CS 4340 - Project 5

Austin Hester

December 05, 2017

Introduction

The goal of this project is to implement linear regression using validation and ridge regularization.

I did linear regression by matrix multiplication, so regularization does not seem to help much, if at all.

Our training data is:

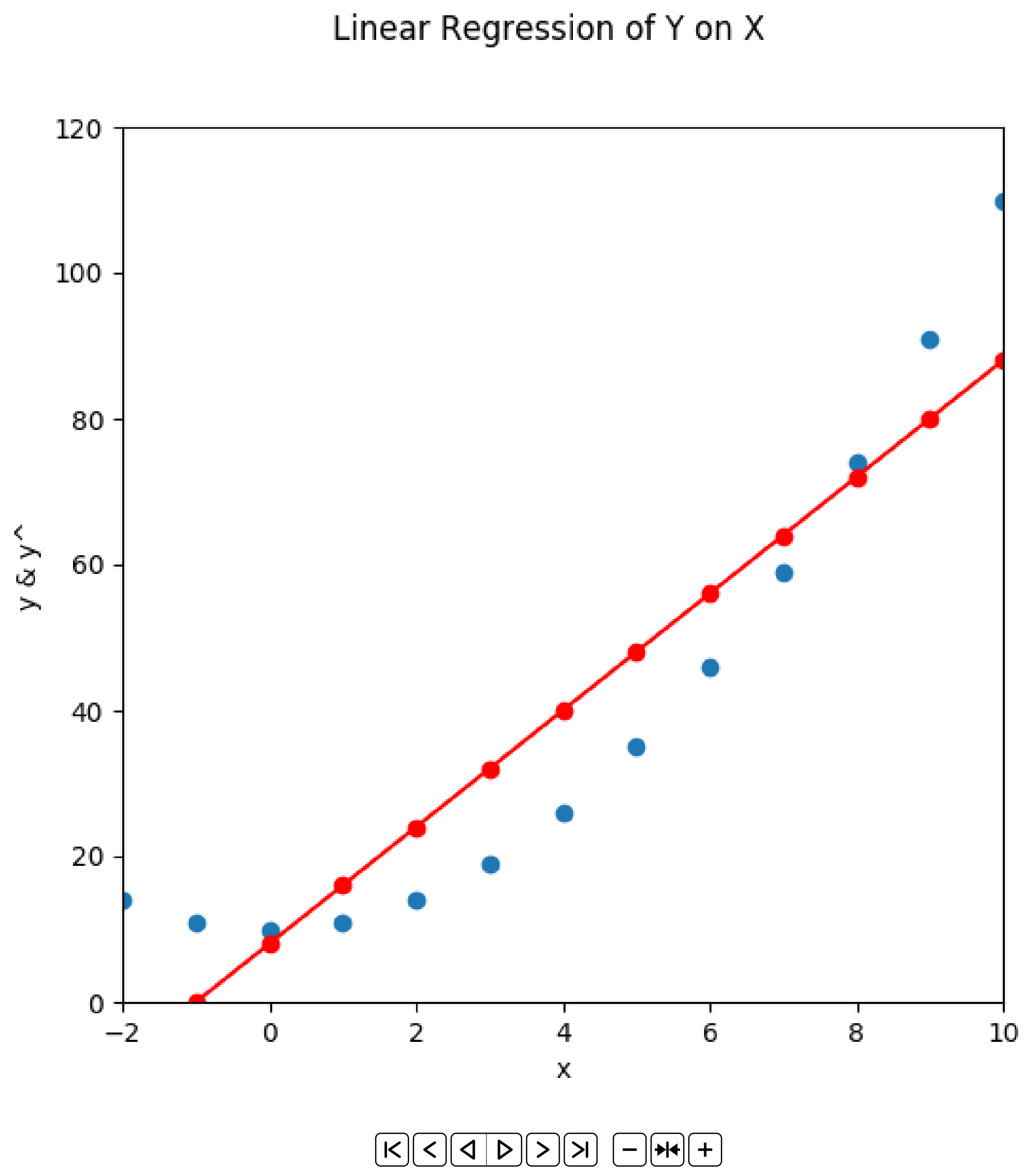
Training Data	
X	**y**
-2	14
-1	11
0	10
1	11
2	14
3	19
4	26
5	35
6	46
7	59
8	74
9	91
10	110

Our y is given by

$$y = f(x) = x^2 + 10$$

.

