Neutrino - Nucleon cross section calculations at high energies.

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Outline

Main Objective

To calculate the differential $\frac{d^2\sigma}{dxdy}$ and total σ cross sections for the following neutral current (NC) reactions

$$\nu_l + \mathrm{N} \longrightarrow \nu_l' + \mathrm{X} \tag{1}$$
$$\overline{\nu_l} + \mathrm{N} \longrightarrow \overline{\nu_l}' + \mathrm{X}, \tag{2}$$

and for the next charged current (CC) reactions

$$\nu_l + \mathrm{N} \longrightarrow l^- + \mathrm{X} \tag{3}$$
$$\overline{\nu_l} + \mathrm{N} \longrightarrow l^+ + \mathrm{X}. \tag{4}$$

νP processes of interest

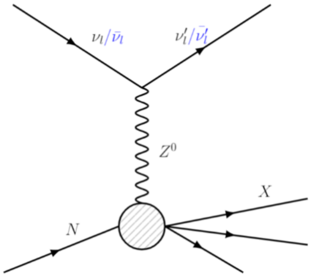


Figure 1: Feynman diagram for the neutral current reactions of interest (Eq. (1) and Eq. (2)).

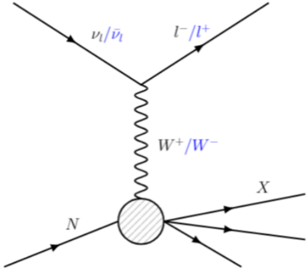


Figure 2: Feynman diagram for the charged current reactions of interest (Eq. (3) and Eq. (4)).

Specific Objectives

1. To study the parton model and its applications to high-energy neutrino interactions: to study the parton distribution functions (PDFs).

2. To obtain the proton PDFs extrapolations for high Q^2, using the DGLAP evolution equations within the QCDNUM software.

3. To obtain analytic expressions for the differential and total cross sections of the interactions mentioned.

Specific Objectives

4. To implement a program that calculates the differential and total cross sections of these interactions.

5. To compare these results with those coming from other extrapolation schemes found in the literature.

Motivation

- PDFs are a key tool to cross section calculations in deep inelastic scattering (DIS) processes involving hadrons [1].
- It is important to:
  1. Produce high-precision PDFs sets.
  2. Give a flexible functional form of the PDFs at low x scale.
- High-precision PDFs can give insights to physics beyond the SM and more accurate predictions of QCD [2].

Methodology

1. A PDF parametrization at an initial energy scale was considered.
2. The PDFs were evolved using the DGLAP evolution equations within QCDNUM.
3. All the evolved PDFs were fitted using `scipy.odr.ODR`.
4. Graphs of the parameters and parametrizations on $\log\left(\frac{Q^2}{2.56^2}\right)$ were performed.
5. Integration of $\frac{d^2\sigma}{dxdy}$ CC and NC were made using `scipy.integrate.nquad` to obtain the total cross sections $\sigma(E_\nu)$.

Initial energy scale parametrization of PDFs

The general form is [2]:

$$xf(x,Q^2) = A\,x^B(1-x)^C(1+Dx+Ex^2+F\log x+G\log^2 x+H\log^3 x), \tag{5}$$

but for each parton [2]:

$$xu_v(x,Q^2) = A_{u_v}\,x^{B_{u_v}}(1-x)^{C_{u_v}}(1+E_{u_v}x^2+F_{u_v}\log x+G_{u_v}\log^2 x) \tag{6}$$

$$xd_v(x,Q^2) = A_{d_v}\,x^{B_{d_v}}(1-x)^{C_{d_v}} \tag{7}$$

$$x\bar{u}(x,Q^2) = A_{\bar{u}}\,x^{B_{\bar{u}}}(1-x)^{C_{\bar{u}}}(1+D_{\bar{u}}x+F_{\bar{u}}\log x) \tag{8}$$

$$x\bar{d}(x,Q^2) = A_{\bar{d}}\,x^{B_{\bar{d}}}(1-x)^{C_{\bar{d}}}(1+D_{\bar{d}}x+F_{\bar{d}}\log x) \tag{9}$$

$$xg(x,Q^2) = A_g\,x^{B_g}\,(1-x)^{C_g}\,(1+F_g\log x+G_g\log^2 x). \tag{10}$$

Initial energy scale parametrization of PDFs

The PDFs parametrizations were taken from [2], since they:

- The polynomials in $\log$ bring good flexibility to the small x regions.
- The χ^2 is smaller than the one of the xFitter parametrization.
- The number of parameters is not very large.

