Neutrino - Nucleon cross section calculations at ultra-high energies.

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Outline

Main Objective

Calculate the differential $\frac{d^2\sigma}{dxdy}$ and total σ cross sections for the following neutral current reactions

$$\nu_l + \mathrm{P} \longrightarrow \nu_l' + \mathrm{X} \tag{1}$$

$$\overline{\nu_l} + \mathrm{P} \longrightarrow \overline{\nu_l}' + \mathrm{X}, \tag{2}$$

and for the next charged current reactions

$$\nu_l + \mathrm{P} \longrightarrow l^- + \mathrm{X} \tag{3}$$

$$\overline{\nu_l} + \mathrm{P} \longrightarrow l^+ + \mathrm{X}. \tag{4}$$

νP processes of interest

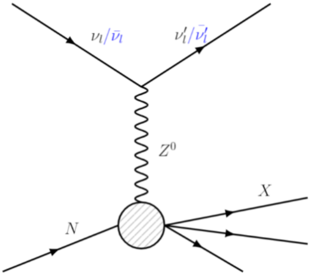


Figure 1: Feynman diagram for the neutral current reactions of interest (Eq. (1) and Eq. (2)).

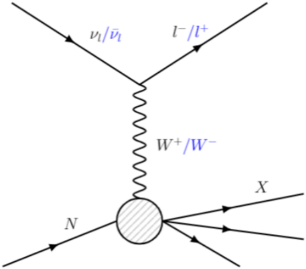


Figure 2: Feynman diagram for the charged current reactions of interest (Eq. (3) and Eq. (4)).

Specific Objectives

- Study the parton model and its applications to high-energy neutrino interactions.
- Obtain the proton PDFs extrapolations for high Q^2, using the DGLAP evolution equations with QCDNUM.
- Obtain analytic expressions for the differential and total cross sections for the interactions mentioned.

Specific Objectives

- Implement a C++ program that calculates the differential and total cross sections of these interactions.
- Compare these results with those coming from other extrapolation schemes found in the literature.

Motivation

- PDFs are a key tool to cross section calculations in DIS processes involving hadrons [1].
- It is important to:
  1. Produce high-precision PDFs sets.
  2. Give a flexible functional form of the PDFs at low x scale.
- High-precision PDFs can give insights to physics beyond the SM and more accurate predictions of QCD [2].

Methodology

- To have xpdf parametrization at initial scale (from Ref [2]).
- Evolution using DGLAP evolution equations $\rightarrow$ QCDNUM.
- Fitting $\rightarrow$ ROOT.
- Extrapolation of PDFs to any Q^2.
- Calculation of $\frac{d^2\sigma}{dxdy}$ and σ cross sections to any Q^2.

Parton Distribution Functions (PDFs)

The PDFs parametrizations were taken from [2] and in general take the form:

$$
\begin{aligned}
xf(x, Q^2) = A\, x^B (1-x)^C (1 + Dx + Ex^2 + F \log x \\
+ G \log^2 x + H \log^3 x)
\end{aligned}
$$

Parton Distribution Functions (PDFs)

For every parton [2]:

$$
\begin{aligned}
xg(x,Q^2) &= A_g\, x^{B_g}(1-x)^{C_g}(1+D_g x+E_g x^2+F_g \log x+G_g \log^2 x)\\
xu_v(x,Q^2) &= A_{u_v}\, x^{B_{u_v}}(1-x)^{C_{u_v}}(1+E_{u_v} x^2+F_{u_v} \log x+G_{u_v} \log^2 x)\\
xd_v(x,Q^2) &= A_{d_v}\, x^{B_{d_v}}(1-x)^{C_{d_v}}\\
x\bar{u}(x,Q^2) &= A_{\bar{u}}\, x^{B_{\bar{u}}}(1-x)^{C_{\bar{u}}}(1+D_{\bar{u}} x+F_{\bar{u}} \log x)\\
x\bar{d}(x,Q^2) &= A_{\bar{d}}\, x^{B_{\bar{d}}}(1-x)^{C_{\bar{d}}}(1+D_{\bar{d}} x+F_{\bar{d}} \log x)
\end{aligned}
$$

QCDNUM: Fast QCD Evolution and Convolution

- QCDNUM Version 17.01/14.
- Written in Fortran-77, but it has an interface in C++.
- Numerically solves the DGLAP evolution equations on a discrete grid in x and Q^2.
- Evolution of the strong coupling constant and parton densities, up to NNLO order in pQCD [3].