Animated GIF of 3-Fold Validation



The Code

```
# Austin Hester
# Linear Regression with Regularization
# CS 4340 - Intro to Machine Learning
# 12.04.17

import numpy as np
import random
import matplotlib.pyplot as plt

def make_x ():
    X = [ [ 1, i-2 ] for i in range ( 13 ) ]
    return np.array ( X )

def get_y ( X ):
    y = [ x**2 + 10. for x in X ]
    return np.array ( y )

def get_estimated_y ( X, w_lin ):
    return np.array ( [ round ( yi ) for yi in np.dot ( X, w_lin) ] )

def estimate_single ( N, w_lin ):
    return np.dot ( np.array ( [1, N] ), w_lin )

def print_results ( X, y, y_hat, w_lin ):
    print ( "X =\t", X )
    print ( "Wlin =\t", w_lin )
    print ( "y  =\t", y )
    print ( "y^ =\t", y_hat.T )
    slope, intercept = np.polyfit ( X, y_hat, 1 )
    print ( "Equation of linear regression line: ", "y = ",
            slope, "*x + ", intercept )

def plot ( X, y, y_hat, w_lin , turn=0 ):
    fig = plt.figure ( figsize=(6,6) )
    fig.suptitle ( "Linear Regression of Y on X" )
    # Plot training points
    ax = fig.add_subplot ( 1,1,1 )
    ax.scatter ( X, y, cmap='prism' )
    # Plot line learned from linear regression
    slope, intercept = np.polyfit ( X, y_hat, 1 )
    ablines = [ slope * i + intercept for i in X ]
    cx = fig.add_subplot ( 1,1,1 )
    cx.plot ( X, ablines, 'or-' )
    plt.ylabel ( 'y & y^' )
    plt.xlabel ( 'x' )
    plt.xlim ( -2, 10 )
    plt.ylim ( 0, 120 )
    fig.savefig ( "run%d.png" % turn )


def get_MSE ( y, y_hat, w_norm, lam=1 ):
    total = 0
    for i in range ( y.size ):
        total += ( y [i] - y_hat [i] )**2
    return total / y.size + ( lam * w_norm )

def validate ( X, y, turn=0 ):

    if ( turn == 0 ):
        return linear_regression ( X [:8], y [:8])
    if ( turn == 1 ):
        middle = X[:4] + X[8:12]
        ymiddle = y[:4] +  y[8:12]
        return linear_regression ( middle, ymiddle )
        return "asdf", ":asdf"
    if ( turn == 2 ):
        return linear_regression ( X [4:], y [4:] )

def regularize_ridge ( X, y, y_hat, w_lin, lam=1 ):
    w_norm = np.dot ( w_lin.T, w_lin )
    validate ( X, y )
    return get_MSE ( y, y_hat, w_norm, lam )

def linear_regression ( X, y ):
    XxXT = np.dot ( X.T, X )              # Compute X.T * X <- A
    XxXT_inv = np.linalg.inv ( XxXT )    # Compute inverse of A <- B
    Xt = np.dot ( XxXT_inv, X.T )        # Psuedo-inverse of X = B * X.T <-
        C
    w_lin = np.dot ( Xt, y.T )           # Compute weight vector w_lin = C
        * y
    return Xt, w_lin.T

def run ():
    # Get training points
    X = make_x ()
    X1d = X.T [1] [:]
    y = get_y ( X1d )
    # Run linear regression on those points
    Xt, w_lin = linear_regression ( X, y )
    y_hat = get_estimated_y ( X, w_lin )
    plot ( X1d, y, y_hat , w_lin )
    # Regularize and Validate
    print_results ( X1d, y, y_hat, w_lin )
    for i in range(3):
        print ( "————————————————————————————————" )
        print ( "Validation", i+1 )
        Xt, w_lin = validate ( X, y, i )
        print( w_lin )
        y_hat = get_estimated_y ( X, w_lin )
        plot ( X1d, y, y_hat , w_lin, i+1 )
        print_results ( X1d, y, y_hat, w_lin )


run ()
plt.show ()
```

Notes

a) Our training data.

Training Data	
X	**yl**
-2	14
-1	11
0	10
1	11
2	14
3	19
4	26
5	35
6	46
7	59
8	74
9	91
10	110

b) The equation of the line obtained in part (1) is:

$$y = 8x + 8$$

c) For the following data: turn 1 is the first 8 points, turn 2 is all but the middle 4 points, and turn 3 is the last 8 points. I know there are 13 points total, but 13 is prime and we can ignore 1 here or there.