Approximations

It is usual to make an approximation of the form [1]

$$u_s(x,Q^2) = d_s(x,Q^2) = s_s(x,Q^2) = \overline{u}(x,Q^2) = \overline{d}(x,Q^2) = \overline{s}(x,Q^2) \equiv S(x), \quad (11)$$

but through this thesis the following approximations were considered:

$$s(x,Q^2) = \overline{s}(x,Q^2) = \overline{d}(x,Q^2) \quad (12)$$

$$u_s(x,Q^2) = \overline{u}(x,Q^2) \quad (13)$$

$$d_s(x,Q^2) = \overline{d}(x,Q^2) \quad (14)$$

$$c(x,Q^2) = b(x,Q^2) = t(x,Q^2) = \overline{c}(x,Q^2) = \overline{b}(x,Q^2) = \overline{t}(x,Q^2) = 0. \quad (15)$$

QCDNUM: Fast QCD Evolution and Convolution

- QCDNUM Version 17.01/14[1].
- Written in Fortran-77, but it has an interface in C++.
- Numerically solves the DGLAP evolution equations on a discrete grid in x and Q^2.
- Evolution of the strong coupling constant and parton densities, up to NNLO order in pQCD [3].
- QCDNUM interpolation based on quadratic spline interpolation.

[1] https://www.nikhef.nl/~h24/qcdnum/QcdnumDownload.html

QCDNUM PDFs evolution

The evolution of the PDFs is given by the DGLAP equations [3]:

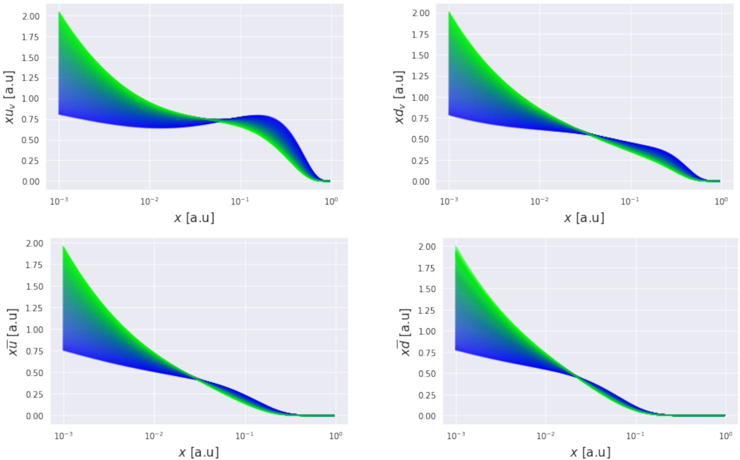
$$\frac{df_i(x, Q^2)}{d\log Q^2} = \frac{\alpha_s}{2\pi} \sum_{j=q,\bar{q},g} \int_x^1 \frac{dy}{y}\ f_i(y, Q^2)\, P_{ij}\left(\frac{x}{y}\right), \tag{16}$$

where:

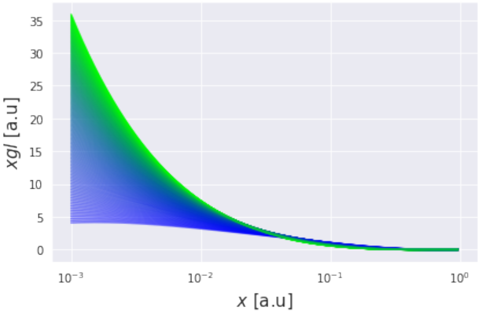
- α_s is the strong coupling constant.
- P_{ij} are the QCD splitting functions.
- x (y) is the fraction of the proton's momentum carried by a final (initial) parton.

