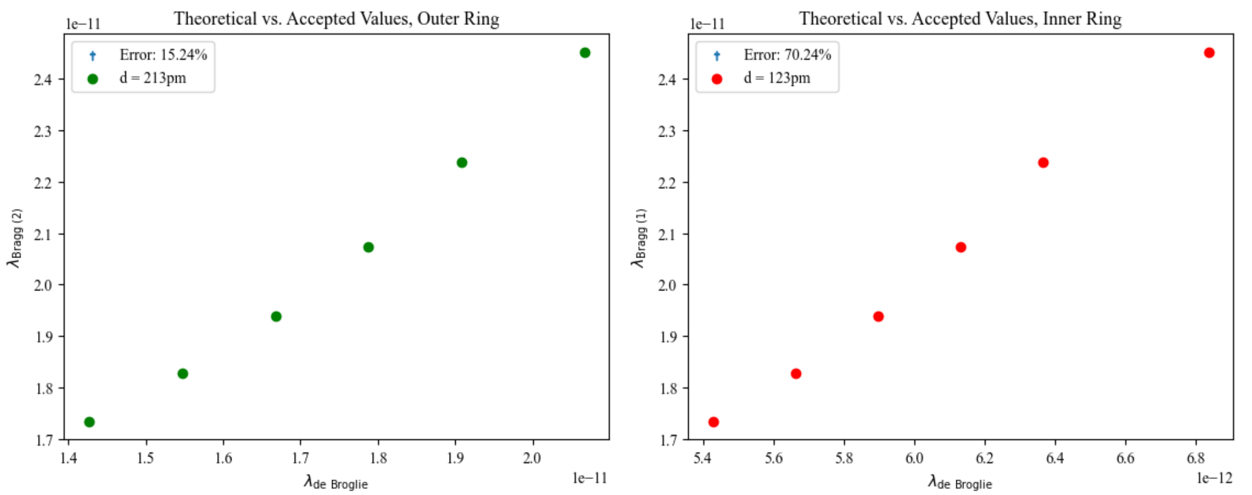

Figure 4: Plot of values of theoretical and accepted values for de Broglie wavelength.

Bafflingly, however, the values for λ_1 are way off: with a whopping $6.05 \pm 0.46 \times 10^{-12}$ m, an error of 70.24%! I could not figure out where this could be coming from, given that I was using the same Python function to calculate both values, and inputting all of the correct values. Since the human error was minimized by calculating everything with a single function in Python, I genuinely have no idea where this discrepancy could be coming from, unless we grossly mis-measured the inner ring radius. This is a bizarre possibility, however, given that the inner ring was the brightest, and therefore the easiest to measure the diameter for.

Finally, we also used equation (7) from theory (7.2.2) for assigned (hkl) planes, comparing them to the manual and finding the one with the most agreement. Our closest agreement for the brightest ring was n = 3, with (h,k,l) = (1,1,1), that brought us close to the theoretical value.

4.3 Calculations of Physical Constants

As described in the theory section, we can measure Planck's constant using our values for λ. For this we obtain a value of 6.90×10^{-34} J·s, which agrees quite well with the accepted value of 6.626×10^{-34} J·s.

5 Discussion

Throughout the three experiments we performed, we were able to obtain measurements of varying degrees of agreement with today's accepted values. One large drawback of our theory is that we did not consider the effects of Lorentz boosting under relativistic conditions. The electrons being accelerated in Experiment 1, for example, were moving at velocities - which we calculated - closer to the speed of light. These conditions should have led to additional considerations due to the added terms to the energy of the electron. This should also affect the calculation of de Broglie wavelengths. In addition, the apparatus we used was older than that described in the manual, which may have affected our ability to accurately measure radii, and to measure more than two diffraction rings. This limits the experiment, but did not take away from our ability to make good measurements regardless (at least for the outer ring, in the case of the graphite sample. For the inner ring, I can only assume that some calculation error must have occurred, though this is bizarre since once again the same Python equation was used for both). We did, however, obtain results somewhat within tolerance for most of the experiments, particularly in the measurement of Planck's constant and the de Broglie wavelength calculated for polycrystalline gold.

6 Conclusion

Though we were met with significant errors in some places, these experiments provide overwhelming evidence in favor of the wave-particle duality of the electron. **Charge to mass ratio** calculations, particularly ones in good agreement with theoretical values, provide strong evidence for the particle nature of the electron, which is already intuitive to the physics enjoyer. Measurements of **wavelength** of the electron provide mathematical evidence toward its wave nature, particularly when combined with the compelling evidence of seeing diffraction patterns on a film. This visual aspect particularly helps to see this duality which has held a profound effect on modern physics ever since its proposition and subsequent demonstration.

Rather than put the code in this report, I have created a temporary GitHub where I have uploaded the Jupyter notebook where I performed each of my calculations. Feel free to check it out!

→ https://github.com/aheranval/lab2

7 Appendix

7.1 Appendix A: Data Tables

Electron Diam. (pixels)	Bulb Diam. (pixels)	Electron Diam. (calculated, m)
372	523	0.110
304	432	0.109
234	403	0.090
411	601	0.106
110	399	0.043

Table 1: Data for Experiment 1. Electron radius calculated using diameter of the bulb, 6.1 inches.