$\lambda = 0.1$
MSE turn 1 = 746.8
MSE turn 2 = 204.4
MSE turn 3 = 550.7

$\lambda = 1$
MSE turn 1 = 907.0
MSE turn 2 = 451.0
MSE turn 3 = 1016.7

$\lambda = 10$
MSE turn 1 = 2509.0
MSE turn 2 = 2917.0
MSE turn 3 = 5676.7

$\lambda = 100$
MSE turn 1 = 18529.0
MSE turn 2 = 27577.0

MSE turn 3 = 52276.7

d) I chose $\lambda = 0.1$ because it gives the lowest MSE (besides zero). I do not really see the point of regularization when using linear regression. Linear regression is a non-iterative process, and it does not use MSE or SSE when obtaining a weight vector.

e) Our final equation of the line is:

$$y = 12x - 19.3$$

Example Run

Here is example output of a run of 3-fold validation

```
X =  [-2 -1  0  1  2  3  4  5  6  7  8  9 10]
Wlin =    [ 8.  8.]
y  =      [  14.   11.   10.   11.   14.   19.   26.   35.   46.   59.   74.   91.
  110.]
y^ =      [ -8.  -0.   8.  16.  24.  32.  40.  48.  56.  64.  72.  80.  88.]
Equation of linear regression line:  y =  8.0 *x +  8.0
---------------------------------------------------
Validation 1
lambda =  0.1
MSE turn  1  =  746.8
[ 13.   3.]
X =  [-2 -1  0  1  2  3  4  5  6  7  8  9 10]
Wlin =    [ 13.   3.]
y  =      [  14.   11.   10.   11.   14.   19.   26.   35.   46.   59.   74.   91.
  110.]
y^ =      [  7.  10.  13.  16.  19.  22.  25.  28.  31.  34.  37.  40.  43.]
Equation of linear regression line:  y =  3.0 *x +  13.0
---------------------------------------------------
Validation 2
lambda =  0.1
MSE turn  2  =  204.4
[ 15.   7.]
X =  [-2 -1  0  1  2  3  4  5  6  7  8  9 10]
Wlin =    [ 15.   7.]
y  =      [  14.   11.   10.   11.   14.   19.   26.   35.   46.   59.   74.   91.
  110.]
y^ =      [  1.   8.  15.  22.  29.  36.  43.  50.  57.  64.  71.  78.  85.]
Equation of linear regression line:  y =  7.0 *x +  15.0
---------------------------------------------------
Validation 3
lambda =  0.1
MSE turn  3  =  550.777777778
[-19.33333333  12.        ]
X =  [-2 -1  0  1  2  3  4  5  6  7  8  9 10]
```

```
Wlin =    [-19.33333333   12.          ]
y  =      [  14.    11.    10.    11.    14.    19.    26.    35.    46.    59.    74.    91.
  110.]
yˆ =      [ -43.   -31.   -19.    -7.     5.    17.    29.    41.    53.    65.    77.    89.
  101.]
Equation of linear regression line:  y =  12.0 *x +  -19.0
--------------------------------------------------
Validation 1
lambda =  1
MSE turn  1  =  907.0
[ 13.    3.]
X =  [-2 -1  0  1  2  3  4  5  6  7  8  9 10]
Wlin =    [ 13.    3.]
y  =      [  14.    11.    10.    11.    14.    19.    26.    35.    46.    59.    74.    91.
  110.]
yˆ =      [  7.  10.  13.  16.  19.  22.  25.  28.  31.  34.  37.  40.  43.]
Equation of linear regression line:  y =  3.0 *x +  13.0
--------------------------------------------------
Validation 2
lambda =  1
MSE turn  2  =  451.0
[ 15.    7.]
X =  [-2 -1  0  1  2  3  4  5  6  7  8  9 10]
Wlin =    [ 15.    7.]
y  =      [  14.    11.    10.    11.    14.    19.    26.    35.    46.    59.    74.    91.
  110.]
yˆ =      [  1.   8.  15.  22.  29.  36.  43.  50.  57.  64.  71.  78.  85.]
Equation of linear regression line:  y =  7.0 *x +  15.0
--------------------------------------------------
Validation 3
lambda =  1
MSE turn  3  =  1016.77777778
[-19.33333333   12.          ]
X =  [-2 -1  0  1  2  3  4  5  6  7  8  9 10]
Wlin =    [-19.33333333   12.          ]
y  =      [  14.    11.    10.    11.    14.    19.    26.    35.    46.    59.    74.    91.
  110.]
yˆ =      [ -43.   -31.   -19.    -7.     5.    17.    29.    41.    53.    65.    77.    89.
  101.]
Equation of linear regression line:  y =  12.0 *x +  -19.0
--------------------------------------------------
Validation 1
lambda =  10
MSE turn  1  =  2509.0
[ 13.    3.]
X =  [-2 -1  0  1  2  3  4  5  6  7  8  9 10]
Wlin =    [ 13.    3.]
y  =      [  14.    11.    10.    11.    14.    19.    26.    35.    46.    59.    74.    91.
  110.]
yˆ =      [  7.  10.  13.  16.  19.  22.  25.  28.  31.  34.  37.  40.  43.]
```

```
Equation of linear regression line:  y =  3.0 *x +  13.0
--------------------------------------------------------
Validation 2
lambda =  10
MSE turn  2  =  2917.0
[ 15.   7.]
X =  [-2 -1  0  1  2  3  4  5  6  7  8  9 10]
Wlin =   [ 15.   7.]
y  =     [  14.   11.   10.   11.   14.   19.   26.   35.   46.   59.   74.   91.
  110.]
yˆ =     [  1.   8.  15.  22.  29.  36.  43.  50.  57.  64.  71.  78.  85.]
Equation of linear regression line:  y =  7.0 *x +  15.0
--------------------------------------------------------
Validation 3
lambda =  10
MSE turn  3  =  5676.77777778
[-19.33333333  12.        ]
X =  [-2 -1  0  1  2  3  4  5  6  7  8  9 10]
Wlin =   [-19.33333333  12.        ]
y  =     [  14.   11.   10.   11.   14.   19.   26.   35.   46.   59.   74.   91.
  110.]
yˆ =     [ -43.  -31.  -19.   -7.    5.   17.   29.   41.   53.   65.   77.   89.
  101.]
Equation of linear regression line:  y =  12.0 *x +  -19.0
--------------------------------------------------------
Validation 1
lambda =  100
MSE turn  1  =  18529.0
[ 13.   3.]
X =  [-2 -1  0  1  2  3  4  5  6  7  8  9 10]
Wlin =   [ 13.   3.]
y  =     [  14.   11.   10.   11.   14.   19.   26.   35.   46.   59.   74.   91.
  110.]
yˆ =     [  7.  10.  13.  16.  19.  22.  25.  28.  31.  34.  37.  40.  43.]
Equation of linear regression line:  y =  3.0 *x +  13.0
--------------------------------------------------------
Validation 2
lambda =  100
MSE turn  2  =  27577.0
[ 15.   7.]
X =  [-2 -1  0  1  2  3  4  5  6  7  8  9 10]
Wlin =   [ 15.   7.]
y  =     [  14.   11.   10.   11.   14.   19.   26.   35.   46.   59.   74.   91.
  110.]
yˆ =     [  1.   8.  15.  22.  29.  36.  43.  50.  57.  64.  71.  78.  85.]
Equation of linear regression line:  y =  7.0 *x +  15.0
--------------------------------------------------------
Validation 3
lambda =  100
MSE turn  3  =  52276.7777778
```

```
[−19.33333333   12.          ]
X =  [−2 −1  0  1  2  3  4  5  6  7  8  9 10]
Wlin =   [−19.33333333   12.          ]
y  =     [  14.   11.   10.   11.   14.   19.   26.   35.   46.   59.   74.   91.
  110.]
y^ =     [ −43.  −31.  −19.   −7.    5.   17.   29.   41.   53.   65.   77.   89.
  101.]
Equation of linear regression line:  y =  12.0 *x +  −19.0
```