Results

Evolved PDFs



Evolved PDFs



Comparing with literature

Dulat & all [4]:

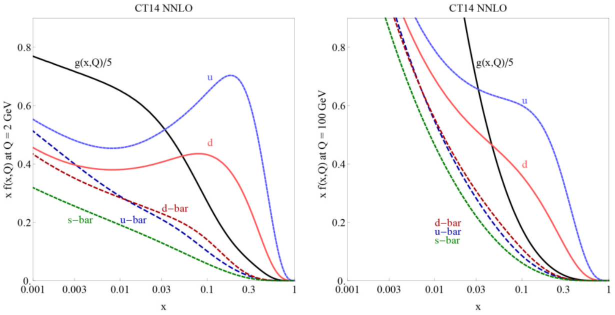


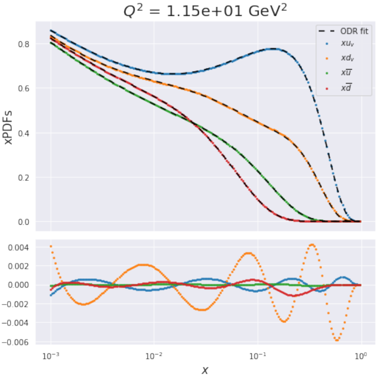
Fig. 2.10: The recent CTEQ CT14 parton distribution functions at $Q = 2\,\mathrm{GeV}$ and $Q = 100\,\mathrm{GeV}$ for u, $\bar{u}$, d, $\bar{d}$, $s = \bar{s}$ and g (from [15]).

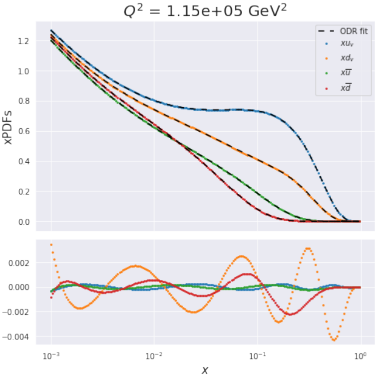
Fitting the evolved PDFs

- The fitting was made using: `scipy.odr.ODR`[2].
- The fitting was made using the same initial PDF parametrizations as the model, and the initial parameters were replaced at each fitting by the new ones the program found.

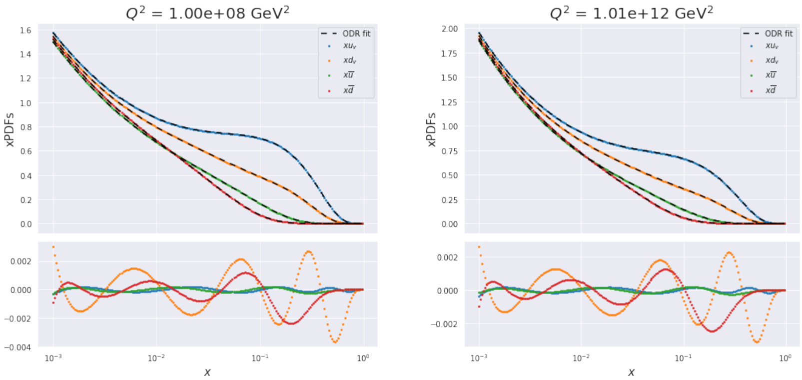
[2] `https://docs.scipy.org/doc/scipy/reference/odr.html`.

