SHG at the DBE

*Conditions to have SHG at a DBE*

The energy and momentum conservation requirements to have SHG are

Where the superscripts differentiate between the two photons at the fundamental frequency. In this study, we want to enhance the SHG process by operating the nonlinear waveguide at a fundamental frequency very close to a DBE, . Our target dispersion diagram is shown below.

A diagram of a graph

Description automatically generated

Figure 1. Target dispersion diagram of the periodic nonlinear waveguide that supports SHG with a DBE at the fundamental.

Near the DBE frequency , the dispersion diagram is approximated as

Where is the flatness parameter. The wavenumbers at the fundamental are

,

Where . At the second harmonic, we want to have an RBE close to . Then, we have confined harmonics at , whose wavenumbers are close to the edge of the second Brillouin Zone. We expect this feature will make it easier to achieve modal phase matching. Near an RBE, the dispersion is approximated as

And the SH wavenumbers are

where . Therefore, we cannot have 2 photons at the fundamental combine into 1 photon at the second harmonic (there is no second harmonic at exactly ). We have 4 photons at the fundamental that combine into 2 photons at the second harmonic, i.e.,

The modal phase matching is not a direct consequence of this discussion because of material dispersion. The modal indices at the fundamental are

Where .

These requirements impose strict conditions on the dispersion of the coupled gratings design. They are

1. Support a DBE at .
2. Support confined modes at .
3. Phase matching between the fundamental and second harmonic modes,

How do we characterize these conditions numerically?

|  |  |
| --- | --- |
| **Dispersion:** | **Near the EPD frequency, we have** |
| DBE |  |
| RBE |  |
| **Group velocity:** |  |
| DBE |  |
| RBE |  |
| **Group velocity dispersion (GVD):** |  |
| DBE |  |
| RBE |  |

1. Support a DBE at .  
     
   The DBE is characterized by having zero group velocity at the edge of the Brillouin Zone. However, the RBE also has zero .   
     
   To ensure that we impose DBE formation only, we minimize the second derivative of the dispersion, the group velocity dispersion (GVD), which is constant for an RBE but tends to zero at a DBE, as shown in the table on the right.  
     
   These concepts are also shown in the figure below, which shows the dispersion, , and GVD around a DBE and an RBE.

A graph of a function

Description automatically generated

1. Support confined modes at .  
     
   We want the modes at the second harmonic to be close to the edge of the second BZ (small ), so that phase matching is easier to accomplish. To do so, we design the coupled gratings to have an RBE at or slightly above twice the DBE frequency, i.e., . This is accomplished by minimizing the group velocity in a small range of frequencies.
2. Phase matching between the fundamental and second harmonic modes,

Achieving this phase matching requires introducing the mismatch difference,

Note that we do not *need* an RBE to obtain phase matching, but keeping the harmonics close to the edge of the second Brillouin Zone is expected to make phase matching easier.