Math 128A, Fall 2014, Wilkening
Programming Assignment 2
due Wed, Oct 22

In this assignment we will explore the accuracy of polynomial interpolation for large n. It turns out that the interpolating polynomial usually does not converge to the function as $n \to \infty$ when uniformly spaced grid points are used, but there is another set of points, the Chebyshev grid points, in which it is guaranteed to converge under very mild assumptoins on f. (f only needs to be Lipshitz continuous.)

1. Write a program that takes as input a function f (such as we used for Newton's method) and two vectors, `xin` and `xout`, containing the interpolation points and the evaluation points, respectively. The program should return `yout`, the values of the interpolating polynomial $P(x)$ at each point in `xout`. Use the formula

$$P(x) = \sum_{j=0}^{n} f(x_j) L_j(x), \qquad L_j(x) = \prod_{k \neq j} \frac{x - x_k}{x_j - x_k},$$

where the x_j are the entries of the vector `xin` and x ranges over the entries of `xout`. Do this with $n = 10, 19, 50, 99$ on two grids:

```
xin = linspace(-1,1,n+1)              (uniform),
xin = cos(linspace(-pi,0,n+1))        (Chebyshev).
```

Note that the Chebyshev grid points are clustered more closely at the endpoints of the interval. For output, use `xout=linspace(-1,1,500)`, and for the function, use `f = @(x) 1.0 ./ (1+9*x.^2)`.

2. Write a second program that does exactly the same thing as above, but using the following (barycentric) formula for the interpolating polynomial:

$$P(x) = \left\{ \begin{array}{ll} f(x_j) & x = x_j \\ \sum_{j=0}^{n} \frac{\lambda_j f(x_j)}{x - x_j} \Big/ \sum_{j=0}^{n} \frac{\lambda_j}{x - x_j}, & x \notin \{x_0, \dots, x_n\} \end{array} \right\}, \qquad \lambda_j = \frac{1}{\prod_{k \neq j}(x_j - x_k)}.$$

In case you are curious, the derivation of barycentric interpolation is given in chapter 5 of Trefethen's book, *Approximation Theory and Approximation Practice*, which will be posted on bCourses.

What to turn in: your two codes and 8 plots. For $n = 10, 19$, plot $f(x)$ and $P(x)$ on top of each other, using either code for $P(x)$. For $n = 50, 99$, plot `semilogy(xout,1.0e-18+abs([yout1-f(xout),yout2-f(xout)]),'linewidth',1)`, where `yout1` and `yout2` are the results of the two codes you wrote. Examples with $n = 14$ and $n = 99$ are given below.