Fitting the evolved PDFs

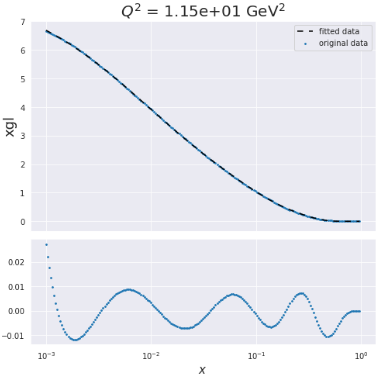


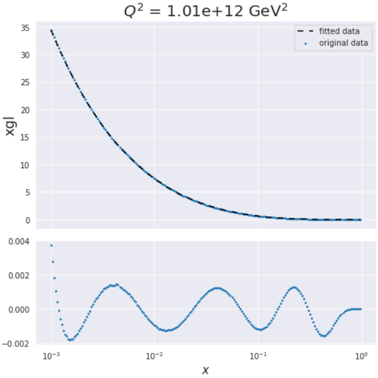


Fitting the evolved PDFs

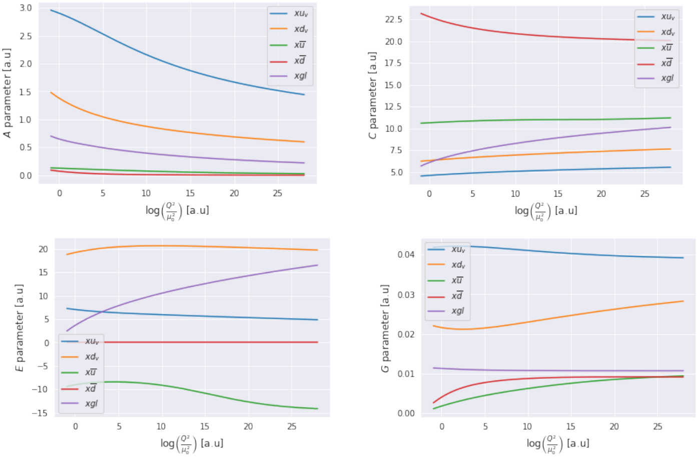


Fitting the evolved PDFs





Parameters Graphs



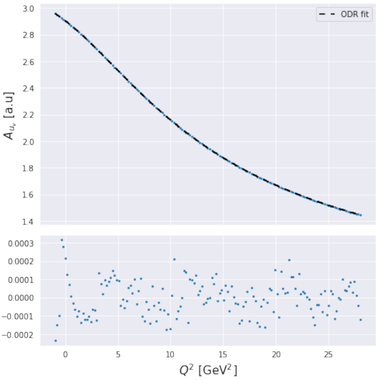
Parameters parametrization

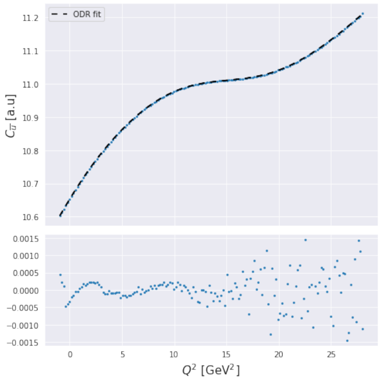
If $P_i \in \{A_{u_v}, A_g, ..., G_{u_v}, G_g\}$, then the optimal parametrization that was found is:

$$P_i = p_0 + p_1 z + p_2 z^2 + p_3 z^3 + p_4 z^4 + p_5 z^5 + p_6 z^6 + p_7 z^7 + p_8 z^8 + p_9 z^9,$$

where $z = \log\left(\frac{Q^2}{\mu_0^2}\right)$ and the p_j values are found in tables [4.1] to [4.5] of the document.

Fitting the parameters





Differential cross sections

The functions to integrate were [5]:

$$\frac{d^2\sigma^{\nu}_{CC}}{dxdy} = \frac{2\,\left(G_F^W\right)^2 m_N E}{\pi}\left(\frac{M_W^2}{M_W^2+Q^2}\right)^2 x\left[q(x,Q^2)+\tilde{q}(x,Q^2)(1-y)^2\right], \qquad (17)$$

$$\frac{d^2\sigma^{\nu}_{NC}}{dxdy} = \frac{\left(G_F^Z\right)^2 m_N E}{2\pi}\left(\frac{M_Z^2}{M_Z^2+Q^2}\right)^2 x\left[q^0(x,Q^2)+\tilde{q}^0(x,Q^2)(1-y)^2\right], \qquad (18)$$

where

$$q(x,Q^2) = \frac{u_v(x,Q^2)+d_v(x,Q^2)}{2} + \frac{u_s(x,Q^2)+d_s(x,Q^2)}{2} + s_s(x,Q^2).$$

