Optimal Flow control of a noise amplifier, using system identification

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ONERA

THE FRENCH AEROSPACE LAB

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Outline

Objectives

Noise amplifier

- Globally stable
- Local amplification of any external perturbation

The challenge

- Control of the unsteadiness
- Model solely based on available data
- The upstream excitations are unknown
- The upstream excitations are alwas present

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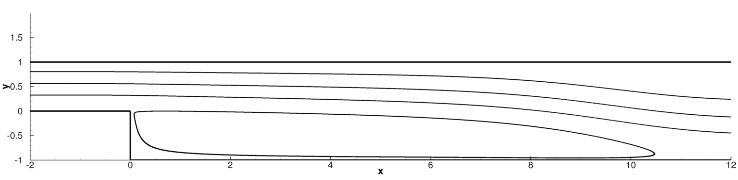
Step configuration & equations

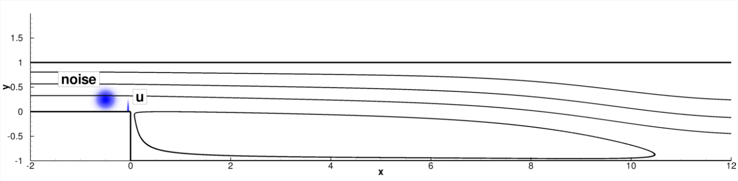


$$Re = 500$$

$$\begin{aligned} \partial_t \mathbf{v} + (\mathbf{v_0} \cdot \nabla)\, \mathbf{v} + (\mathbf{v} \cdot \nabla)\, \mathbf{v_0} &= -\nabla p + \frac{1}{Re}\Delta \mathbf{v} - \varepsilon \left(\mathbf{v} \cdot \nabla\right) \mathbf{v} \\ \nabla \cdot \mathbf{v} &= 0 \end{aligned}$$

$$X(t+1) = AX(t)$$

Step configuration & equations



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$$X(t+1) = AX(t) + Bu(t) + B_w w(t)$$

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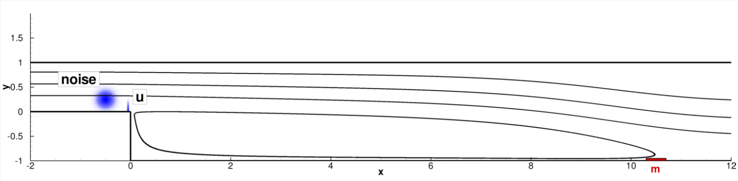

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$$\begin{aligned}
X(t+1) &= AX(t) + Bu(t) + B_w w(t) \\
m(t) &= C_m X(t)
\end{aligned}$$

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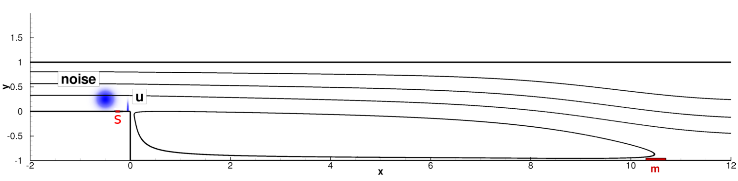

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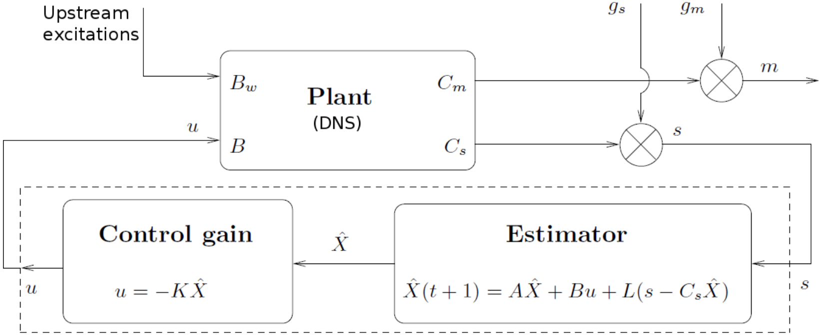
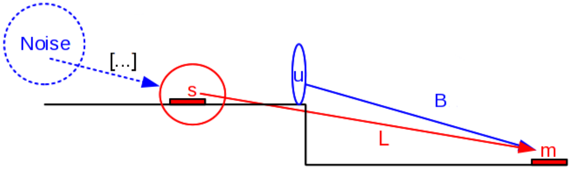
$$\begin{aligned} X(t+1) &= AX(t) + Bu(t) + B_w w(t) \\ m(t) &= C_m X(t) \\ s(t) &= C_s X(t) \end{aligned}$$