QCDNUM Evolution of the Strong Coupling Constant

$a_s = \alpha_s/2\pi$ evolves as [3]:

$$\frac{da_s(\mu^2)}{d\ln\mu^2} = -\sum_{i=0}^{2} \beta_i a_s^{i+2}(\mu^2)$$

where $\mu^2 = \mu_R^2$ and the β functions (Gell-Mann & Low functions) are:

$$\beta_0 = \frac{11}{2} - \frac{1}{3}n_f$$
$$\beta_1 = \frac{51}{2} - \frac{19}{6}n_f$$
$$\beta_2 = \frac{2857}{16} - \frac{5033}{144}n_f + \frac{325}{432}n_f^2,$$

where n_f is the number of quarks with $m < \mu$.

QCDNUM xPDFs evolution

The evolution of the xPDFs is given by [3]:

$$\frac{\partial f_i(x,\mu^2)}{\partial \ln \mu^2} = \sum_{j=q,\bar{q},g} \int_x^1 \frac{dz}{z} P_{ij}\left(\frac{x}{z},\mu^2\right) f_i(z,\mu^2),$$

where P_{ij} are the QCD splitting functions, $\mu^2 = \mu_F^2$ the mass factorisation scale, and $i,j \in \{g,q,\bar{q}\}$, with size $n_f + 1$.

QCDNUM interpolation based on quadratic spline interpolation.

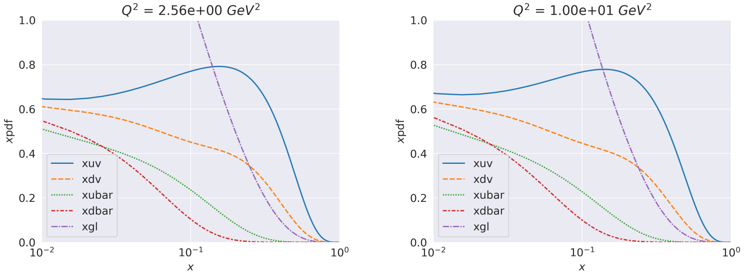
QCDNUM evolution function

QCDNUM::evolfg(pdfset_type, func, pdf_flavour, iq0 ,epsi)

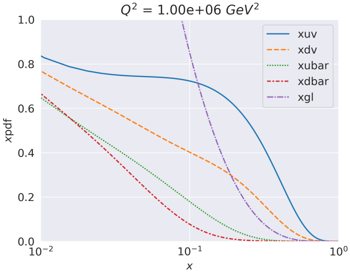
* pdfset_type = 1: unpolarized pdfset.
* func = function that give the pdf parametrizacion at initial energy scale q_0.
* pdf_flavour = array of quark flavours.
* iq0: Q^2 grid index of the q_0 scale.
* epsi:

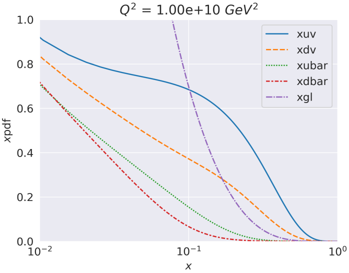
Preliminary results

Evolution up to $10^9 \mathrm{GeV}^2$ using QCDNUM.



Preliminary results





C++ program

```
#include <iostream>
#include <iomanip>
#include <cmath>
#include <fstream>
#include "QCDNUM/QCDNUM.h"
using namespace std;

// x Parton Distribution functions:
//---------------------------------------------------------------------
double xupv(double x)
{
  double A_uv = 10.19304899633023;
  double B_uv = 0.76, C_uv = 4.6, E_uv = 2.6, F_uv = 0.35, G_uv = 0.049;
  double pd = A_uv * pow(x, B_uv) * pow((1 - x), C_uv) * (1 + E_uv * pow(x, 2) + F_uv * log(x) + G_uv * pow(log(x), 2));
  return pd;
}
//---------------------------------------------------------------------
double xdnv(double x)
{
  double A_dv = 5.564255181349489, B_dv = 0.99, C_dv = 4.7;
  double pd = A_dv * pow(x, B_dv) * pow((1 - x), C_dv);
  return pd;
}
//---------------------------------------------------------------------
double xglu(double x)
{
  double A_g = 0.9304776592815227, B_g = -0.52, C_g = 4.5, F_g = 0.217, G_g = 0.0112;
  double pd = A_g * pow(x, B_g) * pow((1 - x), C_g) * (1 + F_g * log(x) + G_g * pow(log(x), 2));
  return pd;
}
//---------------------------------------------------------------------
double xdbar(double x)
{
```