$$\tilde{q}(x,Q^2) = \frac{u_s(x,Q^2)+d_s(x,Q^2)}{2}.$$

Differential cross sections

and

$$q^0(x,Q^2) = \left[\frac{u_v(x,Q^2)+d_v(x,Q^2)}{2} + \frac{u_s(x,Q^2)+d_s(x,Q^2)}{2}\right](L_u^2+L_d^2) + \left[\frac{u_s(x,Q^2)+d_s(x,Q^2)}{2}\right](R_u^2+R_d^2) + s_s(x,Q^2)(L_d^2+R_d^2)$$

$$\tilde{q}^0(x,Q^2) = \left[\frac{u_v(x,Q^2)+d_v(x,Q^2)}{2} + \frac{u_s(x,Q^2)+d_s(x,Q^2)}{2}\right](R_u^2+R_d^2) + \left[\frac{u_s(x,Q^2)+d_s(x,Q^2)}{2}\right](L_u^2+L_d^2) + s_s(x,Q^2)(L_u^2+R_u^2).$$

Cross sections

- The integration was made using: `scipy.integrate.nquad`[3].
- The best integration limits found, for both CC and NC, were:
$$x \in [10^{-3},\ 1], \qquad y \in [0,\ 1].$$

[3] `https://docs.scipy.org/doc/scipy/reference/generated/scipy.integrate.nquad.html`.

Cross sections

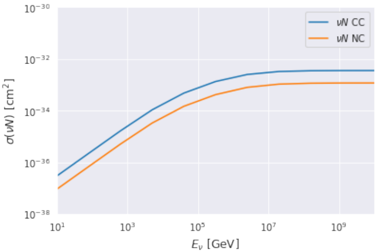


Figure 1: Cross sections for νN CC and NC, at high energies.

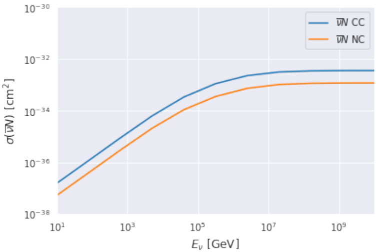


Figure 2: Cross sections for $\overline{\nu}N$ CC and NC, at high energies.

Cross sections

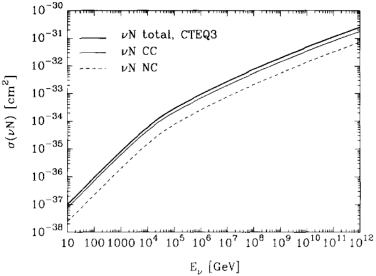


Figure 3: νN CC and NC total cross sections, taken from [5].

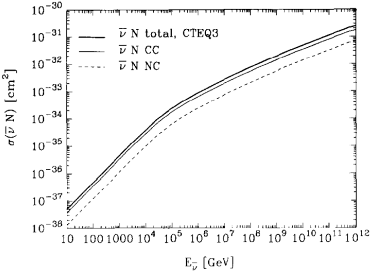


Figure 4: $\overline{\nu}N$ CC and NC total cross sections, taken from [5].

Limitations of the results

1. DGLAP limitations at high energies ($E_\nu \sim 10^8$ GeV, i.e. large x) [6][7] $\longrightarrow$ To include the BFKL approach to the small x region, through the unified CCFM equations [8].
2. The approximations used $\longrightarrow$ To include the heavy quark PDFs contribution.
3. The fit and integration program used.

Conclusions

- A study of the parton model and its QCD corrections at low orders was made.
- Good approximations to the νN, $\overline{\nu} N$ CC and NC were obtained, which can be enhanced by the CCFM equations.
- The parametrization of the parameters is powerful since it allows to skip the step of evolving the PDFs with an evolution software.
- The DGLAP equations are good approximations at not very high energies.

Conclusions

To improve:

- To compare the PDFs evolved using different inputs (heavy masses threshold, different dataset, different parametrizations at initial scale, ...)
- To use different evolution software.
- To include the CCFM equations.
- To include the heavy quark contribution.
- To find or construct a more robust fit and integration software.

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