LQG

Linear Low Order Model

$$
\begin{aligned}
X(t+1) &= AX(t) + Bu(t) + B_w w(t) \\
m(t) &= C_m X(t) + g_m(t) \\
s(t) &= C_s X(t) + g_s(t)
\end{aligned}
$$

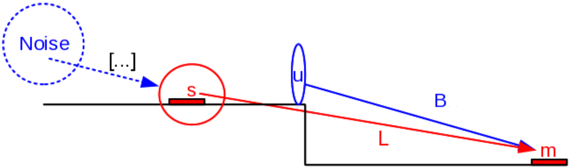




Estimator equation within the classical framework:

$$
\left\{
\begin{array}{rcl}
\hat{X}(t+1) & = & \underbrace{(A - LC_s)}_{A_e}\hat{X}(t) + Bu(t) + Ls(t) \\
\hat{m}(t) & = & C_m\hat{X}(t)
\end{array}
\right.
$$

$$
\Rightarrow \hat{m}(t) = \sum_{k=0}^{\infty} \begin{bmatrix} C_m A_e^k B & C_m A_e^k L \end{bmatrix} \begin{bmatrix} u \\ s \end{bmatrix} (t-k)
$$



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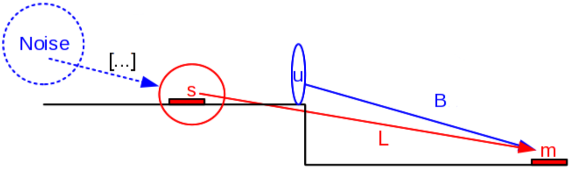
$$

Identifying an estimator



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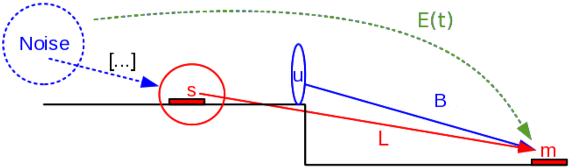
Identification of the transfer function, using ARMAX model



ARMAX : Auto Regressive Moving Average eXogenous

$$\underbrace{m(t) + \sum_{k=1}^{n_a} a_k m(t-k)}_{auto-regressive} = \underbrace{\sum_{k=1}^{n_b} \mathbf{b_k} \begin{bmatrix} u \\ s \end{bmatrix} \left(t - k - \begin{bmatrix} n_{d_u} \\ n_{d_s} \end{bmatrix}\right)}_{eXogenous} + \underbrace{E(t)}_{MovingAverage}$$

Learning dataset

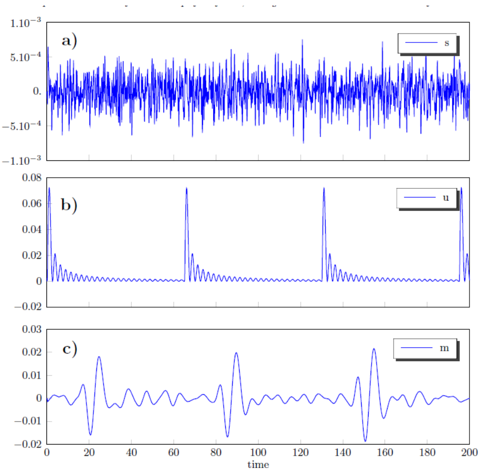


Figure 1: Learning dataset

Testing dataset

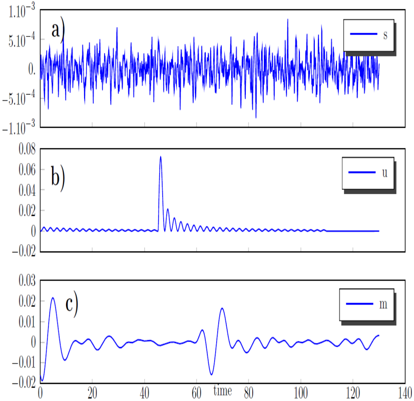


Figure 2: First testing dataset

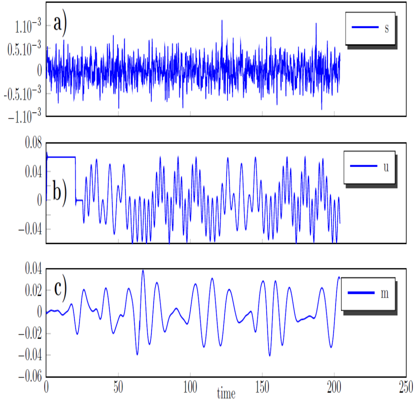


Figure 3: Second testing dataset

Identified model performances

Figure 4: Estimator performances evaluation

From the estimator to a full model

Estimator

$$\hat{X}(t+1) = \underbrace{(A - LC_s)}_{A_e}\hat{X}(t) + Bu(t) + Ls(t)$$

C_s is needed

- s is a measurement $\Rightarrow s \neq$ white noise
- s could be affected by the control u