C++ program

```
//evolve all pdf's.
QCDNUM::evolfg(pdfset_type,func,pdf_flavour,iq0,eps);      //TODO: Enteder esta variable eps.

//cout << "     mu2       x           uv         dv        ubar        dvar         gl" << endl;
cout << setprecision(4) << scientific;

// loop to interpolate all pdf's at different scales of q^2
for(int iq2 = 0; iq2 < nq2; iq2++)
{
  double q2_value = q2[iq2];
  cout << "q2_value = " << q2_value <<endl;

  // to save the output in a file saved in the output file
  ofstream myfile;
  myfile.open("/opt/qcdnum-17-01-14/output/pruebaCxx_q_" + to_string(q2_value) +  ".csv");
  myfile << "x xuv xdv xubar xdbar xgl" << endl;

  for(int ix = 0; ix < nx; ix++) {
    double x_value  = x[ix];
    double uv = QCDNUM::fvalxq(pdfset_type,2,x_value,q2_value,ichk);     // the indexes are accordin
    double dv = QCDNUM::fvalxq(pdfset_type,1,x_value,q2_value,ichk);
    double ubar = QCDNUM::fvalxq(pdfset_type,-2,x_value,q2_value,ichk);
    double dbar = QCDNUM::fvalxq(pdfset_type,-1,x_value,q2_value,ichk);
    double gl = QCDNUM::fvalxq(pdfset_type,0,x_value,q2_value,ichk);
    // the above is the same as QCDNUM::allfxq(pdfset_type,x_value,q_value,pdf,0,ichk); and printing

    myfile << x_value << " " << uv << " " << dv << " " << ubar << " " << dbar << " " << gl << endl;
  }
  myfile.close();
}
```

Comparing with literature

Dulat & all [4]:

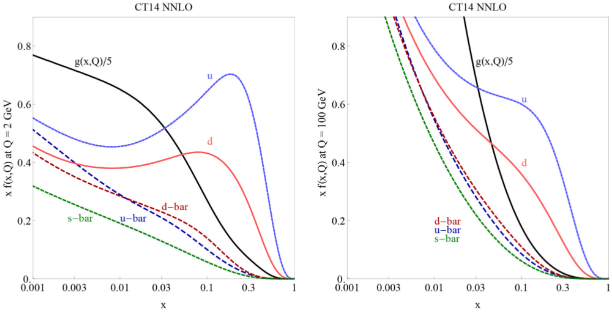
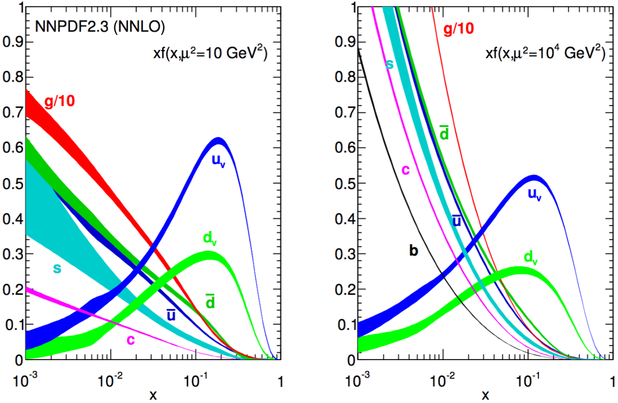


Fig. 2.10: The recent CTEQ CT14 parton distribution functions at $Q = 2\,\mathrm{GeV}$ and $Q = 100\,\mathrm{GeV}$ for u, $\bar{u}$, d, $\bar{d}$, $s = \bar{s}$ and g (from [15]).

Comparing with literature

Stefano Carrazza [5]:



Preliminary results

1. Theoretical study of the DIS and the parton model.
2. An C++ program was created to evolve de xPDFs, using QCDNUM.
3. Correct xPDFs evolution in Q^2, to $Q^2 = 10^9 GeV^2$.
4. Correct graphs of these evolutions.

Future Work

1. To finish the theoretical to study the parton model and pQCD [6].
2. Perform the least square fitting method using TGraph::Fit from ROOT [7], for each decade.
3. Evolve the PDFs with QCDNUM for each fitted xPDFs.
4. Extrapolate in Q^2 each xPDFs.
5. Calculate analytically each differential and total cross sections.
6. Writing the document.

References

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