V_e (V)	I_m (A)	R (m)	B (T $\times 10^{-4}$)	v (m/sec)	e/m (Coul/kg)
200.5	1.03	0.055	8.03	8.40×10^6	2.06×10^{11}
225.5	1.12	0.054	8.72	8.91×10^6	2.03×10^{11}
225.5	1.35	0.045	10.5	8.91×10^6	2.01×10^{11}
250.0	1.20	0.053	9.35	9.38×10^7	2.04×10^{11}
250.0	2.81	0.022	21.9	9.38×10^7	2.15×10^{11}

Table 2: Table with calculations for experiment 1.
The average value for e/m is found to be 2.06 $\times 10^{11} \pm$ 0.05 C/kg.

Acc. Voltage (kV)	Radius (cm)	Experimental λ (m $\times 10^{-11}$)	Theoretical λ (m)
12.0	3.7	1.14	1.12 $\times 10^{-11}$
14.0	3.4	1.05	1.04 $\times 10^{-11}$
16.0	3.2	0.97	9.68 $\times 10^{-12}$
18.0	3.0	0.92	9.12 $\times 10^{-12}$
20.0	2.8	0.86	8.66 $\times 10^{-12}$

Table 3: Data for Experiment 2. Our average value is $9.92 \pm 0.097 \times 10^{-11}$ m, and the average error between experimental and theoretical values is 1.23%.

7.2 Appendix B: Equations for Theory

Note: for definitions of each symbol and associated SI units, please see section 7.3.

7.2.1 Experiment 1

The following equations are, numbered exactly as within the corresponding manual, the equations for the experiment detailed in section 3.1.

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Voltage (kV)	Inner Ring Radius (cm)	Outer Ring Radius (cm)
5.0	2.3	3.5
4.5	2.4	3.8
4.0	2.5	4.1
3.5	2.6	4.4
3.0	2.7	4.7
2.5	2.9	5.1

Table 4: Data for Experiment 3.

Voltage (kV)	λ_{dB} (m)	Inner R (cm)	λ_1 (m)	Outer R (cm)	λ_2 (m)
5.0	1.73×10^{-11}	2.3	5.43×10^{-12}	3.5	1.43×10^{-11}
4.5	1.83×10^{-11}	2.4	5.66×10^{-12}	3.8	1.55×10^{-11}
4.0	1.94×10^{-11}	2.5	5.90×10^{-12}	4.1	1.67×10^{-11}
3.5	2.07×10^{-11}	2.6	6.13×10^{-12}	4.4	1.79×10^{-11}
3.0	2.24×10^{-11}	2.7	6.37×10^{-12}	4.7	1.91×10^{-11}
2.5	2.45×10^{-11}	2.9	6.84×10^{-12}	5.1	2.07×10^{-11}

Table 5: Calculations for Experiment 3. Values of λ_2, on average $1.73 \pm 0.21 \times 10^{-11}$m, are in good agreement with the theoretical values, which average 2.04×10^{-11}m. The error here is on average 15.25%. Values for λ_1 are way off: $6.05 \pm 0.46 \times 10^{-12}$ m, an error of 70.24%.

$$B = \frac{8\mu_0 N I_m}{5a\sqrt{5}} \tag{1}$$

$$F_B = \frac{mv^2}{R} \tag{2a}$$

$$F_B = evB \tag{2b}$$

$$eV = \frac{mv^2}{2} \tag{2c}$$

7.2.2 Experiment 2

$$\lambda = \frac{h}{p} = \frac{h}{mv} \tag{1}$$

$$\frac{1}{2}mv^2 = eV \tag{2}$$

$$\lambda = \frac{h}{\sqrt{2meV}} \tag{3}$$

$$\lambda \approx \left(\frac{1.50}{V}\right)^{1/2} \tag{4}$$

$$n\lambda = 2d\sin\theta \tag{5}$$

$$d = \frac{a}{(h^2 + k^2 + l^2)^{1/2}} \tag{6}$$

$$\lambda = \frac{2a\sin\theta}{(H^2 + K^2 + L^2)^{1/2}} = \frac{r}{D} \times \frac{a}{(H^2 + K^2 + L^2)^{1/2}} \tag{7}$$

7.2.3 Experiment 3

This experiment has the same equations as Experiment 2, with the addition of the following:

$$D = 2L\tan 2\theta \tag{2}$$

7.3 Appendix C: Definitions of Symbols and SI Units in Equations

Symbol	Definition	SI Unit
λ	De Broglie wavelength	m
e	Elementary charge of an electron	C (Coulomb)
m	Mass of an electron	kg
v	Velocity of the electron	m/s
V	Accelerating voltage	V (Volt)
B	Magnetic field strength	T (Tesla)
R	Radius of electron trajectory	m
h	Planck's constant	J·s
d	Spacing in crystal	m
θ	Bragg diffraction angle	degrees (°)

Table 6: Definitions of symbols and SI units used in the experiment.