Full model:

$$ROM : \begin{cases} X(t+1) = AX(t) + Bu(t) + Lw(t) \\ s(t) = C_s X(t) + w(t) \\ m(t) = C_m X(t) + E(t) \end{cases}$$

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Linear regression to obtain C_s:

$$\begin{aligned} s(t) &= C_s\hat{X}(t) + w(t) \\ \Leftrightarrow s(t) &= f\left[\underbrace{s(t-1), s(t-2)}_{\text{sensor characterization}} \quad \dots \underbrace{u(t-1), u(t-2)}_{\text{control effects}} \dots\right] + w(t) \end{aligned}$$

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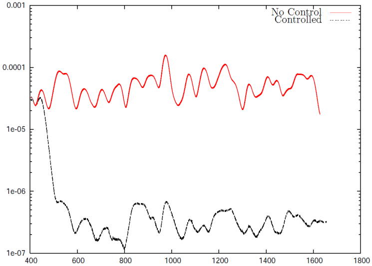
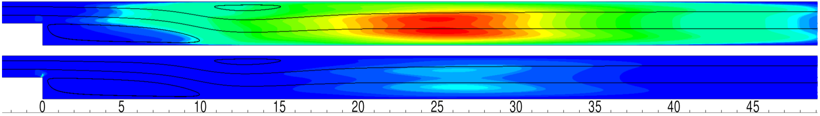
Perturbations energy reduction



Total energy reduction (log scale)

- Total energy reduction: 99%
- Maximal kinetic energy reduction: 96% (at $x \approx 25$)
- The compensator consists of a 17x17 matrix



Mean kinetic energy reduction

Noise measurement influence onto the control performances

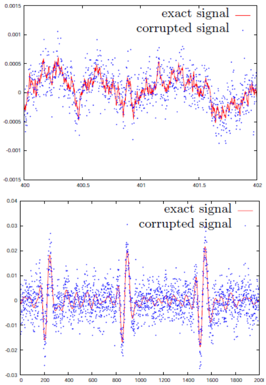


Figure 5: Noisy signals

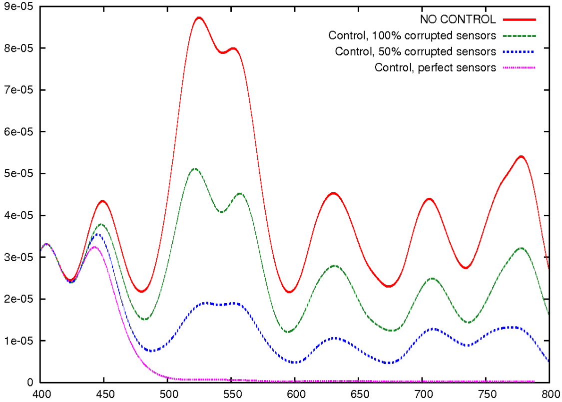
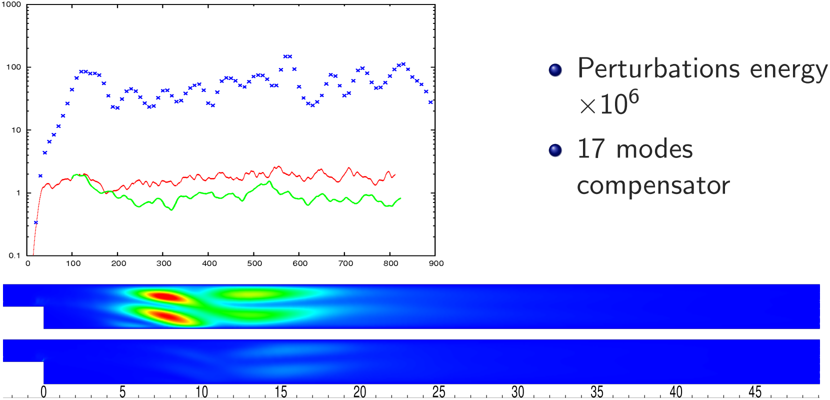