Variations

8) Initial Choice of Weights

Step size, $c = 0.01$.

Zeros

Zeros	
Iterations	**Misclassified Test Pts**
37	0
14	1
91	0
9	0
46	0

Our "control." Weights initialized at zeros gives the quickest, most accurate results.

Ones

Ones	
Iterations	**Misclassified Test Pts**
258	1
225	0
224	0
230	2
376	0

Weights initialized at ones takes much longer, with similar accuracy.

random.uniform(-1,1)

random.uniform(-1,1)	
Iterations	**Misclassified Test Pts**
45	1
185	0
122	1
94	4
203	3

Randomly chosen weights run somewhere in between, with much lower accuracy.

9) Initial Choice of Step Size Constant (c)

Weights initialized at zeros. Step size matters less than you'd think because the Perceptron line is drawn depending on the ratio the weights rather than the values themselves.

Step size: 0.01

c=0.01	
Iterations	**Misclassified Test Pts**
15	0
31	0
27	2
30	0
80	1

Our "control." Quick results, fairly accurate.

Step size: 0.1

c=0.1	
Iterations	**Misclassified Test Pts**
27	0
165	0
335	0
13	1
16	0

May take much longer, however very accurate.

Step size: 1.0

c=1.0	
Iterations	**Misclassified Test Pts**
7	1
94	0
33	1
20	1
133	2

Usually quick, but not as accurate.

10) Order of Picking Points

Weights initialized at zeros. Step size, $c = 0.01$.

Random Misclassified Point

Random	
Iterations	**Misclassified Test Pts**
21	0
19	0
45	0
19	3
64	0

Our "control." Quick results, fairly accurate.

First Misclassified Point

First Point	
Iterations	**Misclassified Test Pts**
25	1
77	0
17	0
21	1
341	0

May take much longer due to an outlier, however very accurate.

Last Misclassified Point

Last Point	
Iterations	**Misclassified Test Pts**
47	0
45	1
20	2
45	1
60	1

Very steady quickness, but not as accurate.