7.4 Appendix D: Images

Here are some images from the experiment! This one was quite a fun one.

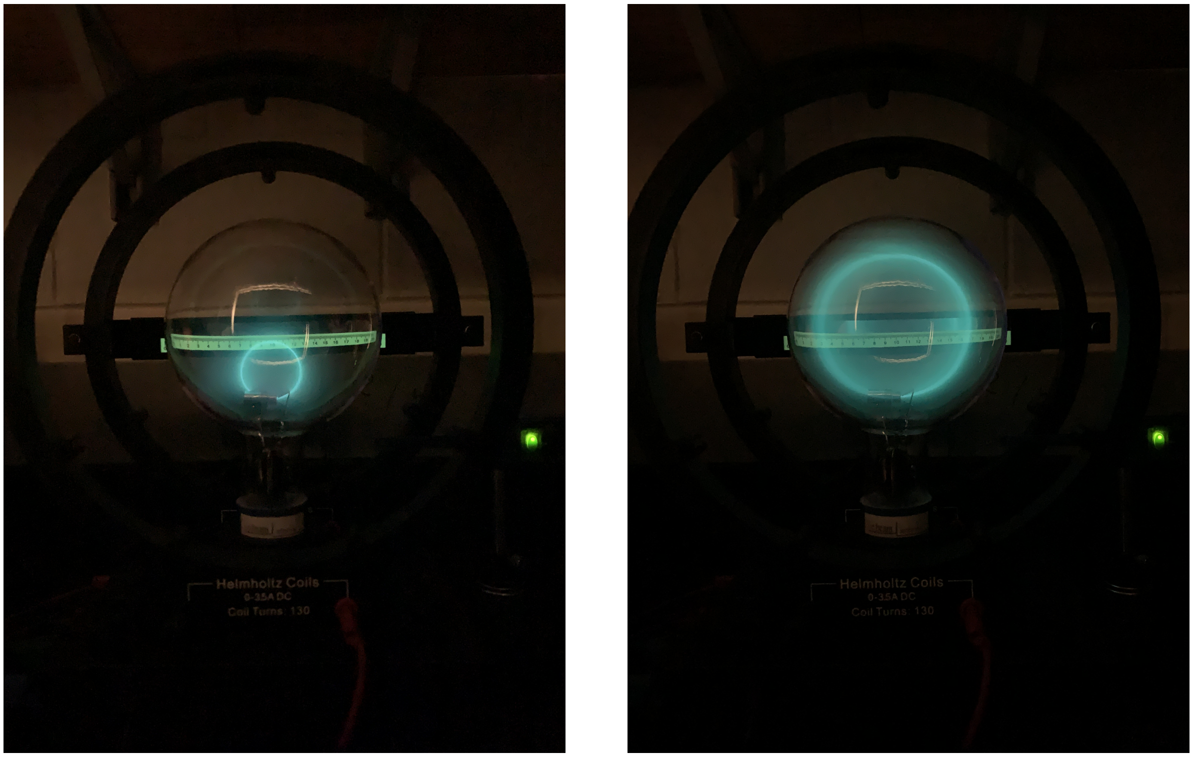

Figure 5: Images of electron beams from experiment 1.

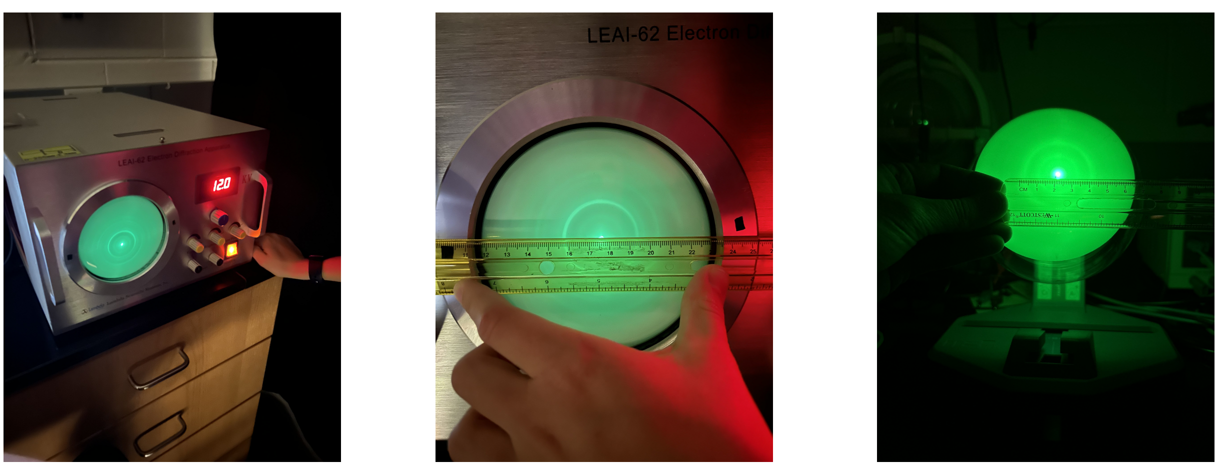

Figure 6: Images of setup and measurement from experiments 2 and 3.

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