Figure 6: Influence of the noise measurement onto the final performances

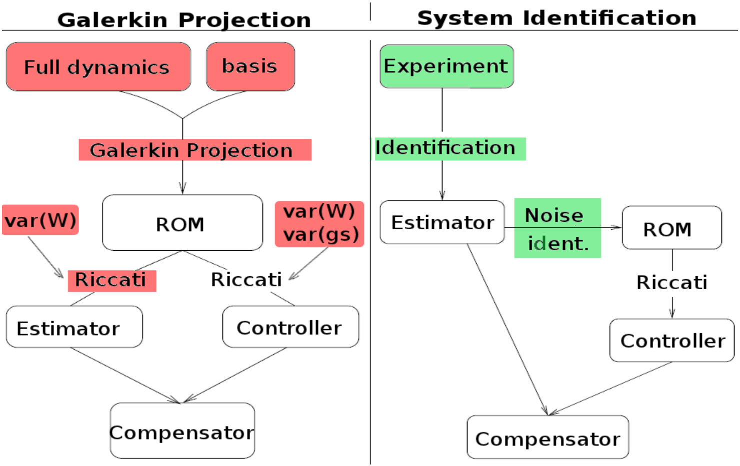
Effects of the non-linearities onto the control performances



- Perturbations energy: $\times 10^6$
- 17 modes compensator

Figure 7: The mean turbulent kinetic energy is reduced by a factor of 98%, at $x \approx 8$

Comparison between classical Galerkin/POD method, & Identification method



Conclusions

Summary

- The model is identified rather than projected
- The process only requires data which can readily be extracted from a lab experiment
- Low dimensional controller
- The downstream sensor is only used for the identification process
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- Good robustness against noise measurement, and against high amplitude excitations
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The end

- Thank you for your attention

Appendix: Realization of the ARMAX equation

General Space state form

$$(ARMAX\ Estimator):\begin{cases} X(t+1) &= A_e X(t) + Bu(t) + Ls(t) \\ m(t) &= C_m X(t) + E(t). \end{cases}$$

State-Space Realization of ARMAX equation

$$\underbrace{\begin{bmatrix} m(t) \\ m(t-1) \\ \cdots \\ m(t-n_a+1) \\ \mathbf{U}^t_{t-n_{bu}+1} \\ \\ \mathbf{S}^t_{t-n_{bs}+1} \end{bmatrix}}_{X(t+1)} = \underbrace{\begin{bmatrix} --- & C_{ARMAX} & --- \\ 1 & 0 & 0 \\ & \cdots & \\ \cdots & 1 & 0 \\ & Shift(n_{bu}) & \\ & & \\ & Shift(n_{bs}) & \end{bmatrix}}_{A_e} \underbrace{\begin{bmatrix} m(t-1) \\ m(t-2) \\ \cdots \\ m(t-n_a) \\ \mathbf{U}^{t-1}_{t-n_{bu}} \\ \\ \mathbf{S}^{t-1}_{t-n_{bs}} \end{bmatrix}}_{X(t)} + \underbrace{\begin{bmatrix} 0 \\ \cdots \\ \cdots \\ \cdots \\ 1 \\ \\ 0 \\ \cdots \end{bmatrix}}_{B} u(t) + \underbrace{\begin{bmatrix} 0 \\ \cdots \\ \cdots \\ \cdots \\ 0 \\ \\ 1 \\ \cdots \end{bmatrix}}_{L} s(t) + \begin{bmatrix} 1 \\ 0 \\ \cdots \\ \\ \\ \\ \\ \end{bmatrix} E(t)$$

$$Shift(n) = \begin{pmatrix} 0 & 0 & 0 & 0 & \cdots \\ 1 & 0 & 0 & 0 & \cdots \\ 0 & 1 & 0 & 0 & \cdots \\ 0 & 0 & 1 & 0 & \cdots \\ & & \cdots & & \end{pmatrix} \in \mathbf{R}^{n\times n}, \quad \mathbf{M}^b_a = \begin{bmatrix} m(t=b) \\ \cdots \\ m(t=a+1) \\ m(t=a) \end{bmatrix}, \mathbf{U}^b_a = \begin{bmatrix} u(t=b) \\ \cdots \\ u(t=a+1) \\ u(t=a) \end{bmatrix},$$